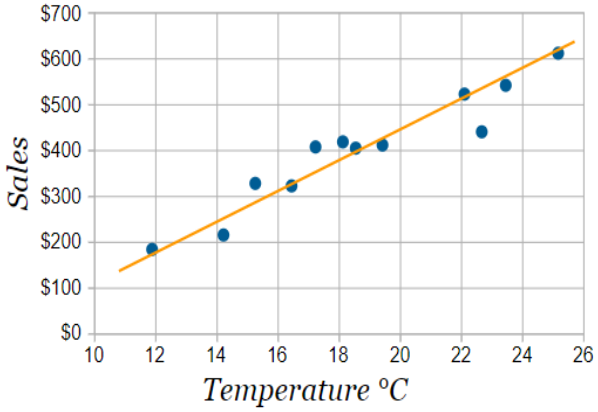
LINEAR REGRESSION

Line of Best Fit

Imagine you have some points, and want to have a line that best fits them like this:



We can place the line "by eye": try to have the line as close as possible to all points, and a similar number of points above and below the line.

But for better accuracy let's see how to calculate the line using Least Squares Regression.

The Line

Our aim is to calculate the values m (slope) and b (y-intercept) in the equation of a line :

y = mx + b

Where:

- y = how far up
- x = how far along
- m = Slope or Gradient (how steep the line is)
- b = the Y Intercept (where the line crosses the Y axis)

Steps

To find the line of best fit for N points:

Step 1: For each (x,y) point calculate x2 and xy

Step 2: Sum all x, y, x2 and xy, which gives us Σx, Σy, Σx2 and Σxy (Σ means "sum up")

$$\mathbf{m} = \frac{N\,\Sigma(xy) - \Sigma x\,\Sigma y}{N\,\Sigma(x^2) - (\Sigma x)^2}$$

LINEAR REGRESSION (Cont...)

Step 3: Calculate Slope m:

(N is the number of points.)

Step 4: Calculate Intercept b: $b = \frac{\Sigma y - m \Sigma x}{N}$

Step 5: Assemble the equation of a line :y = mx + b

Example :Let's have an example to see how to do it !

Sam found how many hours of sunshine vs how many ice creams were sold at the shop from Monday to Friday:

"x" Hours of Sunshine	"y" Ice Creams Sold
2	4
3	5
5	7
7	10
9	15

Let us find the best m (slope) and b (y-intercept) that suits that data

y = mx + b

Step 1: For each (x,y) calculate x2 and xy:

x	y	x^2	xy
2	4	4	8
3	5	9	15
5	7	25	35
7	10	49	70
9	15	81	135

Step 2: Sum x, y, x2 and xy (gives us Σx, Σy, Σx2 and Σxy):

LINEAR REGRESSION (Cont...)

x	y	x^2	xy
2	4	4	8
3	5	9	15
5	7	25	35
7	10	49	70
9	15	81	135
Σx: 26	**Σy: 41**	**$Σx^2$: 168**	**Σxy: 263**

Also **N** (number of data values) = **5**

Step 3: Calculate Slope **m**:

m = $N\,\Sigma(xy) - \Sigma x\Sigma y / N\,\Sigma(x^2) - (\Sigma x)^2$

= $5 \times 263 - 26 \times 41 / 5 \times 168 - 26^2$

= $1315 - 1066 / 840 - 676$

= $249 / 164$

= 1.5183.

Step 4: Calculate Intercept **b**:

b = $\Sigma y - m\,\Sigma x / N$ = $41 - 1.5183 \times 26 / 5$ = 0.3049.

Step 5: Assemble the equation of a line: y = mx + b

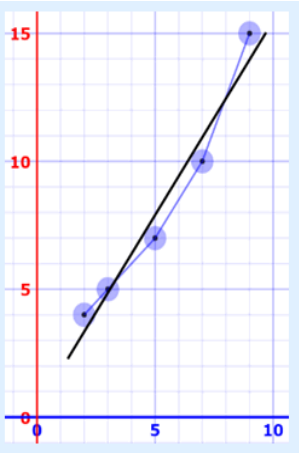
y = 1.518x + 0.305

Let's see how it works out:

x	y	y = 1.518x + 0.305	error
2	4	3.34	−0.66
3	5	4.86	−0.14
5	7	7.89	0.89
7	10	10.93	0.93
9	15	13.97	−1.03

LINEAR REGRESSION (Cont...)

Here are the (x,y) points and the line y = 1.518x + 0.305 on a graph:



Sam hears the weather forecast which says "we expect 8 hours of sun tomorrow", so he uses the above equation to estimate that he will sell

y = 1.518 x 8 + 0.305 = 12.45 Ice Creams

Sam makes fresh waffle cone mixture for 14 ice creams just in case.

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