Matrix Algebra

Professor Seema Singh

Use of Mathematical Economics

- To study economic problems with the formal tools of Mathematics.
- To understand a mathematical economics problems,
  i. State the unknown,
  ii. the data and
  iii. the restrictions/conditions.
- To plan solutions to these problems by finding a connection between the data and the unknown.
- To carry out your plans for solving mathematical economics problems
- It is done for general insights into current and future problems

Use of Matrix Algebra in Econometrics

- “In 1973 Wessily Leontiff won the Noble Prize in Economics for his work in input-output analysis. His seminal work allowed for a greater quantification of economic models. Input-output analysis, also called Inter−Industry Analysis, creates an environment where the user can predict the consumption and demand for a system.”

Theil Henery (1983), “Chapter-1: Linear Algebra and Matrix Methods in Econometrics” Handbook of Matrices, Volume-1, pp. 3-65; https://www.sciencedirect.com/science/article/pii/S1573441283010053

- Vectors and matrices played a minor role in the econometric literature published before Second World War, but they have become an indispensable tool in the past several decades.
- Part of this development results from the importance of matrix tools for the statistical component of econometrics;
- another reason is the increased use of matrix algebra in the economic theory underlying econometric relations.

Theil Henery (1983), “Chapter-1: Linear Algebra and Matrix Methods in Econometrics” Handbook of Matrics, Volume-1, pp. 3-65; https://www.sciencedirect.com/science/article/pii/S1573441283010053

- The paper reviews the concepts of linear dependence and orthogonality (operations change just one thing without affecting others) (In mathematics, orthogonality is the generalization of the notion of perpendicularity to the linear algebra of bilinear forms. Two elements u and v of a vector space with bilinear form B are orthogonal when B(u, v) = 0. Depending on the bilinear form, the vector space may contain nonzero self-orthogonal vectors) of vectors and the rank of a matrix.
- A major reason related to the usefulness of matrix methods is that many topics in econometrics have a multivariate character.
- A general method of estimation is maximum likelihood (ML) that can be shown to have certain optimal properties for large samples under relatively weak conditions. The derivation of the ML estimates and their large sample covariance matrix involves the *information matrix,* which is (apart from sign) the expectation of the matrix of second-order derivatives of the log-likelihood function with respect to the parameters.
- The prominence of ML estimation in recent years has greatly contributed to the increased use of matrix methods in econometrics.

Beginning……

- The beginnings of matrices and determinants goes back to the second century BC to the fourth century BC.
- But, the ideas did not make it to mainstream Mathematics until the late 16th century.
- The Babylonians around 300 BC studied problems which lead to simultaneous linear equations.
- The Chinese, between 200 BC and 100 BC, came much closer to matrices than the Babylonians. Indeed, the text Nine Chapters on the Mathematical Art written during the Han Dynasty gives the first known example of matrix methods.
- In Europe, 2x2 determinants were considered by Cardano at the end of the 16th century and larger ones by Leibniz and, in Japan, by Seki about 100 years later.

Application of Econometrics problem

- In econometrics, we have data, say
- Depandent variable - Y
- No of observations= n,
- So Matricx of dependent variable = Ynx1
- on a dependent variable, Y, and on n explanatory variables, X.

Applications

- it provides an important base to many of the principles and practices. Some of the things Matrices is used for are to solve systems of linear format, to find least-square best fit lines to predict future outcomes or find trends, to encode and decode messages.

Origin……

- With the turn into the 19th century Gauss introduced a procedure to be used for solving a system of linear equations. His work dealt mainly with the linear equations and had yet to bring in the idea of matrices or their notations. His efforts dealt with equations of differing numbers and variables as well as the traditional pre-19th century works of Euler, Leibnitz, and Cramer. Gauss' work is now summed up in the term Gaussian elimination. This method uses the concepts of combining, swapping, or multiplying rows with each other in order to eliminate variables from certain equations. After variables are determined, the student is then to use back substitution to help find the remaining unknown variables.

Origin.....

- matrix multiplication or matrix algebra came from the work of Arthur Cayley in 1855.

Models

MODELLING: THE INDIAN EXPERIENCE (1/2)

- Essentially the models have been of two categories - (i) econometric models based on Keynesian theory of effective demand, and (ii) programming and input-output models incorporating essentially the supply side o f the economy. The first category of models were o f Klein - Tinbergen variety. Most o f these models were constructed from data which overlapped in time. Many of these models assumed supply to be in fin ite ly elastic. The second category of models broadly fall in the following categories; (i) growth models for planning purposes, (ii) multisectoral comparative static consistency planning models; (iii) in ter-

MODELLING: THE INDIAN EXPERIENCE (2/2)

- temporal consistency planning models; (iv) comparative sta tic linear programming optimising planning models; (v) inter-temporal linear programming optimising planning models. These models analysed the direct and indirect effects o f a combinations o f policy instruments and constraints on several v ita l aspects o f the economy. Pew have also studied the implications of alternative policies / on growth and structural change. Some have also analysed the impact o f alternative development strategies on the distribution of income and removal of poverty

Work which has been done in the recent times

- **Structural Changes in Indian Economy: An Input-Output Analysis**
  POONAM MUNJAL
  Indian Economic Review
  New Series, Vol. 42, No. 1 (January-June 2007), pp. 77-95
  Published by: Department of Economics, Delhi School of Economics, University of Delhi; Springer
  https://www.jstor.org/stable/29793876

- As the Indian economy has grown, it has witnessed several changes in its structure. The 1990s have been a period of transition and structural change for the Indian industrial economy. In this paper, it is attempted to study the structural changes in the Indian economy over a period of ten years using input-out analysis, which provides the tools necessary to evaluate industries, including their relationships to the rest of the economy. The present study adopts the technique called "Multiplier Product Matrix" to study these structural changes. The variations observed in the visual presentation of MPM analysis, denominated economic landscapes, show the changing linkages between the industries

Magtibay-Ramos Nedelyn, Estrada Gemma, **Felipe Jesus, (2010)"Exploring the Philippine Economic Landscape and Structural Change Using the Input-Output Framework",** WORKING PAPER NO. 631 | October, Levy Economics Institute of Bard College, http://www.levyinstitute.org/publications/exploring-the-philippine-economic-landscape-and-structural-change-using-the-input-output-framework 01.9.2020

- This paper explores the degree of structural change of the Philippine economy using the input-output framework. It examines how linkages among economic sectors evolved over 1979–2000, and identifies which economic sectors exhibited the highest intersectoral linkages. We find that manufacturing is consistently the key sector in the Philippine economy. Specifically, resource-intensive and scale-intensive manufacturing industries exhibit the highest linkages. We also find a growing impact on the economy of private services and transportation, communication, and storage sectors, probably due to the globalization of these activities. Overall, however, the services sector exhibits lower intersectoral linkages than the manufacturing sector. We conclude that the Philippines cannot afford to leapfrog the industrialization stage and largely depend on a service-oriented economy when the potential for growth still lies primarily in manufacturing.

Sonis Michael and Hewings Geoffrey J.D.(1999)ECONOMIC LANDSCAPES= MULTIPLIER PRODUCT MATRIX ANALYSIS FOR MULTIREGIONAL INPUT-OUTPUT SYSTEMS", Hitotsubashi Journal of Economics 40 (1999), pp.59-74. https://core.ac.uk/download/pdf/6835519.pdf 01.9.2020

- Earlier work (Sonis and Hewings, 1993, 1995; Sonis, Hewings and Miyazawa 1997a) has explored new ways of examining the structure of regional economies using input-output and social accounting systems. In this paper, attention is focused on a new approach to the interpretation of Miyazawa's concepts of left and right multipliers in the decomposition of multiregional input-output systems. Using the technique of the multiplier product matrix (Sonis et al., 1997c), the hierarchical decomposition proposed exploits the insights offered by the fields of influence theory and provides a way of interpreting Miyazawa's left and right multipliers in terms of multiregional feedback loops.

- Thanks