Knowledge representation

Inference in First-Order Logic

- Inference in First-Order Logic is used to deduce new facts or sentences from existing sentences.
- Basic terminologies :

1. **Substitution:** Substitution is a fundamental operation performed on terms and formulas. It occurs in all inference systems in first-order logic. The substitution is complex in the presence of quantifiers in FOL. If we write **F[a/x]**, so it refers to substitute a constant "**a**" in place of variable “**x**".
2. **Equality** : First-Order logic does not only use predicate and terms for making atomic sentences but also uses another way, which is equality in FOL. For this, we can use **equality symbols** which specify that the two terms refer to the same object.

Example: Brother (John) = Smith.

As in the above example, the object referred by the **Brother (John)** is similar to the object referred by **Smith**. The equality symbol can also be used with negation to represent that two terms are not the same objects.

$\neg$ (x=y) which is equivalent to x $\neq$y.

Inference rules of FOL

Universal Generalisation

- Universal generalisation is a valid inference rule which states that if premise P(c) is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as ∀ x P(x).
- It can be represented as: $\dfrac{P(c)}{\forall x\, P(x)}$.
- This rule can be used if we want to show that every element has a similar property.
- In this rule, x must not appear as a free variable.
- **Example:**

  Let's represent, P(c): "**A byte contains 8 bits**", so for ∀ **x P(x)** "**All bytes contain 8 bits**.", it will also be true.

Universal Elimination

- Universal instantiation is also called as universal elimination or UI is a valid inference rule. It can be applied multiple times to add new sentences.
- As per UI, we can infer any sentence obtained by substituting a ground term for the variable.
- The UI rule state that we can infer any sentence P(c) by substituting a ground term c (a constant within domain x) from ∀ **x P(x) for any object in the universe of discourse**.
- It can be represented as: $\dfrac{\forall x\, P(x)}{P(c)}$.
- Example:

  IF "Every person like ice-cream"=> ∀x P(x) so we can infer that
  "John likes ice-cream" => P(c)

Existential Elimination

- It can be applied only once to replace the existential sentence.
- The new KB is not logically equivalent to old KB, but it will be satisfiable if old KB was satisfiable.
- This rule states that one can infer P(c) from the formula given in the form of $\exists x\, P(x)$ for a new constant symbol c.
- The restriction with this rule is that c used in the rule must be a new term for which P(c) is true.
- It can be represented as: $\dfrac{\exists x\, P(x)}{P(c)}$
- Example:

  From the given sentence: $\exists x$ Crown(x) $\wedge$ OnHead(x, John),

  So we can infer: **Crown(K)** $\wedge$ **OnHead(K, John),** as long as K does not appear in the knowledge base.
- The above used K is a constant symbol, which is called **Skolem constant**.
- The Existential instantiation is a special case of **Skolemization process**.

Existential Introduction/Generalization

- An existential introduction is also known as an existential generalization, which is a valid inference rule in first-order logic.
- This rule states that if there is some element c in the universe of discourse which has a property P, then we can infer that there exists something in the universe which has the property P.
- It can be represented as: $\dfrac{P(c)}{\exists x P(x)}$
- **Example: Let's say that,**

  "Priyanka got good marks in English.”

  "Therefore, someone got good marks in English."