Second Leture

1. Various concepts used in Economic Analysis

Like any other subject of science, economics is concerned with the explanation and prediction of observed phenomena which can only be done on the basis of theories. However, economic theories like any other subject are applicable only under certain circumstances. Following are some of the concepts which we consider useful in explaining and predicting economic phenomena:

1.1 Assumption

Economics analysis explains economic behaviour of a human being which depends on several factors. These factors are highly unpredictable and uncertain. Thus, each individual behaves and responds differently under particular circumstances. Not only that, each factor is dependent on other factors along with several other factors. In that circumstance, no linear or non-linear relationship can be established between two variables. However, when two variables are considered keeping others as constant, they show certain relationship. As in case of demand and price, they are inversely related when other variables are assumed constant. If any of the variables also changes, the inverse relationship does not hold good. For example, if income of the household increases, demand for a particular product increases even if its price also increases. Similarly, during winter season, demand for woollen clothes increase even if its price also increases.

1.2 Stock dimension and Flow dimension of an Economic Variable

In case of stock dimension, value of a variable has no time dimension. Physical quantities which exist at any point of time are measured through stock dimension. Some of the examples of stock dimensions are stock of finished product of a firm, stock of raw material with a firm. On the contrary, value of variable which has time dimension is considered under flow dimension. It is discussed in reference of time generally a year. Requirement of raw material per year by a firm or national income of a country during a year are some of the examples.

1.3 Equilibrium: Statics and Dynamics

Equilibrium refers to a market condition where demand of and supply for the product is same. Once the situation of equilibrium by a firm or industry is achieved, it tends to persist. Equilibrium is achieved by the balancing of market forces. In comparative static studies, equilibrium and other positions are considered without discussing the transitional period and process in between these two situations. On the other hand, dynamics deals with the time path and the process of adjustment in course of achieving equilibrium. Engineering students whose mathematics is undoubtedly very good, can discuss dynamic equilibrium also.

2. Model:

Model in economics may be representation of a theory or part of a theory which is made with the application of statistical and mathematical techniques. It is used to gain an understanding in cause-and-effect relationship between variables and to measure the phenomena more accurately. It is similar to a situation where an automobile engineer makes a model of car before making the actual car. The model may be similar to actual car but simple in comparison of actual car. The engineer may use the model to do experiment regarding various performance indicator of the actual machine. Similarly, models in Economics are very helpful in understanding the state through which the firm is undergoing and to make appropriate decision for the benefit of the firm while dealing in microeconomics and how high economic growth may be achieved by an economy or inflation may be controlled in macroeconomics.

3 Mathematical Tools of Economic Model

3.1 Graphs

In Economics, Management and Business, cause and effect relationship is omnipresent. When price increases, a consumer is tending to demand less. For better understanding of the relationship. So relationship between price and demand can be shown as a Graph.

3.2 Functions

A function is a mathematical relationship in which the values of a dependent variable are determined by the values of one or more independent variables.

Functions with a single independent variable are called Simple Univariate functions. There is a one to one correspondence. Functions, with more than one independent variable, are called Multivariate functions. The independent variable is often designated by X. The dependent variable is often designated by Y. For example, Y is function of X which means Y depends on X or the value of Y is determined by the value of X. Mathematically one can write Y = f(X).

3.3 Equation

A function has at least 2 variables: an output variable and one or more input variables. An equation states that two expressions are equal, and it may involve any number of variables (none, one, or more). A function can often be written as an equation, but not every equation is a function.

3.4 Type of Function

3.2.1 Polynomial Function

Polynomial is made up of two words, poly, and nomial. "Poly" means many, and "nomial" means the term, and hence when they are combined, we can say that polynomials are "algebraic expressions with many terms". A polynomial function is a function that involves only non-negative integer powers or only positive integer exponents of a variable in an equation like the quadratic equation, cubic equation, etc. In Economics, it is used to model economic growth patterns.

C = a + b Y
Where C = consumption

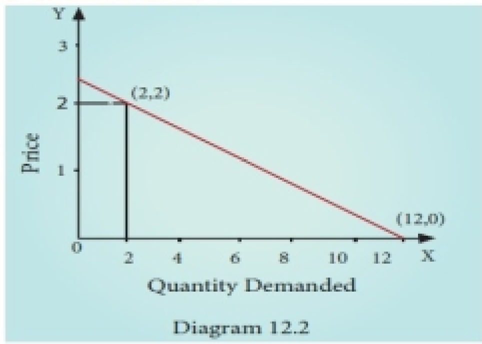
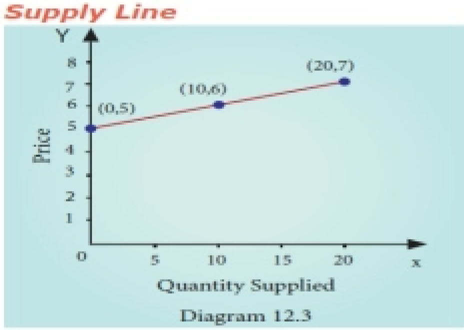
3.2.2 Linear Function

A statement of relationship between two quantities is called an equation. In an equation, if the largest power of the independent variable is one, then it is called as Linear Equation. Such equations when graphed give straight lines.

For example Y = 100+10X.

Where Y = Total Cost;

100 = Faxed Cost and X= Variable Cost

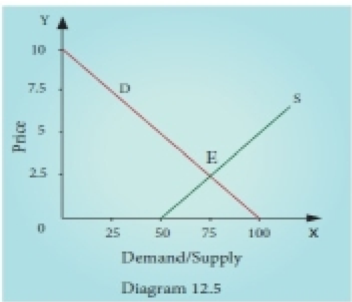
Demand Function	Q_d □f (P_x) where Q_d stands for Quantity demand of a commodity and P_x is the price of that commodity.	Demand Line 
Supply Function	Qs □ f (P_x) where 'Q_s' stands for Quantity supplied of a commodity and Px is the price of that commodity.	Supply Line 

4. Equilibrium

The point at which demand line and supply line cut each other is known as point of equilibrium which can be obtained the values of two unknowns with two equations. At equilibrium point,

Example 2.1 If demand and supply functions Qd □ 100̃5P and Qs □ 5P respectively.

Demand = Supply



<u>Solution: At</u> Equilibrium, Qs □ Qd;

Or, 5 P □ 100̃ 5 P

Or, 10P □ 100; P □ 10

When P □ 10, In supply function, Qs □ 5 P □ 5 x 10 □ 50

In demand function, Qd □ 100̃ 5 P □ 100̃ 5(10) □ 50

Hence at

P □ 10, Qd □ 50, Qs □ 50. So, Quantity demanded is equal to supply at 50 units when price is **Rs.**10

Example: 2.2: The market demand curve is given by D □ 50 ̃ 5P. Find the maximum price beyond which nobody will buy the commodity.

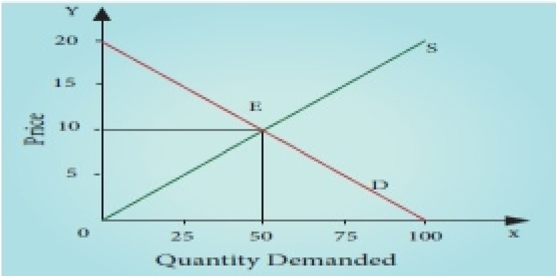


Diagram 12.6

Solution:

Given, Qd □ 50 ̃ 5P; 5P □ 50 ̃ Qd; 5P □ 50 when Qd is zero.

P=50/5; P □ 10 When P □ 10, Demand is 0

Hence P □ 10, which is the maximum price beyond which nobody will demand the commodity.

1.3.2 Quadratic Functions

A function of the general form $y=ax^2+bx+c$ $y = a x^2 + b x + c$ where a , b and c are real numbers and $a\neq0$ is called a Quadratic function. These functions are called quadratic as x is raised to the power of 2.

Example- 2.3 In economics, a quadratic equation may be used where as a product of two linear expressions.

TR = x . p

Where,
TR =Total Revenue;
X= quantity Sold
P= Price of the product

Because the quantity of a product sold often depends on the price, you sometimes use a quadratic equation to represent revenue as a product of the price and the quantity sold.

Because the quantity of a product sold often depends on the price, you sometimes use a quadratic equation to represent revenue as a product of the price and the quantity sold. Quadratic forms are important in testing the second order conditions that distinguish maxima from minima in economic optimization problems, in checking the concavity of functions that are twice continuously di§erentiable and in the theory of variance in statistics.

Example 1: You have designed a sports T-shirts, now you want to make it a business venture.

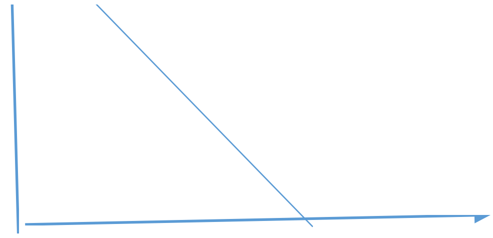
The Cost, Fixed Cost = Rs. 700,000

Variable Cost = Rs. 110/shirt

Based on existing market demand, the demand curve will be,

Q_D = 70,000- 200p where p = price and the demand curve will be

If p = 0, 70,000 is the demand and P =Rs. 350, demand is 0	Price

P= 300; demand will be10,000 t-shirts	

Unit sale = X = 70,000- 200p

Total revenue = PX = (70,000- 200p) P = 70,000 P – 200P^2

Total Cost= 700,000 + 110 (70,000- 200 P)= 700,000 + 7700000 – 22000P

= 8400000 – 22000P

Profit (π) = TR = TC = 70,000 P – 200P^2 - 8400000 + 22000P

Or, -200P^2 + 92000 P – 8400000 = 0 (QUADRATIC EQUATION)

Divide both side by -200,

Or, P^2 – 460 P +42000 = 0

Or, P^2 – 460 P = - 42000

Add 52900 on both sides

Or, P^2 – 460 P + 52900 = - 42000 + 52900

Or (P – 230)^2 = 10900

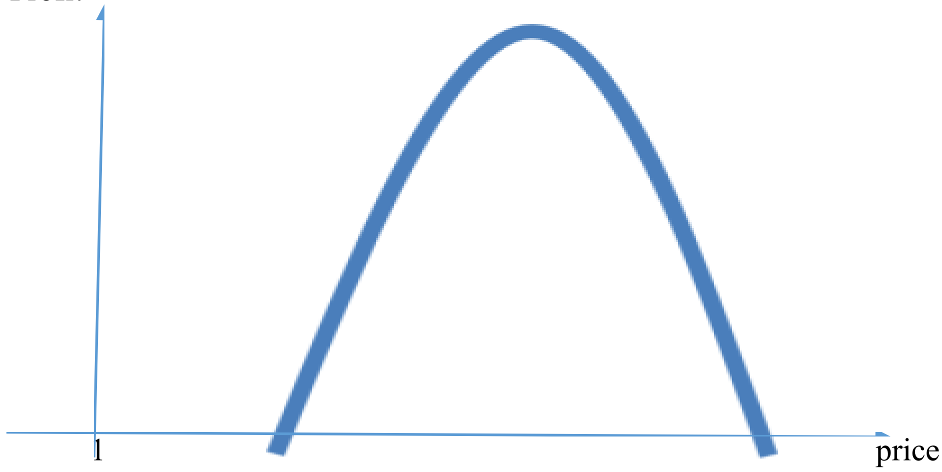
Take square root on both side of the equation

Or, P- 230 = +/- 104 (to the nearest whole number)

Or P = 230 +/- 104 = 334 or 126

At, both points, profit is zero. But in Economic analysis, we want to know maximum profit which is exactly halfway.

Profit



2.3.3 Power Function vs. Exponential Function

$$F(x) = Kx^r \quad \text{where } K \neq 0$$

Where x, the independent variable is base which is raised to a. This is called power function.

Again,

G(x) = a^x where x, the independent variable s raised to the base ‘a’.

2.3.4 Logarithmic Functions

B. Sequences and Series: convergence, algebraic properties and applications

In mathematics, a sequence is an enumerated collection of objects in which repetitions are allowed and order matters while a series is, roughly speaking, a description of the operation of adding infinitely many quantities, one after the other, to a given starting quantity.

One of the main example of sequence and series in macroeconomics is theory of population. Models to depict Population Growth

The Exponential Equation is a Standard Model Describing the Growth of a Single Population

The population doubles every day, or in this population the individuals divide once per day.

Individual	1	2	4	8	16	32	64
Day	0	Second day	3^{rd}	4th	5th	6th	7th

We can see here that, on any particular day, the number of individuals in the population is simply twice what the number was the day before, so it can be written

N(today) = $2N$(yesterday).

Or $N(t) = 2N(t - 1)$ where t = positive value.

Or, $N(6) = 2N(5)$, or $N(5) = 2N(4)$ or $N(4) = 2N(3)$, etc.

So, $N(6) = 2[2N(4)\}$, or $N(6) = 2[2\{2N(3)\}]$ or $N(6) = 2.3N(3)\}]$, or $N(6) = 2.6N(0)\}]$,

Thus we can see a relatively simple generalization,
namely $N(t) = 2^t [N(0)]$
where t stands for any time at all (e.g., if $t = 6$, $N(6) = 2^6[N(0)]$).
Finally we note that this equation was derived from the specific situation, where one division per day was the hard and fast rule. However, the division rate could be anything. If there were two divisions per day but one cell always died, we would expect three individuals from each single individual and Equation would be $N(t) = 3^t N(0)$.
So the division rate could be any number at all and the general equation becomes,

$N(t) = R^t N(0)$ ----------------- **(Exponential Function)**

where R = the finite rate of population increase
In Figure 2 we illustrate this equation for various values of R. It is normally referred to as the exponential equation, and the form of the data in Figure 2 is the general form called exponential.

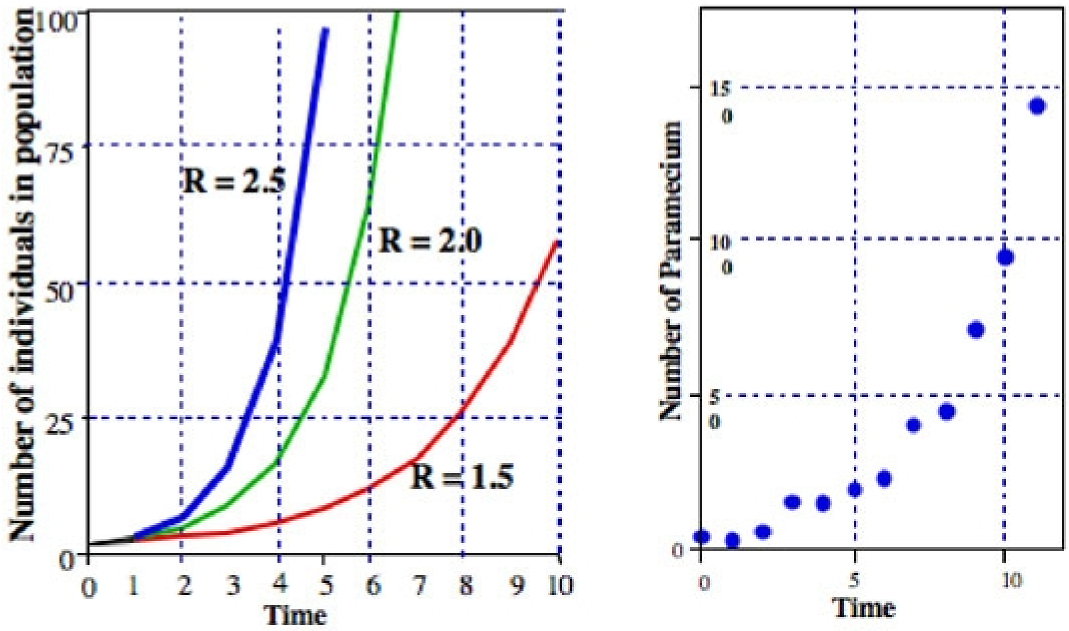


Figure 2: Left: general form of exponential growth of a population (equation 2). Right: actual numbers of Paramecium in a 1 cc sample of a laboratory culture.

Any value of R can be represented in an infinite number of ways (e.g., if R = 16, we could write R = 8 x 2, or R = 4.2, or R = 32/2, or R = 2.718282.77 (Euler's constant).
Expressing whatever value of R as Euler's constant raised to some power is actually extremely useful — it brings the full power of calculus into the picture. As Euler's constant denoted by *e,*

$N(t) = N(0)\, e^{rt}$ …………………………… (iii)

If the natural log of both sides of Equation iii

ln [*N*(*t*)] = *ln* [*N*(0)] + *rt* --(*logarithmic equation)*

Here the population began with a single individual,

So, $\ln(N(t)) = rt$ ………………………………………(iv)

from which we see that the natural log of the population, at any particular time, is some constant, times that time. Constant r = the intrinsic rate of natural increase.

The pattern of growth is very close to the pattern of the exponential equation.

Another way of writing the exponential equation is as a differential equation, that is, representing the growth of the population in its dynamic form. Rather than asking what is the size of the population at time t, we ask, what is the rate at which the population is growing at time t.
The rate is symbolized as dN/dt
which simply means "change in N relative to change in t,".
the rate of growth by differentiating Equation 4,

$\frac{d\ln N(t)}{dt} = r$ ………………………………………(equation v)

it says that the rate of growth of the log of the number in the population is constant. That constant rate of growth of the log of the population is the intrinsic rate of increase.

Recall that the rate of change of the log of a number is the same as the "per capita" change in that number, which means we can write Equation 5 as

$$\frac{d\ln N}{dt} = \frac{dN}{Ndt} = r$$

As it is obvious where the variable t goes, and the equation is rearranged a bit,

$$\frac{dN}{dt} = rN \text{ ....(equation vi)}$$

where the parameter r = the intrinsic rate of natural increase.
The basic relationship between finite rate of increase and intrinsic rate is
$r = ln(R)$
where ln = the natural logarithm.
Note that Equation 6 and Equation 3 are just different forms of the same equation (Equation 3 is the integrated form of Equation 6; Equation 6 is the differentiated form of Equation 3), and both may be referred to simply as the exponential equation.

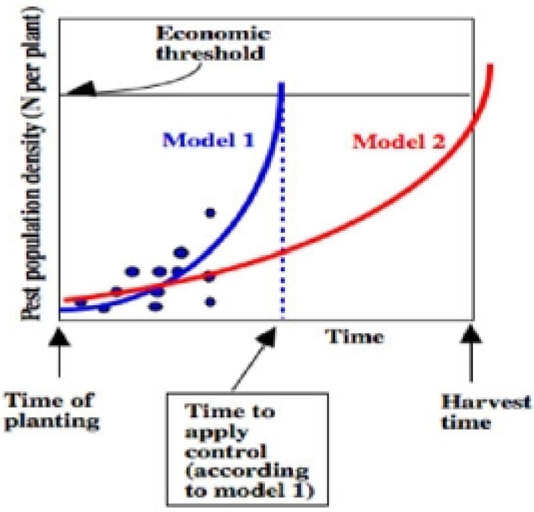


Figure 3: Hypothetical case of a pest population in an agroecosystem

According to model 1 (which has a relatively large estimate of R), the farmer needs to think about applying a control procedure about half way through the season. According to model 2 (which has a relatively small estimate of R), the farmer need not worry about controlling the pest at all, since its population exceeds the economic threshold only after the harvest. Clearly, it is important to know which model is correct. In this case, according to the available data (blue data points), either model 1 or 2 appears to provide a good fit, leaving the farmer still in limbo.

The exponential equation is a useful model of simple populations, at least for relatively short periods of time. For example, if a laboratory technician needs to know when a bacterial culture reaches a certain population density, the exponential equation can be used to provide a prediction as to exactly when that population size will be reached. Another example is in the case of agricultural pests. Herbivores are always potentially major problems for plants. When the plants subjected to such outbreaks are agricultural, which is to say crops, the loss can be very significant for both farmer and consumer. Thus, there is always pressure to prevent such outbreaks. Since WWII the major weapon in fighting such pest outbreaks has been chemical pesticides, such as DDT. However, in recent years we have come to realize that these pesticides are extremely dangerous over the long run, both for the environment and for people. Consequently there has been a movement to limit the amount of pesticides that are sprayed to combat pests. The major way this is done is to establish an economic threshold, which is the population density of the potential pest below which the damage to the crop is insignificant (i.e., it is not really necessary to spray). When the pest population increases above that threshold, the farmer needs to take action and apply some sort of pesticide, or other means of controlling the pest. Given the nature of this problem, it is sometimes of utmost importance to be able to predict when the pest will reach the economic threshold. Knowing the *R* for the pest species enables the farmer to predict when it will be necessary to apply some sort of control procedure (Figure 3).

The exponential equation is also a useful model for developing intuitive ideas about populations. The classic example is a pond with a population of lily pads. If each lily pad reproduces itself (two pads take the place of where one pad had been) each month, and it took, say, three years for the pond to become half filled with lily pads, how much longer will it take for the pond to be completely covered with lily pads? If you don't stop to think too clearly, it is tempting to say that it will take just as much time, three years, for the second half of the pond to become as filled as the first. The answer, of course, is one month.
Another popular example is the proverbial ancient Egyptian (or sometimes Persian) mathematician who asks payment from the king in the form of grains of wheat (sometimes rice). One grain on the first square of a chess board, two grains on the second square, and so forth, until the last square. The Pharaoh cannot imagine that such a simple payment could amount to much, and so agrees. But he did not fully appreciate exponential growth. Since there are 64 squares on the chess board, we can use Equation 2 to determine how many grains of wheat will be required to pay on the last square (*R* raised to the 64th power, which is about 18,446,744,074,000,000,000 — a lot of wheat indeed, certainly more than in the whole kingdom). These examples emphasize the frequently surprising way in which an exponential process can lead to very large numbers very rapidly.

The Idea of Density Dependence Modifies the Exponential Equation

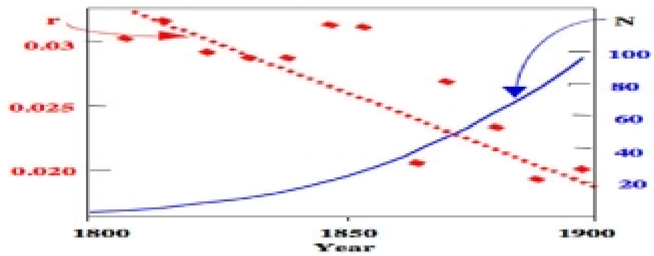


Figure 4: Growth of the human population of the United States of America during the nineteenth century (blue curve), and estimates of the intrinsic rates of increase during that period (red data points)

Note the general tendency for r to decrease throughout the century even while the overall population is increasing.

While the exponential equation is a useful model of population dynamics (i.e., changes in population numbers over time), in the real world we are not swimming in bacteria, or *Paramecium*, or slime moulds. That is, something happens to stop the growth of organisms, be they cells in the body, ciliates in ponds or lions in the savanna. That something else is usually referred to as intraspecific competition, which means that the performance of the individuals in the population depends on how many individuals are in it, more usually referred to as density dependence. This is a complicated issue, one that has inspired much debate and acrimony in the past, and one that still forms an important base for more modern developments in ecology.

The idea was originally associated with the human population, and was brought to public attention as early as the eighteenth century by Sir Thomas Malthus. His writings influenced Darwin's thinking, and formed an important component of the development of the idea of evolution by natural selection. Unfortunately, it is only the more exaggerated claims of Malthus that are normally cited, usually in the context of the growth of the human population. Malthus talked quite a lot about what in fact limits population growth. Indeed, an analysis of the growth of the US human population in the nineteenth century reveals an interesting pattern, one that is repeated with many other organisms. If we estimate the intrinsic rate of increase (the *r* in Equation 6) for short periods of time during the nineteenth century, we find that there is a general decrease in those estimates during the century, at the same time there is an increase in the numbers in the population (Figure 4).

These data for the US human population suggest something general about populations — as population numbers increase, the estimate of the intrinsic rate of natural increase; over a short period of time, tends to decrease. This means that there is a general relationship between the intrinsic rate of increase and population density. We can plot the data from population studies like the one shown in Figure 4 as a graph of population density versus the estimate of intrinsic rate of increase. The general appearance of such a graph is illustrated in Figure 5, for the same laboratory population of *Paramecium* that we looked at before. (Figure 5).

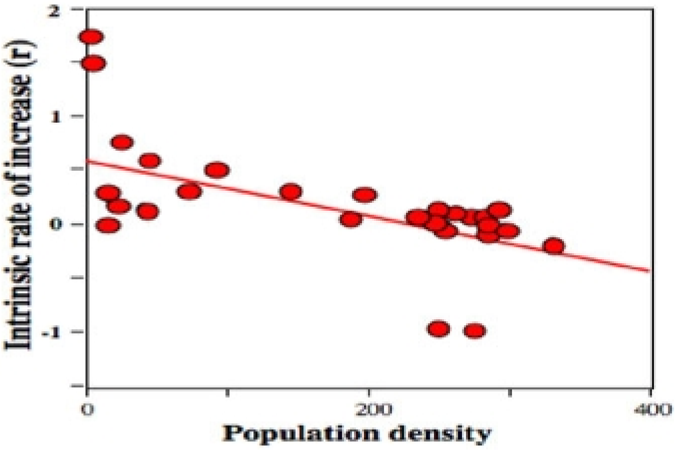


Figure 5: Representing the intrinsic rate of increase as a function of population density for a laboratory population of Paramecium

The higher the population density, the lower the intrinsic rate of increase.

The pattern in Figure 5 is very typical of many populations in both laboratory and natural settings. As with most data from real populations, there is considerable variation. But the trend is clear — with higher population densities, the intrinsic rate of increase tends to be smaller. So, ecologists have made a kind of generalization about this common pattern. The intrinsic rate of increase, as defined in the exponential equation, is not a constant number at all but rather is itself a function of the density of the population. As with many initial attempts at quantifying a concept, we assume a linear approximation (e.g., the straight line drawn through the data points in Figure 5).

If we presume the r of the exponential equation is a function of N (the population density), rather than writing

$$\text{Equation 7:} \quad \frac{dN}{dt} = rN$$

as we did before (we keep the equation numbering system the same, so this equation is still Equation 3), we must modify it in accordance with the observation that r is a function of N (which we write $r = f[N]$), which makes Equation 6 turn into

$$\frac{dN}{dt} = [f(N)]N$$

But if we assume $f(N)$ is linear, we must write $f(N) = a - bN$, where a and b are two constants. So Equation 6 turns into

$$\frac{dN}{dt} = [f(N)]N = (a-bN)N = an-bN^2$$

from which you can see that the differential equation of population growth takes on a quadratic form, which means it has a shape that looks sort of exponential when N is very small, at the beginning of a population trajectory, and levels off as N becomes large. The basic form of this equation is shown in Figure 6 when fitted to data from the same laboratory population of *Paramecium* used to compute the data plotted in Figure 5 (Figure 6).

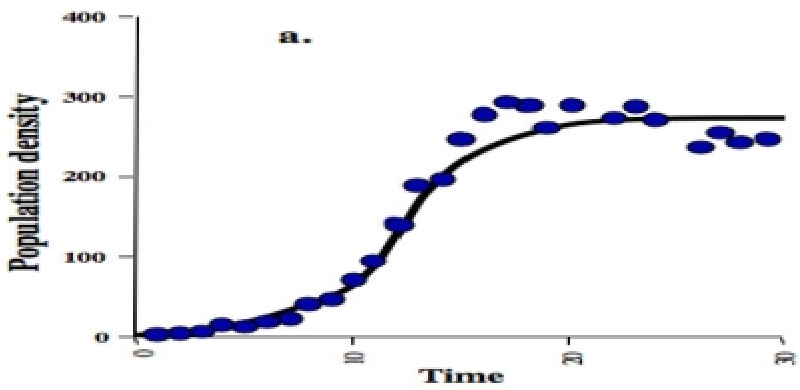


Figure 6: A laboratory population of Paramecium

Note how the density first looks exponential (indeed these are the same data presented in figure 2, but over a longer time frame), but later, after the population gets to around 300 cells per cc, it levels off. This is the classic logistic pattern.

There is an alternative way of dealing with this same problem, one that usually makes more intuitive sense since it provides the arbitrary constants with biological meaning. Consider the situation in Figure 7. This is a picture of a caterpillar that lives in the tropical rain forests of Central America. The really interesting part of the picture is all the small white cocoons on its back. These are cocoons filled with small parasitic wasps. The adult wasp lays eggs on the caterpillar when it is young and the larval wasps eat the tissue of the growing caterpillar until they reach pupating size, at which time they emerge through the cuticle of the caterpillar's skin and form a cocoon, as you see in Figure 7. Here we have a situation that would seem to create a natural limit for the population density of the wasps. If you count the cocoons on the caterpillar in Figure 7, you will find there are 80 cocoons on this one hapless caterpillar. Suppose there are 100 caterpillars in the environment. If this caterpillar represents the highest number of wasps that could possibly successfully complete their life cycle on a single caterpillar (most likely the case), we can then say that there is a ceiling on the potential population density of the wasps of 100 x 80 = 8000. Thus this environment for this species of wasp has a carrying capacity of 8000 (Figure 7).

Now we go back to the exponential equation and ask what needs to be changed about it to take into account of this kind of population ceiling, the carrying capacity. As before we can speculate that what happens in a population is that its effective intrinsic rate of increase actually changes as the density of the population changes. But now we have a new biological concept we can use in developing this theory — the carrying capacity. Let us suppose that the rate at which a population is increasing at any given point in time is proportional to the relative amount of space available to the population — a population with very few individuals will have a lot of space left and will thus grow rapidly, whereas a population with many individuals will have less space left and will thus grow more slowly. That is, to take the concrete example of the parasitic wasps in Figure 7, if there are 100 caterpillars in the environment, and there is space for 80 wasps at the most in each of them, then there is space for 8000 wasps in this environment. The actual amount of space that remains available to the wasp population is 8000 - N, where N is the current population density of wasps. Thus the relative amount of space left will be (8000 - N)/8000 (divide by

8000 because we wish to calculate the relative amount of space left — the proportion or percentage of what could be maximally available). Population ecologists usually use the symbol K for the carrying capacity, so the intrinsic rate would be multiplied by $(K - N)/K$. Substituting this Figure for the $f(N)$ (which is the function that the intrinsic rate of increase is) gives us our final result, the famous logistic equation that describes logistic population growth.

$$\text{Equation 8: } \frac{dN}{dt} = rN\left(\frac{K-N}{K}\right)$$

Its basic form has already been introduced (see Figure 6), and note that with a little bit of algebra the arbitrary constants, a and b of Equation 7, take on intuitive biological meaning (i.e., $a = r$ and $b = r/K$). For many smaller organisms such as bacteria, ciliates, various amoeboid organisms, diatoms, and others, this equation describes population growth reasonably well. For larger organisms such as elephants, humans, trees, or even mice, it is usually thought to be too simple. More complicated models have been developed that take into account more aspects of an organism's life, but these are beyond the scope of an elementary introduction. For now, all that is necessary to know is that much of population ecology, population genetics, and related fields have, as part of their founding rules, the exponential equation and the logistic equation.

Different theories have been discussed below:

Population Theory One # The Malthusian Theory of Population:

Thomas Robert Malthus enunciated his views about population in his famous book, Essay on the Principle of Population as it affects the Future Improvement of Society, published in 1798.

The Malthusian doctrine is stated as follows:

(1) Due to a natural instinct to increase at a fast rate among human being, population increases in geometrical progression and if unchecked doubles itself every 25 years. Thus starting from 1, population in successive periods of 25 years will be 1, 2, 4, 8, 16, 32, 64, 128, 256 (after 200 years).

(2) On the other hand, the food supply increases in a slow arithmetical progression due to the operation of the law of diminishing returns based on the supposition that the supply of land is constant. Thus the food supply in successive similar periods will be 1, 2, 3, 4, 5, 6, 7, 8, and 9 (after 200 years).

(3) Since population increases in geometrical progression and the food supply in arithmetical progression, population tends to outrun food supply. Thus an imbalance is created which leads to over-population. This is depicted in Figure 1.

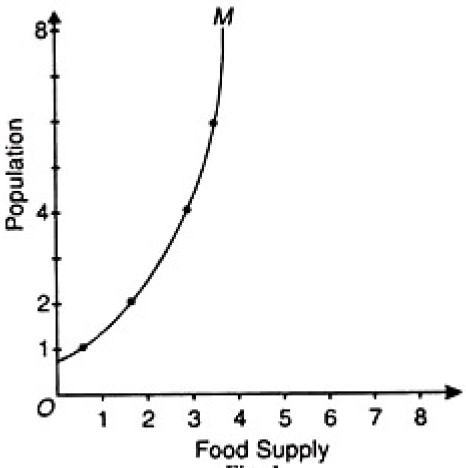
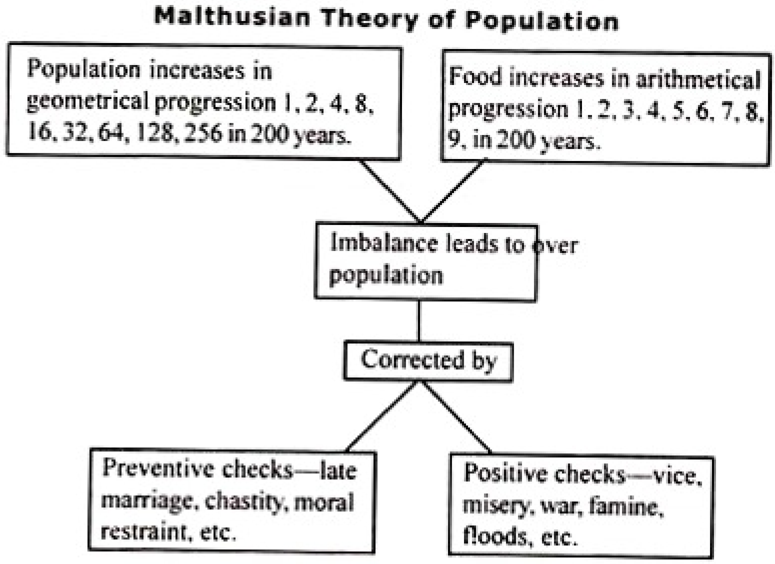


Fig. 1

The food supply in arithmetical progression is measured on the horizontal axis and the population in geometrical progression on the vertical axis. The curve M is the Malthusian population curve which shows the relation between population growth and increase in food supply. It rises upward swiftly.

(4) To control over-population resulting from the imbalance between population and food supply, Malthus suggested preventive checks and positive checks. The preventive checks are applied by a man to control the birth rate. They are foresight, late marriage, celibacy, moral restraint, etc.

Malthus doctrine is illustrated below.



If people fail to check growth of population by the adoption of preventive checks, positive checks operate in the form of vice, misery, famine, war, disease, pestilence, floods and other natural calamities which tend to reduce population and thereby bring a balance with food supply.

Criticisms of the Malthusian Doctrine:

1. Based on assumption of static world that there is no change in knowledge due to agricultural production will increase and the law of diminishing marginal utility is operating..

2. Neglected the human capital aspect in Population growth. As rightly pointed out by Seligman "The problem of population is not merely one of mere size but of efficient production and equitable distribution." Thus the increase in population may be necessary.

3. Empirical Evidence proves this Theory Wrong:

Empirically, it has been proved by demographists that population growth is a function of the level of per capita income. When per capita income increases rapidly, it lowers the fertility rate and the rate of population growth declines. Dumont's "social capillarity thesis" has proved that with the increase in per capita incomes, the desire to have more children to supplement parental incomes declines.

When people are accustomed to a high standard of living, it becomes a costly affair to rear a large family. Population tends to become stationary because people refuse to lower their standard of living. This has actually happened in the case of Japan, France and other western countries.

Marx's Response to Malthus' Thesis:

The debate about the Malthusian theory has continued down to the present. Economists such as J.S. Mill and J.M. Keynes supported his theory whereas others, especially, sociologists, have argued against it. According to them, the widespread poverty and misery of the working class people was, not due to an eternal law of nature as propounded by Malthus but to the misconceived organization of society.

Karl Marx went one step further and argued that starvation was caused by the unequal distribution of the wealth and its accumulation by capitalists. It has nothing to do with the population. Population is dependent on economic and social organization. The problems of overpopulation and limits to resources, as enunciated by Malthus, are inherent and inevitable features associated with the capitalist system of production.

Marx's contention that food production could not increase rapidly was also debated when new technology began to give farmers much greater output. French sociologist E. Dupreel (1977) argued that an increasing population would spur rapid innovation and development to solve problems, whereas a stable population would be complacent and less likely to progress.

During the depression of the 1930s, the debate changed somewhat because the birth rate fell sharply in industrial (western) nations. Some predicted that human species would die out. Schemes were proposed to encourage families to have more children by giving them allowances for each child born. The birth rate rose sharply after World War II, especially in the underdeveloped nations like India, Africa and Bangladesh. Birth control programmes were instituted to control the population so as to eliminate starvation.

Despite the criticisms, the Malthusian thesis gained widespread currency during his lifetime. His ideas had profound effects on public policies, on the classical and neo-classical economists, on demographers and evolutionary biologists led by Charles Darwin.

Population Theory#2 Theory of Demographic Transition:

Demographic transition is a term, first used by Warren S. Thompson (1929), and later on by Frank W. Notestein (1945), referring to a historical process of change which accounts the trends in births, deaths and population growth that occurred in today's industrialized societies, especially European societies. This process of demographic change began for the most part in the later 18th century.

Demographic transition should not be regarded as a 'law of population growth', but as a generalized description of the evolutionary process. In simple terms, it is a theory which attempts to specify general laws by which human populations change in size and structure during industrialization. It is frequently accepted as a useful tool in describing the demographic history of a country.

The theory postulates a particular pattern of demographic change from a high fertility and high mortality to a low fertility and low mortality when a society progresses from a largely rural agrarian and illiterate society to a dominant urban, industrial, literate and modern society.

It is typically viewed as a three-stage process:

(i) That the decline in immortality comes before the decline in fertility,

(ii) that the fertility eventually declines to match mortality, and

(iii) that socio-economic transformation of a society takes place simultaneously with its demographic transformation.

The demographic transition theory is characterized by conspicuous transition stages.

The transition from high birth and death rates to low rates can be divided into three stages (some scholars like Haggett, 1975 have divided into four or five stages):

i. Stage I:

In the first stage of transition, death rates (especially the infant deaths) begin to fall as a result of advances in public health and sanitation as well as improvements in nutrition and food supply. Since the birth rate continues to remain high relative to the declining death rate, there is a rapid 'transitional' growth as we find in India today.

ii. Stage II:

In the second stage, changes in social attitudes, the introduction of cheap forms of contraception and increases in life expectancy create social pressures for smaller families and for a reduction of fertility.

The diffusion of knowledge and cheap medical technology has brought many non-industrial societies into this stage of the demographic transition however, these societies have been unable to enter the third stage. The result has been very high rates of population growth in countries that are not experiencing corresponding economic growth.

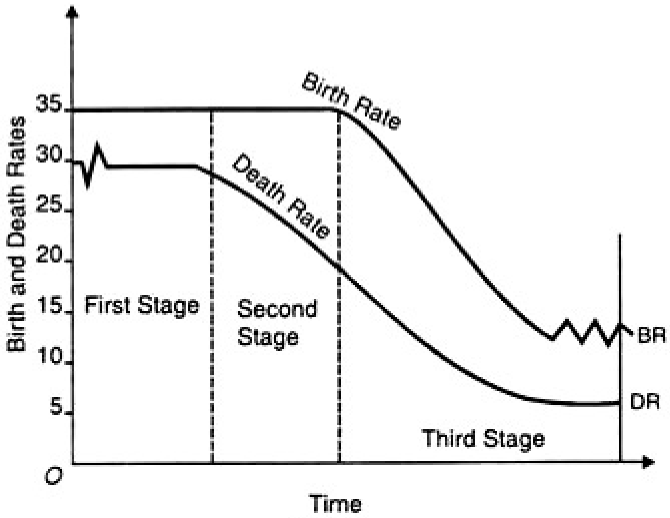


Fig. 5

iv. Stage III:

Birth and death rates both decline appreciably leading to zero population growth. The theory holds that pre-industrial societies were characterized by stable populations which had both a high death rate and birth rate. It postulates a little and slows population growth. The theory states that the high mortality rates characteristic of undeveloped areas will decline before fertility rates which are also high.

In the last (third) stage of demographic transition birth and death rates decline appreciably which eventually becomes approximately equal, and in time it will result in zero population growth. Before this stage begins, there can be one more stage in which low birth and death rates lead to slow population growth.

The populations of advanced, urban industrial societies, which have entered the last stage, are now stable with low birth and death rates. In some cases (e.g., Eastern and Central Europe) birth rates have fallen so slow that the rate of natural increase was actually zero or negative. In this stage, the technical know-how is abundant, the deliberate controls on family planning are common and the literacy and education levels are also very high.

The growth pattern of human populations is thus held to be S-shaped, involving a transition from one type of demographic stability with high death rates to another type of plateau with low death and birth rates. Among the later demographers, Coale and Hoover further elaborated upon the role of development and modernization in the process of transition in demographic behaviour, maintained that a society characterized by peasant economy is marked with very high birth and death rates.

Death rates are high because of lack of adequate nutritive food, primitive sanitary conditions and absence of any preventive and curative measures of control over diseases. A high birth rate, on the other hand, is a functional response to high death rates, particularly among infants and children.

In the present-day world, as would be true of any point in time, different countries of the world are at different stages of the demographic transition. In the opinion of Glenn Trewartha (1969), this is largely due to the dual nature of man. According to him, biologically, man is same everywhere and is engaged in the process of reproduction but culturally man differs from one part of the world to another. It is the cultural diversity of man that gives rise to varying fertility patterns in different areas resulting in different stages of demographic transition discussed above.

Criticism:

Although the theory of demographic transition has been appreciated widely by the demographers, it has been criticized on many grounds also. There are even critics who have gone to the extent of saying that it cannot be called a theory.

The main points of criticism are:

Firstly, this theory is merely based upon the empirical observations or the experiences of Europe, America and Australia.

Secondly, it is neither predictive nor its stages are segmental and inevitable.

Thirdly, the role of man’s technical innovations cannot be underrated, particularly in the field of medicine, which can arrest the rate of mortality.

Fourthly, neither does it provide a fundamental explanation of the process of fertility decline, nor does it identify the crucial variables involved in it.

Fifthly, it does not provide a time frame for a country to move from one stage to another.

Finally, it does not hold good for the developing countries of the world, which have recently experienced unprecedented growth in population due to drastic decline in death rates.

In spite of these criticisms and shortcomings, the demographic transition theory does provide an effective portrayal of the world's demographic history at macro level of generalizations. As an empirical generalization developed on the basis of observing the demographic trend in the West, the transition process for any country can easily be understood.

Population: Theory # 3. The Optimum Theory of Population:

The optimum theory of population was propounded by Edwin Cannan in his book Wealth published in 1924 and popularized by Robbins, Dalton and Carr-Saunders. Unlike the Malthusian theory, the optimum theory does not establish relationship between population growth and food supply. Rather, it is concerned with the relation between the size of population and production of wealth. Thus, it is more realistic.

Definitions:

But what is optimum population? The optimum population is the ideal population which combined with the other available resources or means of production of the country will yield the maximum returns or income per head.

The concept of optimum population has been defined differently by Robbins, Carr- Saunders and Dalton. Robbins defines it as "the population which just makes the maximum returns possible is the optimum population or the best possible population." Carr-Saunders defines it as "that population which produces maximum economic welfare". To Dalton, "Optimum population is that which gives the maximum income per head." If we were to examine these views, we find that Dalton's view is more scientific and realistic which we follow.

Assumptions:

This theory is based on the following assumptions:

1. The natural resources of a country are given at a point of time but they change over time.

2. There is no change in techniques of production.

3. The stock of capital remains constant.

4. The habits and tastes of the people do not change.

5. The ratio of working population to total population remains constant even with the growth of population.

6. Working hours of labour do not change.

7. Modes of business organisation are constant.

The Theory:
Given these assumptions, the optimum population is that ideal size of population which provides the maximum income per head. Any rise or diminution in the size of the population above or below the optimum level will diminish income per head.

Given the stock of natural resources, the technique of production and the stock of capital in a country, there is a definite size of population corresponding to the highest per capita income. Other things being equal, any deviation from this optimum-sized population will lead to a reduction in the per capita income.

If the increase in population is followed by the increase in per capita income, the country is under-populated and it can afford to increase its population till it reaches the optimum level. On the contrary, if the increase in population leads to diminution in per capita income, the country is over- populated and needs a decline in population till the per capita income is maximised. This is illustrated in Figure 2.

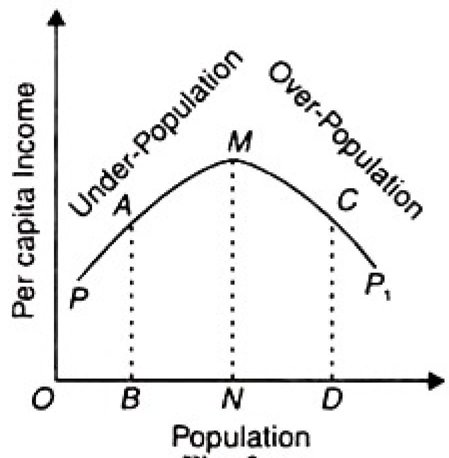


Fig. 2

In the figure, OB population is measured along the horizontal axis and per capita income on the vertical axis. In the beginning, there is under-population and per capita income increases with population growth. The per capita income is BA population which is less than the maximum per capita income level NM. The ON size of population represents the optimum level where per capita income NM is the maximum.

If there is a continuous increase in population from ON to OD then the law of diminishing returns applies to production. As a result, the per capita production is lowered and the per capita income also declines to DC due to increase in population. Thus ND represents over-population. This is the static version of the theory. But the optimum level is not a fixed point.

It changes with a change in any of the factors assumed to be given. For instance, if there are improvements in the methods and techniques of production, the output per head will rise and the optimum point will shift upward.

What the optimum point for the country is today, may not be tomorrow if the stock of natural resources increases and the optimum point will be higher than before. Thus the optimum is not a fixed but a movable point.

According to Cannan, **"At any given time, increase of labour up to a certain point is attended by increasing proportionate returns and beyond that point further increase of o labour is attended by diminishing proportionate returns."** The per capita income is the highest at the point where the average product of labour starts falling. This point of maximum returns is the point of optimum population.

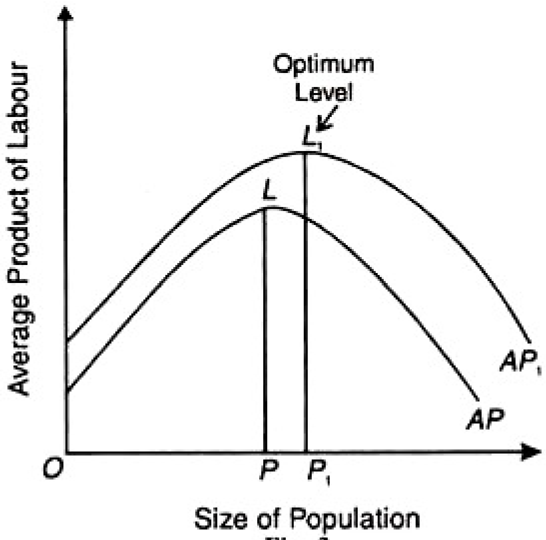


Fig. 3

This is illustrated in Figure3. The size of population is measured on the horizontal axis 2 and the average product of labour on the vertical axis. AP is the average product of labour or income per head curve. Up to OP level, increases in population lead to a rise in the average product of labour and per capita income.

Beyond OP, the average product of labour and per capita income falls. Hence when population is OP, the per capita income is the highest at point L. Thus, OP is the optimum level of population. To the left of OP, the country is under-populated and beyond OP, it is over-populated.

However, OP is not a fixed point. If due to inventions there are improvements in the techniques of production, the average product of labour might increase and push the level of per capita income upward so that the optimum point rises. This is shown in the figure where the AP_1 curve represents the higher average product of labour and point L shows the maximum per capita income at the new optimum level of population OP_1.

Dalton's Formula:

Dalton has deduced over-population and under-population which result in the deviation from the optimum level of population in the form of a formula. The deviation from the optimum, he calls maladjustment. Maladjustment (M) is a function of two variables, the optimum level of population O and the actual level of population A.

The maladjustment is M = A-0/0

When M is positive, the country is over-populated, and if it is negative, the country is under-populated. When M is zero, the country possesses optimum population. Since it is not possible to measure O, this formula is only of academic interest.

It's Criticisms:
Despite the superiority of the optimum theory over the Malthusian theory of population, it has serious weaknesses.

(1) No Evidence of Optimum Level:
The first weakness of the optimum theory is that it is difficult to say whether there is anything like an optimum population. There is no evidence about g the optimum population level in any country.

(2) Impossible To Measure Optimum Level:
It is impossible to measure the optimum level quantitatively. As pointed out by 2 Prof. Bye, it is "impossible to calculate it with any semblance of m exactness for any country at any time."

(3) Optimum Level Vague:
Optimum population implies a £ qualitative as well as a quantitative ideal population for the country. The qualitative ideal implies not only physique, knowledge and intelligence, but also the best age composition of population. These variables are subject to change and are related to an environment. Thus the optimum level of population is vague.

(4) Correct Measurement of Per Capita Income not Possible:
Another difficulty pertains to the measurement of per capita income in the country. It is not an easy task to measure changes in the per capita income. The data on per capita income are often inaccurate, misleading and unreliable which make the concept of optimum as one of doubtful validity.

(5) Neglects the Distributional Aspect of increase in Per Capita Income.:
Even if it is assumed that per capita income can be measured, it is not certain that the increase in population accompanied by the increase in per capita income would bring prosperity to the country. Rather, the increase in per capita income and population might prove harmful to the economy if the increase in per capita income has been the result of concentration of income in the hands of a few rich. Thus the optimum theory of population neglects the distributional aspect of increase in the per capita income.

(6) Optimum Level not fixed but oscillating:
The concept of the optimum population assumes that the techniques of production, the stock of capital and natural resources, the habits and tastes of the people, the ratio of working population to total population, and the modes of business organisation are constant. But all these factors are constantly changing. As a result, what may be the optimum at a point of time might become less or more than the optimum over a period of time. This is illustrated in Figure 4.

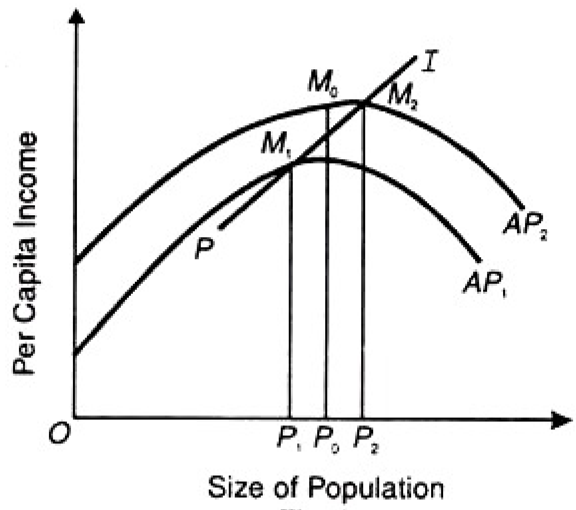


Fig. 4

AP is the average product of labour or per capita income curve. Suppose there is an innovation which brings a change in the techniques of production. It shifts the per capita income curve to AP_1 As a result, the optimum level of population rises from OP_1 to OP_2 with the increase in per capita income from P_1M_1 to P_2M_2. If the per capita income rises further due to a change" in any of the above assumed factors, the AP_2 curve will shift upward.

The AP_2 or AP_1 curve can also shift downward if, for instance, the capita income falls due to an adverse change in the given factors. If the locus of all such points like M_1 M_2 etc., are joined by a line, we have the PI curve which represents the path of the movement of the optimum population as a result of changes in the economic factors.

If, however, the actual level of population is assumed to be OP_0 and the optimum level OP_1 then the country is over-populated. If OP_2 is the optimum level, then the country is under-populated. Thus the optimum is not a fixed level but an oscillating one.

(9) Does not explain Determinants of Population Growth:

It does not explain the reasons for rise or fall in birth and death rates, the influence of urbanisation and migration on population growth, etc.

(10) The theory fails to explain about the nature of an optimum path of population growth.

(11) It does not explain how the optimum level once reached is maintained.

Continuous Function

There are many natural examples of discontinuities from economics, In fact economists often adopt continuous functions to represent economic relationships when the use of discontinuous functions would be a more literal interpretation of reality. It is important to know when the simplifying assumption of continuity can be safely made for the sake of convenience and when it is likely to distort the true relationship between economic variables too much. Our first example illustrates a class of situations in which it is usual to use a model with continuous functions even though this is a distortion of reality in a literal sense. In most such cases the assumption is not a harmful one. However, as many of the other examples illustrate. the idea of discontinuity may be inherent in an economic model itself, with the solution hinging entirely on the existence of some point of discontinuity of the relevant function.

Divisibility and the Production Function

The first step in modeling the decisions of a firm is usually the analysis of die available technology. This relationship between inputs used and outputs generated is generally presumed to be represented by some production function. In the case of one input, call it x, and one output, call it y, we can write y = f (x).

What does it mean to say that this function is continuous on some range of values (usually x > 0. x e R)? In the first place, to assume that /(x) is continuous at a point .v - a implies that /(x) is defined on some open interval of real numbers containing the point a. This means that x must be infinitely divisible. That is. one can choose x to be a value that deviates even by infinitesimal amounts from ,r = a.

An example of an input {and an output) that would not be infinitely divisible would be holts used in the production of an automobile. Since one would not use a fraction like a half of a bolt, it would only make literal sense to treat bolts as integer valued. Therefore it does not make sense to contemplate an open interval of points including some value x = a bolts. However, if a manufacturer producer 20,000 vehicles per year using 1,050 bolts in each vehicle, it seems reasonable tc simply treat bolts and vehicles as infinitely divisible and represent the relationship between them as y = x/1.050. where v is the number of bolts used and y is the number of vehicles produced. (Of course, we have ignored all the other inputs.)