Graph Theory Visualization

A Visual Guide to Graph Theory Concepts

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December 5, 2024

Abstract

This document provides visual representations of fundamental graph theory concepts using TikZ. It includes examples of basic graphs, Euler paths, network flows, and spanning trees with detailed explanations and properties.
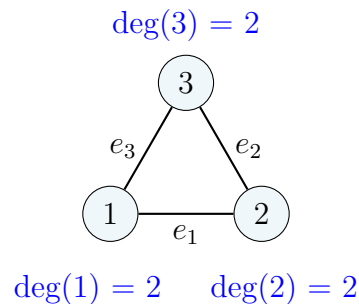
Contents

1   Basic Graph Concepts

Definition 1.1 (Graph)**.** A graph G = (V, E) consists of a set V of vertices and a set E of edges, where each edge connects two vertices.

**Property 1.2** (Degree)**.** The degree of a vertex is the number of edges incident to it.

Theorem 1.3 (Handshaking Lemma)**.** The sum of degrees of all vertices in a graph is equal to twice the number of edges.

2   Graph Properties

Important properties of graphs include:

- **Connectivity**: A graph is connected if there exists a path between any two vertices.

- **Planarity**: A graph is planar if it can be drawn without edge crossings.

- **Bipartiteness**: A graph is bipartite if its vertices can be divided into two disjoint sets.

Example 2.1 (Bipartite Graph)**.**
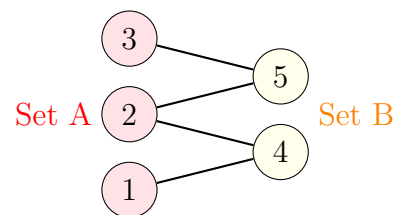
3   Graph Coloring

A proper vertex coloring is an assignment of colors to vertices such that no adjacent vertices share the same color.
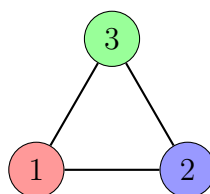
Chromatic number = 3

Property 3.1 (Chromatic Number)**.** The chromatic number of a graph is the minimum number of colors needed for a proper vertex coloring.