Trajectory Model of a Table Tennis Ball

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1. Abstract:

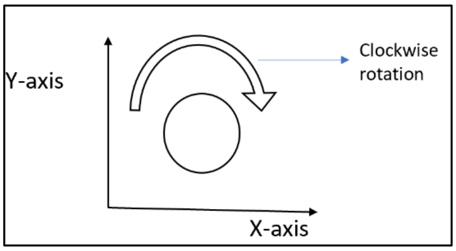
Table tennis is a high intensity racquet sport played with a light 2.7g ball of 40mm diameter. Players manage to bring in drastic changes in the trajectory of the ball by inducing spin. The report studies the difference in trajectory of a table tennis ball hit with top-spin or back-spin with a flat hit of the ball with no spin. A trajectory model is modelled on MATLAB based on the aerodynamics of a table tennis ball in air. It is seen from the model that a ball with back-spin travels further than a flat hit ball, and a flat hit ball travels further than a ball hit with top-spin.

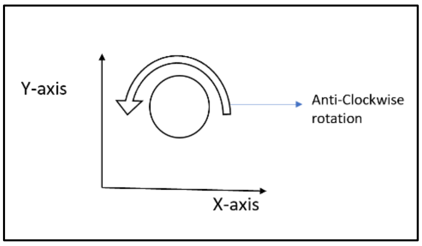
2. Research Question:

To identify the difference in trajectories of a table tennis ball of 2.7g and 38mm diameter when it has top-spin or back spin and a flat shot on the ball with no spin.

3. Introduction:

A table tennis ball with top-spin when seen from the side view will rotate clockwise in the direction of the trajectory of the ball and for a ball with backspin, the ball will rotate anti-clockwise [1]. To hit a ball with top-spin the ball will have to be hit on the upper half allowing it to be hit with a larger velocity than a ball being hit with back-spin.





A 2D trajectory model is implemented to show the difference in the ball's trajectories. Since the lift force acts perpendicular to the axis of rotation only the X-Y 2D plane will suffice to show the effect of top-spin and back-spin on a ball rotating with the X-axis as the axis of rotation.

4. The trajectory model:

To model the trajectory the forces acting on the ball are identified and the position of the ball is updated at time intervals based on the effect of the forces. For a ball travelling through air the multiple forces acting on it need to be resolved to determine its position.

4.1 Forces acting on the ball:

The total sum of forces acting on the ball can be given by the following equation, where F_g is force due to gravity, F_b is air buoyancy force, F_d is drag force, F_m is the magnus force (Schneider, Lewerentz, Lüskow, Marschall, & Kemnitz, 2018).

$$\sum F = F_g + F_b + F_d + F_m \tag{1}$$

<u>Force of Gravity:</u>

$$F_g = -mg \tag{2}$$

Force of gravity is the product of mass of the table tennis ball (m=2.7g) and the gravitational acceleration of the earth ($g=9.8m/s^2$). It always acts downwards towards the centre of the earth.

Air Buoyancy Force:

$$F_b = m_b g \tag{3}$$

Air buoyancy force is the product of mass of the air in the ball (m_b) and the gravitational acceleration of the earth ($g=9.8m/s^2$). Since $m_b \ll m$ the force due to air buoyancy is not considered.

Drag Force:

Drag force acts in a direction against the direction of motion of the ball and is directly proportional to the cross-sectional area(A) and velocity of the ball (U) (Schneider, Lewerentz, Lüskow, Marschall, & Kemnitz, 2018).

$$F_d = -\frac{1}{2} C_d \rho A v \tag{4}$$

ρ is the density of air(ρ = 1.225kg/m^3), C_d is the drag coefficient which can be calculated given the Reynolds number (Re) of the ball. Reynolds number is the ratio of the inertial forces and the viscous forces.

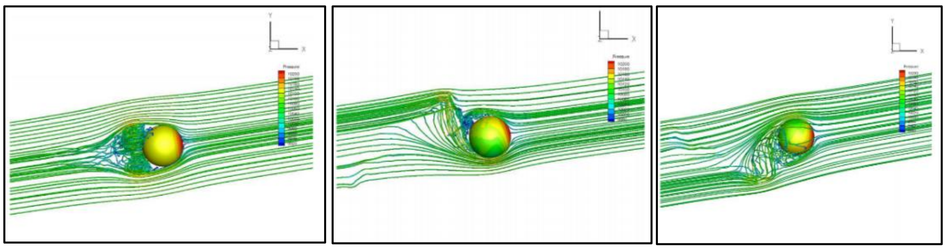
$$C_d = \frac{24}{Re * (1 + (0.15 * Re^{0.687}))} \tag{5}$$

Reynolds number is calculated using the below equation where μ is the viscosity of air(μ = 1.789e^{-5}), D is the diameter of the ball(40mm) (Ou, Castonguay, & Jameson, 2011).

$$Re = \frac{\rho U D}{\mu} \tag{6}$$

Lift Force:

This is the force that is responsible for change in path of a travelling ball when it has spin on it. The spin on the ball which causes an air layer to rotate around the surface of the ball. This brings about a pressure difference around the ball which sways the direction of the ball.



(a) (b) (c)

Figure 2: (a) airflow field of a ball with no spin showing no difference in pressure between the upper and lower face of the ball. (b) airflow field of a ball with top-spin showing pressure difference causing the ball to move downward. (c) airflow field of a ball with back-spin showing pressure difference causing the ball to move upward.

Lift forces acts perpendicular to axis of rotation and can be calculated using the formula, where C_m is the lift coefficient (C_m= 0.028), *A* is the cross sectional area of the ball , V_{spin} the velocity of rotation of the ball in meter per second and ρ the density of air. The value of C_m is taken to be 0.028. (Miyazaki, Sakai, Komatsu, Takahashi, & Himeno, 2016)

$$F_m = \frac{1}{2} C_m \rho A V_{spin} \tag{7}$$

The below diagram shows all the forces acting on table tennis ball with spin. (Nonomura, Nakashima, & Hayakawa, 2010). Lift force acts perpendicular to the axis of rotation and drag force opposite to the direction of motion.

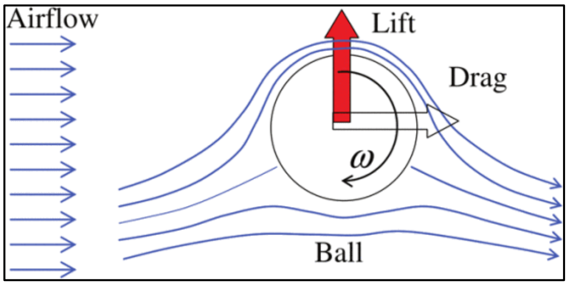


Figure 3: Diagram showing the direction of lift and drag force.

4.2 Modelling the Trajectory:

The trajectory is modelled to find out the position of the ball at every instance of time. The chosen time interval is 0.001seconds. For every time stamp the forces acting on the ball are among the X and Y axis direction are calculated. Given the resolved forces (F_x and F_y) and the mass of the ball(m=2.7g) the acceleration of the ball in both the directions (A_x and A_y) can be calculated by Newtons second law of motion.

$$F = ma \tag{8}$$

Equation (8) can be re-written as the following to find the acceleration:

$$a = \frac{F}{m} \tag{9}$$

Position of the ball is calculated from its initial coordinates to the point of it hitting the table. The total flight time of the ball can be calculated by keeping of track of the number of time intervals the ball has stayed in the air before it hits the table.

Input Parameters:

Symbol and units	**Parameter**
U (meter per second)	Initial velocity of the ball
V_{spin} (meter per second)	Velocity of spin of the ball
X	Initial X coordinate of the ball
Y	Initial Y coordinate of the ball
Theta (degrees)	Launch angle of the ball

Constants:

Parameter	Value
Air Density at sea level (ρ)	1.255kg/m^3
Mass of the ball (m)	0.0027 kg
Radius of the ball (r)	0.038m
Acceleration due to gravity (g)	9.8 m/s^2
Viscosity of Air (μ)	1.81 × 10^{-5} kg/(m·s)

Resolution of Drag and Lift force into its horizontal and vertical components:

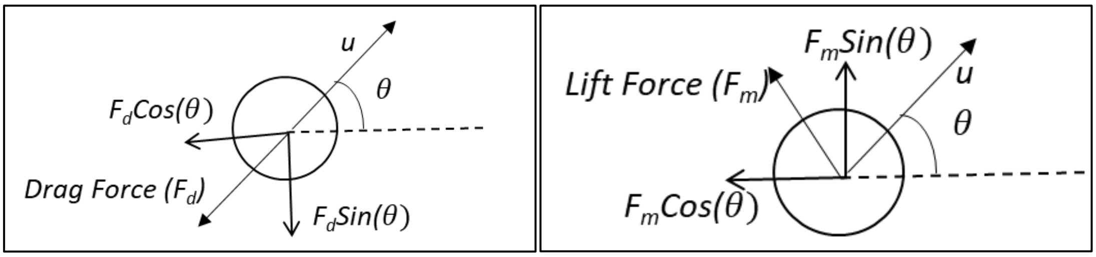


Figure 4: Resolution of drag force and lift force into its horizontal and vertical components.

Using equation (8) and the resolved forces we can obtain the acceleration of a ball with no spin in the X and Y directions by the following equations:

$$A_x = (-F_d Cos(\theta))/m \tag{10}$$

$$A_y = ((-F_d Cos(\theta) - mg)/m \tag{11}$$

For a ball with spin, the lift force will be included in the net aerodynamic forces on the ball , therefore A_x and A_y will be:

$$A_x = (-F_d Sin(\theta) + F_m Sin(\theta))/m \tag{12}$$

$$A_y = ((-F_d Sin(\theta) + F_m Sin(\theta) - mg)/m \tag{13}$$

The velocity of the ball changes with every time interval. Using incremental modelling we can calculate the horizontal and vertical components of velocity using the equations:

$$V_x = A_x \times 0.001 + V_{x-1} \tag{14}$$

$$V_y = A_y \times 0.001 + V_{y-1} \tag{15}$$

$$V_{resultant} = \sqrt{V_x^2 + V_y^2} \tag{16}$$

Similarly, the position of the ball can be calculated at different instances of the time stamp with Vx and Vy values, by the following equations:

$$X = X_{i-1} + V_x \times 0.001 \tag{17}$$

$$Y = Y_{i-1} + V_y \times 0.001 \tag{18}$$

The angle of the ball keeps changing along with its position and with respect to the effect of the aerodynamic forces on the ball. The angle (theta) will be the angle between the velocity vector and the X-axis, this can be calculated with the horizontal and vertical components of velocity:

$$\theta = \tan^{-1}(\frac{V_y}{V_x})$$

The position of the ball at all instances of time are stored in an array and plotted to display the trajectory of the ball.

User Interface and Inputs:

A graphical user interface using MATLAB Apps allows to provide inputs to the trajectory model and visualise the trajectory.

The limits of the input parameters are decided upon based on the study done for trajectory modelling of a 38mm and 40mm table tennis ball. (Xie, Teh, & Qin, 2002)

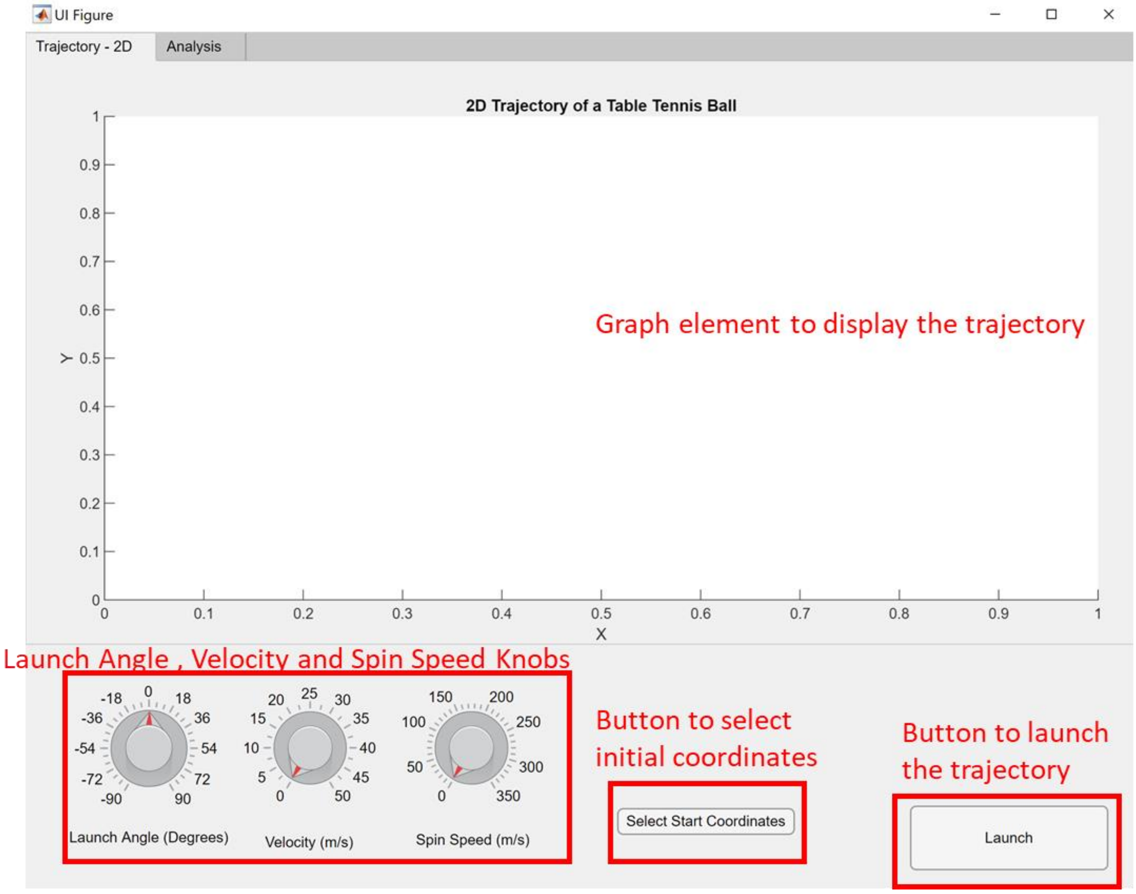


Figure 5: GUI of the trajectory model

Steps of Usage:

1. Use launch angle velocity knob to input angle theta.
2. Use the velocity and Spin speed knob to input velocity and spin speed.

3. Click on Select Start coordinates button and left click to input the X,Y start coordinates and then hit enter.
4. Click launch to visualise the trajectory.

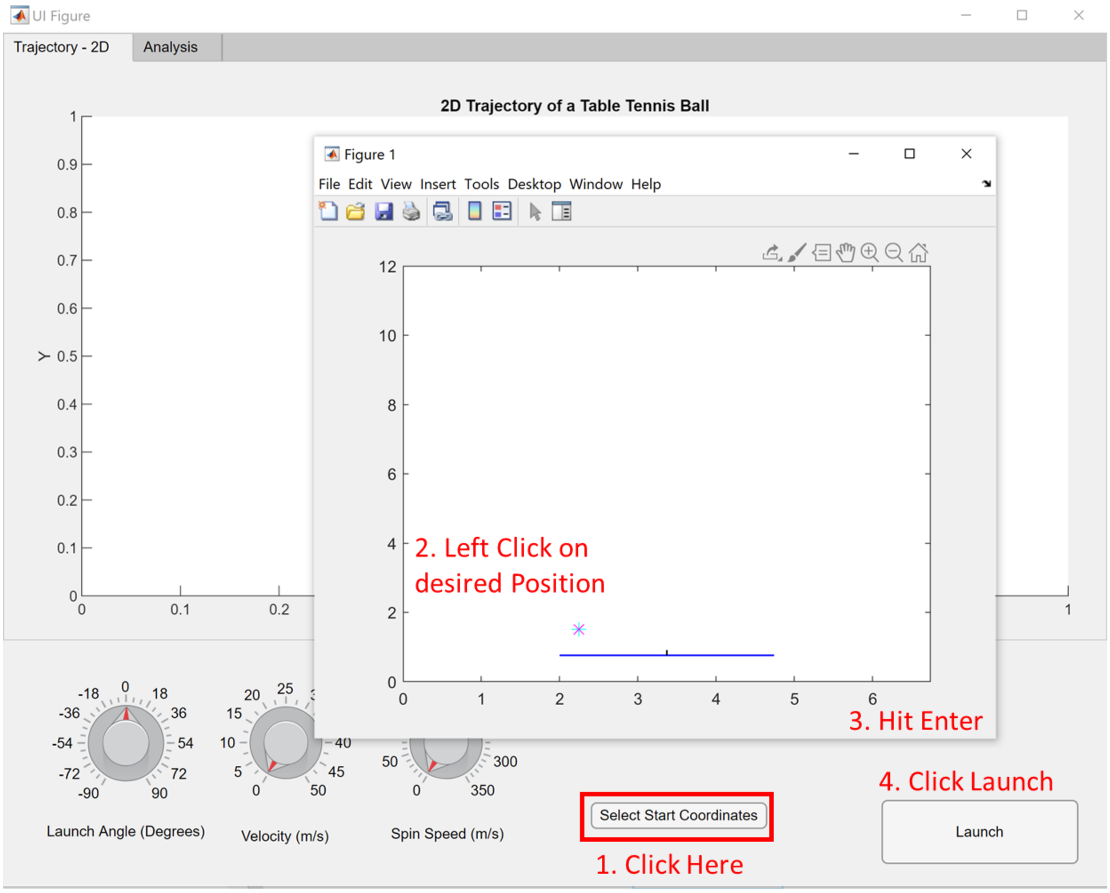


Figure 6: Steps to select initial coordinates

Results:

Based on the input the MATLAB GUI plots the trajectories of the ball. It is seen the ball with back-spin travels the furthest and spends more time in the air. The ball with top spin travels the least and spends the least time in the air.

The magnus effect on a ball with back spin causes the ball to stay up in the air for longer, whereas on a ball with top spin it causes the ball to move downward.

The below screenshot shows the trajectories of a ball with no spin(black ball) , top spin(green ball) and back spin(blue ball).We can conclude from the results of the model that a ball with backspin will travel further and a ball with top spin will travel lesser than a ball with no spin.

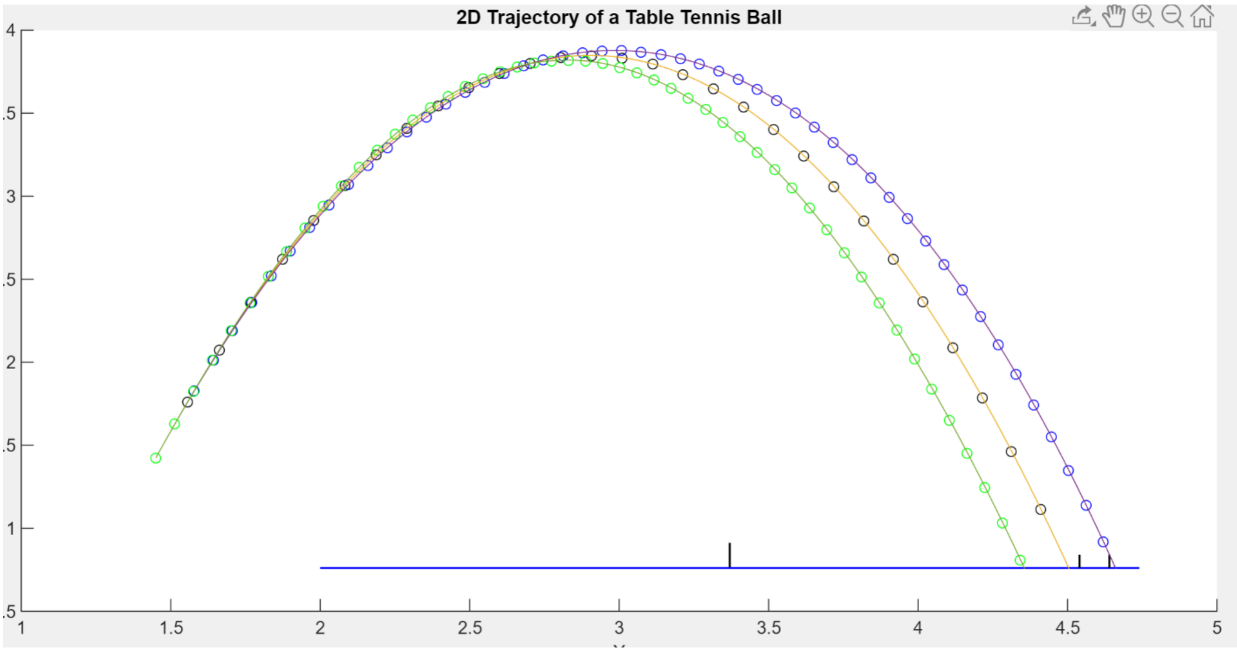


Figure 7: Trajectory of a ball with no spin, back spin and top spin.

Analysis of the shot is available on Tab 2 of the GUI.

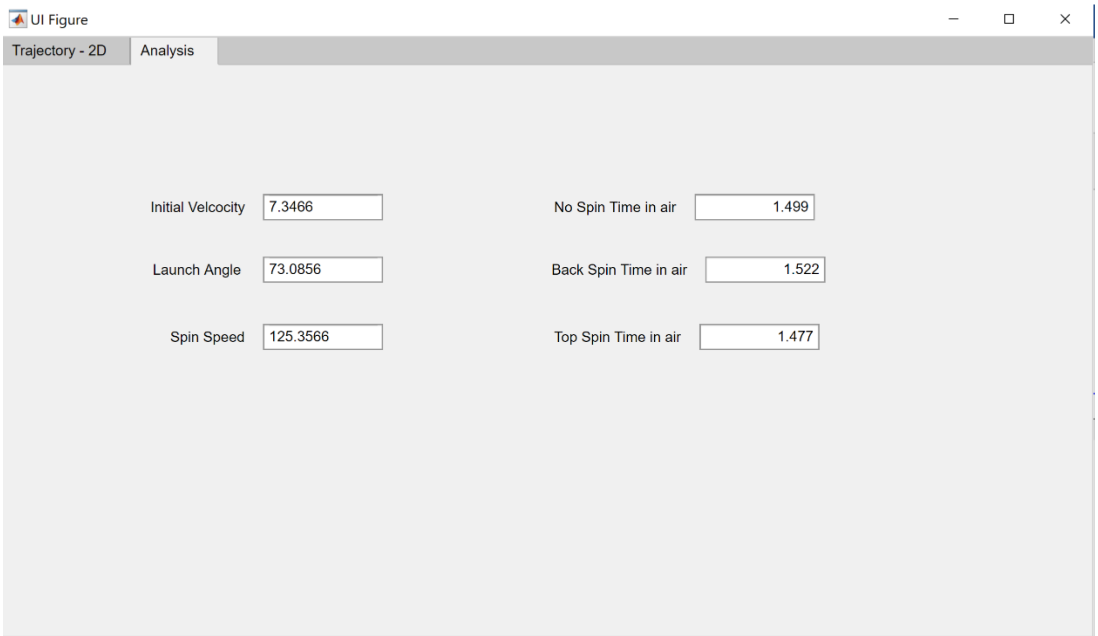


Figure 8: Analysis tab showing the time the ball spends in the air. A ball with backspin spends more time than a ball with top spin or no spin in air.

References

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Word Count without References: 1583 words