

# Vehicle dynamic modeling and stability program active front steering sliding mode integrated control

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**Abstract**—This document describes the simulation and results of the research conducted by Xiaobin Fan et al [1], titled Vehicle dynamic modeling and stability program active front steering sliding mode integrated control. we tried to use the models and controllers in this paper to establish an ESP and AFS controller for the vehicle. Based on the equations on the paper, the vehicle model and controllers are designed. although the information about simulation conditions is not enough, we could find similar patterns for results through simulation.

## I. Introduction

**A**CTIVE front steering (AFS) and electronic stability program (ESP) are active safety devices for vehicles to improve vehicle handling and stability. In the linear range of the tire lateral force, the vehicle can intervene more directly and quickly through the AFS, and the energy required and tire wear is so small as to be difficult for a driver to detect. This differs from the nonlinear range of automobile tires, where the control effect of ESP is obvious. Xiaobin Fan et al focused on the implementation of the algorithm and real vehicle verification. they tried to reach to satisfy the robustness of the control algorithm. So, in their study, a precise vehicle high-dimensional nonlinear dynamic model and a tire dynamic friction model were established. AFS, ESP, and coordinated control algorithms were designed. However, this paper focuses on controller parts which contain a simulation of AFS and ESP.

## II. VEHICLE DYNAMICS MODELING

In this section, the vehicle nonlinear multi-degree of freedom dynamics model is developed, which includes the body, steering system, braking system, and tire subsystem model. we will discuss each one of these subjects briefly.

### A. Vehicle body dynamics model

The body dynamics model was established as Fig. 1, which includes longitudinal motion, lateral motion, yaw motion, and roll motion. longitudinal, lateral, and vertical forces, yaw moment, lateral and longitudinal load transfer, etc computed by vehicle body dynamic equations. One of the most important parameters which are used to design AFS and ESP controllers is Yaw angle  $\psi$  and lateral acceleration  $A_y$ .

### B. Wheel dynamics model

The dynamic equations of the driving wheel are as follows.

$$\dot{\omega}_i = \frac{1}{I_{Wi}} [T_i - RF_{ti} - T_{brki} - d_i F_{zi}] \quad (1)$$

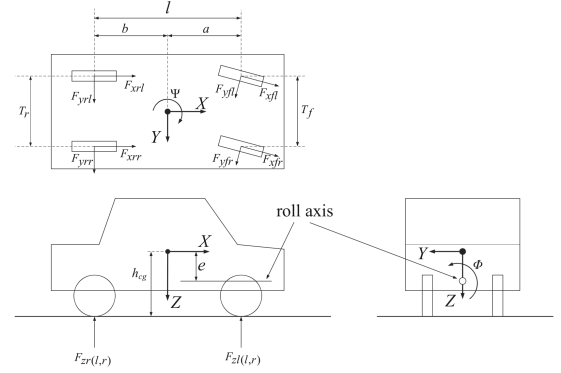


Fig. 1: Vehicle body dynamics model.

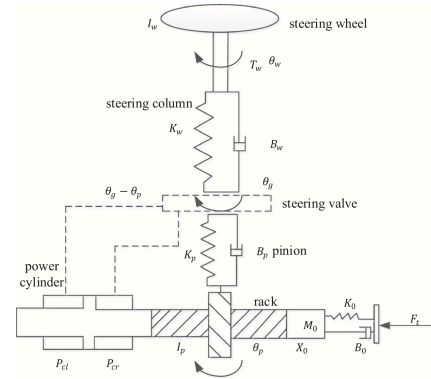


Fig. 2: Electric hydraulic power-assisted steering.

these formulas are used to calculate the wheels' side slip angle ( $\alpha_i$ ) and longitudinal slip ( $\rho_i$ ) for each tire. These values will be used to determine vehicle steering behavior.

### C. Electronically controlled hydraulic power steering system model

In this paper, the steering system is designed by using an electro-hydraulic power system as it is shown in figure 2. The dynamic equation of this system is used to make the steering angle function of velocity and driver input pressure. with this method, the vehicle can obtain higher stability in high-velocity maneuvers and higher steering angle in low velocity.

### D. Tire dynamic friction model

The tire's mechanical properties have an important influence on vehicle handling stability, braking safety,

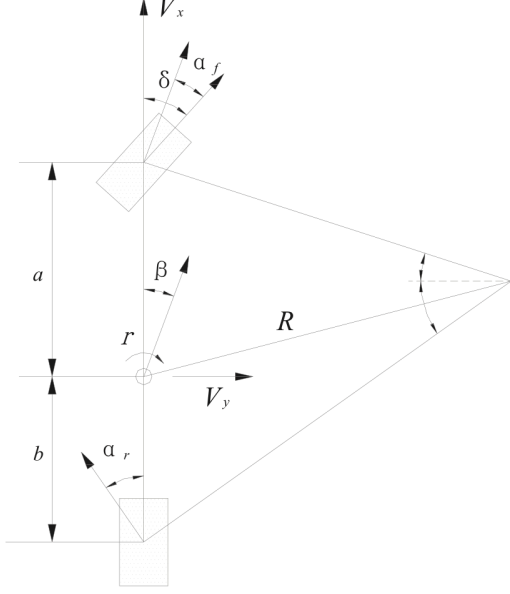


Fig. 3: Linear two degrees of freedom model.

and ride comfort. The development of modern automobile dynamics needs to establish an accurate tire model that reflects the tire's physical essence. three different models are used to simulate tire behavior as real as possible. the models are:

- 1) Dugoff tire model
- 2) Magic formula tire model
- 3) Tire brush model
- 4) Tire LuGre dynamic friction model

To compare the results of the mentioned model, a reference model(205/55R16) is used and other models are used to simulate the reference model. The simulations were conducted in different situations. in the end, the LuGre dynamic friction tire model was found preferable because it reflects the tire response characteristics not even in linear sections, but also the transient behavior is well simulated using this model.

### III. AFS/ESP INTEGRATED CONTROL SYSTEM

In order to calculate the vehicle ideal state, a linear two degrees of freedom model is established, which includes lateral motion and yaw motion, and is shown in figure 3. The paper has used the single track model dynamic equations to find  $\beta$  and  $\dot{r}$ . the equations are following.

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \delta \quad (2)$$

where  $\delta$  is steering angle,  $\beta$  is side slip angle and  $r$  is

yaw rate. Also, the parameters of equation 2 are:

$$\begin{aligned} a_{11} &= \frac{-2 \cdot Caf - 2 \cdot Car}{m \cdot u} \\ a_{12} &= -1 + \frac{2 \cdot Car \cdot b - 2 \cdot Caf \cdot a}{m \cdot u^2} \\ a_{21} &= \frac{2 \cdot Car \cdot b - 2 \cdot Caf \cdot a}{izz} \\ a_{22} &= \frac{-2 \cdot Caf \cdot a^2 - 2 \cdot Car \cdot b^2}{izz \cdot u} \\ b_1 &= \frac{2 \cdot Caf}{m \cdot u}, b_2 = \frac{2 \cdot Caf \cdot a}{izz} \end{aligned}$$

From this model, we can compute the steady-state model by putting  $\dot{\beta} = 0$  and  $\dot{r} = 0$ . When doing circular motion, the yaw speed of the vehicle is limited by the condition of the road surface adhesion, that is, there is the maximum allowable yaw angular velocity. The maximum yaw rate can be expressed by the approximate formula  $r_{uplim} = 0.85|\frac{u}{V_x}|$ . In this way, the ideal yaw rate  $r_d$  is equal to:

$$r_d = \min[|r_{ss}|, |r_{uplim}|] \cdot \text{sign}(\delta) \quad (3)$$

The same approach can be used to find  $\beta_d$ . The empirical formula of limit side slip angle is  $\beta_{uplim} = \tan^{-1}(0.02\mu.g)$ . , so the ideal side slip angle is as follows.

$$\beta_d = \min[|\beta_{ss}|, |\beta_{uplim}|] \quad (4)$$

#### A. AFS control logic

This paper tries to implement the AFS system by tracking the error between the actual and desired yaw rate. The tracking error is  $e_c = r - r_d$  and its rate is  $\dot{e}_c = \dot{r} - \dot{r}_d$ . Also, The sliding surface can be defined as  $s = e_c$ . The sliding mode dynamics of the system are as follows.

$$\dot{s} = -\lambda \cdot s \quad (5)$$

where,  $\lambda$  is positive constant. Now we can use this equation into vehicle model equations (equation 2) and find the control rule for steering angle which is the following.

$$\delta = \left( \frac{1}{b_2} \right) (-a_{21}\beta - a_{22}r + \dot{r}_d - \lambda(s)) - \text{sign}(s) \cdot X \quad (6)$$

where,  $\chi$  is the control gain, which determines the speed of the system when it reaches the slip surface. In this paper, we assumed  $\chi = 1$ . Then the additional input angle of AFS is  $\Delta\delta = \rho_c \cdot (\delta - \delta_d)$ ,  $\rho_c$  is the coordination control parameter,  $\delta_d$  is front wheel desired angle by the driver.

#### B. coordinated control algorithm

The switching control algorithm is adopted in this paper. For a convenient handle in engineering, setting  $\beta_d = 0$ . we design The value of  $\rho_c$  as a function of  $\beta$  and  $\dot{\beta}$ . So we defined a Condition function that switches the controller works based on the output of this condition. The condition is the following.

$$Cond = |\kappa_1 \dot{\beta} - \kappa_2 \beta| \quad (7)$$

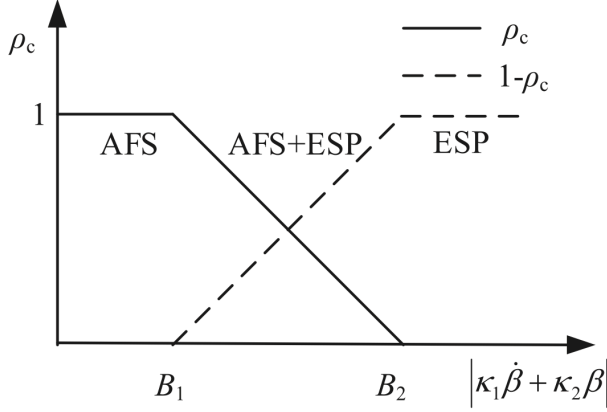


Fig. 4: Coordination control characteristics.

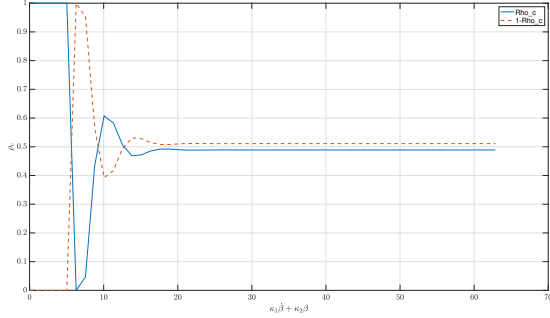


Fig. 5: Coordination control characteristics for Step steer input at  $u = 100 \text{ km/h}$

$\kappa_1$  and  $\kappa_2$  can be tuned to modify the effect of  $\dot{\beta}$  and  $\beta$  on the switching control algorithm. In this paper, we tried to determine the range of  $\beta$  and  $\dot{\beta}$  and based on the value of these ranges, select  $\kappa_1$  and  $\kappa_2$ . after testing different values of parameters, we decided to put  $\kappa_1 = 50$  and  $\kappa_2 = 100$ .

There is the following formula that determines the switching controller behavior based on equation 7 and other tunable parameters. The value of  $\rho_c$  is shown in the following formula.

$$f(z) = \begin{cases} \rho_c = 0 & \text{for } Cond < B1 \\ \rho_c = \frac{B2 - Cond}{B2 - B1} & \text{for } B1 \leq Cond \leq B2 \\ \rho_c = 1 & \text{for } Cond > B2 \end{cases} \quad (8)$$

$B1$  and  $B2$  represent the definition of a stable region and can be obtained by experimentation. After testing different values of parameters, we put  $B1 = 23$  and  $B2 = 5$ . In figure 5, we can see the actual activation function for vehicle with Step steer input at  $u = 100 \text{ km/h}$ . Of course, the activation function figure is different for every scenario.

### C. ESP

the ESP system applies selective braking to individual wheels to counteract oversteer or understeer situations, helping the driver maintain control and prevent skidding or sliding. By actively intervening in critical moments, ESP enhances overall vehicle stability and reduces the risk of accidents, particularly during sudden maneuvers or slippery surfaces. The simple selection principle of the active braking wheel is shown in Table. In this paper, a simple control rule is used, that is, this control method can provide the compensation torque well so as to control the vehicle stability.

$\delta$	EC	Direction of Steering Wheel	Steering Characteristics	Brake Wheel Selection
+	+	Right	Oversteer	Left front wheel
+	-	Right	Understeer	Right rear wheel
-	+	Left	Oversteer	Right front wheel
-	-	Left	Understeer	Left rear wheel

TABLE I: Coordination control characteristics.

To simulate the table I, we used a State machine in Matlab Simulink. The steering angle and  $\delta\alpha$  ( $\alpha_r - \alpha_f$ ) are used as the input. As soon as  $\delta\alpha$  takes a value other than 0, the ESP tries to lead  $\delta\alpha$  to 0 again so the vehicle can naturally steer again.

The ESP approach in this paper, is based on the required yaw moment ( $M_z$ ) to keep the vehicle in natural steering behavior. the  $M_z$  can be calculated based on the  $\frac{dF_{xi}}{dt}$  and  $\frac{dF_{yi}}{dt}$ . The longitudinal and lateral load transfer of each wheel produces yaw moment w.r.t vehicle COM. According to Table I, if there is oversteer and it is necessary to brake on the outside front wheel, the yaw moment  $\Delta M_z$  is obtained as follows.

$$\Delta M_z = \delta F_x \left( \frac{1}{2} \cdot Tw \cdot \cos(\delta) + a \cdot \sin(\delta) \right) - \delta F_y \left( a \cdot \cos(\delta) - \frac{1}{2} \cdot Tw \cdot \sin(\delta) \right) \quad (9)$$

When understeer occurs, the inner rear wheel is braked, and the yaw moment  $\Delta M_z$  is obtained as follows.

$$\Delta M_z = (\delta F_x \cdot 0.5 \cdot Tw) - (\delta F_y \cdot b) \quad (10)$$

By applying  $\Delta M_z$  to the vehicle, we can keep the vehicle in a stable situation.

## IV. Simulation comparison

### A. Paper results

According to the above AFS, ESP, and integrated control logic, simulation analysis is carried out. In the main Paper, the simulations were conducted as follows.

- Sine wave steering input

- 1) High adhesion coefficient road ( $\mu=0.9$ , initial speed  $u_0=100 \text{ km/h}$ )

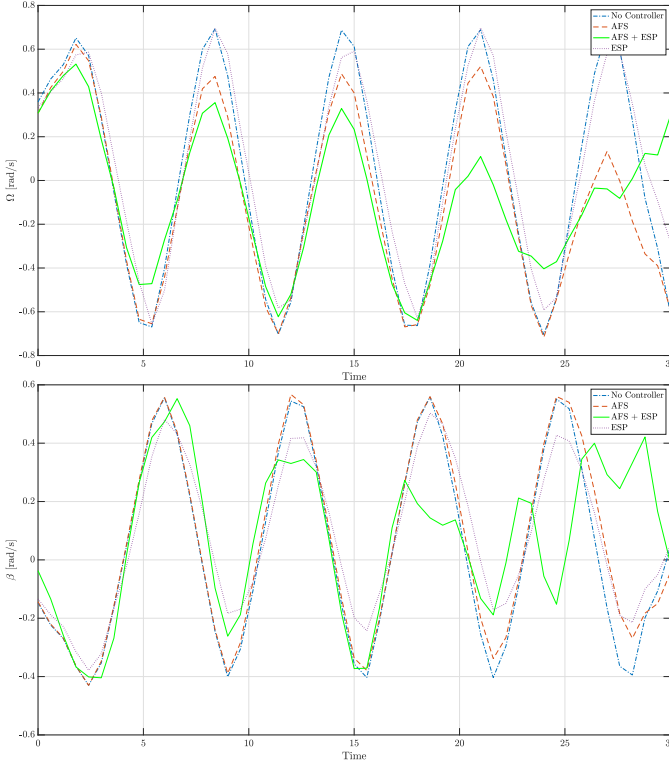


Fig. 6: Test 1: Serpentine simulation of high friction road.

- 2) Low adhesion coefficient road ( $\mu=0.2$ , initial speed  $u_0=60$  km/h)

- Step function steering input

- 1) High adhesion coefficient road ( $\mu=0.9$ , initial speed  $u_0=100$  km/h)
- 2) Low adhesion coefficient road ( $\mu$ , initial speed  $u_0=60$  km/h)

Based on the figures in the paper for the mentioned tests, in the first test, the vehicle without any controller doesn't slip much and it seems the vehicle is in a stable situation all the time. However, there is no information about vehicle parameters or selected parameters for AFS or ESP controllers.

We have tried to conduct the same tests but since we are using different vehicle parameters, the results are not as smooth as it is in the main paper. However, as it's obvious, the controllers are helping to keep the vehicle as stable as possible.

## V. Conclusion

The multi-degree of freedom nonlinear dynamic vehicle model including body movement, wheel movement and electronically controlled hydraulic power steering system was built first. A comparative study was conducted with the magic formula tire model, Dugoff tire model, brush tire model, and LuGre dynamic friction tire model and the steady state tire model based on LuGre. A comparative study was conducted between the simulation results and the real vehicle test results, during which the precision of the model was verified. However, the pre-defined Matlab

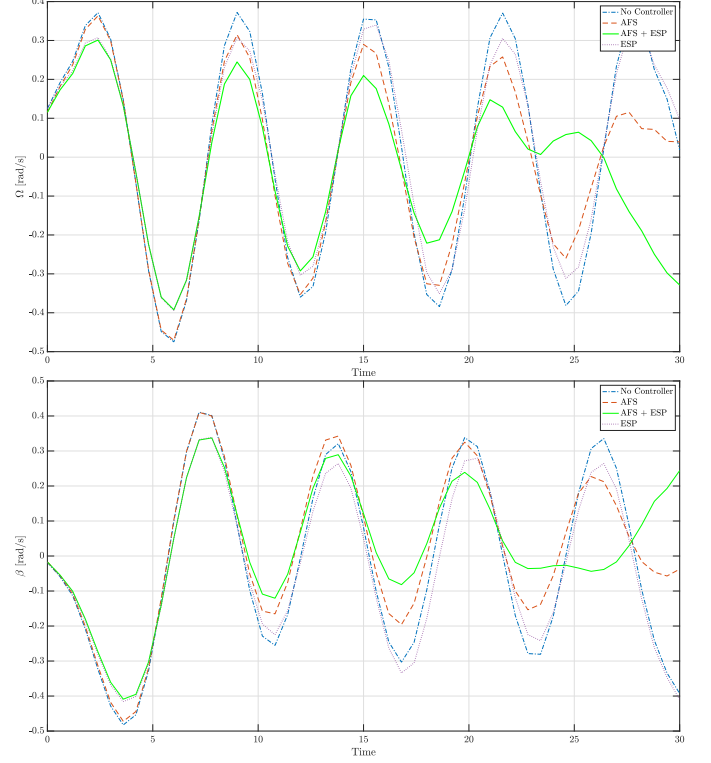


Fig. 7: Test 2: Serpentine simulation of low friction road.

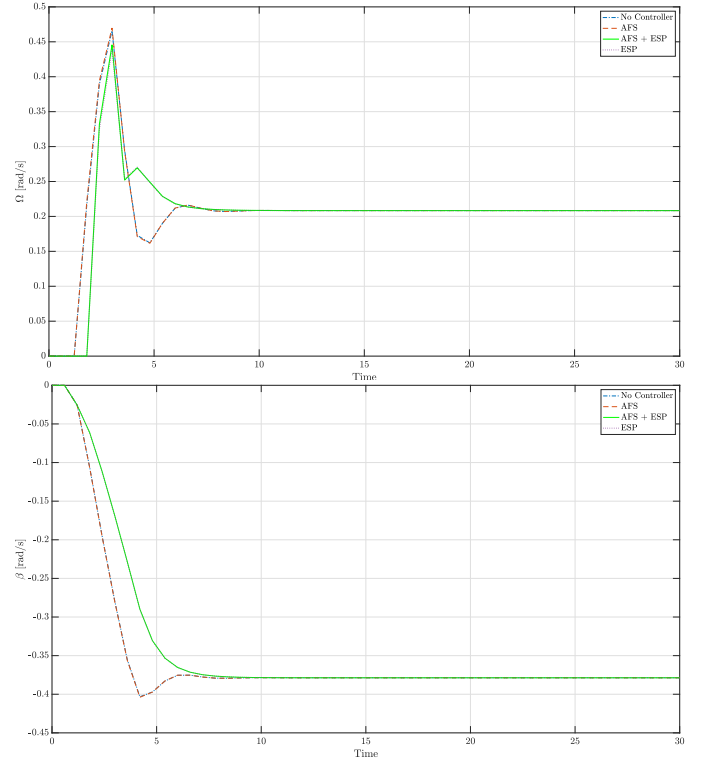


Fig. 8: Test 3: Step angle input of high friction road.

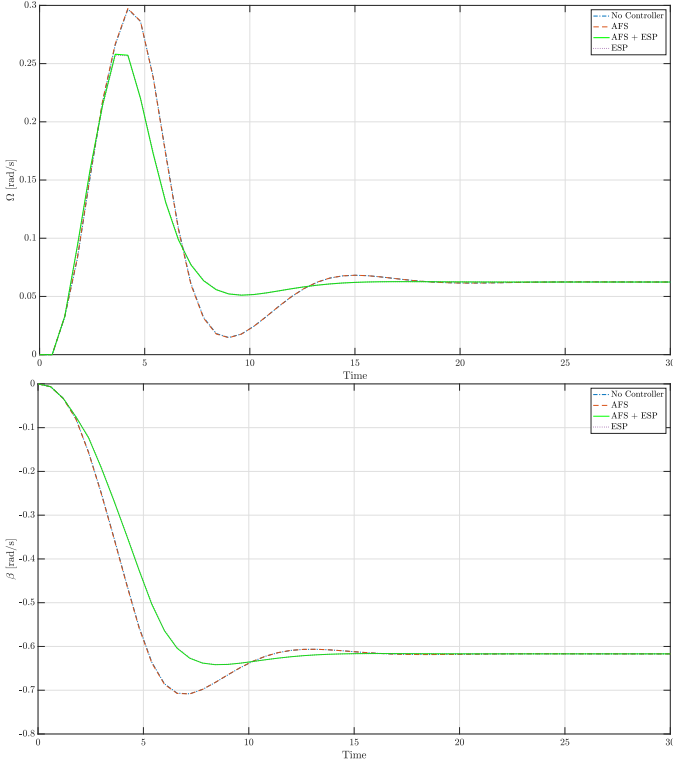


Fig. 9: Test 3: Step angle input of low friction road.

double-track vehicle model is used for designing AFS and ESP controllers. an AFS controller was designed based on sliding mode variable structure control, and the AFS and ESP integrated coordination controller was proposed. The sinusoidal and steering angle step input simulation analysis was proposed based on a multi-DOF vehicle model and nonlinear tire model. The simulation results show that, compared with the action of AFS or ESP alone, the vehicle response characteristics are better under a coordinated control strategy, so the method can obviously improve the handling stability and safety of the vehicle.

### References

- [1] Xiaobin Fan and Zixiang Zhao VEHICLE DYNAMICS MODELING AND ELECTRONIC STABILITY PROGRAM/ ACTIVE FRONT STEERING SLIDING MODE INTEGRATED CONTROL. 2018 Chinese Automatic Control Society and John Wiley Sons Australia, Ltd.
- [2] The MathWorks, Inc. (2022). Optimization Toolbox version: 9.4 (R2022b). 2018 Chinese Automatic Control Society and John Wiley Sons Australia, Ltd.