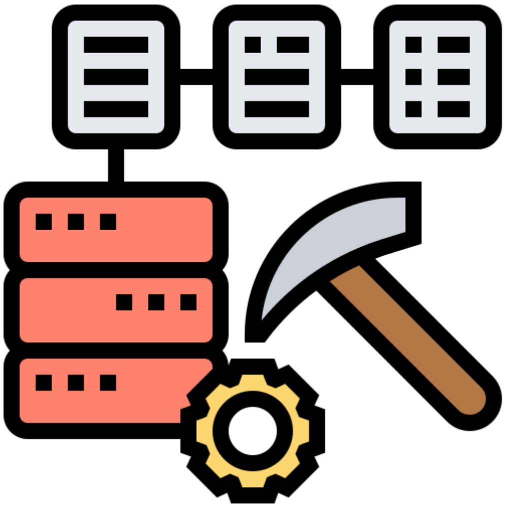

Data Mining

Homework 1

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Problem 1

1. I choose the probability space such that each element in Ω corresponds to a unique ordering of the cards in the deck.
   Mathematically, $\Omega = \{\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{52!}\}$, where each σ_i is a distinct permutation.
   Since each permutation in Ω is equally likely when we shuffle a standard deck of cards, we have a uniform probability distribution.
   The probability measure P assigns equal probabilities to all permutations in Ω. Therefore, for any event $A \subseteq \Omega$, we have:

   $$P(A) = \frac{|A|}{52!}$$

   Specifically, for each individual permutation σ_i in Ω:

   $$P(\sigma_i) = \frac{1}{52!}$$

2. Probability of the each event is:

   (a) **Finding the probability that the first two cards include at least one ace**
   We can use the complement rule:

   $$\text{Probability (at least one ace in the first two cards)} =$$

   $$1 - \text{ Probability (no aces in the first two cards)}$$

   Now, let's calculate the probability of not getting any aces in the first two cards. There are 52 cards in the deck initially, and 48 of them are not aces. When we pick the first card, there are 48 non-ace cards out of 52 possibilities. After picking the first card, there are now 51 cards left in the deck, with 47 of them being non-ace cards. Therefore, the probability of not getting an ace on the second card, given that the first card was not an ace, is:

$$\frac{48}{52} \cdot \frac{47}{51}$$

So, the probability that the first two cards contain at least one ace is:

$$1 - \left(\frac{48}{52} \cdot \frac{47}{51}\right) \approx 0.1494$$

(b) **Finding the probability that the first five cards include at least one ace**

Same as item (a) we can use the complement rule:

$$\text{Probability (at least one ace in the first five cards)} =$$

$$1 - \text{ Probability (no aces in the first five cards)}$$

Same as previous item, the probability of not getting an ace on the first 5 cards, is:

$$\frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50} \cdot \frac{45}{49} \cdot \frac{44}{28}$$

So, the probability that the first five cards include at least one ace:

$$1 - \left(\frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50} \cdot \frac{45}{49} \cdot \frac{44}{28}\right) \approx 0.3147$$

(c) **The first two cards are a pair of the same rank**

(d) **The first five cards are all diamonds.**

There are 13 diamonds in a standard deck.

The probability of drawing a diamond on the first draw is $\frac{13}{52}$ because there are 13 diamonds out of 52 cards.

Similarly, for the second, third, fourth, and fifth draws, the probabilities are as follows:

Second draw: $\frac{12}{51}$

Third draw: $\frac{11}{50}$

Fourth draw: $\frac{10}{49}$

Fifth draw: $\frac{9}{48}$

Because these are independent events, we can multiply these probabilities together:

$$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \approx 0.00018184$$

So, the probability of drawing the first five cards as diamonds is approximately 0.00018184, or about 0.0182

(e) **The first two cards are a pair of the same rank**

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

Problem 7