Dependable Distributed Systems
Master of Science in Engineering in Computer Science

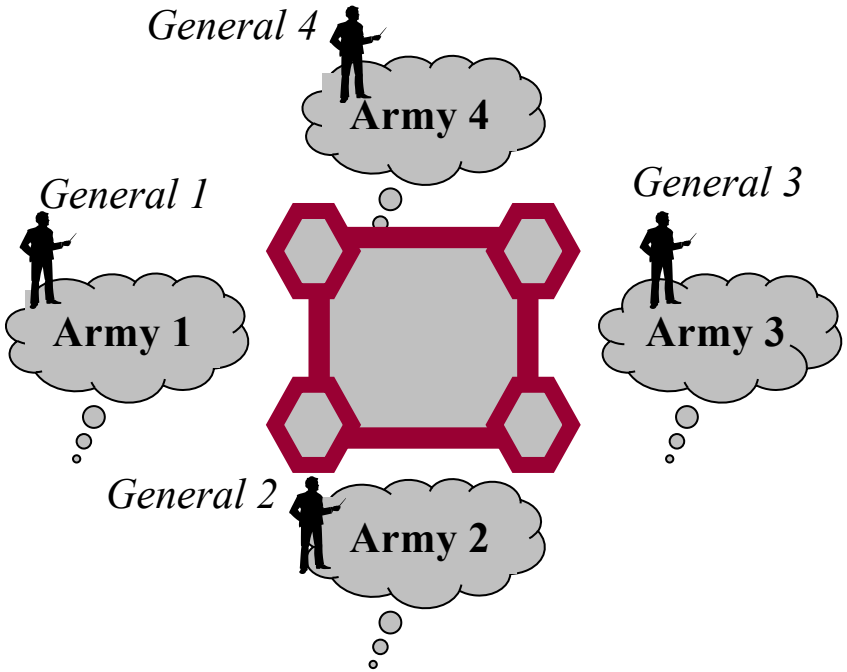
AA 2022/2023

LECTURE 10: CONSENSUS

Consensus Problem

- A group of processes must agree on a value that has been proposed by one of them (e.g., commit/abort of a transaction).
- It is an abstraction of a class of problems where processes start with their opinion and then converge on one of them
- It is a fundamental problem
- We study algorithms working on weak models

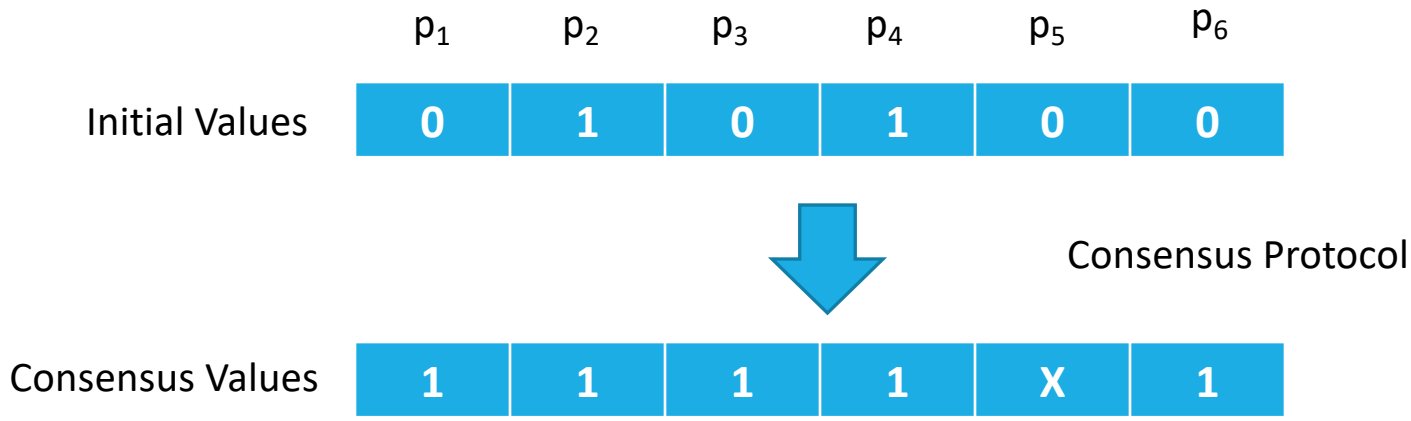
Consensus Example



Generals have to reach consensus between attack and get back

Consensus Definition

- Set of initial values ∈ {0,1}.
- Every process has to decide the same value ∈ {0,1} based on the initial proposals.



Consensus Specification

Module 5.1: Interface and properties of (regular) consensus

Module:

Name: Consensus, **instance** c.

Events:

Request: $\langle\ c, \mathit{Propose} \mid v\ \rangle$: Proposes value v for consensus.

Indication: $\langle\ c, \mathit{Decide} \mid v\ \rangle$: Outputs a decided value v of consensus.

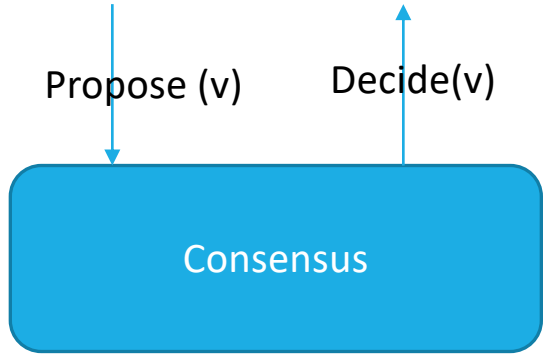
Properties:

C1: *Termination:* Every correct process eventually decides some value.

C2: *Validity:* If a process decides v, then v was proposed by some process.

C3: *Integrity:* No process decides twice.

C4: *Agreement:* No two correct processes decide differently.



FLP Impossibility Result

no algorithm can guarantee to reach consensus in an asynchronous system, even with one process crash failure.

Fisher, Lynch e Patterson (FLP result)
Ref: Journal of the ACM, Vol. 32, No. 2, April 1985.

A Consensus Implementation in Synchronous Systems: Flooding Consensus

Basic Idea:

- Processes exchange their values
- when all the proposals from correct processes are available a one value can be chosen

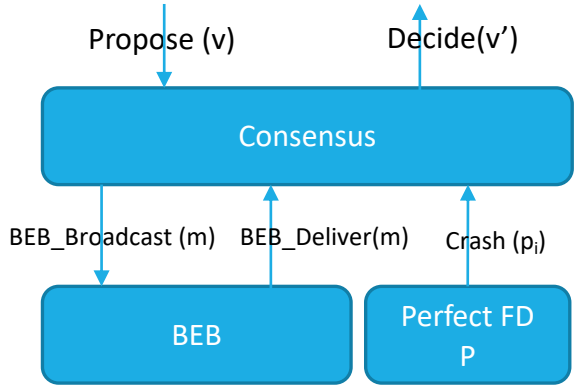
Problem:

- due to failures, some values can be lost

Solution:

- A value can be selected only when no failures happen during the communication

A Consensus Implementation in Synchronous Systems: Flooding Consensus



Algorithm 5.1: Flooding Consensus

Implements:
Consensus, **instance** c.

Uses:
BestEffortBroadcast, **instance** *beb*;
PerfectFailureDetector, **instance** $\mathcal{P}$.

upon event $\langle\ c, Init\ \rangle$ **do**
$correct := \Pi$;
$round := 1$;
$decision := \bot$;
$receivedfrom := [\emptyset]^N$;
$proposals := [\emptyset]^N$;
$receivedfrom[0] := \Pi$;

upon event $\langle\ \mathcal{P}, Crash \mid p\ \rangle$ **do**
$correct := correct \setminus \{p\}$;

upon event $\langle\ c, Propose \mid v\ \rangle$ **do**
$proposals[1] := proposals[1] \cup \{v\}$;
trigger $\langle\ beb, Broadcast \mid [\text{PROPOSAL}, 1, proposals[1]]\ \rangle$;

upon event $\langle\ beb, Deliver \mid p, [\text{PROPOSAL}, r, ps]\ \rangle$ **do**
$receivedfrom[r] := receivedfrom[r] \cup \{p\}$;
$proposals[r] := proposals[r] \cup ps$;

upon $correct \subseteq receivedfrom[round] \wedge decision = \bot$ **do**
if $receivedfrom[round] = receivedfrom[round-1]$ **then**
$decision := \min(proposals[round])$;
trigger $\langle\ beb, Broadcast \mid [\text{DECIDED}, decision]\ \rangle$;
trigger $\langle\ c, Decide \mid decision\ \rangle$;
else
$round := round + 1$;
trigger $\langle\ beb, Broadcast \mid [\text{PROPOSAL}, round, proposals[round-1]]\ \rangle$;

upon event $\langle\ beb, Deliver \mid p, [\text{DECIDED}, v]\ \rangle$ **such that** $p \in correct \wedge decision = \bot$ **do**
$decision := v$;
trigger $\langle\ beb, Broadcast \mid [\text{DECIDED}, decision]\ \rangle$;
trigger $\langle\ c, Decide \mid decision\ \rangle$;

Example of Flooding Consensus

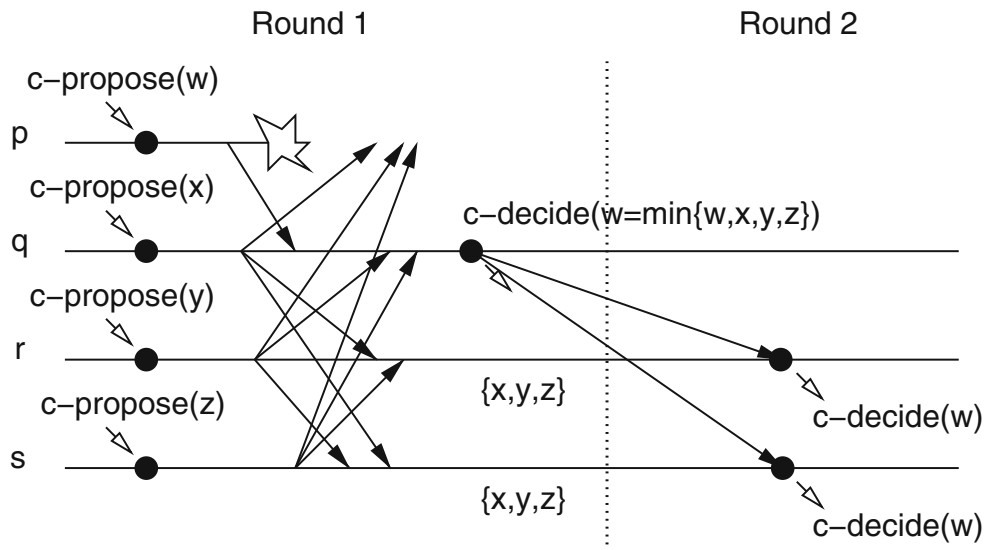
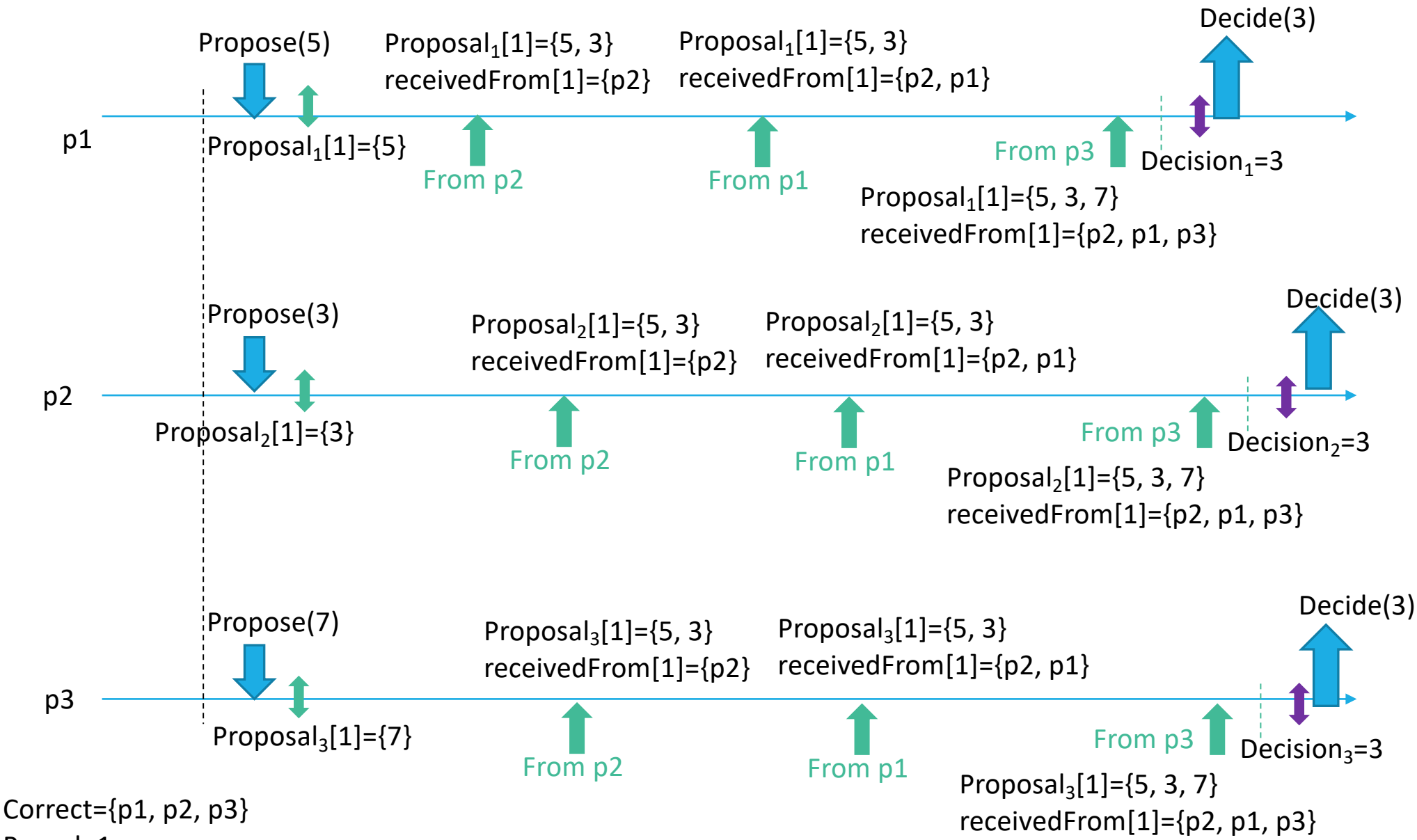
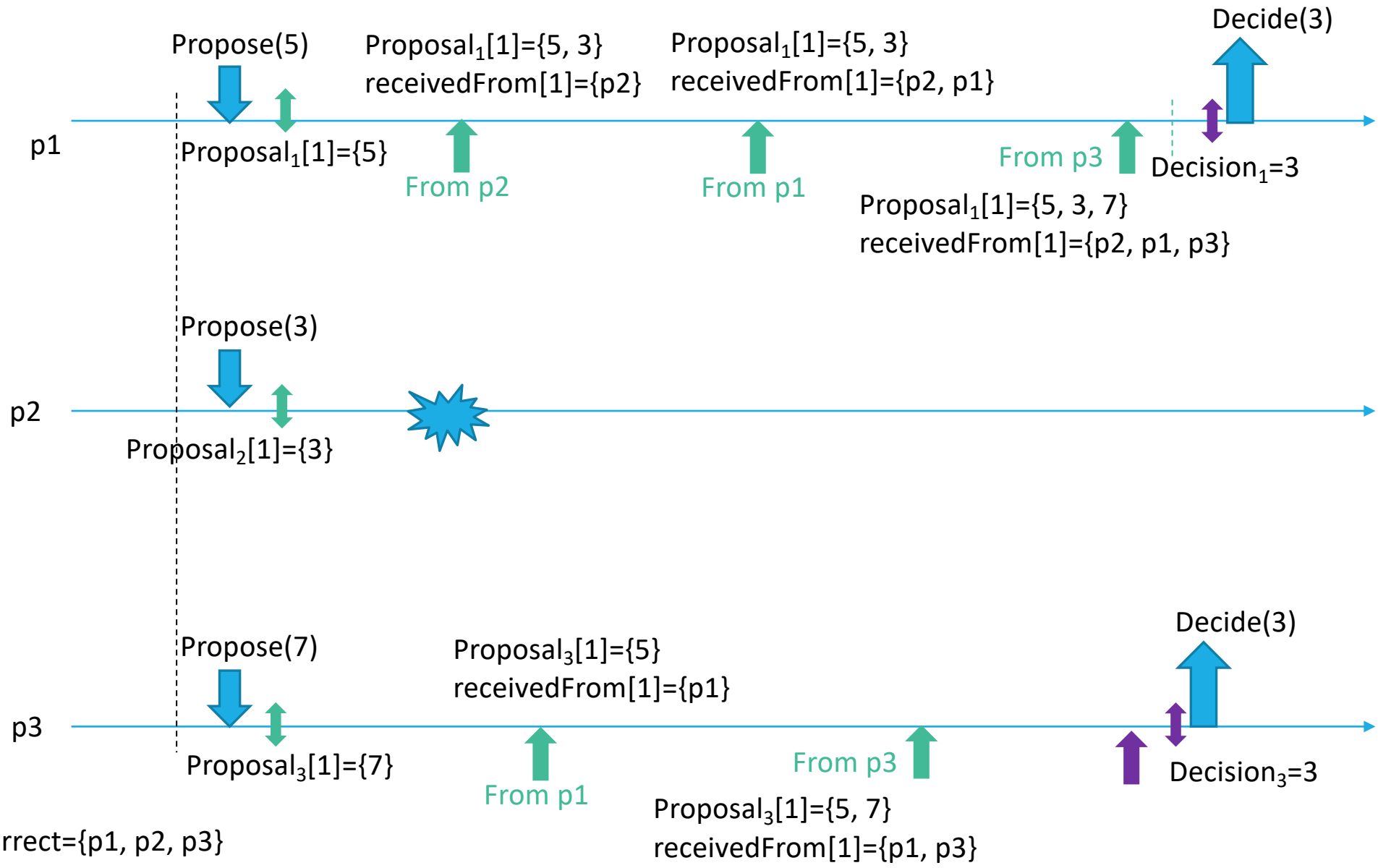


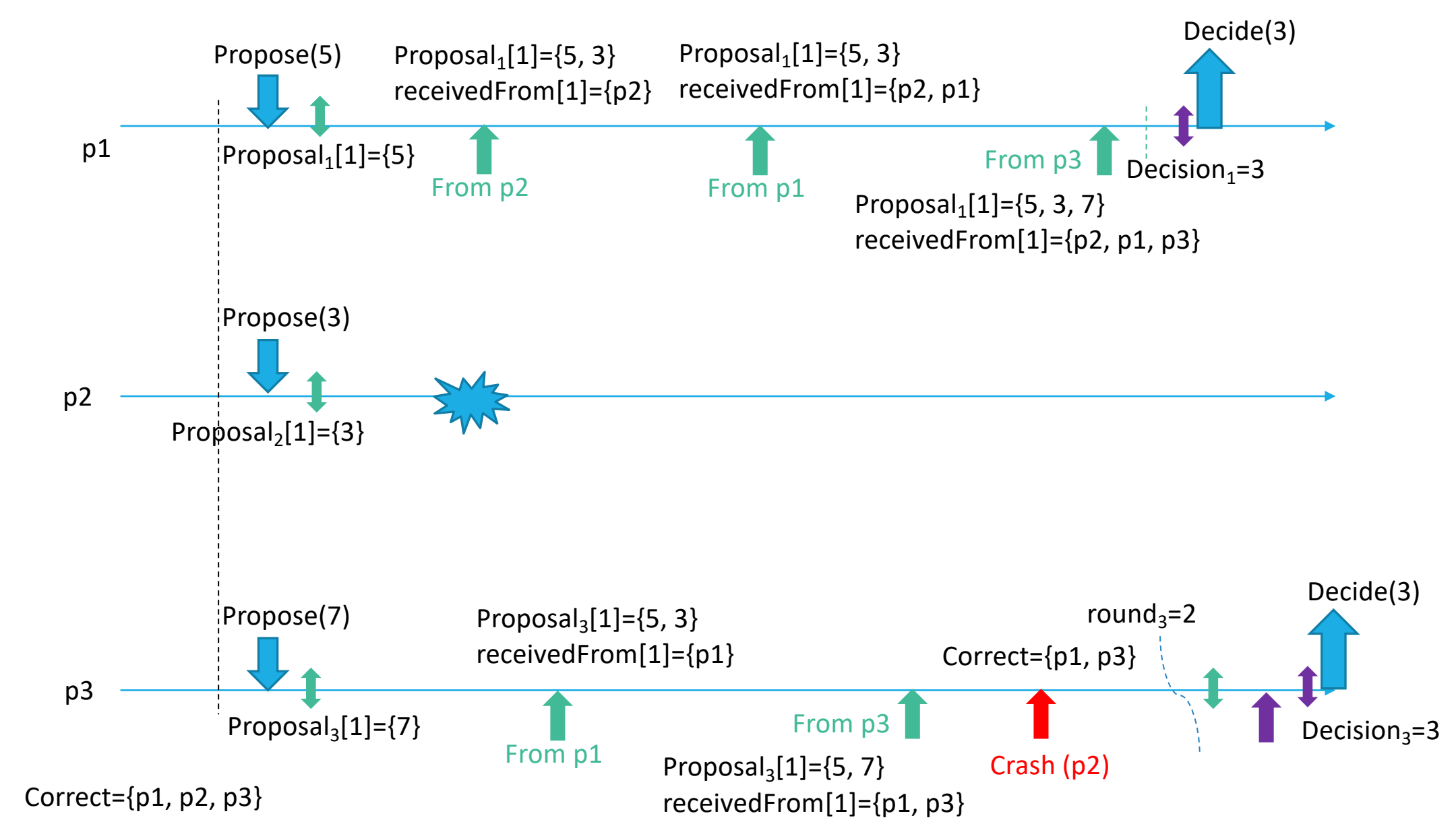
Figure 5.1: Sample execution of flooding consensus



Correct={p1, p2, p3}
Round=1
Decision=⊥
receivedFrom[0]={p1, p2, p3}
Proposals[0]={}



Correct={p1, p2, p3}
Round=1
Decision=⊥
receivedFrom[0]={p1, p2, p3}
Proposals[0]={}



Correct={p1, p2, p3}
Round=1
Decision=⊥
receivedFrom[0]={p1, p2, p3}
Proposals[0]={}

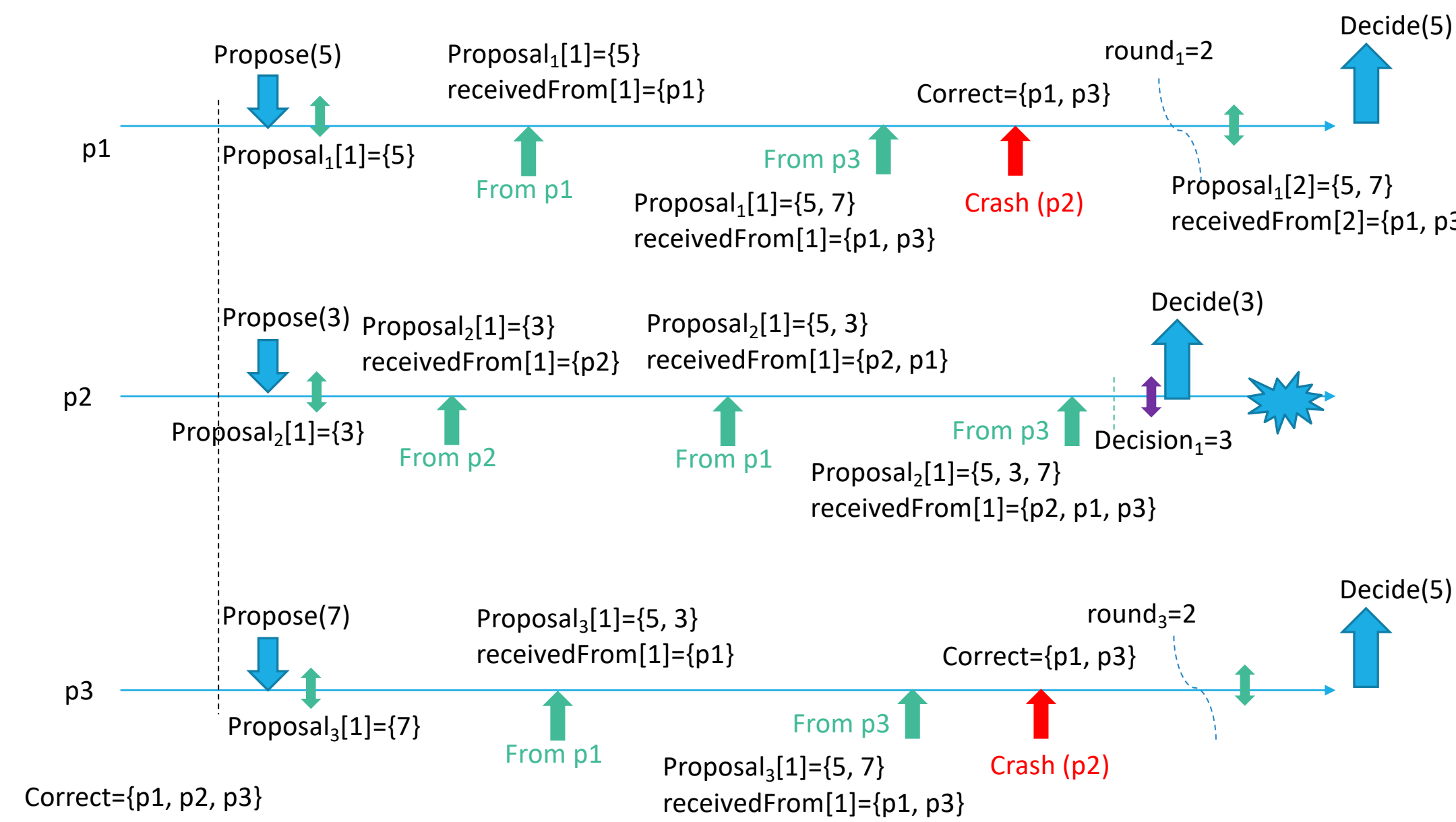
Correctness and Performance

Correctness

- Validity and integrity follow from the properties on the communication channels
- Termination. At most after N rounds processes decide
- Agreement. The same deterministic function is applied to the same values by correct processes

Performance

- Best Case (No failures). One communication round ($2N^2$ messages)
- Worst case (n-1 failures). N^2 messages exchanged for each communication step and at most N rounds => N^3 messages



Correct={p1, p2, p3}
Round=1
Decision=⊥
receivedFrom[0]={p1, p2, p3}
Proposals[0]={}

Uniform Consensus Specification

Module 5.2: Interface and properties of uniform consensus

Module:

Name: UniformConsensus, **instance** *uc*.

Events:

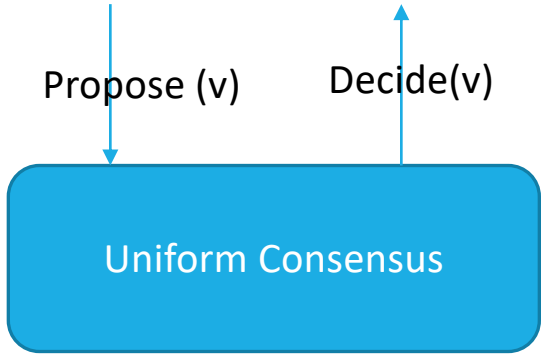
Request: ⟨ *uc*, *Propose* | *v* ⟩: Proposes value *v* for consensus.

Indication: ⟨ *uc*, *Decide* | *v* ⟩: Outputs a decided value *v* of consensus.

Properties:

UC1–UC3: Same as properties C1–C3 in (regular) consensus (Module 5.1).

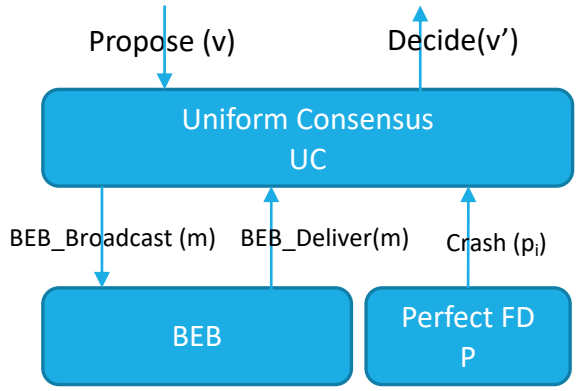
UC4: *Uniform agreement:* No two processes decide differently.



Uniform Consensus and Flooding Consensus algorithm

Does the flooding consensus algorithm (described in the previous slides) satisfy the Uniform Consensus specification?

Uniform Consensus Implementation in Synchronous Systems



Algorithm 5.3: Flooding Uniform Consensus

Implements:
UniformConsensus, **instance** *uc*.

Uses:
BestEffortBroadcast, **instance** *beb*;
PerfectFailureDetector, **instance** $\mathcal{P}$.

upon event $\langle$ *uc*, *Init* $\rangle$ **do**
correct := Π;
round := 1;
decision := $\perp$;
proposalset := $\emptyset$;
receivedfrom := $\emptyset$;

upon event $\langle$ $\mathcal{P}$, *Crash* | p $\rangle$ **do**
correct := *correct* $\setminus$ $\{p\}$;

upon event $\langle$ *uc*, *Propose* | v $\rangle$ **do**
proposalset := *proposalset* $\cup$ $\{v\}$;
trigger $\langle$ *beb*, *Broadcast* | [PROPOSAL, 1, *proposalset*] $\rangle$;

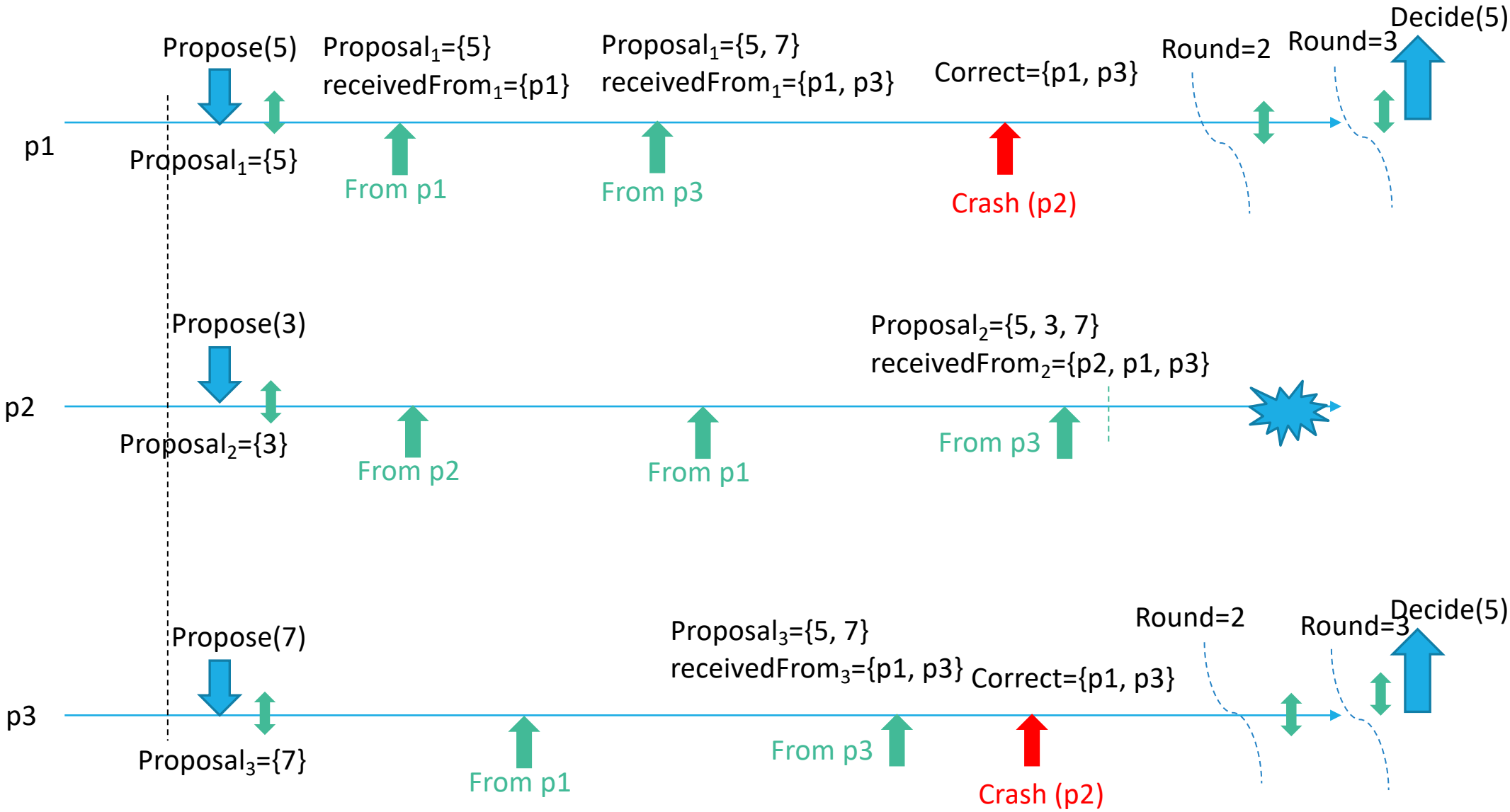
No more related to the round

upon event $\langle$ *beb*, *Deliver* | p, [PROPOSAL, r, ps] $\rangle$ **such that** r = *round* **do**
receivedfrom := *receivedfrom* $\cup$ $\{p\}$;
proposalset := *proposalset* $\cup$ ps;

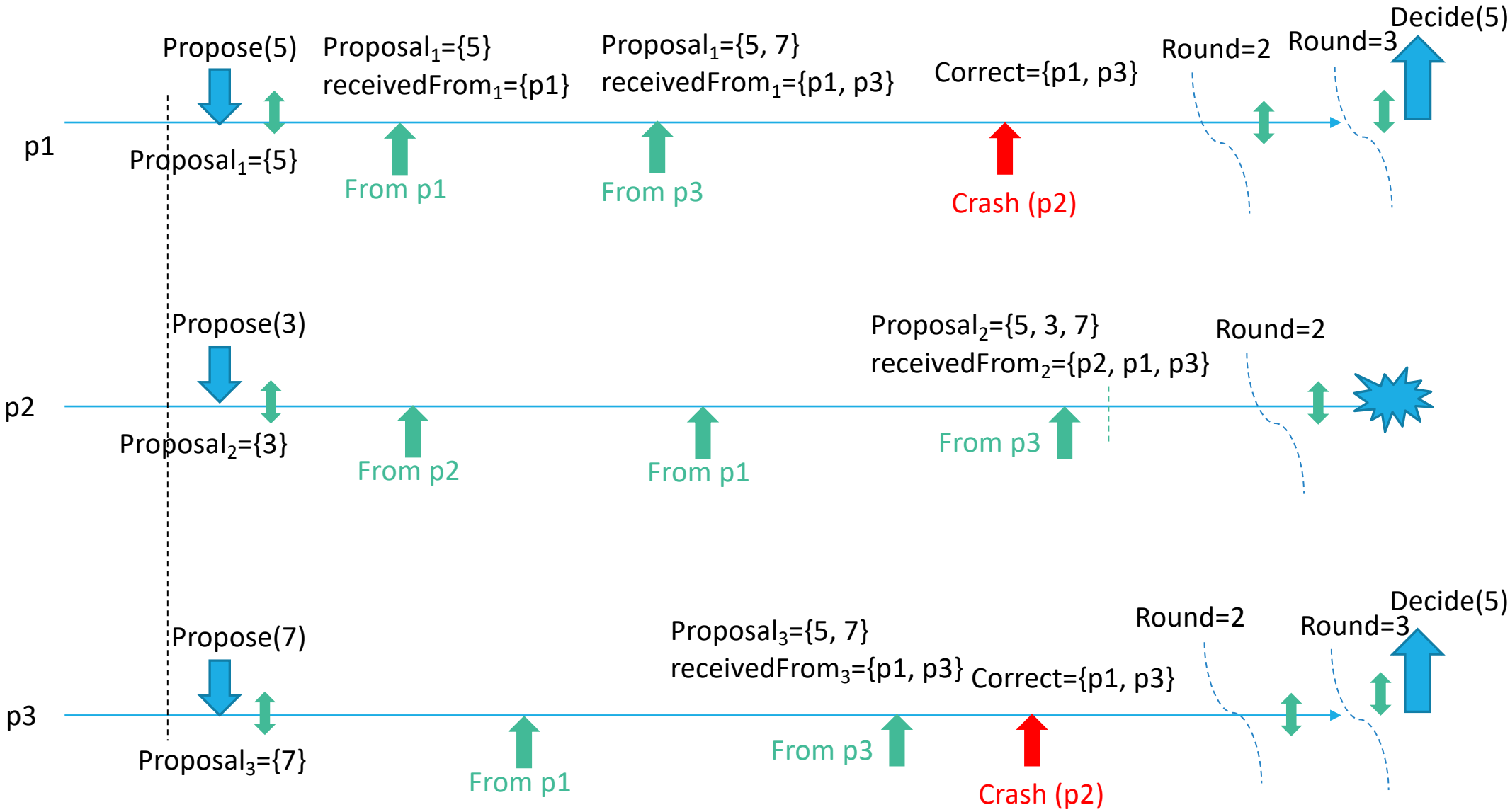
upon *correct* $\subseteq$ *receivedfrom* $\wedge$ *decision* = $\perp$ **do**
if *round* = N **then**
decision := min(*proposalset*);
trigger $\langle$ *uc*, *Decide* | *decision* $\rangle$;
else
round := *round* + 1;
receivedfrom := $\emptyset$;
trigger $\langle$ *beb*, *Broadcast* | [PROPOSAL, *round*, *proposalset*] $\rangle$;

Decision only at the end

Cleaned at the beginning of each round



Correct={p1, p2, p3}
Round=1
Decision=⊥
receivedFrom={}
Proposals={}



Correct={p1, p2, p3}
Round=1
Decision=⊥
receivedFrom={}
Proposals={}

Correctness and Performance

Correctness

- The *validity* and *integrity* properties follow from the algorithm and from the properties of best-effort broadcast
- The *termination* property is ensured as all correct processes reach round N and decide in that round
  - the *strong completeness* property of the failure detector implies that no correct process waits indefinitely for a message from a process that has crashed, as the crashed process is eventually removed from *correct*.
- The *uniform agreement* holds because all processes that reach round N have the same set of values in their variable *proposalset*.

Performance

- N communication steps and O(N3) messages for all correct processes to decide

References

C. Cachin, R. Guerraoui and L. Rodrigues. Introduction to Reliable and Secure Distributed Programming, Springer, 2011

- Chapter 5, Sections 5.1.1, 5.1.2, 5.2.1, 5.2.2