In the name of God

Digital Communication Lab

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PreLab#6

اگر کل مجموعه سیگنال‌ها متعامد باشند :

$$\{ s_1(t), s_2(t), \dots, s_M(t) \}$$

$$\langle s_i(t), s_j(t) \rangle = \begin{cases} \mathcal{E} & i=j \\ 0 & i \neq j \end{cases} \qquad \int |s_i(t)|^2 dt = \mathcal{E}$$

$$\phi_k(t) = \frac{s_k(t)}{\sqrt{\mathcal{E}}} \qquad s_k(t) = \sqrt{\mathcal{E}}\, \phi_k(t) \Rightarrow \underline{s}_k = \begin{bmatrix} 0 \\ \vdots \\ \sqrt{\mathcal{E}} \\ \vdots \\ 0 \end{bmatrix}_{M \times 1} \leftarrow \text{سطر } k \text{ این}$$

$dim: N = M$

در سیگنال‌های متعامد احتمال خطا را با استفاده از قاعده ML محاسبه کنیم :

$$P[C] = \sum_{i=1}^{M} P[C|H_i]\, P[H_i] \qquad \hat{m} = \arg\max P\{\underline{r}|H_m\} = \arg\min \{ \|\underline{r} - \underline{s}_m\|^2 \}$$

$$\|\underline{r} - \underline{s}_m\|^2 = \underbrace{\|\underline{r}\|^2}_{\text{مستقل}} + \underbrace{\|\underline{s}_m\|^2}_{\mathcal{E}} - 2\underline{r}^T \underline{s}_m$$

$$\Rightarrow \hat{m} = \arg\max \{ \underline{r}^T \underline{s}_m \}$$

$$\begin{cases} H_1: \underline{r} = \underline{s}_1 + \underline{n} \\ \vdots \\ H_M: \underline{r} = \underline{s}_M + \underline{n} \end{cases} \qquad \underline{n} = \begin{bmatrix} n_1 \\ \vdots \\ n_M \end{bmatrix} \qquad \underline{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_M \end{bmatrix}$$

$$r_k = \langle r(t), \phi_k(t) \rangle = \int_{-\infty}^{\infty} r(t)\, \phi_k^*(t)\, dt$$

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$$\rightarrow P[C|H_1] = P[\underbrace{n_1 - n_2 > -\sqrt{E},\ n_1 - n_3 > -\sqrt{E},\ \dots,\ n_1 - n_M > -\sqrt{E}}_{A} \mid H_1]$$

$$= E_{n_1}\left\{ P\left[A \mid n_1 = x, H_1\right]\right\}$$

$$P(A \mid H_1, n_1 = x) = P\left[n_2 < n_1 + \sqrt{E},\ n_3 < n_1 + \sqrt{E},\ \dots,\ n_M < n_1 + \sqrt{E} \mid H_1, n_1 = x\right]$$

$$= P\left[n_2 < x + \sqrt{E},\ \dots,\ n_M < x + \sqrt{E} \mid H_1, n_1 = x\right]$$

n_i $i = 2, \dots, M$ از n_1, H_1 مستقل از [illegible]

$$= P\left[n_2 < x + \sqrt{E}\right] \cdots P\left[n_M < x + \sqrt{E}\right]$$

مستقل از

$$= \prod_{k=2}^{M} P\left[n_k < x + \sqrt{E}\right]$$

$$= \prod_{k=2}^{M} Q\left(-\frac{x+\sqrt{E}}{\sqrt{N_0/2}}\right) = \prod_{k=2}^{M}\left(1 - Q\left(\frac{x+\sqrt{E}}{\sqrt{N_0/2}}\right)\right)$$

$$\rightarrow P[C|H_1] = E_{n_1}\left\{\left(1 - Q\left(\frac{n_1+\sqrt{E}}{\sqrt{N_0/2}}\right)\right)^{M-1}\right\}$$

$$= \int_{-\infty}^{\infty}\left(1 - Q\left(\frac{n_1+\sqrt{E}}{\sqrt{\frac{N_0}{2}}}\right)\right)^{M-1}\frac{e^{-n_1^2/N_0}}{\sqrt{\pi N_0}}\,dn_1$$

$$P[C|H_1] = \dots = P[C|H_M] \Rightarrow P[C] = P[C|H_1]$$

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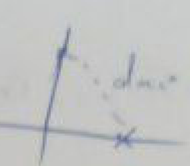
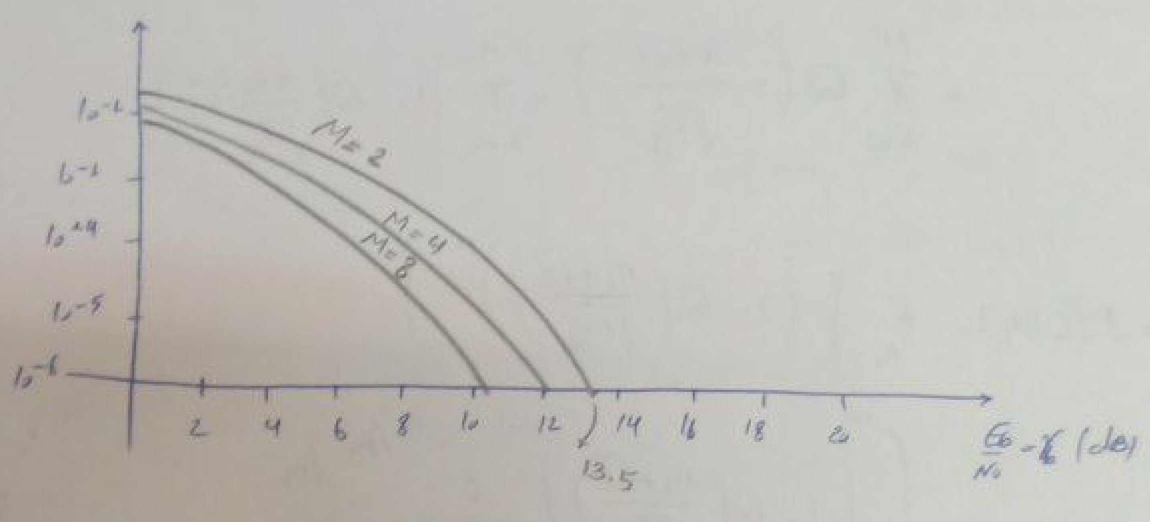
$$\frac{\sqrt{\mathcal{E}} + x}{\sqrt{N_0/2}} = u \rightarrow P[E] = \boxed{\int_{-\infty}^{\infty} \left[1 - \left(1 - Q(u)\right)^{M-1}\right] \frac{e^{-\left(u - \sqrt{\frac{2\mathcal{E}}{N_0}}\right)^2 / 2}}{\sqrt{2\pi}} \, du}$$

$*$ $M=2$



$$P[E] = Q\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right) = Q\left(\frac{d}{\sqrt{2N_0}}\right)$$

در سیگنال‌های متعامد : $\mathcal{E}_{b,avg} = \frac{\mathcal{E}_{avg}}{\log M} = \frac{\mathcal{E}}{\log M} \rightarrow \mathcal{E} = \mathcal{E}_{b,avg} \log_2 M$



در سیگنال‌های متعامد با افزایش نرخ (M) P_e کاهش می‌یابد یعنی برای داشتن P_e ثابت نیاز به توان کمتری داریم

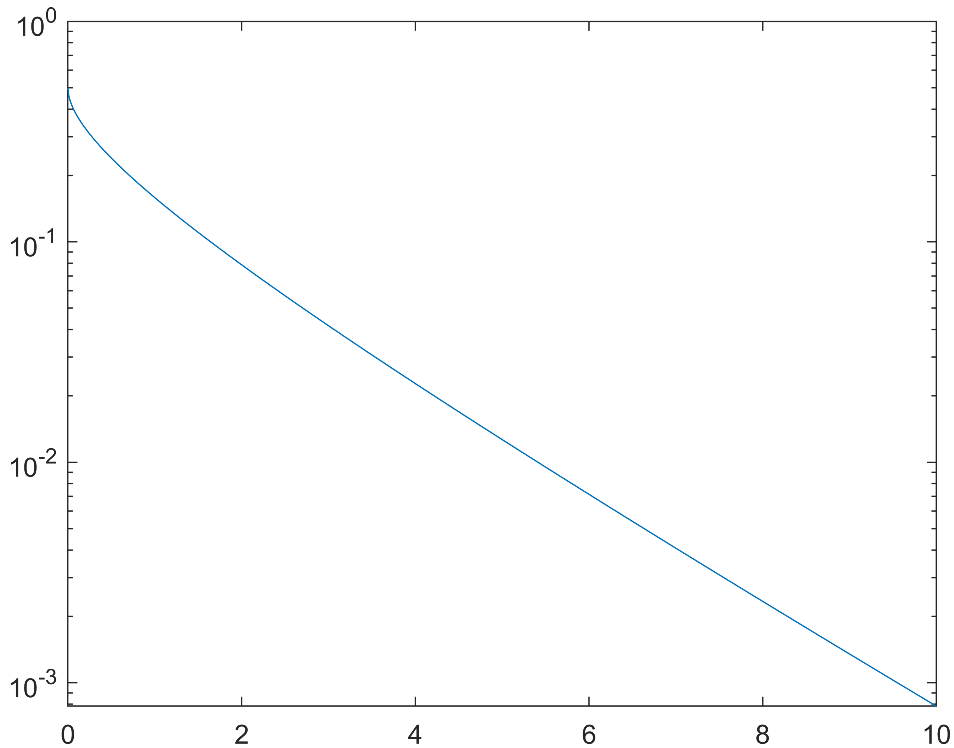
است (توان مصرفی) $P = R_b \mathcal{E}_b = 10^4$ $\quad \mathcal{E}_b = 1 \quad R_b = 10^{\,kbit/sec} \quad M=2$

2×2

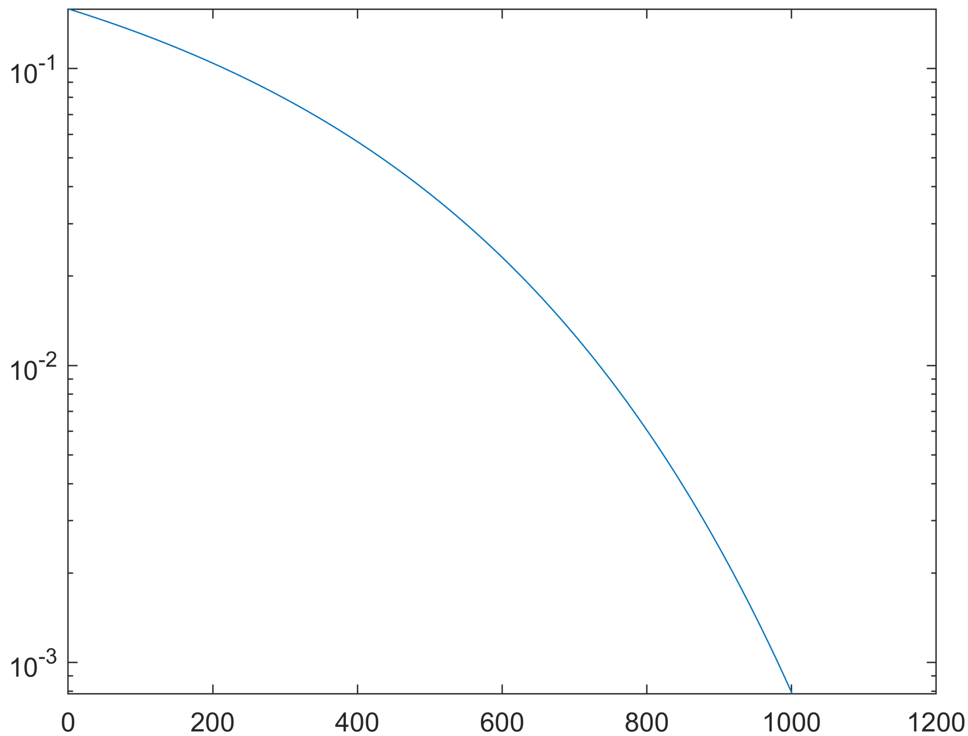
$$P_e = \int_{-\infty}^{\infty} \left(1-(1-Q(u))^{M-1}\right) \frac{e^{-\left(u-\sqrt{\frac{2\epsilon}{N_0}}\right)}}{\sqrt{(2\,\pi)}}\,\mathrm{du}$$

$$P_e = Q\left(\frac{E_b}{N_0}\right), M = 2$$

```
snr = 0:0.01:10;
Q = qfunc(sqrt(snr));
semilogy(snr,Q)
```




```
semilogy(berawgn(snr,'fsk',2,'coherent'))
```




```
semilogy(berawgn(snr,'fsk',2,'noncoherent'))
```