Homework 2

ISyE 6420
Fall 2019

> Due September 22, 2019, 11:55pm. HW2 is not time limited except the due date. Late submissions will not be accepted.
>
> Use of all available electronic and printed resources is allowed except direct communication that violates Georgia Tech Academic Integrity Rules.

1. Cell Clusters in 3D Petri Dishes. The number of cell clusters in a 3D Petri dish has a Poisson distribution with mean $\lambda = 5$. Find the percentage of Petri dishes that have (a) 0 clusters, (b) at least one cluster, (c) more than 8 clusters, and (d) between 4 and 6 clusters inclusive.

2. Silver-Coated Nylon Fiber. Silver-coated nylon fiber is used in hospitals for its anti-static electricity properties, as well as for antibacterial and antimycotic effects. In the production of silver-coated nylon fibers, the extrusion process is interrupted from time to time by blockages occurring in the extrusion dyes. The time in hours between blockages, T, has an exponential $\mathcal{E}(1/10)$ distribution, where $1/10$ is the rate parameter.

Find the probabilities that
(a) a run continues for at least 10 hours,
(b) a run lasts less than 15 hours, and
(c) a run continues for at least 20 hours, given that it has lasted 10 hours.
If you use software, be careful about the parametrization of exponentials.

3. 2-D Density Tasks. If

$$f(x, y) = \begin{cases} \lambda^2 e^{-\lambda y}, & 0 \le x \le y, \ \lambda > 0 \\ 0, & \text{else} \end{cases}$$

Show that:
(a) marginal distribution $f_X(x)$ is exponential $\mathcal{E}(\lambda)$.
(b) marginal distribution $f_Y(y)$ is Gamma $\mathcal{G}a(2, \lambda)$.
(c) conditional distribution $f(y|x)$ is shifted exponential, $f(y|x) = \lambda e^{-\lambda(y-x)}, \ y \ge x$.
(d) conditional distribution $f(x|y)$ is uniform $\mathcal{U}(0, y)$.

4. Nylon Fiber Continued. In the Exercise 2, the times (in hours) between blockages of the extrusion process, T, had an exponential $\mathcal{E}(\lambda)$ distribution. Suppose that the rate parameter λ is unknown, but there are three measurements of interblockage times, $T_1 = 3$, $T_2 = 13$, and $T_3 = 8$.

(a) How would classical statistician estimate λ?

(b) What is the Bayes estimator of λ if the prior is $\pi(\lambda) = \frac{1}{\sqrt{\lambda}},\ \lambda > 0$.

Hint: In (b) the prior is not a proper distribution, but the posterior is. Identify the posterior from the product of the likelihood from (a) and the prior, no need to integrate.