Homework 3

ISyE 6420
Fall 2019

Due September 29, 2019, 11:55pm. HW3 is not time limited except the due date. Late submissions will not be accepted.

Use of all available electronic and printed resources is allowed except direct communication that violates Georgia Tech Academic Integrity Rules.

Estimating the Precision Parameter of a Rayleigh Distribution. If two random variables X and Y are independent of each other and normally distributed with variances equal to σ^2, then the variable $R = \sqrt{X^2 + Y^2}$ follows the Rayleigh distribution. Parameterized with precision parameter $\xi = \frac{1}{\sigma^2}$, the Rayleigh random variable R has a density

$$f(r) \quad = \quad \xi r \exp\left\{-\frac{\xi r^2}{2}\right\}, \quad r \geq 0,\ \xi > 0.$$

An example of such random variable would be the distance of darts from the target center in a dart-throwing game where the deviations in the two dimensions of the target plane are independent and normally distributed.

(a) Assume that the prior on ξ is exponential with the rate parameter λ. Show that the posterior is gamma $\mathcal{G}a\left(2, \lambda + \frac{r^2}{2}\right)$.

(b) Assume that $R_1 = 3, R_2 = 4, R_3 = 2$, and $R_4 = 5$ are Rayleigh-distributed random observations representing the distance of a dart from the center. Find the posterior in this case for the same prior form (a), and give a Bayesian estimate of ξ.

(c) For $\lambda = 1$, numerically find 95% Credible Set for ξ .

Hint: In (b) show that if $r_1, r_2, \ldots, r_n$ are observed, and the prior on ξ is exponential $\mathcal{E}(\lambda)$, then the posterior is gamma $\mathcal{G}a\left(n+1, \lambda + \frac{1}{2}\sum_{i=1}^n r_i^2\right)$.

2. Estimating Chemotherapy Response Rates. An oncologist believes that 90% of cancer patients will respond to a new chemotherapy treatment and that it is unlikely that this proportion will be below 80%. Elicit a beta prior on proportion that models the oncologist's beliefs.

Hint: For elicitation of the prior use $\mu = 0.9,\ \mu - 2\sigma = 0.8$ and expressions for μ and σ for beta.

During a trial, in 30 patients treated, 22 responded.

(a) What are the likelihood and posterior distributions? What is the Bayes estimator of the proportion?

(b) Using Octave, R, or Python, find 95% Credible Set for p.

(c) Using Octave, R, or Python, test the hypothesis $H_0 : p \geq 4/5$ against the alternative $H_1 : p < 4/5$.

(d) Using WinBUGS, find the Bayes estimator and Credible Set and conduct the test. Compare WinBUGS results with (a-c).