Patter Recognition
CS - 669

ASSIGNMENT 1

Bayes Classifier

Group Number 8

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Contents

List of Plots

List of Tables

1. Problem Description

Classification is the problem of identifying to which of a set of categories (sub-populations) a new observation belongs on the basis of a training set of data containing observations (or instances) whose category membership is known.

Data-sets:

- Data-set 1: 2-dimensional artificial data of 3 classes:
  - Linearly separable data set
  - Non-linearly separable data set
- Data-set 2: Real world data set

Data-set-1 and Data-set-2, 75% of data of a class is to be used as training data for that class, and the remaining data is to be used as test data for that class.

Classifiers:

- Co-variance matrix for all the classes is the same and is $\sigma^2 I$.
- Full Co-variance matrix for all the classes is the same and is Σ.
- Co-variance matrix is diagonal and is different for each class.
- Full Co-variance matrix for each class is different.

Objective:

1. Build Bayes classifier to classify data points of given data-sets on the basis of specified classifiers.
2. For each classifier and each data-set we do :
   - Classification accuracy, precision for every class, mean precision, recall for every class, mean recall, F-measure for every class and mean F-measure on test data.
   - Confusion matrix based on the performance for test data.
   - Constant density contour plot for all the classes together with the training data superposed.
   - Decision region plot for every pair of classes together with the training data superposed.
   - Decision region plot for all the classes together with the training data superposed.

2. Solution Approach

First, it is assumed that data-set for each class follow a Gaussian distribution. Now, for each class, mean vector and co-variance matrix is calculated for every classifier. Given a data point, we use value of discriminant function specific to that class as the basis for classifying the point to one of the classes. Our discriminant function for class i is given by:

$$g_i(\mathbf{x}) \;=\; \mathbf{x}^t\mathbf{W}_i\mathbf{x} \;+\; \mathbf{w}_i^t\mathbf{x} \;+\; w_{i0}, \tag{2..1}$$

where,

$$\mathbf{W}_i \;=\; -\frac{1}{2}\,\mathbf{\Sigma}_i^{-1}, \tag{2..2}$$

$$\mathbf{w}_i \;=\; \mathbf{\Sigma}_i^{-1}\mu_i \tag{2..3}$$

and

$$w_{i0} \;=\; -\frac{1}{2}\,\mu_i^t\,\mathbf{\Sigma}_i^{-1}\,\mu_i - \frac{1}{2}\,ln|\mathbf{\Sigma}_i| \;+\; lnP(\omega_i) \tag{2..4}$$

Here, $\mathbf{x}$ is our data point(feature vector) and μ_i, $\mathbf{\Sigma}_i$ are our mean vector and co-variance matrix respectively for class i. The class for which the discriminant function value for a given data point is maximum across all classes, is assigned to the data point.

Contours are plotted with feature-1 and feature-2 on x, y axes respectively and the z-value is taken as maximum Gaussian density for the for the data point.

For a multivariate Normal with dimension k, for the density equal to some level l:

$$(2\pi)^{-k/2}\;|\Sigma|^{-1/2}\;exp(-\frac{1}{2}\;(x-\mu)^{'}\;\Sigma^{-1}\;(x-\mu)) \;=\; l \tag{2..5}$$

$$\Rightarrow\; exp(-\frac{1}{2}\;(x-\mu)^{'}\;\Sigma^{-1}\;(x-\mu)) \;=\; l^{'} \tag{2..6}$$

$$\Rightarrow\; (x-\mu)^{`}\;\Sigma^{-1}\;(x-\mu) \;=\; l^{''} \tag{2..7}$$

where, μ is mean vector and Σ is the covariance matrix. Equation 2..7 represents an ellipsoid centered at μ.

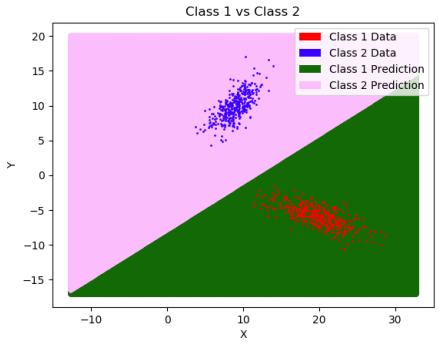
Note: In all the plots, feature 1 and feature 2 are represented on the X-axis and the Y-axis respectively.

3. Results and Plots

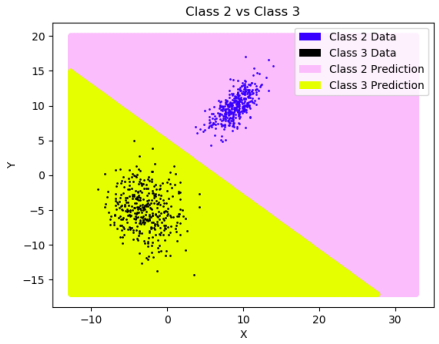
1 Data Set 1 : Linear Data

1.1 Classifier 1 : $\Sigma = \sigma^2 I$

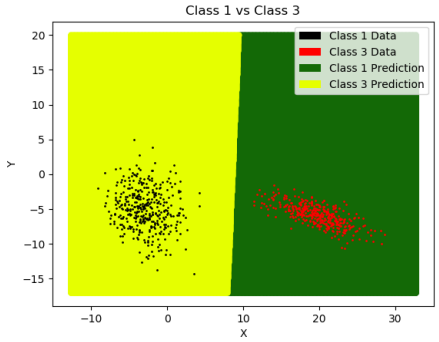
Decision Regions, Training Data and Contours



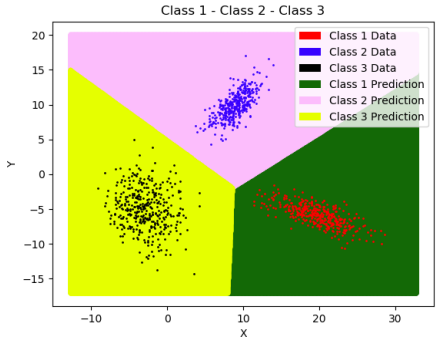
(a) Decision regions and Training data Class 1 and 2



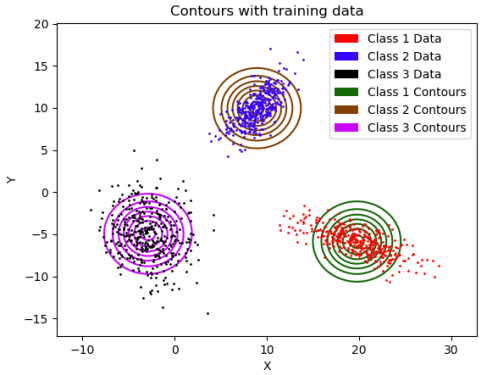
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2 and 3



(e) Contours: Class 1, 2, 3

Figure 3..1. Linear Data - Classifier 1

Confusion Matrix, Precision, Recall and F-measure

Accuracy=100%

	Class1	Class 2
Class1	125	0
Class2	0	125

(a) Confusion Matrix

	Class1	Class2
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..1. Linear Data - Classifier 1 : Class 1 and Class 2

Accuracy=100%

	Class2	Class 3
Class2	125	0
Class3	0	125

(a) Confusion Matrix

	Class2	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..2. Linear Data - Classifier 1 : Class 2 and Class 3

Accuracy=100%

	Class1	Class 3
Class1	125	0
Class3	0	125

(a) Confusion Matrix

	Class1	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..3. Linear Data - Classifier 1 : Class 1 and Class 3

Accuracy=100%

	Class1	Class2	Class 3
Class1	125	0	0
Class2	0	125	0
Class3	0	0	125

(a) Confusion Matrix

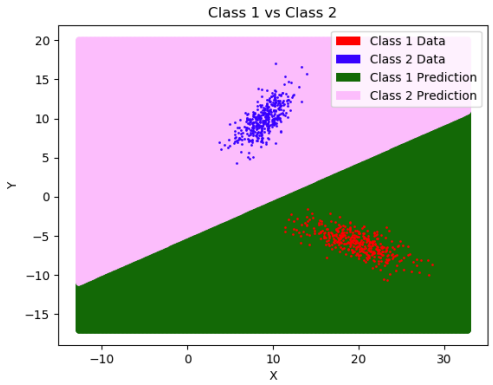
	Class1	Class2	Class3
Precision	1.0	1.0	1.0
Recall	1.0	1.0	1.0
F-Measure	1.0	1.0	1.0

(b) Analysis

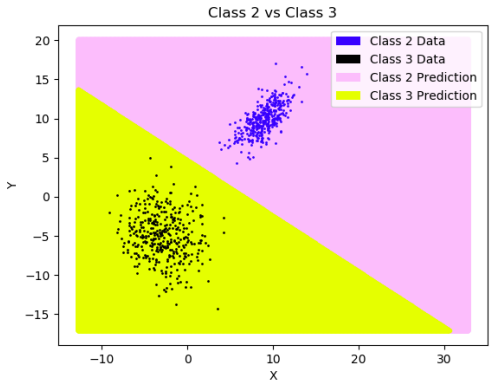
Table 3..4. Linear Data - Classifier 1 : Class 1 ,Class 2 and Class 3

1.2 Classifier 2: $\Sigma_i = \mathbf{\Sigma}$

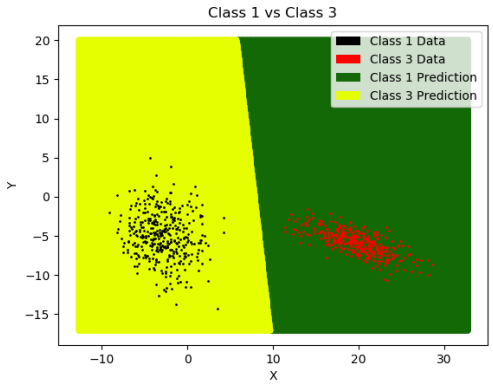
Decision Regions, Training Data and Contours



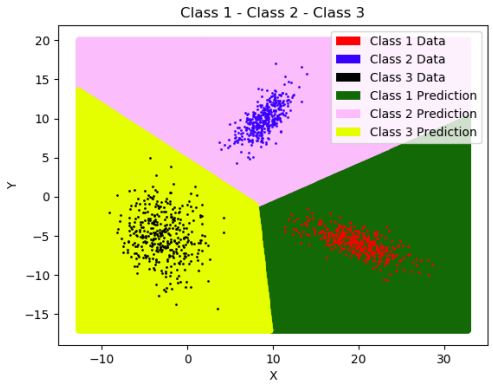
(a) Decision regions and Training data Class 1 and 2



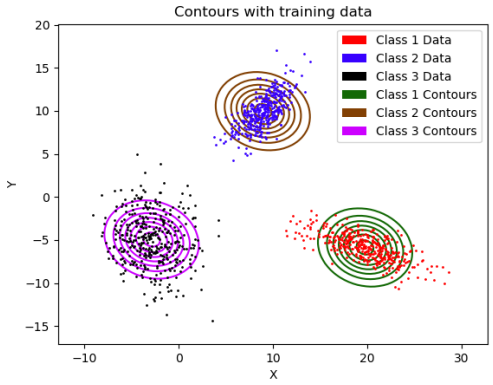
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3



(e) Contours: Class 1, 2, 3

Figure 3..2. Linear Data - Classifier 2

Confusion Matrix, Precision, Recall and F-measure

Accuracy=100%

	Class1	Class 2
Class1	125	0
Class2	0	125

(a) Confusion Matrix

	Class1	Class2
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..5. Linear Data - Classifier 2: Class 1 and Class 2

Accuracy=100%

	Class2	Class 3
Class2	125	0
Class3	0	125

(a) Confusion Matrix

	Class2	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..6. Linear Data - Classifier 2: Class 2 and Class 3

Accuracy=100%

	Class1	Class 3
Class1	125	0
Class3	0	125

(a) Confusion Matrix

	Class1	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..7. Linear Data - Classifier 2: Class 1 and Class 3

Accuracy=100%

	Class1	Class2	Class 3
Class1	125	0	0
Class2	0	125	0
Class3	0	0	125

(a) Confusion Matrix

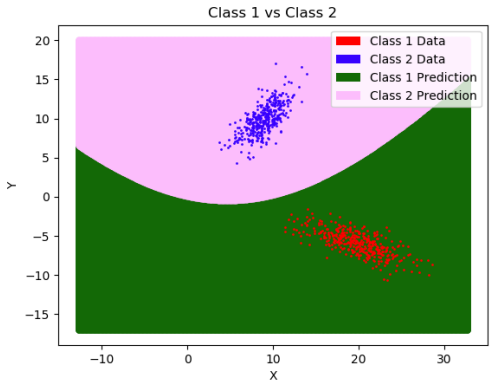
	Class1	Class2	Class3
Precision	1.0	1.0	1.0
Recall	1.0	1.0	1.0
F-Measure	1.0	1.0	1.0

(b) Analysis

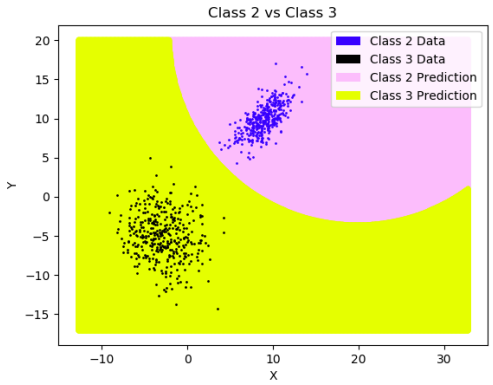
Table 3..8. Linear Data - Classifier 2: Class 1, Class 2 and Class 3

1.3 Classifier 3 : Σ_i is a diagonal matrix

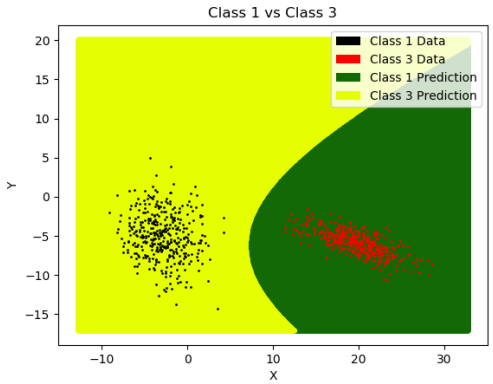
Decision Regions, Training Data and Contours



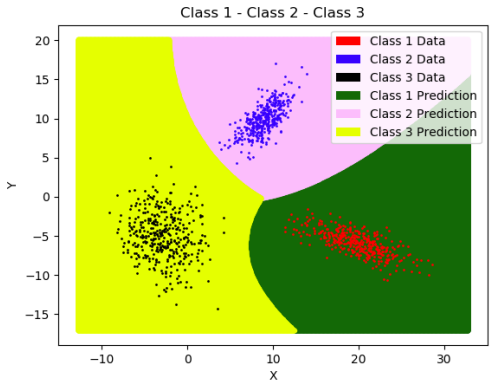
(a) Decision regions and Training data Class 1 and 2



(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3

(e) Contours: Class 1, 2, 3

Figure 3..3. Linear Data - Classifier 3

Confusion Matrix, Precision, Recall and F-measure

Accuracy=100%

	Class1	Class 2
Class1	125	0
Class2	0	125

(a) Confusion Matrix

	Class1	Class2
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..9. Linear Data - Classifier 3: Class 1 and Class 2

Accuracy=100%

	Class2	Class 3
Class2	125	0
Class3	0	125

(a) Confusion Matrix

	Class2	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..10. Linear Data - Classifier 3: Class 2 and Class 3

Accuracy=100%

	Class1	Class 3
Class1	125	0
Class3	0	125

(a) Confusion Matrix

	Class1	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..11. Linear Data - Classifier 3: Class 1 and Class 3

Accuracy=100%

	Class1	Class2	Class 3
Class1	125	0	0
Class2	0	125	0
Class3	0	0	125

(a) Confusion Matrix

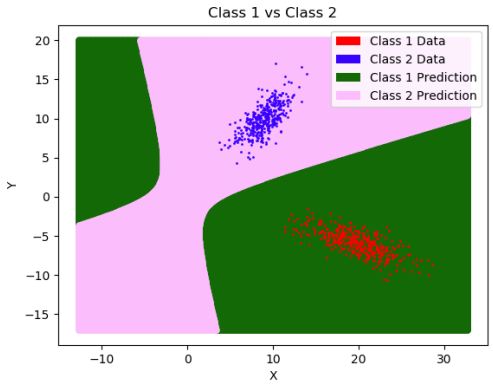
	Class1	Class2	Class3
Precision	1.0	1.0	1.0
Recall	1.0	1.0	1.0
F-Measure	1.0	1.0	1.0

(b) Analysis

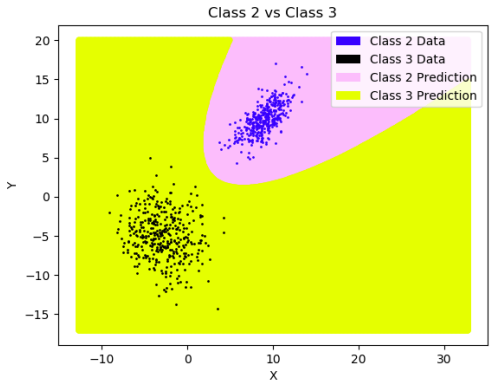
Table 3..12. Linear Data - Classifier 3: Class 1, Class 2 and Class 3

1.4 Classifier 4 : Σ_i is a unique

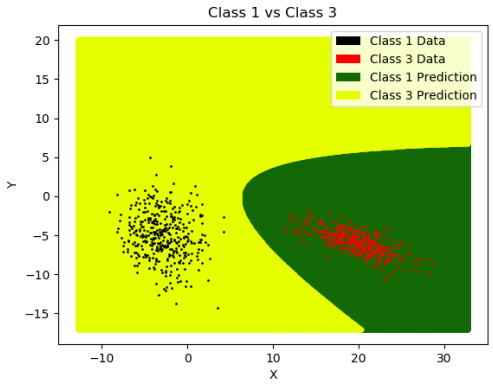
Decision Regions, Training Data and Contours



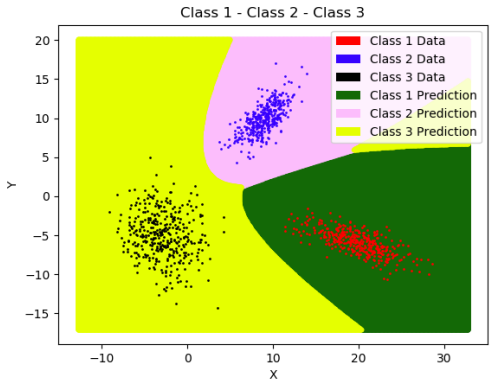
(a) Decision regions and Training data Class 1 and 2



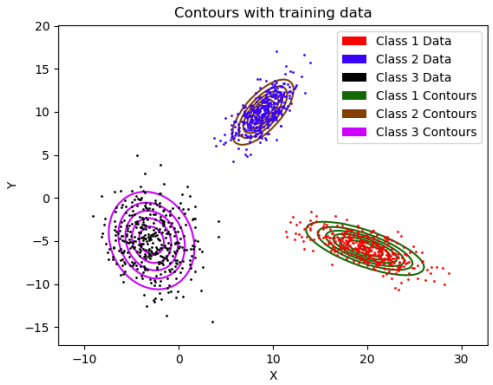
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3



(e) Decision regions and Training data Class 1, 2, 3

Figure 3..4. Linear Data - Classifier 4

Confusion Matrix, Precision, Recall and F-measure

Accuracy=100%

	Class1	Class 2
Class1	125	0
Class2	0	125

(a) Confusion Matrix

	Class1	Class2
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..13. Linear Data - Classifier 4: Class 1 and Class 2

Accuracy=100%

	Class2	Class 3
Class2	125	0
Class3	0	125

(a) Confusion Matrix

	Class2	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..14. Linear Data - Classifier 4: Class 2 and Class 3

Accuracy=100%

	Class1	Class 3
Class1	125	0
Class3	0	125

(a) Confusion Matrix

	Class1	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..15. Linear Data - Classifier 4: Class 1 and Class 3

Accuracy=100%

	Class1	Class2	Class 3
Class1	125	0	0
Class2	0	125	0
Class3	0	0	125

(a) Confusion Matrix

	Class1	Class2	Class3
Precision	1.0	1.0	1.0
Recall	1.0	1.0	1.0
F-Measure	1.0	1.0	1.0

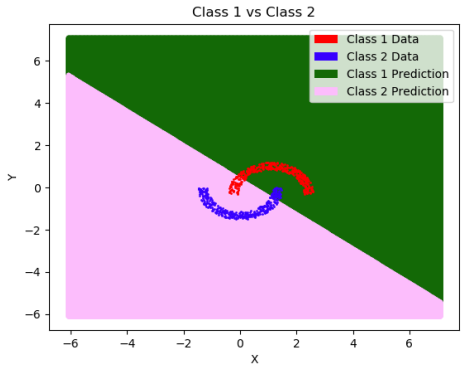
(b) Analysis

Table 3..16. Linear Data - Classifier 4: Class 1, Class 2 and Class 3

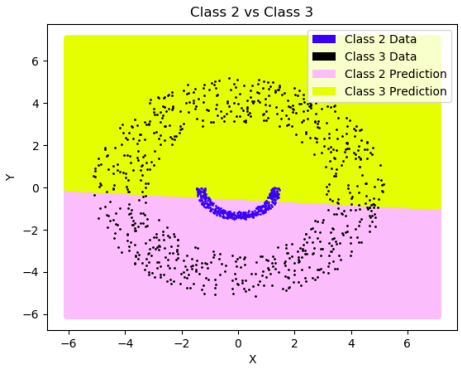
2 Data Set2 : Non Linear Data

2.1 Classifier 1 : $\Sigma = \sigma^2 I$

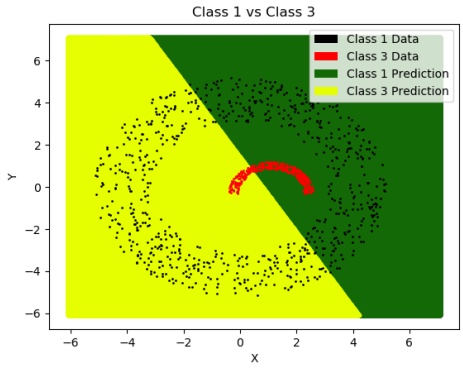
Decision Regions, Training Data and Contours



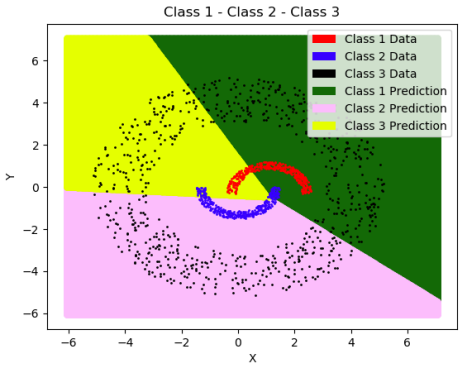
(a) Decision regions and Training data Class 1 and 2



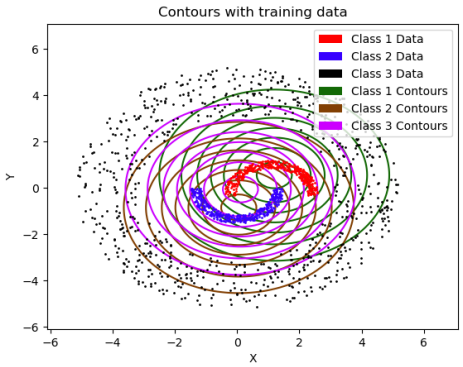
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3



(e) Contours: Class 1, 2, 3

Figure 3..5. Non-Linear Data - Classifier 1

Confusion Matrix, Precision, Recall and F-measure

Accuracy=85.59%

	Class1	Class 2
Class1	110	15
Class2	21	104

(a) Confusion Matrix

	Class1	Class2
Precision	0.8396	0.8739
Recall	0.88	0.832
F-Measure	0.8593	0.8524

(b) Analysis

Table 3..17. Non Linear Data - Classifier 1: Class 1 and Class 2

Accuracy=59.19%

	Class2	Class 3
Class2	93	32
Class3	121	129

(a) Confusion Matrix

	Class2	Class3
Precision	0.4345	0.8012
Recall	0.744	0.516
F-Measure	0.5486	0.6277

(b) Analysis

Table 3..18. Non Linear Data - Classifier 1: Class 2 and Class 3

Accuracy=57.06%

	Class1	Class 3
Class1	95	30
Class3	131	119

(a) Confusion Matrix

	Class1	Class3
Precision	0.4203	0.7986
Recall	0.76	0.476
F-Measure	0.5413	0.5965

(b) Analysis

Table 3..19. Non Linear Data - Classifier 1: Class 1 and Class 3

Accuracy=45.00%

	Class1	Class2	Class 3
Class1	95	0	30
Class2	21	82	22
Class3	114	88	48

(a) Confusion Matrix

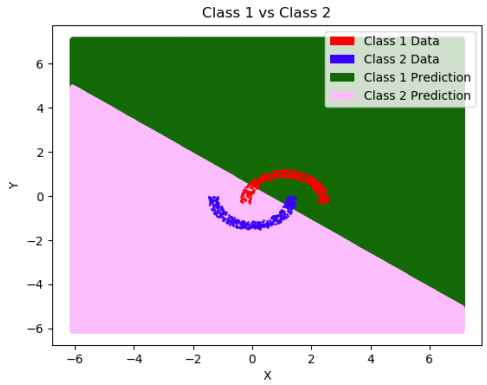
	Class1	Class2	Class3
Precision	0.4130	0.4823	0.48
Recall	0.76	0.656	0.192
F-Measure	0.5352	0.5559	0.2742

(b) Analysis

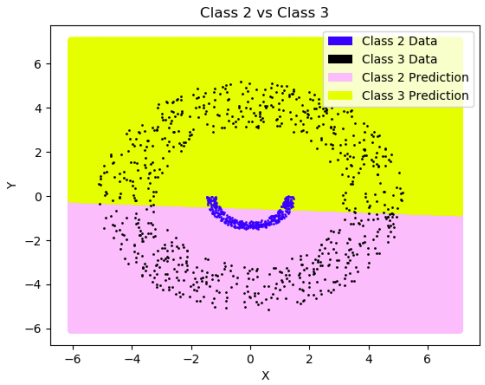
Table 3..20. Non Linear Data - Classifier 1: Class 1, Class 2 and Class 3

2.2 Classifier 2: $\Sigma_i = \mathbf{\Sigma}$

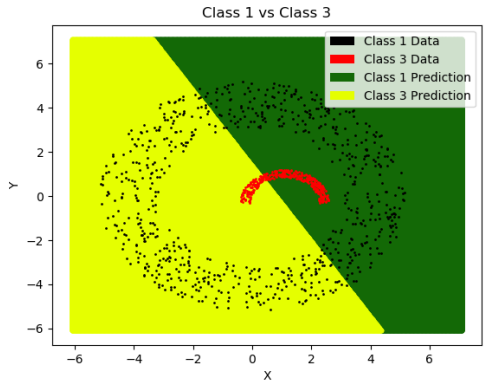
Decision Regions, Training Data and Contours



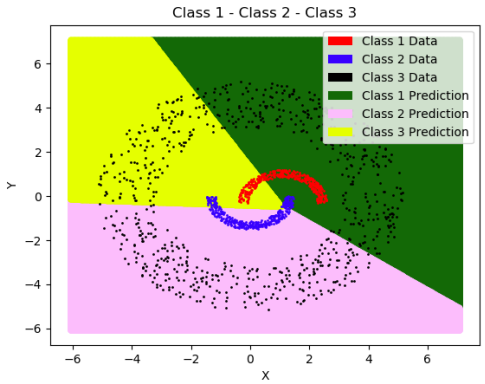
(a) Decision regions and Training data Class 1 and 2



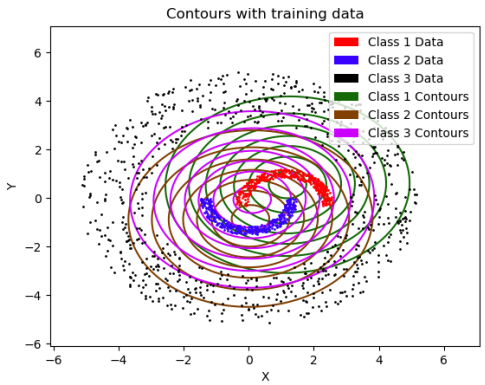
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3



(e) Contours: Class 1, 2, 3

Figure 3..6. Non-Linear Data - Classifier 2

Confusion Matrix, Precision, Recall and F-measure

Accuracy=95.59%

	Class1	Class 2
Class1	121	4
Class2	7	118

(a) Confusion Matrix

	Class1	Class2
Precision	0.9453	0.9672
Recall	0.968	0.944
F-Measure	0.9565	0.9554

(b) Analysis

Table 3..21. Non Linear Data - Classifier 2: Class 1 and Class 2

Accuracy=59.46%

	Class2	Class 3
Class2	95	30
Class3	122	128

(a) Confusion Matrix

	Class2	Class3
Precision	0.4377	0.8101
Recall	0.76	0.512
F-Measure	0.5555	0.6274

(b) Analysis

Table 3..22. Non Linear Data - Classifier 2: Class 2 and Class 3

Accuracy=57.06%

	Class1	Class 3
Class1	95	30
Class3	131	119

(a) Confusion Matrix

	Class1	Class3
Precision	0.4203	0.7986
Recall	0.76	0.476
F-Measure	0.5413	0.5964

(b) Analysis

Table 3..23. Non Linear Data - Classifier 2: Class 1 and Class 3

Accuracy=45.00%

	Class1	Class2	Class 3
Class1	95	0	30
Class2	21	82	22
Class3	113	89	48

(a) Confusion Matrix

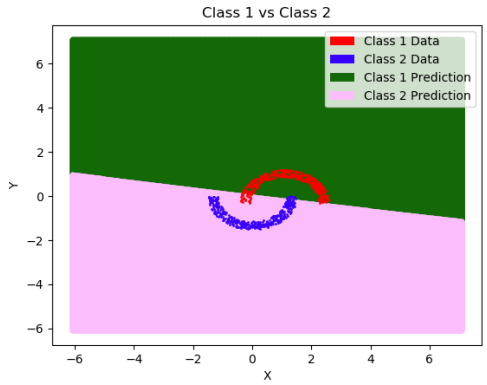
	Class1	Class2	Class3
Precision	0.4148	0.4795	0.48
Recall	0.76	0.656	0.192
F-Measure	0.5367	0.5540	0.2742

(b) Analysis

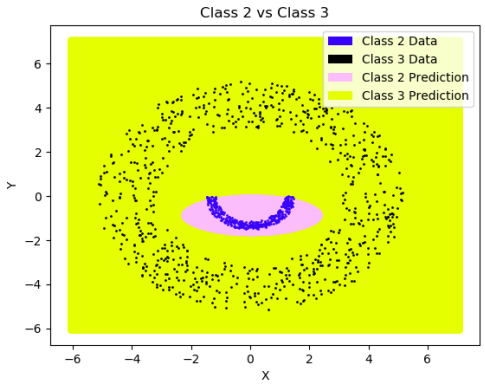
Table 3..24. Non Linear Data - Classifier 2: Class 1, Class 2 and Class 3

2.3 Classifier 3 : Σ_i is a diagonal matrix

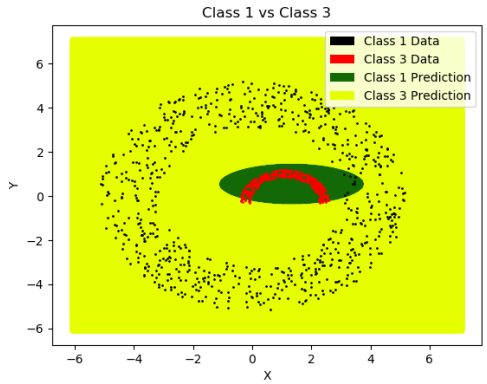
Decision Regions, Training Data and Contours



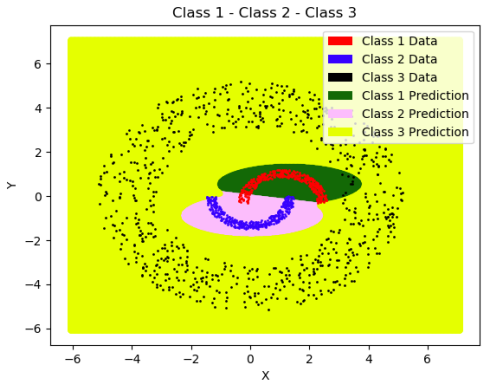
(a) Decision regions and Training data Class 1 and 2



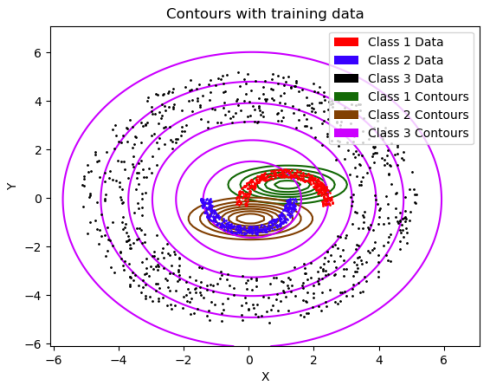
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3



(e) Contours: Class 1, 2, 3

Figure 3..7. Non-Linear Data - Classifier 3

Confusion Matrix, Precision, Recall and F-measure

Accuracy=96.39%

	Class1	Class 2
Class1	121	4
Class2	5	120

(a) Confusion Matrix

	Class1	Class2
Precision	0.9603	0.9677
Recall	0.968	0.96
F-Measure	0.9641	0.9638

(b) Analysis

Table 3..25. Non Linear Data - Classifier 3: Class 1 and Class 2

Accuracy=100%

	Class2	Class 3
Class2	125	0
Class3	0	250

(a) Confusion Matrix

	Class2	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..26. Non Linear Data - Classifier 3: Class 2 and Class 3

Accuracy=98.66%

	Class1	Class 3
Class1	125	0
Class3	5	245

(a) Confusion Matrix

	Class1	Class3
Precision	0.9615	1.0
Recall	1.0	0.98
F-Measure	0.9803	0.9898

(b) Analysis

Table 3..27. Non Linear Data - Classifier 3: Class 1 and Class 3

Accuracy=97.19%

	Class1	Class2	Class 3
Class1	121	4	0
Class2	5	120	0
Class3	5	0	245

(a) Confusion Matrix

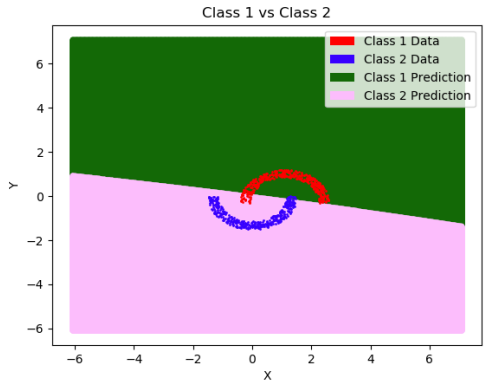
	Class1	Class2	Class3
Precision	0.9236	0.9677	1.0
Recall	0.968	0.96	0.98
F-Measure	0.9453	0.9638	0.9898

(b) Analysis

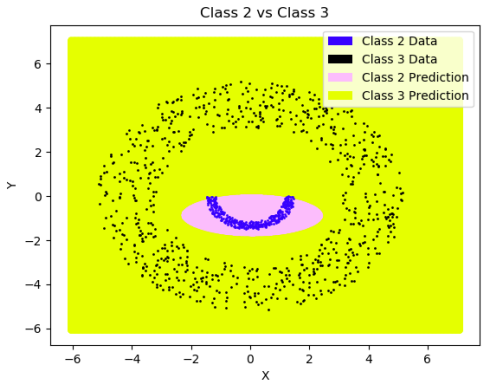
Table 3..28. Non Linear Data - Classifier 3: Class 1, Class 2 and Class 3

2.4 Classifier 4 : Σ_i is a unique

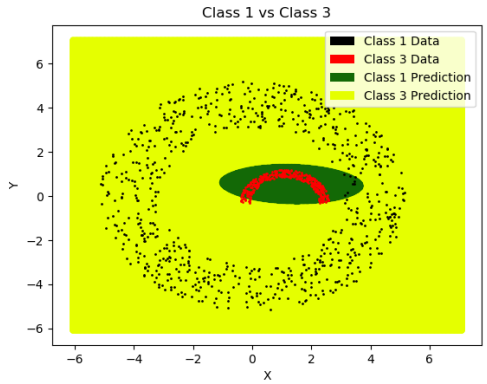
Decision Regions, Training Data and Contours



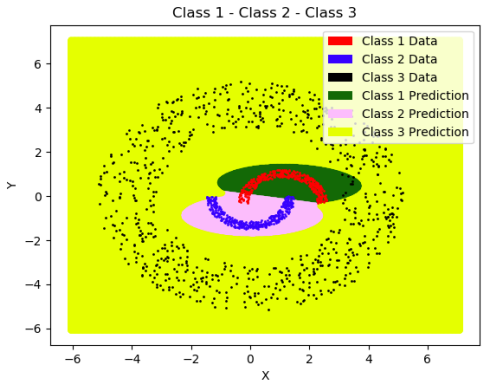
(a) Decision regions and Training data Class 1 and 2



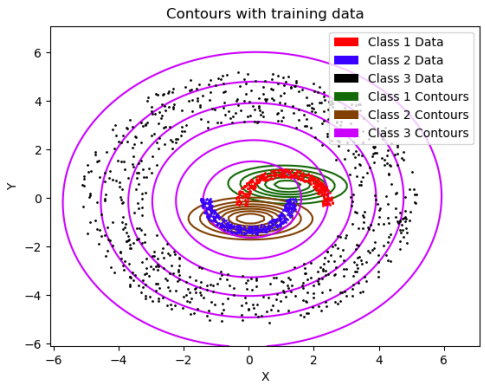
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3



(e) Contours: Class 1, 2, 3

Figure 3..8. Non-Linear Data - Classifier 4

Confusion Matrix, Precision, Recall and F-measure

Accuracy=95.99%

	Class1	Class 2
Class1	120	5
Class2	5	120

(a) Confusion Matrix

	Class1	Class2
Precision	0.96	0.96
Recall	0.96	0.96
F-Measure	0.96	0.96

(b) Analysis

Table 3..29. Non Linear Data - Classifier 4: Class 1 and Class 2

Accuracy=100%

	Class2	Class 3
Class2	125	0
Class3	0	250

(a) Confusion Matrix

	Class2	Class3
Precision	1.0	1.0
Recall	1.0	1.0
F-Measure	1.0	1.0

(b) Analysis

Table 3..30. Non Linear Data - Classifier 4: Class 2 and Class 3

Accuracy=98.399%

	Class1	Class 3
Class1	124	1
Class3	5	245

(a) Confusion Matrix

	Class1	Class3
Precision	0.9612	0.9959
Recall	0.992	0.98
F-Measure	0.9763	0.9879

(b) Analysis

Table 3..31. Non Linear Data - Classifier 4: Class 1 and Class 3

Accuracy=96.99%

	Class1	Class2	Class 3
Class1	120	5	0
Class2	5	120	0
Class3	5	0	245

(a) Confusion Matrix

	Class1	Class2	Class3
Precision	0.9230	0.96	1.0
Recall	0.96	0.96	0.98
F-Measure	0.9411	0.96	0.9898

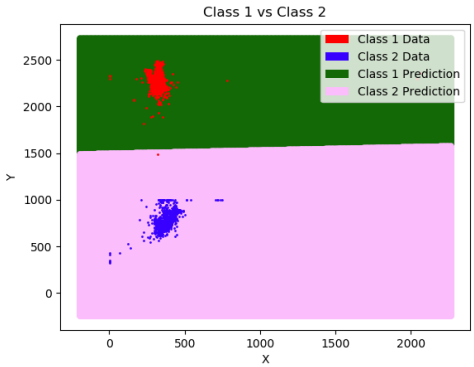
(b) Analysis

Table 3..32. Non Linear Data - Classifier 4: Class 1, Class 2 and Class 3

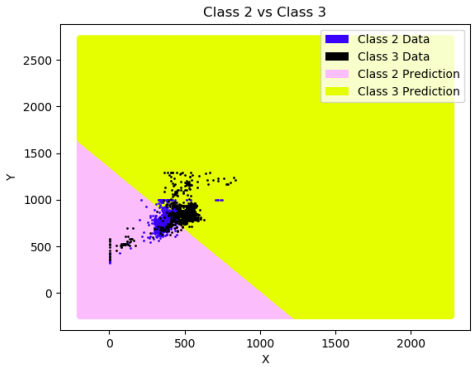
3 Real World Data

3.1 Classifier 1 : $\Sigma = \sigma^2 I$

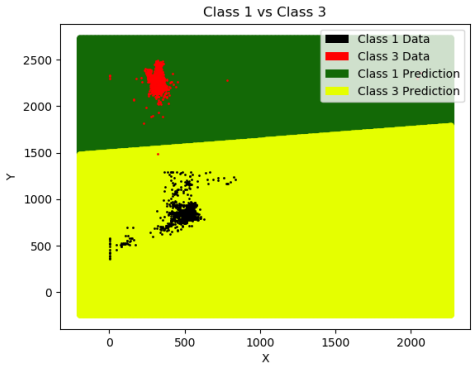
Decision Regions, Training Data and Contours



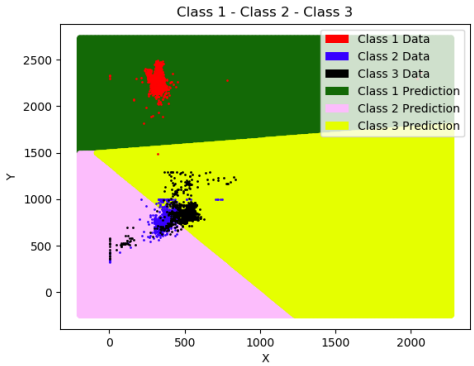
(a) Decision regions and Training data Class 1 and 2



(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3

(e) Contours: Class 1, 2, 3

Figure 3..9. Real Data - Classifier 1

Confusion Matrix, Precision, Recall and F-measure

Accuracy=99.75%

	Class1	Class 2
Class1	594	3
Class2	0	622

(a) Confusion Matrix

	Class1	Class2
Precision	1.0	0.9952
Recall	0.9949	1.0
F-Measure	0.9974	0.9975

(b) Analysis

Table 3..33. Real World Data - Classifier 1: Class 1 and Class 2

Accuracy=82.03%

	Class2	Class 3
Class2	572	50
Class3	172	442

(a) Confusion Matrix

	Class2	Class3
Precision	0.7688	0.8983
Recall	0.9196	0.7198
F-Measure	0.8374	0.7992

(b) Analysis

Table 3..34. Real World Data - Classifier 1: Class 2 and Class 3

Accuracy=99.67%

	Class1	Classs3
Class1	593	4
Class3	0	614

(a) Confusion Matrix

	Class1	Class3
Precision	1.0	0.9935
Recall	0.9932	1.0
F-Measure	0.9966	0.9967

(b) Analysis

Table 3..35. Real World Data - Classifier 1: Class 1 and Class 3

Accuracy=87.67%

	Class1	Class2	Classs3
Class1	593	3	1
Class2	0	572	50
Class3	0	172	442

(a) Confusion Matrix

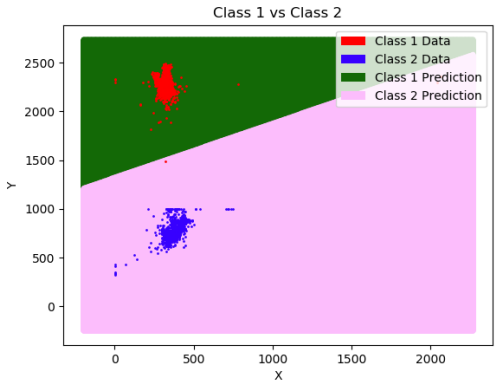
	Class1	Class2	Class3
Precision	1.0	0.7657	0.8965
Recall	0.9932	0.9196	0.7198
F-Measure	0.9966	0.8356	0.7985

(b) Analysis

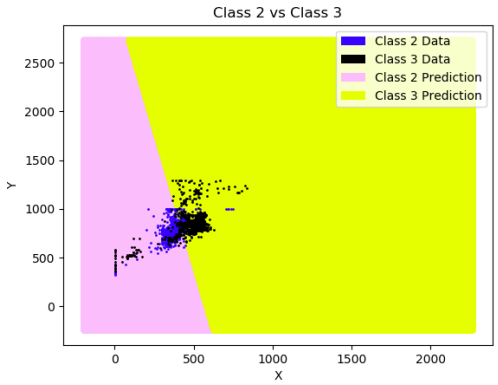
Table 3..36. Real World Data - Classifier 1: Class 1, Class 2 and Class 3

3.2 Classifier 2: $\Sigma_i = \mathbf{\Sigma}$

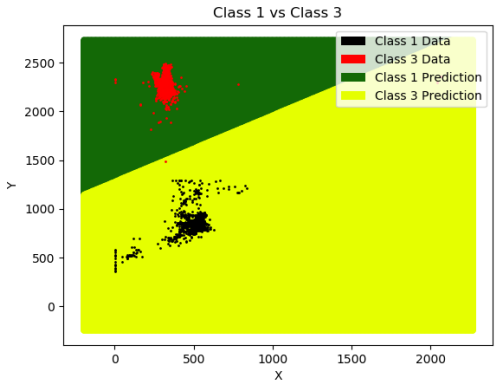
Decision Regions, Training Data and Contours



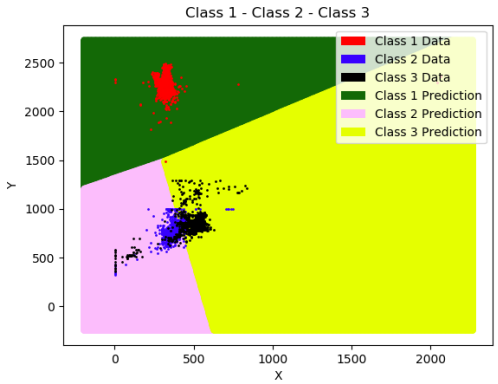
(a) Decision regions and Training data Class 1 and 2



(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3

(e) Contours: Class 1, 2, 3

Figure 3..10. Real Data - Classifier 2

Confusion Matrix, Precision, Recall and F-measure

Accuracy=99.67%

	Class1	Class2
Class1	593	4
Class2	0	622

(a) Confusion Matrix

	Class1	Class2
Precision	1.0	0.9936
Recall	0.9932	1.0
F-Measure	0.9966	0.9967

(b) Analysis

Table 3..37. Real World Data - Classifier 2: Class 1 and Class 2

Accuracy=81.63%

	Class2	Classs3
Class2	601	21
Class3	206	408

(a) Confusion Matrix

	Class2	Class3
Precision	0.7447	0.9510
Recall	0.9662	0.6644
F-Measure	0.8411	0.7823

(b) Analysis

Table 3..38. Real World Data - Classifier 2: Class 2 and Class 3

Accuracy=98.84%

	Class1	Classs3
Class1	583	14
Class3	0	614

(a) Confusion Matrix

	Class1	Class3
Precision	1.0	0.9777
Recall	0.9765	1.0
F-Measure	0.9881	0.9887

(b) Analysis

Table 3..39. Real World Data - Classifier 2: Class 1 and Class 3

Accuracy=87.17%

	Class1	Class2	Classs3
Class1	581	3	13
Class2	0	592	30
Class3	0	189	425

(a) Confusion Matrix

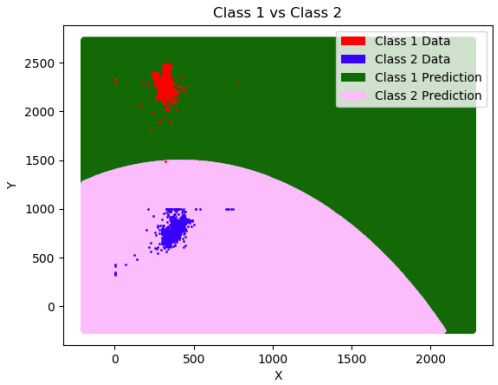
	Class1	Class2	Class3
Precision	1.0	0.7551	0.9081
Recall	0.9731	0.9517	0.6921
F-Measure	0.9864	0.8421	0.7855

(b) Analysis

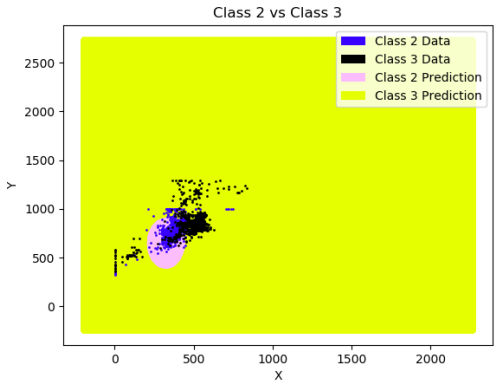
Table 3..40. Real World Data - Classifier 2: Class 1, Class 2 and Class 3

3.3 Classifier 3 : Σ_i is a diagonal matrix

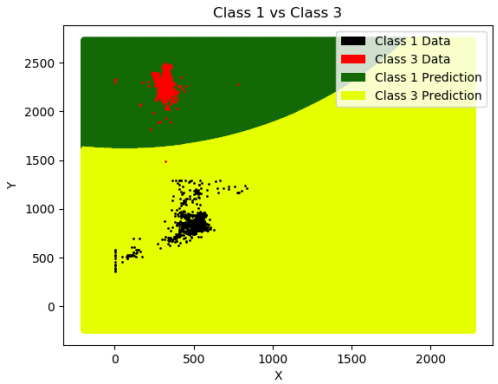
Decision Regions, Training Data and Contours



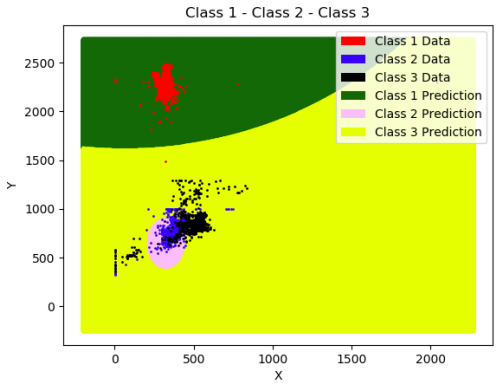
(a) Decision regions and Training data Class 1 and 2



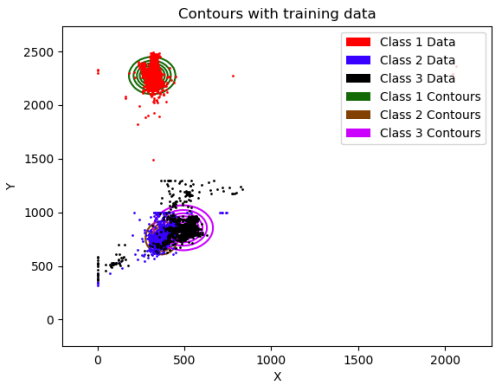
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3



(e) Contours: Class 1, 2, 3

Figure 3..11. Real Data - Classifier 3

Confusion Matrix, Precision, Recall and F-measure

Accuracy=99.75%

	Class1	Class 2
Class1	594	3
Class2	0	622

(a) Confusion Matrix

	Class1	Class2
Precision	1.0	0.9952
Recall	0.9949	1.0
F-Measure	0.9974	0.9975

(b) Analysis

Table 3..41. Real World Data - Classifier 3: Class 1 and Class 2

Accuracy=78.83%

	Class2	Class 3
Class2	546	76
Class3	185	429

(a) Confusion Matrix

	Class2	Class3
Precision	0.7469	0.8495
Recall	0.8778	0.6986
F-Measure	0.8070	0.7667

(b) Analysis

Table 3..42. Real World Data - Classifier 3: Class 2 and Class 3

Accuracy=98.59%

	Class1	Class 3
Class1	580	17
Class3	0	614

(a) Confusion Matrix

	Class1	Class3
Precision	1.0	0.9730
Recall	0.9715	1.0
F-Measure	0.9855	0.9863

(b) Analysis

Table 3..43. Real World Data - Classifier 3: Class 1 and Class 3

Accuracy=84.83%

	Class1	Class2	Class 3
Class1	580	2	15
Class2	0	546	76
Class3	0	185	429

(a) Confusion Matrix

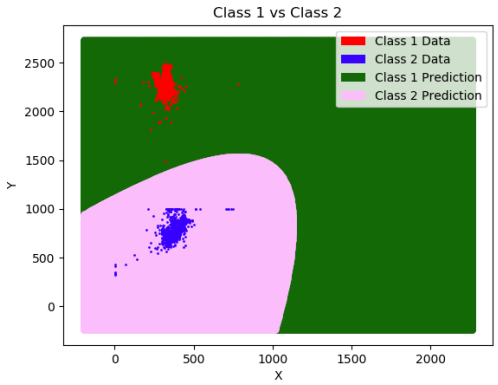
	Class1	Class2	Class3
Precision	1.0	0.7448	0.825
Recall	0.9715	0.8778	0.6986
F-Measure	0.9855	0.8059	0.7566

(b) Analysis

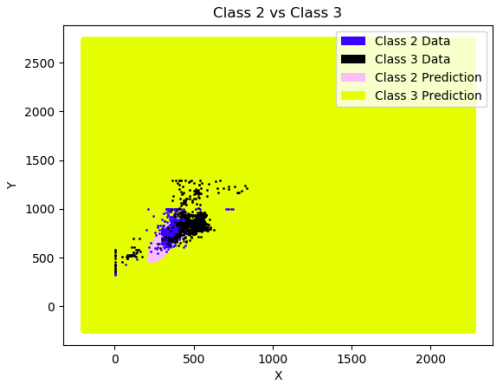
Table 3..44. Real World Data - Classifier 3: Class 1, Class 2 and Class 3

3.4 Classifier 4 : Σ_i is a unique

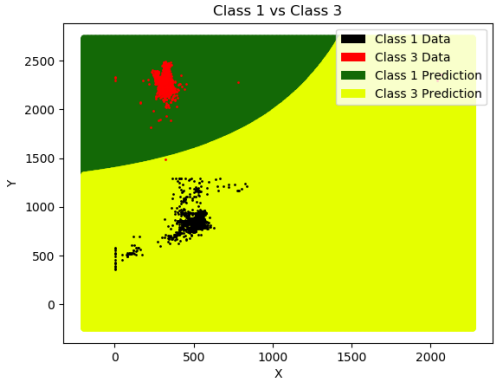
Decision Regions, Training Data and Contours



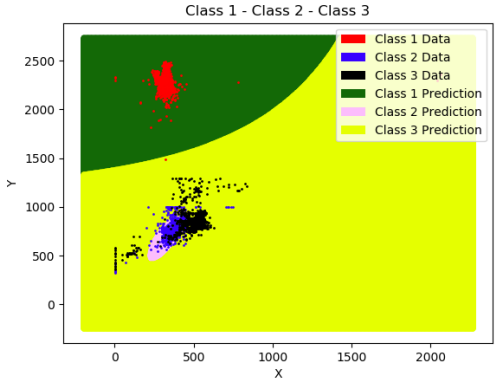
(a) Decision regions and Training data Class 1 and 2



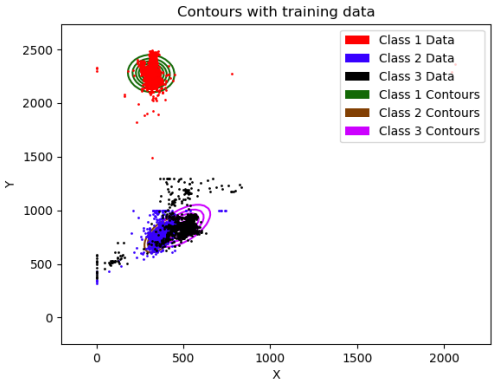
(b) Decision regions and Training data Class 2 and 3



(c) Decision regions and Training data Class 1 and 3



(d) Decision regions and Training data Class 1, 2, 3



(e) Contours: Class 1, 2, 3

Figure 3..12. Real Data - Classifier 4

Confusion Matrix, Precision, Recall and F-measure

Accuracy=99.75%

	Class1	Class 2
Class1	594	3
Class2	0	622

(a) Confusion Matrix

	Class1	Class2
Precision	1.0	0.9952
Recall	0.9949	1.0
F-Measure	0.9974	0.9975

(b) Analysis

Table 3..45. Real World Data - Classifier 4: Class 1 and Class 2

Accuracy= 76.21%

	Class2	Class 3
Class2	531	91
Class3	203	411

(a) Confusion Matrix

	Class2	Class3
Precision	0.7234	0.8187
Recall	0.8536	0.6693
F-Measure	0.7831	0.7365

(b) Analysis

Table 3..46. Real World Data - Classifier 4: Class 2 and Class 3

Accuracy=98.67%

	Class1	Class 3
Class1	581	16
Class3	0	614

(a) Confusion Matrix

	Class1	Class3
Precision	1.0	0.9746
Recall	0.9731	1.0
F-Measure	0.9864	0.9871

(b) Analysis

Table 3..47. Real World Data - Classifier 4: Class 1 and Class 3

Accuracy=83.08%

	Class1	Class2	Class 3
Class1	581	2	14
Class2	0	531	91
Class3	0	203	411

(a) Confusion Matrix

	Class1	Class2	Class3
Precision	1.0	0.7214	0.7965
Recall	0.9731	0.8536	0.6693
F-Measure	0.9864	0.7820	0.7274

(b) Analysis

Table 3..48. Real World Data - Classifier 4: Class 1, Class 2 and Class 3

4. Observations and Inferences

1. When co-variance matrix is diagonal and all the diagonal entries are same, then the shape of contours for all classes will be circular with same radius. Also, its axes will be parallel to the co-ordinate axes. This can be observed for classifier-1 contours : Figure 3..1e, 3..5e, 3..9e.

2. When co-variance matrix is same for all classes with off-diagonal entries not necessarily zero, then the shape of contour will be ellipse, with length of major and minor axes equal for all the classes and its axes not parallel to the co-ordinate axes. This can be observed for classifier-2 contours : Figure 3..2e, 3..6e, 3..10e.

3. When co-variance matrix is diagonal and different for each class, then the shape of ellipses for each class is not same, but their axes are parallel to co-ordinate axes. This can be observed for classifier-3 contours : Figure 3..3e, 3..7e, 3..11e.

4. When co-variance matrix for each class is different, then shape and orientation of ellipse for each class will be different, and none of the ellipse will have axes parallel to co-ordinate axes. This can be observed for classifier-4 contours : Figure 3..4e, 3..8e, 3..12e.

5. Conclusions

1. The decision boundary between classes comes out to be straight line when classes have same co-variance matrix, whereas when co-variance matrices are different, the decision boundary is hyper quadratic in nature.

2. Bayes classifier is simple yet powerful model and return not only the prediction but also the degree of certainty associated, which can be very useful.

3. Bayes classifier is scalable in the sense that it scales linearly with number of features and data points.

4. Bayes classifier makes strong assumption on shape of data distribution. Due to this, the result can be (potentially) very bad.

5. Bayes Classifier fails to determine decision boundaries which are complex than hyper quadratic in nature.

6. Real-world data, due to more randomness has non-linear decision boundaries.

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