Vector Network Analyzer (VNA) based Synthetic Aperture Radar (SAR) Imaging

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***Abstract*—Synthetic Aperture Radar (SAR) is an all-weather, day and night imaging tool. It can penetrate ground and foliage. In SAR imaging, a small antenna is mounted on platforms like aircraft, UAV or satellite and range profiles are collected along flight path and after applying SAR reconstruction algorithm to the recorded data, SAR image is obtained. In this work an X-band Step Frequency Continuous Wave (SFCW) based SAR imaging system has been developed using Rhode & Schwarz ZVB20 Vector Network Analyzer (VNA) and antenna linear positioning system. The motor drive of antenna linear positioning system moves transmit and receive antennas to a cross-range position, stops and the antennas acquire range profile in frequency domain. This process is repeated at regular intervals along the guide-rail until the antennas reach the end of the rail. The recorded data is in the form of 2D data matrix. After the application of Range Migration Algorithm (RMA) to the recorded phase history data of range profiles, SAR image is reconstructed.**

***Index Terms*— Radar, Synthetic Aperture Radar (SAR), Step Frequency Continuous Wave (SFCW), Vector Network Analyzer (VNA), Range Migration Algorithm (RMA), Radar Absorbent Material (RAM),Matched filter, Phase history, Linear positioner.**

I. Introduction

Conventional radar provides only range information. But image requires cross range information as well. The cross range resolution of radar is given by:

$$\Delta y = \frac{R\lambda}{D}$$

where R is the distance of radar from object, λ is the wavelength and D is the aperture diameter. Hence the larger the aperture diameter, the finer will be cross range resolution. But to mount such a large antenna on a small platform like UAV is not possible.

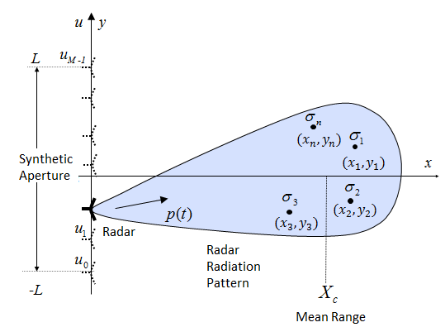
In Synthetic Aperture Radar (SAR) imaging a small antenna is mounted on a moving platform. The antenna records the range profiles of the target area along the path by illuminating it with microwaves by emitting series of pulses. Effectively, the series of observations can be combined using SAR imaging algorithm. This process creates a synthetic aperture that is much larger than the actual radar antenna aperture. The simplest way to look at two-dimensional SAR imaging is via the principle that range imaging, the x domain information, is constructed via the variations of the radar signal frequency (bandwidth), and that the cross-range imaging, the y-domain information, is formed through the variations of the radar radiation pattern (synthetic aperture or radar aspect angle) [1-5].

The aim of this work is proof of concept for SAR at National University of Sciences & Technology (NUST) using Rhode & Schwarz ZVB20 Vector Network Analyzer (VNA) and antenna linear positioning system.

II. Generic SAR Reconstruction

The generic reconstruction for SAR can be developed as follows. Consider a swath (stationary targets) region composed of a set of point reflectors with reflectivity σ_n located at the coordinates (x_n, y_n) $(n = 1,2,3, \dots \dots)$ in the spatial (x, y) domain; see Fig. 1 SAR imaging system geometry. The variable x is used for the range domain (or fast-time domain), and the variable y identifies the cross-range (also called slow-time domain)

Fig. 1 SAR imaging system geometry



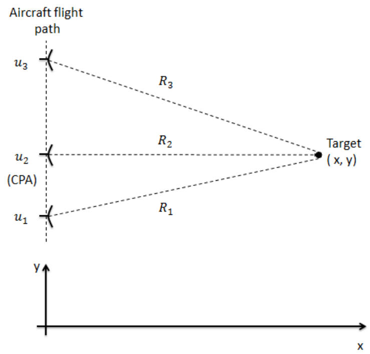
The transmitting/receiving antennas of the radar located at $(0, u)$ in the spatial domain illuminates the target area with a multi-frequency (large-bandwidth) signal $p(t)$.

The measured echoed signal is in the form of

$$s(t,u) = \sum_n \sigma_n p[t - \frac{2\sqrt{x_n^2 + (y_n - u)^2}}{c}]$$

where $\frac{2\sqrt{x_n^2 + (y_n - u)^2}}{c}$ is the round trip delay from the radar to the nth target.

Fig. 2 Target SAR signature is in (t ,u) domain; radar and target in spatial domain



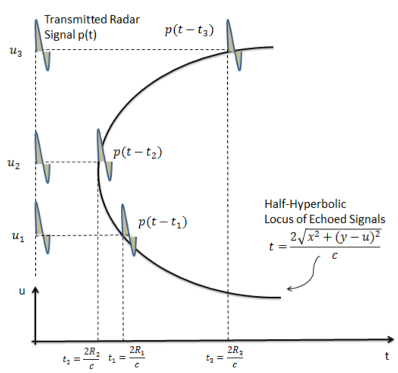
The Fourier transform of the generic SAR signal $s(t, u)$ with respect to the fast time t is

$$s(\omega, u) = P(\omega) \sum_n \sigma_n \exp\left[-j2k\sqrt{x_n^2 + (y_n - u)^2}\right]$$

where $k = \omega/c$ is the wavenumber. As one can see in the above model that the SAR signal is composed of linear combination of the spherical PM signals, that is,

$$\exp\left[-j2k\sqrt{x_n^2 + (y_n - u)^2}\right]$$

Fig. 3 Target SAR signature in (t, u) domain: half-hyperbolic target signature in SAR measurement domain



Our next operation involves a Fourier transformation with respect to the slow time or the synthetic aperture u domain. For the generic SAR model, we assume the measurements are made for $u \in (-\infty, \infty)$. The Fourier transform of the spherical PM signal with respect to the slow time u is (constants and amplitude functions are suppressed in the following)

$$\mathcal{F}_{(u)}\left[\exp\left[-j2k\sqrt{x_n^2 + (y_n - u)^2}\right]\right] = \exp\left(-j\sqrt{4k^2 - k_u^2}x_n - jk_u y_n\right)$$

for $k_u \in [-2k, 2k]$. k_u is referred to as the synthetic aperture frequency domain or slow-time frequency domain.

Using the slow-time Fourier property of the spherical PM signal, the Fourier transform of $s(\omega, u)$ with respect to the slow-time u is:

$$S(\omega, k_u) = P(\omega) \sum_n \sigma_n \exp\left(-j\sqrt{4k^2 - k_u^2}x_n - jk_u y_n\right).$$

The fact that the phase function

$$\exp\left(-j\sqrt{4k^2 - k_u^2}x_n - jk_u y_n\right.$$

is a linear phase function of (x_n, y_n) play a key role in formulating the practical reconstruction algorithm in SAR imaging systems.

We can rewrite the SAR signal in the following form by defining two new functions of (ω, k_u):

$$S(\omega, k_u) = P(\omega) \sum_n \sigma_n \exp\left(-jk_x(\omega, k_u)x_n - jk_y(\omega, k_u)y_n\right)$$

where the two new functions are defined to be:

$$k_x(\omega, k_u) = \sqrt{4k^2 - k_u^2}$$
$$k_y(\omega, k_u) = k_u$$

We call these two functions the SAR spatial frequency mapping or transformation.

We define the ideal target function in the spatial domain via

$$f_0(x, y) = \sum_n \sigma_n \delta(x - x_n, y - y_n),$$

which has the following 2D spatial Fourier transform:

$$F_0(k_x, k_y) = \sum_n \sigma_n \exp\left(-jk_x x_n - jk_y y_n\right)$$

Note that $F_0(k_x, k_y)$ is also composed of a linear combination of a linear phase function of (x_n, y_n), $n = 1,2, \ldots$, which is due to the Fourier shift property.

Next we use the expression for $F_0(k_x, k_y)$ in the SAR signal $S(\omega, k_u)$; this yields

$$S(\omega, k_u) = P(\omega)F_0\left(k_x(\omega, k_u), k_y(\omega, k_u)\right)$$

where $\left(k_x(\omega, k_u), k_y(\omega, k_u)\right)$ are governed by the SAR spatial frequency mapping. For the reconstruction, that is, imaging $f_0(x, y)$ or $F_0(k_x, k_y)$ from the Fourier transform of the measured signal $S(\omega, k_u)$, we have

$$F_0\left(k_x(\omega, k_u), k_y(\omega, k_u)\right) = \frac{S(\omega, k_u)}{P(\omega)}$$

III. Experimental Setup

The experimental setup consists of Rhode & Schwarz ZVB20 Vector Network Analyzer (VNA) which is connected to the transmitting and receiving horn antennas via RF cables. Two different methods are used to interface the VNA with the workstation PC: General Purpose Interface Bus (GPIB) and Ethernet. The horn antennas are mounted on antenna linear positioning system about 5 ft long. The antenna linear positioning system consists of a small cart on which the antennas are mounted and a rail on which the cart moves. Radar Absorbent Material (RAM) is placed between the transmitting and receiving horn antennas in order to reduce the spillover. A motor drive is installed at one of the ends of the rail. A pedal controller is used to control the movement of the motor and hence the cart. The antennas acquire SAR phase history data of the target scene at evenly spaced increments across the rail. The transmitting antenna is connected to Port 1 and the receiving antenna is connected to Port 2 of the VNA.

For a 2-port S-parameter measurement of the Device Under Test (DUT) the signal flow is shown in Fig. 4.The scattering matrix links the incident waves a_1 and a_2 to the outgoing waves b_1 and b_2 according to the following linear equation:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

The forward transmission coefficient, S_{21} , defined as the ratio of the wave quantities b_2/a_1(forward measurement with matched output and $a_2 = 0$), is measured by the VNA. Hence the S_{21} parameter can be expressed as

$$S_{21} = F_0\left(k_x(\omega, k_u), k_y(\omega, k_u)\right) = \frac{S(\omega, k_u)}{P(\omega)}$$

Fig. 4 Two-port network

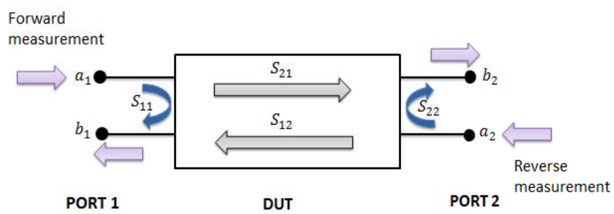


Fig. 5 Block diagram of experimental setup

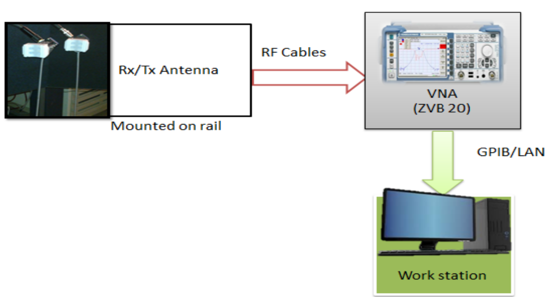
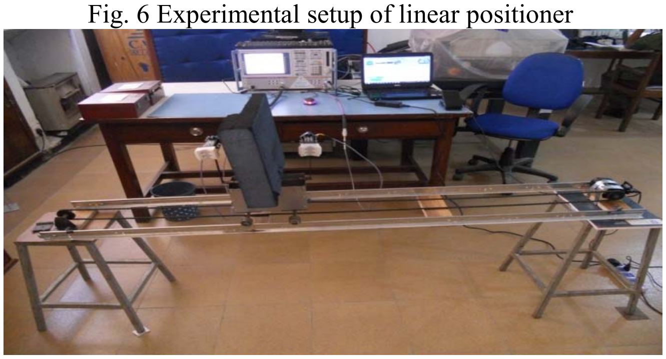


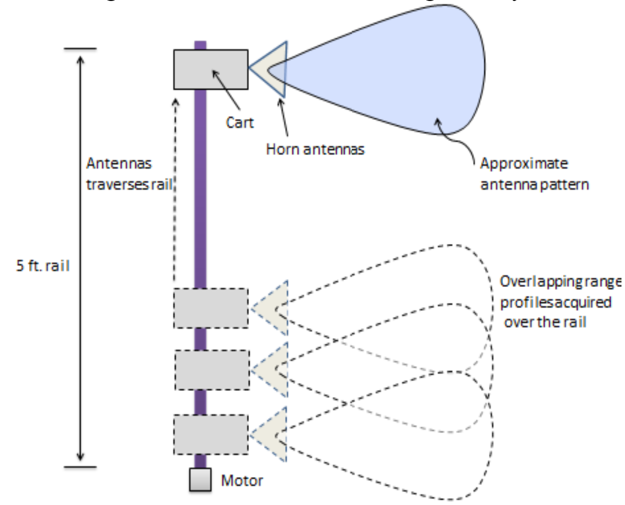
Fig. 6 Experimental setup of linear positioner



IV. DATA COLLECTION GEOMETRY

The antenna linear positioning system moves the transmitting and receiving horn antennas to a cross range location and the radar acquires the range profile. After the SAR phase history data of range profile is complete the antennas are moved to the next cross range position and another range profile is acquired. This process is repeated at regular intervals along the guide rail until the antennas reach the end of the guide rail. The range profiles over each cross range position produce a 2D data matrix as shown in Fig. 7. The cross range steps are equally spaced for uniform sampling.

Fig. 7 Rail SAR data collection geometry



V. RANGE MIGRATION ALGORITHM (RMA)

The reconstruction algorithm is summarized in the following [6]:

Step 1 .FFT with respect to slow-time is performed on the collected data. This converts the data matrix from (ω, u) domain to (ω, k_u) domain.

Step 2. The following reference signal is generated in software:

$$S_0(\omega, k_u) = \exp\left(j\sqrt{4k^2 - k_u^2}X_c\right)$$

Step 3. Match filtering is performed with the reference signal. This gives us the target function:

$$F\left(k_x, k_y\right) = S(\omega, k_u)\exp\left(j\sqrt{4k^2 - k_u^2}X_c\right)$$

where $k_x = \sqrt{4k^2 - k_u^2}$ and $k_y = k_u$.

For the available discrete samples of SAR data in the (ω, k_u) domain, labeled (ω_n, k_{um}), this results in a set of unevenly spaced samples of the target function in the spatial frequency $\left(k_x, k_y\right)$ domain at:

$$k_{xmn} = \sqrt{4k_n^2 - k_{um}^2}, \text{ and } k_{ymn} = k_{um}$$

Step 4. The support of the coverage of SAR data in the spatial frequency $\left(k_x, k_y\right)$ domain is identified:

$$k_x \in [k_{xmin}, k_{xmax}]$$

where

$$k_{xmin} = \min\,[k_{xmn}]$$
$$k_{xmax} = \max\,[k_{xmn}]$$

and

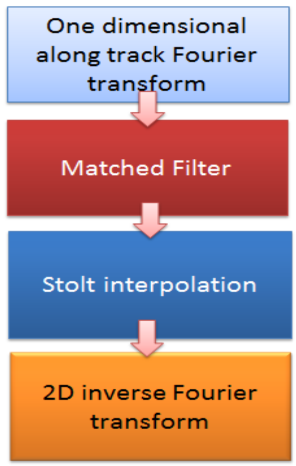
$$k_{ymin} = \min\,[k_{ymn}]$$
$$k_{ymax} = \max\,[k_{ymn}]$$

A uniform grid is identified in this region

Step 5. The samples of the target function spectrum $F(k_x, k_y)$ on the uniform grid are interpolated from its available unevenly spaced data $F(k_{xmn}, k_{ymn})$.

Step 6. Two-dimensional inverse FFT is performed on the evenly spaced samples of the target function spectrum $F(k_x, k_y)$ on the uniform grid. This yields evenly spaced samples of the spatial domain target function $(f(x, y)$ on uniform grid.

Fig. 8 Range Migration Algorithm (RMA)



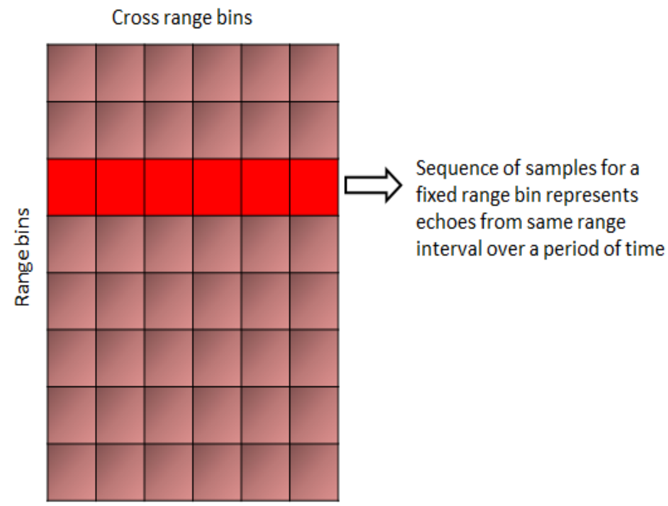
VI. Radar Parameters

The various Radar parameters are listed in Table 1.

Table 1 Radar Parameters

Parameter	Value
Transmitted Power	10 dBm
Modulation Type	Step Frequency Continuous Wave (SFCW)
Operating Frequency Band	X-band
Bandwidth	4 GHz
Center Frequency	10 GHz
Start Frequency	8 GHz
Stop Frequency	12 GHz
Number of frequency steps	512, 1024, 2048
Cross range step size	2 cm
Synthetic Aperture length	128 cm
Number of cross range steps	64
Theoretical range resolution	3.75 cm
Theoretical cross range resolution (@ 10 GHz & R=100cm)	2.34 cm
Antenna type (2 are required)	Horn antennas
Antenna bandwidth	6-16 GHz
VSWR (@ 10GHz)	1.24

Fig. 9 Data collection matrix for linear SAR imaging system



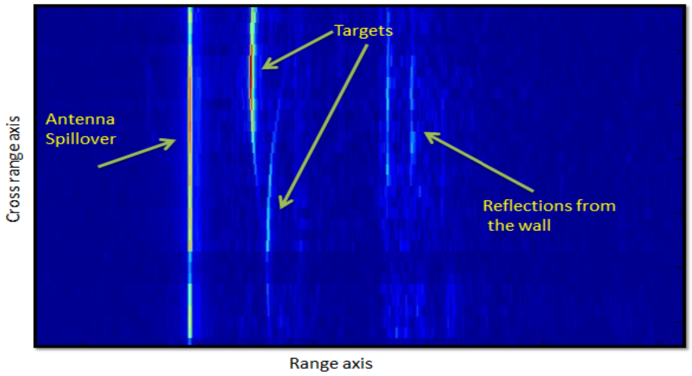
VII. Results

Sample targets of different shapes like cylindrical, flat plate, point targets, scaled model of aircraft and corner reflectors have been fabricated in Industrial Engineering (IE) workshop for the sake of analysis. The reason behind analyzing different targets of different shapes is that a target of complex shape is composed of these basic shapes.

Fig. 10 Sample targets fabricated in Industrial Engineering (IE) Workshop

These targets are analyzed using the Radar system and the results are plotted.

Fig. 11 Collected SAR data in time domain (two targets)



VIII. CONCLUSION

In this work, the proof of concept phase for Synthetic Aperture Radar (SAR) imaging system has been successfully completed using Vector Network Analyzer (VNA) and antenna linear positioning system. Image reconstruction has been done by implementing Range Migration Algorithm (RMA) in MATLAB®.

IX. FUTURE WORK

The future work include developing the lab model for SAR at NUST, implementation of Backprojection Algorithm (BPA), implementation of "through the wall" imaging system [7], Circular SAR (CSAR) imaging and implementation of motion compensation algorithm etc.

Fig. 12 Flat Plate

Fig. 13 SAR image of flat plate

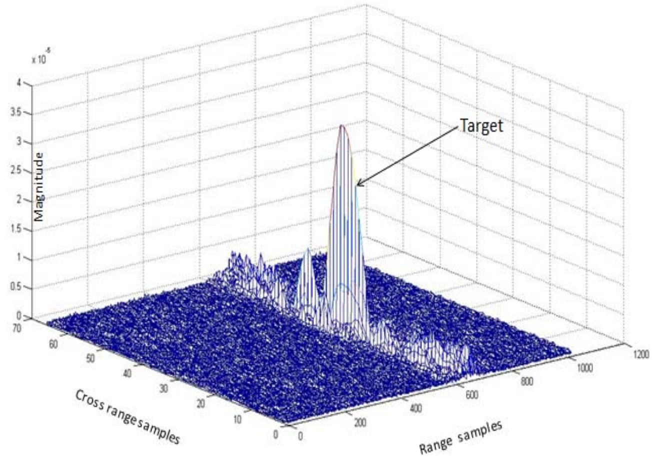


Fig. 14 2D SAR image of flat plate

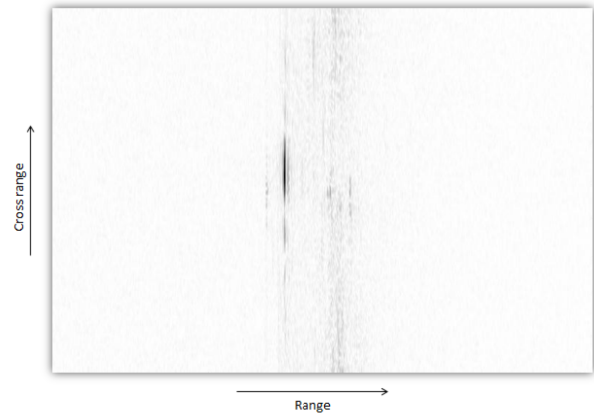


Fig. 15 Metal Cylinder

Fig. 16 SAR image of Cylinder

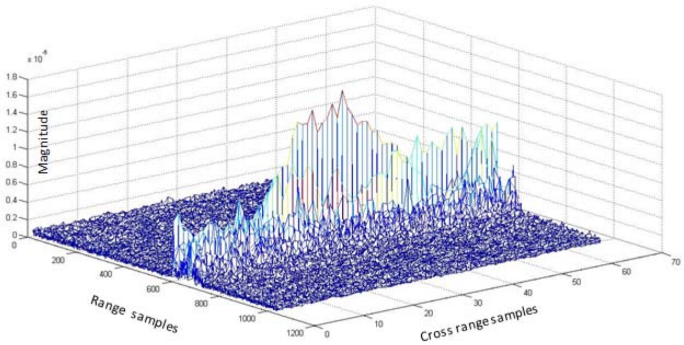


Fig. 17 SAR image of metallic model of aircraft

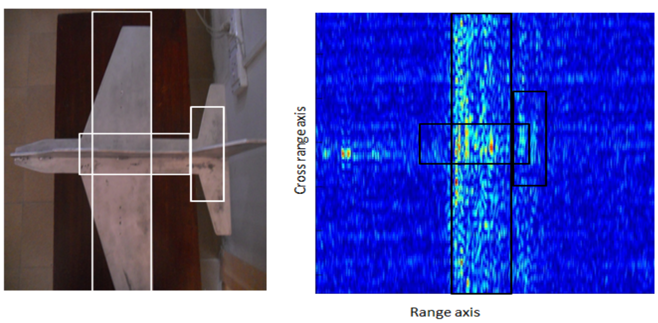
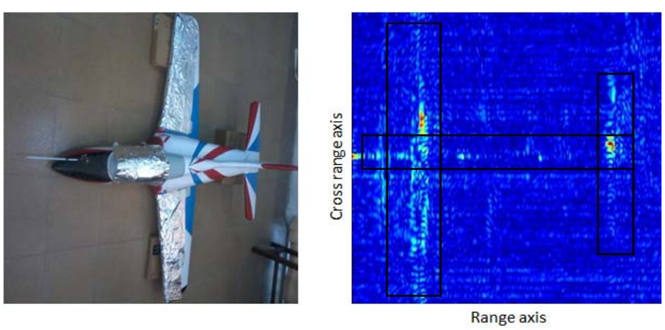


Fig. 18 SAR image of wooden model of K8 aircraft after wrapping with aluminum foil



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