Systems of Linear Equations

The examples in this MATLAB Live Script are adapted from examples from the Systems of linear equations section from M. W. Liberatore, Material and Energy Balances ZyBook. Electronic, interactive textbook: ZyBooks, 2019.

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Learning Objectives

- Solve a system of linear equations using the backslash operator in MATLAB.

Systems of Linear Equations in MATLAB

Systems of linear equations consist of coefficients, constants, and variables, which can be arranged into matrices. The matrix form to express a system of linear equations is matrix A for the coefficients, matrix x for the variables, and matrix b for the constants, which yields the formula $\mathrm{Ax} = b$. Solving for the variables involves taking the transpose of A multiplied by b, or $A^{-1}b = x$. Solving for the variables of a system of linear equations in MATLAB involves using the backslash operator, \

$x = A \setminus b$ will return the solution to $x = A^{-1}b$.

Worked Example

For example, let's look at the following system of linear equations.

$$-3x_1 + 4x_2 - 9 = -2x_3$$
$$x_1 - 5 = 4x_3 - 6x_2 + 2$$
$$-2x_1 - 5x_2 - 8x_3 - 4 = 0$$

First, separate the constant terms and the variable terms resulting the following system of linear equations.

$$-3x_1 + 4x_2 + 2x_3 = 9$$
$$x_1 + 6x_2 - 4x_3 = 7$$
$$-2x_1 - 5x_2 - 8x_3 = 4$$

Second, create the coefficient matrix A. Each row of the coefficient matrix represents the coefficients of one of the linear equations and each column represents the coefficients of one of the variables across all three linear equations. A has the same number of rows and columns, which is equal to the total unknown variables and the number of equations in the system.

```
A = [-3  4  2;
      1  6 -4;
     -2 -5 -8]
```

```
A = 3×3
    -3     4     2
     1     6    -4
    -2    -5    -8
```

Next, create column vector of constant terms, matrix b. Each row of the column vector represents the isolated constant term of a linear equation.

Note: The order of the represented linear equations is arbitrary but should match the order of the rows seen in the coefficient matrix.

```
b = [9;
     7;
     4]
```

```
b = 3×1
     9
     7
     4
```

Finally, use the backslash operator to solve for the values of x_1, x_2, and x_3. The solution will take the form of a column vector, x, with the first, second, and third rows representing the values of x_1, x_2, and x_3 respectively.

```
x = A\b
```

```
x = 3×1
   -2.0213
    1.0638
   -0.6596
```

Note: If the first and second column of the coefficient matrix a were exchanged, the solution would take the form of a column vector, x, with the first, second, and third rows representing the values of x_2, x_1, and x_3 respectively. The order of the columns in the coefficient matrix determines the order of the rows in the solution vector.

```
A = [ 4 -3  2;
      6  1 -4;
     -5 -2 -8];

b = [9;
     7;
     4];

x = A\b
```

```
x = 3×1
    1.0638
   -2.0213
   -0.6596
```

Interactive Example / Template

This example can be used as a template to solve system of linear equations of any size. Using the same procedure as the worked example, solve the following system of linear equations.

$$2x_2 + 7x_4 = 5 - 5x_1 - 8x_3$$
$$2x_1 + 6x_2 = 2 + 4x_3 + 4x_4$$
$$-4x_1 - 3x_2 - 7x_3 + x_4 - 1 = 0$$
$$-9x_2 + 6x_3 - 3 = 2x_1 - 2x_4 + 4$$

The system of linear equations above can be rearranged as follows:

$$5x_1 + 2x_2 + 8x_3 + 7x_4 = 5$$
$$2x_1 + 6x_2 - 4x_3 - 4x_4 = 2$$
$$-4x_1 - 3x_2 - 7x_3 + x_4 = 1$$
$$-2x_1 - 9x_2 + 6x_3 + 2x_4 = 7$$

As there are four equations and four unknowns, A will have four rows and four columns, b will have four rows, and x will return the solutions to the four unknown variables. Enter in the corresponding values below (expanding the number of rows or columns, if needed for new problems) and press run.

```
A = [  5   2   8   7;
       2   6  -4  -4;
      -4  -3  -7   1;
      -2  -9   6   2]
```

```
A = 4×4
     5     2     8     7
     2     6    -4    -4
    -4    -3    -7     1
    -2    -9     6     2
```

```
b = [5;
     2;
     1;
     7]
```

```
b = 4×1
     5
     2
     1
     7
```

```
x = A\b
```