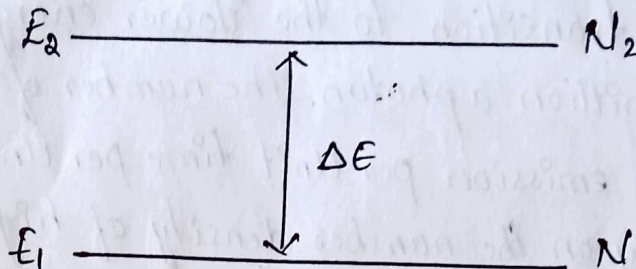


## Assignment - 4

1. Derive the expression for energy density using Einstein's Coefficients.



Consider two energy states  $E_1$  and  $E_2$  of a system of atom ( $E_1 < E_2$ ) with  $N_1$  and  $N_2$  number per unit volume of the system respectively. Let  $U_\nu d\nu$  be the energy of the incident radiation per unit volume of the system where, radiations lie in the frequency range  $\nu$  to  $\nu + d\nu$ , then ' $U_\nu$ ' is called the energy density of frequency " $\nu$ ".

Case (i): Induced absorption

In this case an atom in the energy level,  $E_1$ , can undergo a transition to the level  $E_2$  by absorbing a radiation of suitable frequency.

Case (i): Induced absorption.

The number of such absorptions per unit time per unit volume depends on:

- The number density of lower energy state.
- The energy density ' $U_\nu$ '

$\therefore$  Rate of absorption  $\propto N_1 U_\nu$ .

$$\text{Or Rate of absorption} = B_{12} N_1 U_\nu \quad \text{--- (1)}$$

Where,

$B_{12} \rightarrow$  Einstein's Coefficient of induced absorption



### Case (2): Spontaneous Emission:

In the case, an atom in the higher energy state  $E_2$  undergoes a transition to the lower energy state  $E_1$ , by itself, emitting a photon. The number of such spontaneous emission per unit time per unit volume depends only on the number density of higher energy state i.e.  $N_2$ .

Therefore

$$\text{Rate of Spontaneous emission} = A_2, N_2 \quad \text{--- (2)}$$

Where,  $A_2 \rightarrow$  Einstein's Coefficient of Spontaneous emission.

### Case (3): Stimulated Emission:

In this case an external photon of suitable frequency stimulates an atom for the downward transition and thereby causes emission of stimulated photons. The number of stimulated emission per unit time per unit volume depends on

a) Number density of higher energy state.

b) The energy density ' $U_\nu$ '.

Therefore rate of Stimulated emission  $\propto N_2 U_\nu$ .

$$= B_{21} N_2 U_\nu \quad \text{--- (3)}$$

Where  $B_{21}$  is the Einstein's Coefficient of Stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of Spontaneous emission + Rate of Stimulated emission.



$$\text{i.e. } B_{12} N_1 \mathcal{U}_\nu = A_{21} N_2 + B_{21} N_2 \mathcal{U}_\nu$$

$$\text{i.e. } \mathcal{U}_\nu (B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$\therefore \mathcal{U}_\nu = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$\text{Or } \mathcal{U}_\nu = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right] \rightarrow (4)$$

According to Boltzmann's law we have.

$$N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{kT}\right)} = N_1 e^{-\frac{h\nu}{kT}}$$

$$\therefore \frac{N_1}{N_2} = e^{h\nu/kT} \quad \text{--- (5)}$$

Therefore eqn (4) becomes.

$$\mathcal{U}_\nu = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\frac{B_{12}}{B_{21}} e^{h\nu/kT} - 1} \right] \quad \text{--- (6)}$$

According to Planck's law of Radiation.

$$\mathcal{U}_\nu = \frac{8\pi h\nu^3}{c^3} \left[ \frac{1}{e^{h\nu/kT} - 1} \right] \quad \text{--- (7)}$$

Comparing Eqn (6) and (7) we have.

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \& \quad \frac{B_{12}}{B_{21}} = 1$$

$$\rightarrow B_{12} = B_{21} = B \quad \rightarrow (8) \quad \text{and} \quad A_{21} = A.$$



The identity ③ implies that the probability of induced absorption is equal to the probability of stimulated emission.

Therefore we can write the expression for energy density in terms of Einstein's A & B Coefficients as.

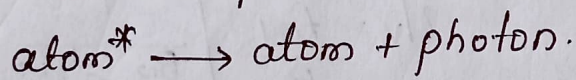
$$U_\nu = \frac{A}{B[e^{h\nu/kT} - 1]}$$

Q. Define

a) Spontaneous Emission:-

The atom after spending its life time in the excited state voluntarily emits a photon of energy  $\Delta E$  equal to  $(E_2 - E_1)$  and falls to the energy state  $E_1$ . The emission where in an atom emits a photon without any aid by external agency is called spontaneous emission.

This process can be represented as.

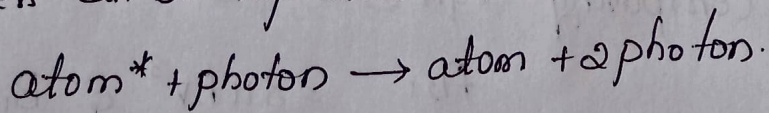


b) Stimulated emission:

The process of the emission of a photon by a system

c) Population Inversion: Under the influence of an incident photon of suitable energy, state to a lower energy state is called stimulated emission.

This process can be represented as:



c) Population Inversion:

It is the state of an atomic system in which the number of atoms in the higher energy level ( $N_2$ ) is



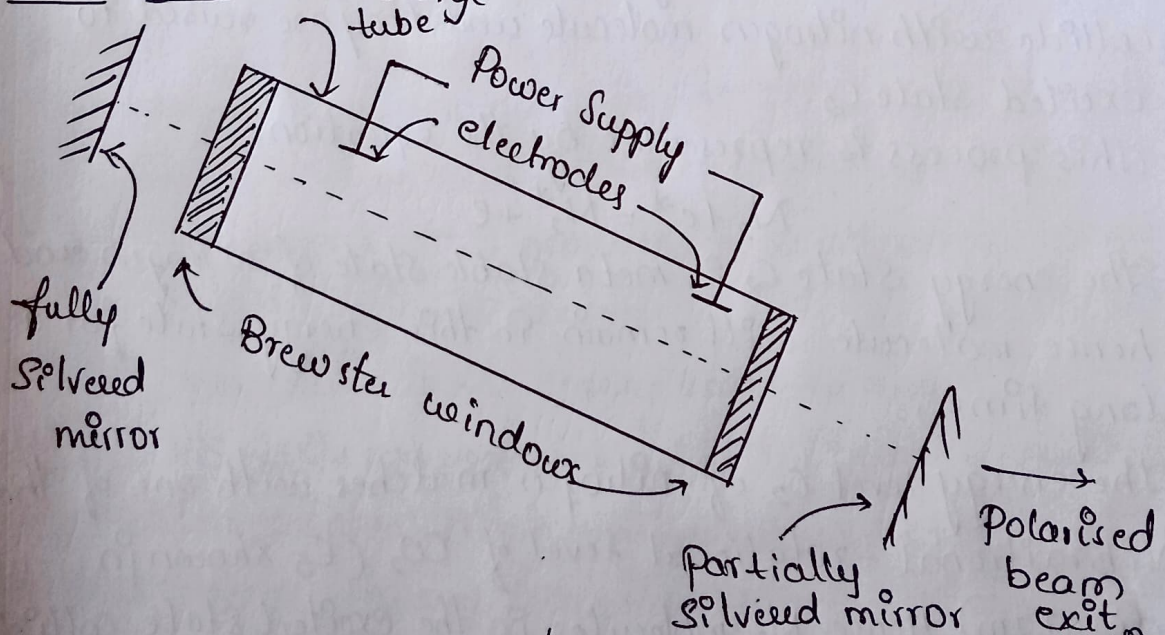
greater than the number of atoms in the lower energy state ( $N_1$ ) when  $N_2 > N_1$ , the rate of stimulated emission exceeds the rate of stimulated absorption hence the intensity of the emitted light beam will be more than that of the incident beam which implies the amplification of light takes place.

#### d) Metastable State:

A Medium in which light gets amplified is known as an active medium. The medium may be solid, liquid or a gas. An active medium must consist of at least 3 energy levels namely, a ground state, a high energy pump state (ordinary excited state) and an intermediate level is called the metastable state.

### 3. Explain Construction and working of $\text{CO}_2$ Laser.

#### Construction:



- It consists of a quartz tube 5m long and 2.5cm in the diameter.
- This discharge tube is filled with gaseous mixture of  $\text{CO}_2$  (active medium), nitrogen and helium in this ratio 1:2:3.



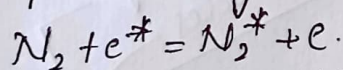
- The terminals of the discharge tube are connected to a high DC power supply with the help of an electrode system. The pumping mechanism based on electric discharge is used to create population inversion.
- The ends of the discharge tube are fitted with NaCl Brewster windows so that the laser light generated will be polarized.
- Two concave mirrors one fully reflecting and the other partially form an optically resonator.

### Working:

The energy level diagram of nitrogen and carbon dioxide molecule is shown in diagram.

When an electric discharge occurs in the gas, the electrons collide with nitrogen molecule and they are raised to excited state  $C_2$ .

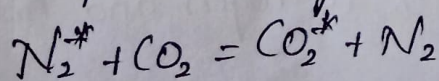
This process is represented by the equation,



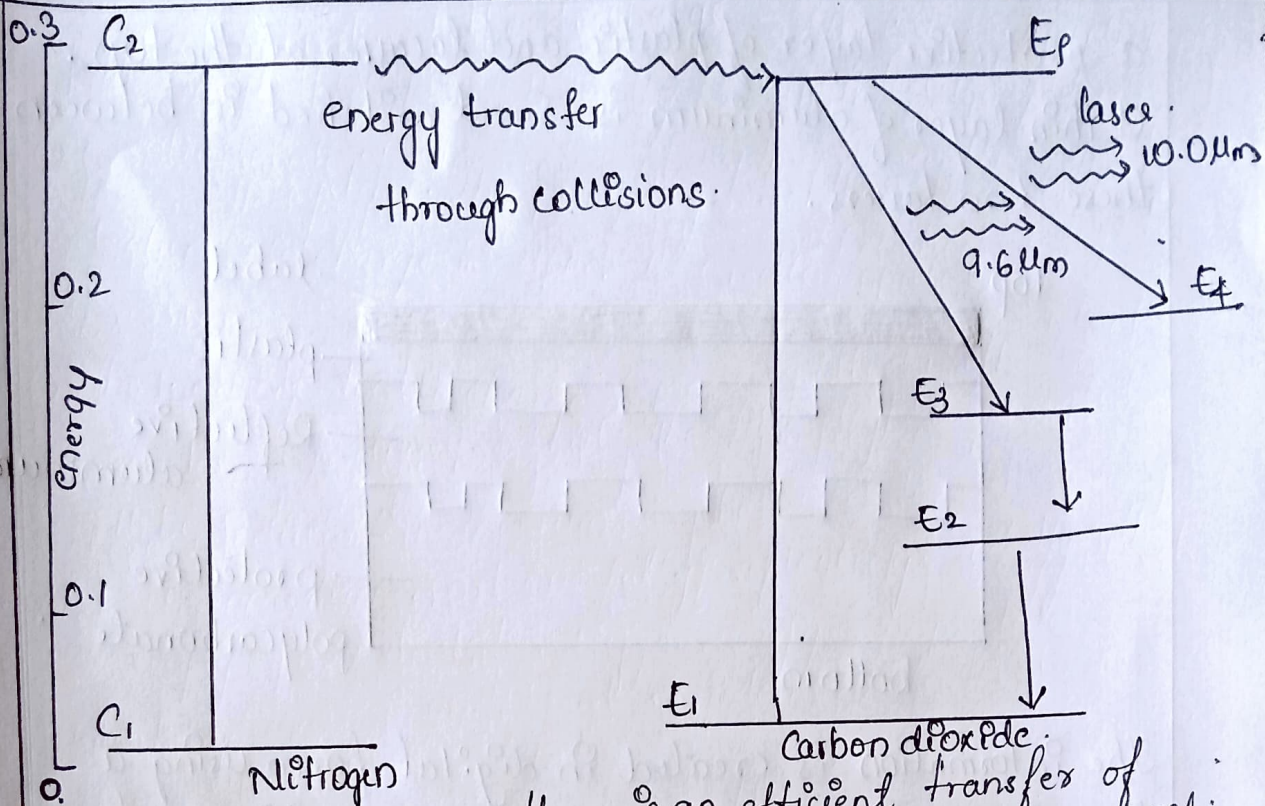
The energy state  $C_2$  is meta stable state of nitrogen and hence molecule will remain in this energy state for a long time.

The energy level  $C_2$  of nitrogen matches with one of the vibrational-rotational level of  $CO_2$  (E5 shown in diagram). Hence  $N_2$  molecules in the excited state collide with  $CO_2$  atoms in ground state and excite to higher vibrational and rotational levels.

This process is represented by the Eqn







Due to this collision, there is an efficient transfer of energy between  $C_2$  level of nitrogen and  $E_5$  level of  $CO_2$ . This kind of energy transfer is called resonant energy transfer.

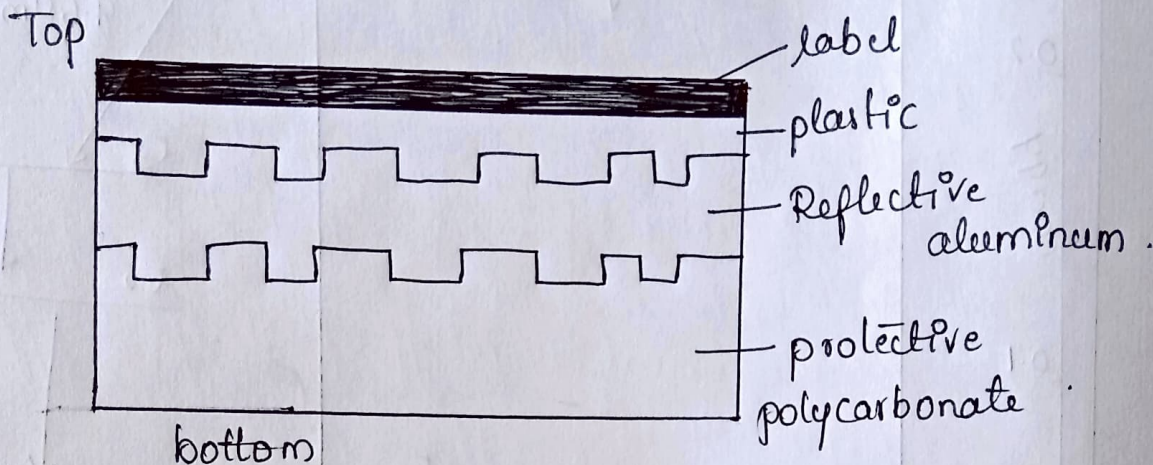
This energy transfer results in the population inversion between  $E_5$  and  $E_4$  and between  $E_5$  and  $E_3$ . The transition from  $E_5$  to  $E_4$  produces a radiation of wavelength  $9.6\mu m$  both lying in IR region.

The  $CO_2$  molecules in the energy state  $E_4$  and  $E_3$  are de-excited to the ground state by the inelastic collision with Helium atoms. These Helium atoms help in carrying the heat energy away from the system.

4. Discuss the use of laser in data storage applications.
- A Compact disc is a thin, circular disc of metal and plastic. It consists of three layers, a tough brittle plastic called polycarbonate at the bottom,



a protective layer of plastic and lacquer at the top..  
A thin layer of aluminum is sandwiched in between these two layers.



The information is created in digital form using a laser beam. A series of microscopic holes/bumps known as pits are formed by burning of the surface at certain specified interval. The presence of this pit or bump in a fixed length indicates the number 'zero'. The unburnt flat area on the disc is called land and it represents the number 'one'. Thus the information is stored by burning some lengths of the surface (zeros) and leaving some length unburnt (ones) in binary form.

While reading, the CD, the surface is scanned by a laser beam. As the beam bounces, it follows the pattern of pits and lands. Laser light reflected from pit represents binary zero and from the land represents binary number one. An electronic circuit generates zeros & ones from these pulses. These binary numbers are converted into an analog electrical signal using decoder.