Metrics for Quantifying Co-development at the Individual Level

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Abstract

Previous research on co-development has focused on modeling the relations at the group level, however, how individuals differ in co-development may provide important information as well. Recent work has used vector plots to visually explore individual differences in co-development, however, these judgements were made based on visual inspection of a vector plot rather than the calculation of metrics. Here we propose two metrics that can be used to quantify co-development at the individual level: the co-development change ratio (CCR) and the angle of co-development metric (ACM). CCR provides information about the symmetry of development, examining whether an individual grew the same pace in one skill relative to peers as compared to growth in the other skill relative to peers. ACM represents the relative amount and direction of change on each skill. This paper provides a tutorial on how to calculate and interpret these two metrics for quantifying co-development at the individual level.

Metrics for Quantifying Co-development at the Individual Level

Co-development involves an exploration of how two skills longitudinally develop in relation to one another. Previous work has focused more on modeling the relations at the group level (e.g., correlations between initial start and growth on each skill), however, how individuals differ in co-development may provide important information as well. For example, recent work on the co-development of word and nonword reading has revealed individual differences in symmetry (relative amount of growth) between growth in word and nonword reading ability such that some children grew more in word reading than nonword reading ability from 1st to 4th grade (asymmetric) whereas other children showed relatively similar amounts of growth in both word and nonword reading (symmetric) (Steacy et al., 2021). Steacy and colleagues used vector plots to visualize the trajectories modeled from a parallel process growth curve model, allowing them to visualize the differences in the angles across the distribution. Vector plots express both the magnitude and direction of change, such that the origination point of a single-headed arrow represents initial status of each individual in the sample, and the tip of the arrow represents final status of each individual. The angle of the arrow then represents the relative amount of change for each variable such that arrows pointed at a 45-degree angle suggests growth patterns that grow proportional to one another, when presented on similar scales. Using vector plots Steacy et al. (2021) showed that the angle of the arrow drawn by word and nonword reading growth varied as a function of initial starting position, such that those who started lower on these skills tended to grow in word reading without concomitant growth in nonword reading (showing a flat vector). Steacy et al. (2021) also added a color gradient representing initial phonological awareness skill to the vectors, visually inspecting the plot for the relation between phonological awareness and growth symmetry. Steacy et al. (2021) found there to be an association between initial phonological awareness skill and vector slopes (relative growth in word and nonword reading) based on visual inspection, however, with no metrics to quantify these vector slopes in a meaningful way (particularly in the presence of negative slopes and differing scales) at the individual level, Steacy et al. (2021) were unable to quantify the magnitude of this association. Although the visual inspection of the vector plot revealed individual differences in co-development, without appropriate metrics to quantify these individual differences, these differences cannot be explored further such as to predict other measures of interest or correlate it with other measures such as phonological awareness. Thus, the creation of such metrics to quantify individual differences in co-development is needed. Here we propose two metrics (angle and change ratio) that can be used to quantify co-development at the individual level. These metrics can be used to provide information about the individual differences in co-development as well as can be used as predictors in other analyses to represent these individual differences.

Despite the plethora of statistical modeling choices for estimating the co-development of correlated processes (e.g., Petscher et al., 2016) a particular shortcoming is the lack of a useful metric to quantify the magnitude and direction of co-development at the individual level. Beyond the information that can be gleaned from parallel process growth models (e.g., intercept and slope coefficients for each trajectory, functional form of growth, correlation between intercepts and slopes across the constructs of interest), *how* individuals differ in co-development may provide important information as well. In a sample of 16,000 second grade students, Petscher et al. (2016) estimated the co-development of reading comprehension and oral reading skills where the primary pattern of growth was marked by greater increases in oral reading relative to their reading comprehension. Such studies of co-development use individual vector plots as a graphical means to express both the magnitude and direction of co-development for each person in the sample. The angle or slope of the vector generally represents the relative growth on each measure. However, the angle on a vector plot depends on the axis scaling and the interpretation of a slope can vary based on scale differences. One of the reasons why just using a simple slope to quantify this is problematic is that differences in scales can cause problems. For example, growth on X has a mean of 5 and a standard deviation of 2 and growth on Y has a mean of 40 and a standard deviation of 10. Based on the change ratio we will describe later, someone who grows the same amount relative to peers on both metrics would be considered having symmetric growth. So someone who grew the mean on X and Y, would be symmetric, and so would someone who grew 1SD above the mean on both X and Y, as well as 1SD below the mean on both X and Y. That is, their amount of growth relative to what is typical for that measure should be the same for both X and Y. Using the change ratio approach these 3 cases would all have a change ratio of 1. However, if you were to calculate just a slope, someone who grew the mean on both would have a slope of 8, someone growing 1SD above the mean on both would have a slope of 7.1, and 1SD below the mean on both would have a slope of 10. Although all growing an amount that would be considered symmetrical, showing growth positions relative to the sample being equal for both X and Y, the slopes do not capture this due to differences in scale for each measure. Using just a simple slope will provide biased estimates of symmetry when either the means or the standard deviations differ, which is likely always the case. Thus, we propose two metrics that account for the fact that scales will differ between measures, allowing for co-development to be quantified at the individual level even with differing scales.

Vector plots can serve an important function in the reporting aspect of bivariate or multivariate growth curve models to show a reader what the general pattern of co-development may look like. Thus, similar to individual growth curve analyses that may use spaghetti plots to provide the reader with a graphical means to visualize estimated trajectories for univariate growth, so do vector plots provide the reader with a parallel means to visualize co-development. In a similar way, just as plots of individual growth curves do not give a reader a metric of individual growth and must instead view the individual level estimated intercepts and slopes, so do vector plots lack individual-level estimates. Judgements in patterns of co-development in published works (e.g., Petscher et al., 2016; Steacy et al., 2021) were made based on visual inspection of a vector plot of estimated growth trajectories rather than the calculation of metrics. These and other recently published studies of co-development (e.g., Larsen et al., 2022; Zhang et al., 2018) could benefit from adding to the estimation and plotting of co-development with metrics that can serve as a source of individual differences in the pattern of co-development.

Exploring individual differences in co-development recently emerged in the area of reading research, however, applications of this method are cross-disciplinary. For example, growth in weight relative to growth in height may be used as a predictor for physical health. Barboza et al. (2017) used parallel process growth curve models to explore the co-development of trauma symptoms and externalizing behavior, finding correlated slopes. However, perhaps at the individual level, more growth on one relative to the other may predict later adjustment outcomes. Kieffer and Lesaux (2012) found morphological awareness growth to be related to vocabulary knowledge growth. However, they only explored group level effects, potentially individuals who grow less on morphological awareness relative to their vocabulary growth may be predictive of future spelling success, showing a similar pattern to the word/nonword reading relationship such that increases in vocabulary without increases in morphological awareness may indicate an asymmetric learning system, not functioning as it should. We propose two metrics to open the door to exploring these individual differences in co-development in a variety of disciplines and introduce the idea of exploring these individual differences to the field at large.

Angle of Co-development Metric and Co-development Change Ratio

We propose two metrics (the co-development change ratio (CCR) and the angle of co-development metric (ACM)) that can be used to quantify co-development at the individual level. These metrics can be used to provide information about the individual differences in co-development as well as can be used as predictors in other analyses. One problem with making judgments based on visual inspection of vector plots is that the perception of relative amount of growth in each depends heavily on the scale of each axis. Growing 10 points on each measure does not necessarily indicate symmetric growth if the measures are on different scales, thus, it is important to take sample based information into account to determine what symmetric growth looks like. CCR takes into account what average growth is for each measure, whereas the ACM situates growth relative to variability in each measure at the first time point. Both metrics provide an estimate of co-development at the individual level, but they do so differently, and provide different information. Neither metric is sufficient to provide all information for every sample and research question, thus we propose two metrics here. Some samples and research questions may be suited for just CCR (e.g., investigations of relative growth with unidirectional growth), just ACM (e.g., samples with multidirectional growth such as some cases increasing and others decreasing), or may need both metrics to be used in tandem to provide the full picture of information (e.g., exploring both relative growth in two measures relative to peer growth as well as relative growth and direction of change relative to variability at the first timepoint).

The first metric is the co-development change ratio (CCR) which provides information about the symmetry of development, examining whether an individual grew the same pace in one skill relative to peers as compared to growth in the other skill relative to peers. One example of asymmetric growth would be an individual who grew above average on X but below average on Y, showing more growth in X relative to peers than in Y relative to peers. However, growth does not need to be on opposite sides of the mean to be considered asymmetric. For example, an individual could have grown 1 SD below the mean on X and 2 SD below the mean on Y, still yielding an asymmetric pattern in which they grew more on X relative to their peers as compared to their growth on Y relative to that of their peers.

The second metric proposed here, angle of co-development metric (ACM), represents the relative amount and direction of change on each skill. The ACM is represented in degrees from -180 to 180 as the angle from a line drawn from the origin to (change on X, change on Y) and the right side of the x-axis (see Figure 1). This angle can also be thought of on a typical vector plot, between the vector drawn from the initial starting position (z-scored) on each variable X and Y to the endpoint on each represented as a developmental scale score (scaled based on the first timepoint) and a horizontal vector representing growth only on X (see Figure 2). The vector is the hypotenuse, the opposite side is the difference between developmental scale scores on Y, and the adjacent side is the change in developmental scale score on X. The angle can be calculated using arctan given change in X and change in Y. For example, an individual who increased 1SD on each skill would have a 45 degree angle, representing positive and equal growth on each skill. It is important to note here that an ACM of 45 is not the same as a CCR of 1 because average change can differ between X and Y. For example, student A has changed 1SD on both X and Y, yielding a 45 degree ACM, however, 1SD was average change for X but average change on Y was 3SD so student A grew the average amount for X but below average for Y, leading to a CCR of <1. The CCR accounts for the amount of growth that is typical for that skill, whereas the

ACM uses the amount of change relative to the sample variability at the first timepoint (i.e., developmental scale score).

ACM vs CCR

Although both metrics represent co-development at the individual level, they do so differently and provide unique information. The CCR uses average growth typical in the sample for each measure to evaluate how much an individual grew relative to peers on each measure. However, the ACM uses change relative to variability in the sample at the first timepoint. The ACM does not take into account that different amounts of growth may be expected for each variable. The CCR does not account for different directions of growth, whereas the ACM will account for this.

Figure 3 shows an example of a vector plot created to demonstrate the differences between ACM and CCR, which are presented in Table 1. Vector B stays the same on X and decreases on Y, which is represented by -90 in the ACM. CCR does not account for negative growth but rather treats it as simply “less positive growth” relative to others. Since B grew .5 SD below the mean growth on X and 1.2 SD below the mean on Y, B has a CCR of .93, indicating more growth on X relative to peers compared to growth on Y relative to peers. The presence of this negative may not be concerning to the use of CCR as a sole measure of co-development since B is the only one to have negative growth in Y and it is small which may be a product of measurement error and thus the negative direction may not be critical to interpretation (i.e., no real difference between negative and low growth), but this should be evaluated based on measure and expectation. However, the presence of C and E, having negative growth on Y but positive growth on X have ACMs greater than 90 degrees indicating this direction on both. The CCR is above 1 for these, indicating less positive growth in X relative to peer as compared to their growth on Y relative to peers. If looking at only the CCR, the fact that X change is negative is not specifically accounted for. Vector D has an ACM of 45 degrees, indicating positive growth on X and Y that is equal with respect to the variability at the first timepoint (which happens to be the same for X1 and Y1 in this case). However, what the ACM doesn't take into account is the fact that average growth on Y is much larger than that on X. So although D grew .6 SD on each measure, the average growth on Y was 3.1 SD. This relative difference in comparison to growth of peers is represented in the CCR which is .86 for D, indicating less growth on X relative to peers as compared to growth on Y relative to peers.

Below we describe how to calculate and interpret the change ratio and angle metric.

Tutorial: Angle and change ratio for exploring co-development

This tutorial describes how to calculate 2 metrics (angle and ratio) for exploring co-development between two measures of interest

d = dataset

X1 = X variable at time 1

X2 = X variable at time 2

Y1 = Y variable at time 1

Y2 = Y variable at time 2

Co-development change ratio

Co-development change ratio, CCR, represents the comparative growth between two skills, quantifying the amount of growth relative to peers in one skill as compared to the amount of growth relative to peers in another skill. The change ratio signifies how well there is a match (symmetry) between the amount of growth in the context of what is typical for peers on that variable compared to how much they grew on another variable in relation to their peers. For example, a student who grew the average amount on both X and Y would have symmetric growth (a change ratio of 1). So too would a student whose amount of growth was 1SD below the mean on both X and Y have symmetric growth because they had the same position compared to their peers growth on each measure. Asymmetric growth occurs when amount of growth relative to peers on one variable is larger than the other variable. The amount of asymmetry is represented by how far away the CCR is from 1. A value of one indicates symmetry, i.e., X and Y grew at the same rate. Values larger than 1 indicate growth on X being smaller than that of Y, with increasingly larger values deviating from 1 indicating greater growth in Y than X. Conversely, smaller values below 1 signify an increasingly discrepant growth in X being larger relative to Y. As a whole, an increase in CCR represents an increase in the comparative growth position on X relative to that of Y. That is, as the CCR increases, growth on X becomes increasingly larger in its relation to Y growth (X growth in comparison to Y growth is increasing). Increases in the CCR can be thought of in terms of either an increase in X growth for a constant Y growth or a decrease in Y growth for a given X growth.

How to calculate CCR

Step 1: Calculate change. Calculate a difference score by subtracting the first timepoint from the second. An excel file can be found in the supplemental materials to follow along with these steps.

$\Delta X = X2 - X1$

$\Delta Y = Y2 - Y1$

R code:

```
d$Xdiff = d$X2-d$X1

d$Ydiff = d$Y2-d$Y1
```

Step 2: Calculate scaled change (z-score of change + 10). Scaled scores are used so that change in X and Y are on the same scale, removing differences due to raw score scale. A mean of 10 is used here rather than zero so that there are no negative scores, which would impact the results when division is employed.

$z\Delta X = \frac{\Delta X - \overline{\Delta X}}{\sigma_{\Delta X}} + 10$

$z\Delta Y = \frac{\Delta Y - \overline{\Delta Y}}{\sigma_{\Delta Y}} + 10$

R code:

```
d$Xzdiff = scale(d$Xdiff)+10
d$Yzdiff = scale(d$Ydiff)+10
```

Step 2a: If average change is negative for one of the variables. If average growth is negative, the z-score should first be inverted.

If $\boldsymbol{\Delta}$X < 0:

$z\boldsymbol{\Delta}X = -\frac{\boldsymbol{\Delta X - \overline{\Delta X}}}{\boldsymbol{\sigma_{\Delta X}}} + 10$

If $\boldsymbol{\Delta}$Y < 0:

$z\boldsymbol{\Delta}Y = -\frac{\boldsymbol{\Delta Y - \overline{\Delta Y}}}{\boldsymbol{\sigma_{\Delta Y}}} + 10$

R code:

```
if(mean(d$Xdiff)<0){d$Xzdiff = (scale(d$Xdiff)/-1)+10}
if(mean(d$Ydiff)<0){d$Yzdiff = (scale(d$Ydiff)/-1)+10}
```

Step 3: Calculate change ratio. Divide the scaled change score of Y by the scaled change score of X.

$CRM = \frac{z\Delta Y}{z\Delta X}$

R code:

```
d$changeratio = d$Yzdiff/d$Xzdiff
```

How to interpret change ratio values

The CCR represents the amount of change in Y relative to peers compared to the amount of change in X relative to peers. A CCR of 1 represents individuals who grew the same pace in X and Y relative to peers. Where a score above 1 represents an individual who grew more in Y relative to peers compared to X growth relative to peers. Scores below 1 represents greater growth in X relative to peers as compared to their growth in Y relative to that of their peers.

Limitations of change ratio

One limitation of the CCR is that it cannot distinguish positive from negative growth. An individual who grows negatively just has "less growth" than a slight positive, with no real distinction for positive vs negative growth.

Angle of Co-development Metric

Angle of Co-development Metric, ACM, represents amount and direction of growth on X as compared to the amount and direction of growth on Y. ACM is represented in degrees ranging from -180 to 180 representing the angle between the vector drawn representing growth on each variable X and Y and the right side of the x-axis. Rather than accounting for how much growth in a given time is typical among peers like the CCR, the ACM represents amount of growth as it relates to the amount of variation in the sample at the first timepoint. That is, growth is represented by the change in the number of standard deviation units based on the sample at the first timepoint (developmental scale score difference). A developmental scale score allows for comparisons of scores across time by representing the score in z-score units of the first timepoint. For example, let's say that the mean of X at time 1 was 100 and the SD was 10. A score of 110 would have a developmental scale score of 1 at both time 1 and 2 (regardless of the mean and standard deviation at time 2). If an individual had a score of 90 at T1 and 120 at T2, there developmental scale scores would be -1 and 2 for T1 and T2 respectively. Therefore, the difference between their developmental scale scores would be 3, meaning that this individual grew 3 T1 standard deviation units between T1 and T2. If this individual also grew 3 SD units on Y, we would see this represented by a 45 degree angle, showing the same amount (as represented by SD units) of positive growth on both X and Y. Unlike the CCR, ACM accounts for the possibility that growth may occur in different directions such as one student may increase on X whether the other decreases.

How to calculate ACM

Step 1: calculate developmental scale scores. Developmental scale scores are scaled based on the first time point such that 0 represents mean performance at the first time point and 1 represents a score 1SD above the mean at the first time point. Note that the formula uses the mean and standard deviation of the first timepoint for both timepoints. An excel file can be found in the supplemental materials to follow along with these steps.

$$\boldsymbol{X1_{dev}} = \frac{\boldsymbol{X1-\overline{X1}}}{\boldsymbol{\sigma_{X1}}}$$

$$\boldsymbol{X2_{dev}} = \frac{\boldsymbol{X2-\overline{X1}}}{\boldsymbol{\sigma_{X1}}}$$

$$\boldsymbol{Y1_{dev}} = \frac{\boldsymbol{Y1-\overline{Y1}}}{\boldsymbol{\sigma_{Y1}}}$$

$$\boldsymbol{Y2_{dev}} = \frac{\boldsymbol{Y2-\overline{Y1}}}{\boldsymbol{\sigma_{Y1}}}$$

R code:

```
d$X1_dev = (d$X1 - mean(d$X1))/sd(d$X1)

d$X2_dev = (d$X2 - mean(d$X1))/sd(d$X1) #note that you are using mean and sd from the first
```

timepoint

```
d$Y1_dev = (d$Y1 - mean(d$Y1))/sd(d$Y1)

d$Y2_dev = (d$Y2 - mean(d$Y1))/sd(d$Y1)
```

Step 2: calculate difference between developmental scale scores. Subtract the developmental scale scores to get a change from time 1 to time 2.

$\Delta X_{dev} = X2_{dev} - X1_{dev}$

$\Delta Y_{dev} = Y2_{dev} - Y1_{dev}$

R code:

```
d$Xdevdiff = d$X2_dev - d$X1_dev

d$Ydevdiff = d$Y2_dev - d$Y1_dev
```

Step 3: calculate angle. Take the arctan of the change in Y developmental scale score and change in X developmental scale score to find the angle. Depending on the program used, the resulting angle may be in radians or degrees, if represented in radians then multiply by 180/ $\boldsymbol{\pi}$ to get the angle in degrees. Note that differences may also occur in whether the angle is represented in -180 to 180 or 0 to 360, which can also be converted by subtracting 360.

ACM = $arctan(\frac{\Delta Y_{dev}}{\Delta X_{dev}}) * \frac{180}{\pi}$

R code:

```
d$angle = atan2(d$Ydevdiff, d$Xdevdiff)*180/pi
```

How to interpret ACM

0 = no growth on Y

90 = no growth on X

45 = grew the same number of standard deviation units on X and Y

-90<angle<0 = decrease on Y, increase on X

0<angle<45 = increase on X and Y, increase more on X than Y

45<angle<90 = increase on X and Y, increase more on Y than X

90<angle<180 = increase on Y, decrease on X

-180<angle<-90 = decrease on X and Y

Note: an angle of 45 is not the same as a change ratio of 1 because average change can differ between X and Y. For example, student A has changed 1SD on both X and Y, yielding a 45 degree angle, however, 1SD was average change for X but average change on Y was 3SD so student A grew the average amount for X but below average for Y, leading to a change ratio of <1.

Limitations of ACM

A 45 degree angle represents growing the same number of standard deviation units of the first timepoint in both X and Y. However, it does not account for what is typical growth for that variable. For example, if on average people grew 2SD on X but 5SD on Y average growth on each would manifest as an angle between 45 and 90 degrees suggesting more growth on Y than X, not accounting for average growth like in the change ratio.

Additional considerations should be taken when aggregating angles across participants. For example, when calculating the average angle, if the closest distance between an individual angle and the mean crosses the -180/180 threshold, taking a simple average will be biased. For example, if one angle is 175 and one is -165, the average should be -175 however, when simply taking the average of the numbers 175 and -165 you would arrive at 5, in a totally different quadrant and interpretation. It is important to examine whether any data cross the -180/180 bound prior to using ACM as a predictor or reporting aggregate data.

Application example of using a change ratio as a predictor

In a recent study, Edwards (2022) used a measure of the change ratio to explore whether individual differences in the relative growth between word and nonword reading contributes to individual differences in reading fluency. Among students demonstrating the same proficiency in accuracy, wide ranges in speed of word reading are observed. Yet, little is known about what contributes to individual differences in reading fluency beyond accuracy. Edwards (2022) investigated whether asymmetric growth in word and nonword reading contributes to individual differences in word reading speed. Here we will walk you through how Edwards (2022) used the change ratio and rigorously tested its impact. Untimed word reading accuracy was measured with word identification (WID), untimed nonword reading accuracy was measured with word attack (WA), and time word reading efficiency was measured with sight word efficiency (SWE). WID is included in the model because untimed word reading ability will predict timed word reading ability in representing word reading accuracy and allow for an investigation of rate after already controlling for word reading accuracy ability. At a given WID score, there is a range of possible SWE scores denoting differences in reading rate since all students have the same untimed accuracy. These differences in SWE with the same WID score is of particular interest here, meaning that something contributes to individual differences in efficiency above simply the ability to read the words accurately.

$$4th\ grade\ SWE = \beta_0 + \beta_1 \frac{scaled\ change\ in\ WID\ from\ 1st\ to\ 4th\ grade}{scaled\ change\ in\ WA\ from\ 1st\ to\ 4th\ grade} + \beta_2\ 4th\ grade\ WID + \varepsilon$$

Results showed that the relative growth in untimed word reading accuracy to untimed nonword reading accuracy from 1st to 4th grade contributed significantly to the prediction of 4th grade word reading efficiency after controlling for word reading accuracy. Specifically, those who grew more in untimed word reading accuracy than untimed nonword reading accuracy from 1st to 4th grade were slower readers compared to others with the same current accuracy levels.

In order to test whether one of the change measures is driving the relationship, two regressions were used to determine whether the change ratio still contributes significantly even in the presence of one of the change measures. A multiple regression with 4th grade WID, the change ratio, and WID change predicting 4th grade SWE will show whether WID change is what is driving the relationship between the change ratio and SWE. If the change ratio is no longer significant in the presence of WID change then the amount of WID change is the important piece in the ratio. The same will be done for WA. If the change ratio is still significant in both cases, this means that it is the relative growth that is important, as opposed to amount of change in either. This is important because it would show that it is not purely the amount of growth that is predictive but rather the growth in WID relative to the growth in WA.

$$\begin{aligned} &4th\ grade\ SWE \\ &= \beta_0 + \beta_1 \frac{scaled\ change\ in\ WID\ from\ 1st\ to\ 4th\ grade}{scaled\ change\ in\ WA\ from\ 1st\ to\ 4th\ grade} \\ &+ \ \beta_2\ 4th\ grade\ WID + \beta_3\ change\ in\ WID\ from\ 1st\ to\ 4th\ grade + \ \varepsilon \end{aligned}$$

$$\begin{aligned} &4th\ grade\ SWE \\ &= \beta_0 + \beta_1 \frac{scaled\ change\ in\ WID\ from\ 1st\ to\ 4th\ grade}{scaled\ change\ in\ WA\ from\ 1st\ to\ 4th\ grade} \\ &+ \ \beta_2\ 4th\ grade\ WID + \beta_3\ change\ in\ WA\ from\ 1st\ to\ 4th\ grade + \ \varepsilon \end{aligned}$$

These models were conducted to evaluate whether change in one of the measures alone can account for the prediction of word reading efficiency by the change ratio, however, results showed the change ratio was still significant in the presence of both change in untimed word reading and change in untimed nonword reading alone. This means that it is in fact the *relative* growth that is important, as opposed to amount of change in either. This is important because it showed that it is not purely the amount of growth that is predictive but rather the growth in untimed word reading relative to the growth in untimed nonword reading.

Furthermore, the robustness of the unique contribution of the change ratio was further explored by including 4th grade WA and 1st grade SWE (autoregressive effect) to the model with 4th grade WID and the change ratio predicting 4th grade SWE. This will explore whether the asymmetry in growth provides more to the prediction than just knowing the end score of WID and WA and having prior information about speeded word reading. This would suggest that the relative growth is what is driving this relation rather than simply skills in either WID or WA or inherent unchanging speed differences (i.e., autoregressive effect).

$$4th\ grade\ SWE = \beta_0 + \beta_1 \frac{scaled\ change\ in\ WID\ from\ 1st\ to\ 4th\ grade}{scaled\ change\ in\ WA\ from\ 1st\ to\ 4th\ grade} + \beta_2\ 4th\ grade\ WID + \beta_3\ 4th\ grade\ WA + \beta_4\ 1st\ grade\ SWE + \varepsilon$$

Results showed the change ratio to provide unique information to the prediction suggesting that the relative growth is what is impactful here rather than simply current skills in untimed word or nonword reading, or inherent unchanging speed differences (i.e., autoregressive effect).

The calculation of the change ratio was critical to the estimation of symmetry in co-development at the individual level, allowing for an exploration of how individuals may differ in their relative growth on two skills. Calculating this metric at the individual level allowed for it to be used in analyses to predict other variables. Here, Edwards found asymmetry in growth in word and nonword reading to be related to reading speed. This exemplifies the utility of such a metric at the individual level as a potential indicator/predictor as well as calls for considering these individual differences in co-development research.

Conclusions

Individual differences in co-development may be an important factor of consideration in reading research and likely other fields as well. Yet to date, little work has explored these individual differences or how these differences could even be quantified for use in analyses. We propose the co-development change ratio and angle of co-development metric as two possible metrics for quantifying co-development at the individual level and when used together, can provide an understanding of the developmental symmetry between two skills. Each metric provides values that are easily interpreted and can provide a basis for understanding individual differences in co-development.

Prior to the development of these metrics, there was no way to quantify individual differences in co-development. Steacy et al. (2021) added a color gradient representing initial phonological awareness skill to a vector plot, visually inspecting the plot for the relation between phonological awareness and growth symmetry. They showed an association between initial phonological awareness skill and vector slopes (relative growth in word and nonword reading) based on visual inspection, however, with no metrics to quantify these vector slopes in a meaningful way (particularly in the presence of negative slopes and differing scales) at the individual level, Steacy et al. (2021) were unable to quantify the magnitude of this association. Applying the calculation of CCR presented here to the data used in the vector plot for Steacy et al. (2021), we are able to quantify that relationship with a correlation of .55 between pretest phonological awareness skill and the change ratio. Future research should explore this relationship further. CCR is the more appropriate choice for the particular research questions and presenting data in Steacy et al. however, we also calculated ACM for illustration purposes here and the correlation between ACM and pretest phonological awareness skill was .53. Both the CCR and ACM showed that individuals who started with low phonological awareness skill were more likely to have lower nonword reading growth in comparison to their word reading growth. This is consistent with the visual inspection by Steacy and colleagues but now we can quantify the magnitude of this association.

These metrics provide a means for quantifying individual differences in co-development. However, future research is needed to determine how these metrics may be impacted by issues such as non-normality, reliability, and sample size. Future research should simulate the conditions which may impact the calculation and interpretation of these metrics to better inform under which conditions they are appropriate to use.

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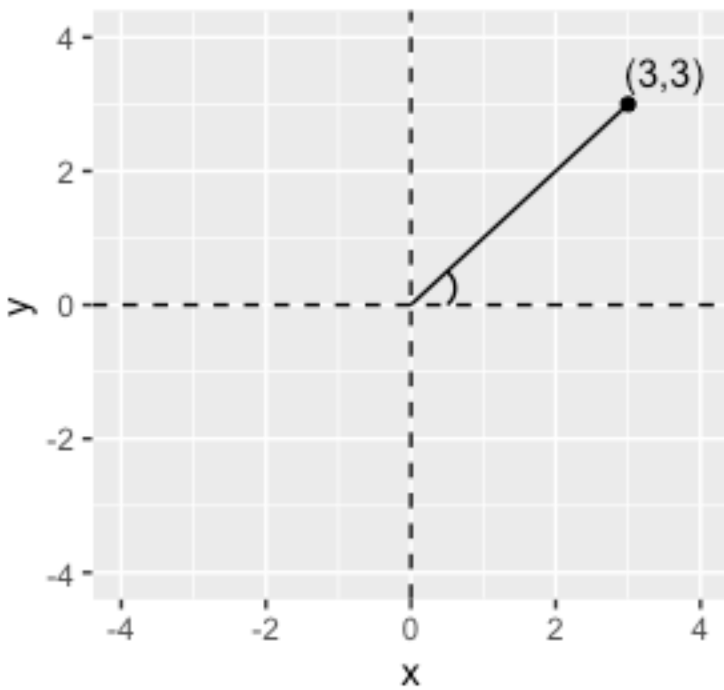
Figure 1

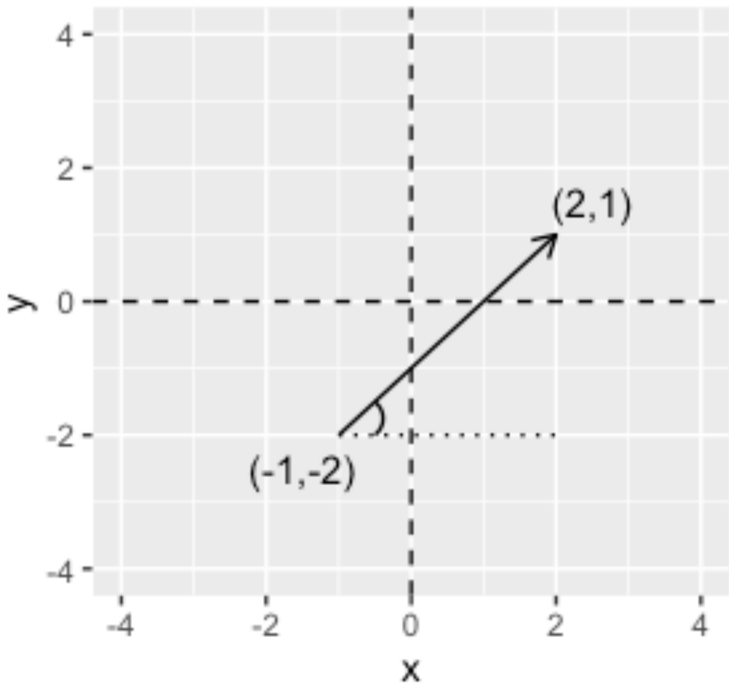
Angle from right side of x-axis to line drawn from the origin to (change on X, change on Y)



Note: Angle is drawn from the right side of the x-axis to a line drawn from the origin to (change on X, change on Y)

Figure 2

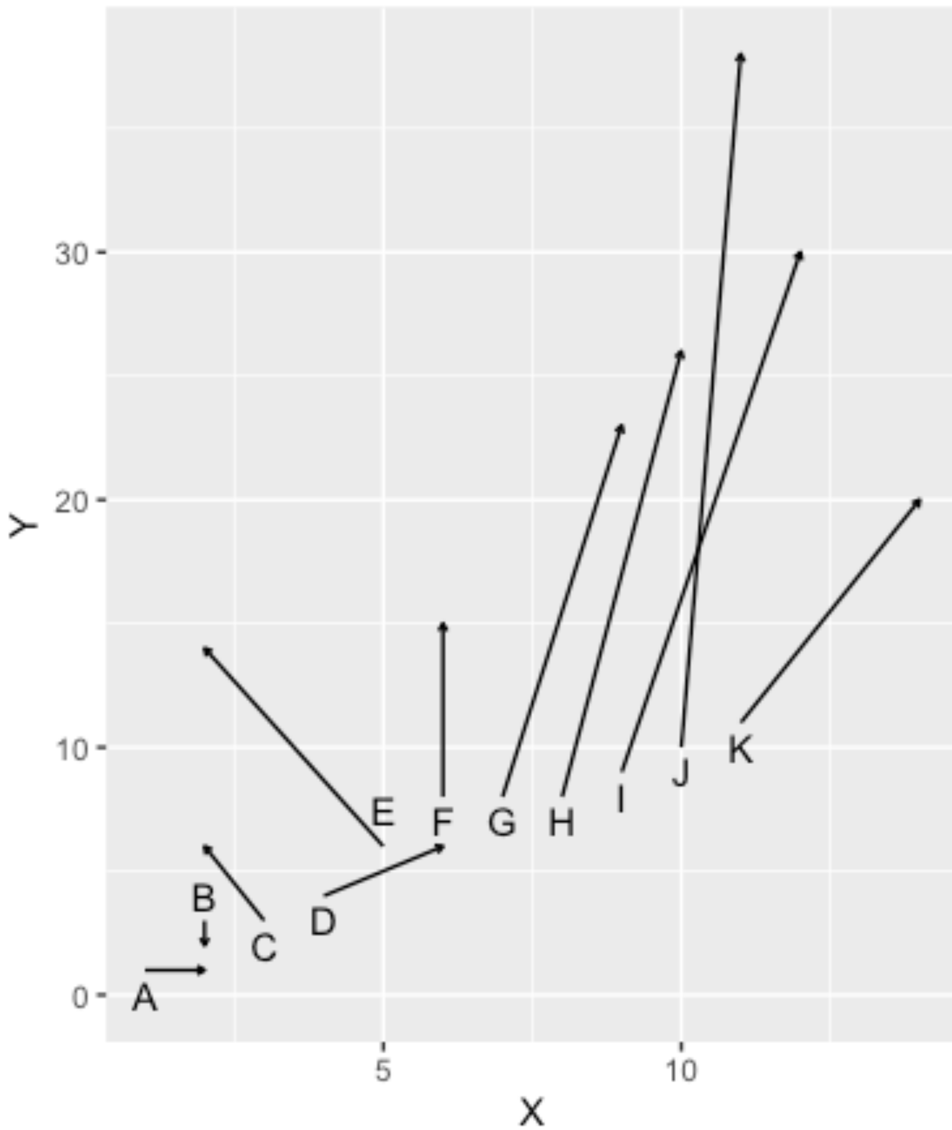
Angle drawn on a typical vector plot



Note: Angle drawn between the vector drawn from the initial starting position (z-scored) on each variable X and Y to the endpoint on each represented as a developmental scale score (scaled based on the first timepoint) and a horizontal vector representing growth only on X (represented by dotted line here).

Figure 3

Example vector plot



Note: The data represented here correspond to the data in Table 1.

Table 1

Data and calculated metrics corresponding to the example in Figure 3

Letter	CCR	ACM	X1	X2	Y1	Y2
A	0.89	0.00	1	2	1	1
B	0.93	-90.00	2	2	2	1
C	1.02	116.57	3	2	3	5
D	0.86	45.00	4	6	4	6
E	1.25	110.56	5	2	5	13
F	1.04	90.00	6	6	6	15
G	1.00	82.87	7	9	7	23
H	1.02	83.66	8	10	8	26
I	1.00	81.87	9	12	9	30
J	1.18	87.95	10	11	10	38
K	0.89	71.57	11	14	11	20