SETS REFERENCE
CSC236 — 2024 FALL

A **set** is an unordered collection of distinct **elements** / **members**.
When a set's elements are themselves sets, the set is sometimes called a **family** / **collection of sets**.
Two common ways of defining a set are by **enumeration** and by **set comprehension**.
For a set with a small number of elements, an enumeration simply lists the elements: $\{e_0, e_1, \ldots\}$.
The **empty set** $\{\}$, which has no elements, is also written as "ϕ".
A **singleton** set is a set containing exactly one element.
For larger sets, including some infinite ones, "..." can be used to indicate elements following an obvious pattern.
A set comprehension has one of the following forms, and in each case the ":" can also be written as "|":

- $\{x : P(x)\}$
  - this introduces a variable name to refer to individual elements of the sets, and a condition / property specifying the elements in terms of that variable
  - it's read as "the set of x such that / with / where / for which / satisfying / with the property that $P(x)$"
- $\{x \in D : P(x)\}$
  - this includes a **domain** set that further restricts the elements
  - it's equivalent to $\{x : x \in D \text{ and } P(x)\}$, and read as "the set of x in D such that [...]"
- $\{f(x_0, \ldots, x_n) : P(x_0, \ldots, x_n)\}$
  - this specifies the elements as a function of some introduced variables, with a condition / property specifying and possibly relating the values of those variables

The **size** of a set S, written symbolically as "$|S|$", is the number of elements in S, which is a natural number or infinity.
Writing "$|S| < \infty$" means S is finite, and writing "$|S| = \infty$" means S is infinite.
To symbolically indicate that x is an element of S we write "$x \in S$", which is read as "x is an element / member of S" or "S contains x [as an element]", or "x belongs to / occurs in / is in S".
A set A is a **subset** of a set B, written symbolically as "$A \subseteq B$" or "$B \supseteq A$", iff every element in A is also in B. In this case B is also called a **superset** of A.
"$A \subseteq B$" is also read as "B contains A [as a subset]". Notice that just "s contains t" can be ambiguous, since it could mean $t \in s$ or it could mean $t \subseteq s$, so it should only be used when the context makes the interpretation clear.
A set A is a **proper** / **strict subset** of a set B , written symbolically as "$A \subsetneq B$" or "$B \supsetneq A$", iff A is a subset of B and $A \neq B$. In this case B is also called a **proper** / **strict superset** of A. "$A \subsetneq B$" is also read as "B properly / strictly contains A" or "B contains A as a proper / strict subset".
The notation $A \subset B$ is sometimes used to mean $\subseteq$ but sometimes used to mean $\subsetneq$, depending on context — in CSC236 we'll try to avoid it.
The **union** of two sets A and B, written symbolically as "$A \cup B$", is $\{x : x \in A \text{ or } x \in B\}$.
The **intersection** of two sets A and B, written symbolically as "$A \cap B$", is $\{x : x \in A \text{ and } x \in B\}$.
Since $(A \cup B) \cup C = A \cup (B \cup C)$, and similarly for intersection, we can write $A \cup B \cup C$ to mean either, and similarly for intersection. This generalizes to any number of sets.
For a set of sets S, the **union** of S, written symbolically as "$\bigcup S$", is $\{x : x \in s \text{ for some } s \in S\}$. For a set I (an "index" set) and sets s_i for each $i \in I$, the **indexed union** $\bigcup_{i \in I} s_i$ is $\bigcup \{s_i : i \in I\}$; if I is a finite set $\{i_0, i_1, \ldots, i_n\}$ then this is equal to $s_0 \cup s_1 \cup \cdots \cup s_n$, and if I is an infinite set $\{i_0, i_1, \ldots\}$ then this is also written as $s_{i_0} \cup s_{i_1} \cup \cdots$. The notation $\bigcup_{i=a}^{b} s_i$ for integers $a \leq b-1$ is also considered an indexed union and is defined as $\cup \{s_i : i \in \mathbb{Z} \wedge a \leq i \leq b-1\}$.
For a *non-empty* set of sets S, the **intersection of** S, written symbolically as "$\bigcap S$", is $\{x : x \in s \text{ for every } s \in S\}$.
There are indexed versions of intersection analogous to union, except that the set of indices must be non-empty.
Two sets A and B are **disjoint** iff their intersection is [the] empty [set].
The **set difference** of two sets A and B, written symbolically as "$A \setminus B$", is $\{x \in A : x \notin B\}$. This is also called the "complement of B in A". Sometimes a default "domain" or "universe" set A is specified first and then for $B \subseteq A$ the complement of B in A is just called the "complement of B" and written symbolically as "B^c".
For a natural number n, an n**-tuple**, or just **tuple**, is an ordered sequence of n elements, written symbolically as "$(x_1, \ldots, x_n)$". A 2-tuple is also called an "[ordered] pair", a 3-tuple is also called a "triple", etc.
For a natural number n, the **Cartesian product** of sets A_1, ..., A_n, written symbolically as "$A_1 \times \cdots \times A_n$" is $\{(a_1, \ldots, a_n) : \text{each } a_i \in A_i\}$. There is an indexed version $\bigtimes_{i=a}^{b} A_i$ analogous to indexed union and intersection.
The **power set** of a set A, written symbolically as "$\mathcal{P}(A)$", is $\{s : s \subseteq A\}$.
A set of non-empty sets P is a **partition** of a set A iff $A = \cup P$ and every pair of distinct sets in P is disjoint (which is also phrased as the sets in P being "pairwise / mutually disjoint").