Vanderbilt University
Leadership, Policy and Organizations
Class Number 9952
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Model Specification

We'll be working today with the wage2 dataset, which includes monthly wages of male earners along with a variety of characteristics. We'll be attempting to esimtate some fairly standard wage models, but we'll also try to answer the most vexing question for many students: what variables should I put in my model?

The most important answer to that question is to use theory. Theory and previous results are our only guide—the data simply can't tell you by themselves what belongs in the model and what doesn't. However, we can use a combination of theory and applied data analysis to come up with a model that fits the data well and says something interesting about theory.

Missing Data

Before we start with all that, let's talk again about how Stata handles missing data. Let's assume that we want to estimate several nested models, first with hours, education and age, then the same model with mother's education, then the same model with father's education, then a final model with all variables. Our results look like this:

```
.
. reg lwage hours educ age

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  3,   931) =   46.91
       Model |  21.7514568     3  7.25048559           Prob > F      =  0.0000
    Residual |  143.904838   931   .15457018           R-squared     =  0.1313
-------------+------------------------------           Adj R-squared =  0.1285
       Total |  165.656294   934  .177362199           Root MSE      =  .39315

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0047011   .0017887    -2.63   0.009    -.0082115   -.0011906
        educ |   .0616404   .0058814    10.48   0.000     .0500981    .0731827
         age |   .0227339   .0041411     5.49   0.000     .0146069    .0308608
       _cons |   5.403279   .1732026    31.20   0.000     5.063366    5.743191
------------------------------------------------------------------------------

.
. reg lwage hours educ age meduc

      Source |       SS       df       MS              Number of obs =     857
-------------+------------------------------           F(  4,   852) =   38.47
       Model |  22.8514162     4  5.71285406           Prob > F      =  0.0000
    Residual |  126.509635   852  .148485487           R-squared     =  0.1530
-------------+------------------------------           Adj R-squared =  0.1490
       Total |  149.361051   856  .174487209           Root MSE      =  .38534

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
```

```
-------------+----------------------------------------------------------------
       hours |  -.0058052   .0018374    -3.16   0.002    -.0094115   -.0021989
        educ |   .0525597   .0064521     8.15   0.000     .0398957    .0652236
         age |   .0243798   .0042747     5.70   0.000     .0159896      .03277
       meduc |   .0184424   .0049725     3.71   0.000     .0086826    .0282022
       _cons |    5.33402   .1776844    30.02   0.000      4.98527    5.682771
------------------------------------------------------------------------------

.
. reg lwage hours educ age feduc

      Source |       SS       df       MS              Number of obs =     741
-------------+------------------------------           F(  4,   736) =   34.17
       Model |  20.2139719     4  5.05349299           Prob > F      =  0.0000
    Residual |  108.836202   736  .147875274           R-squared     =  0.1566
-------------+------------------------------           Adj R-squared =  0.1521
       Total |  129.050173   740  .174392126           Root MSE      =  .38455

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |   -.007041   .0019639    -3.59   0.000    -.0108964   -.0031855
        educ |   .0475258   .0070026     6.79   0.000     .0337785    .0612732
         age |   .0262759   .0045806     5.74   0.000     .0172834    .0352685
       feduc |   .0172076   .0047569     3.62   0.000     .0078689    .0265462
       _cons |   5.421121   .1897058    28.58   0.000     5.048692     5.79355
------------------------------------------------------------------------------

.
. reg lwage hours educ age feduc meduc

      Source |       SS       df       MS              Number of obs =     722
-------------+------------------------------           F(  5,   716) =   27.64
       Model |   20.519095     5  4.10381899           Prob > F      =  0.0000
    Residual |  106.292836   716  .148453682           R-squared     =  0.1618
-------------+------------------------------           Adj R-squared =  0.1560
       Total |  126.811931   721    .1758834           Root MSE      =   .3853

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0071408   .0019821    -3.60   0.000    -.0110321   -.0032495
        educ |    .046006   .0072322     6.36   0.000     .0318071    .0602049
         age |   .0249456   .0046705     5.34   0.000     .0157762     .034115
       feduc |   .0114239   .0055571     2.06   0.040     .0005138     .022334
       meduc |   .0132414   .0063094     2.10   0.036     .0008543    .0256285
       _cons |   5.404781   .1929204    28.02   0.000     5.026024    5.783539
------------------------------------------------------------------------------
```

The results are extermely problematic because each set of results is on a different sample! The first set has 857 observations, the second 741, and down to 722 for the final one. Stata performs casewise deletion when running regressions, and doesn't adjust unless you tell it to. In this case none of the standard tests of model fit are relevant, because it's not the same sample.

The solution is to use the `e(sample)` command to limit the sample to the relevant analysis sample. First, run the model that restricts the data the most (has the most missing data), then limit subsequent models using the statement `if e(sample)==1`.

The natural log transformation

The variable `lwage` is the natural log of wages. This means that it has been transformed by taking the natural log of the underlying variable:

$$log_e(y_i) = x \equiv e^x = y_i$$

Where e is Euler's constant, $e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{1} + \frac{1}{1\times2} + \frac{1}{1\times2\times3} \dots$

The log transformation is used all the time, and particularly in econometrics. It's useful whenever you have a variable that follows some kind of exponential distribution, with widely disparate levels. Earnings, school sizes, revenues of institititions of higher education and state populations are all examples of these kinds of situations.

When the dependent variable is log transformed but the independent variable is not, this is called a log-level regression. In a log-level regression,

$$log(y_i) = \beta_o + \beta_1 x_i + \epsilon_i$$

Which implies that

$$y_i = e^{\beta_o + \beta_1 x_i + \epsilon_i}$$

And . . .

$$\frac{dy}{dx} = \beta e^{\beta_0 + \beta_1 x_1 + \epsilon} = \beta_1 y$$

Which means that the coefficent, β_1

$$\beta_1 = \frac{dy}{dx}\frac{1}{y}$$

This changes our interpretation to mean that for a one unit increase in x, y is predicted to increase by β_1 proportion of y or more commonly by $100 * \beta_1$ percent. It changes the scale of the dependent variable to be on the $1/y$ scale as opposed to the y scale, so everything is about a proportional (or percentage) increase in y.

Quick Exercise

Interpret the coefficients from the basic earnings regression of log wages on years of education.

Selecting Variables: Stepwise Regression OR A Cautionary Tale of Woe

When selecting variables for a model, students are sometimes tempted by the dark side of stepwise regression, which is a step on the path toward the greater evil that is data mining. I will illustrate why this is a bad idea. The basic idea with stepwise regression is to eliminate variables from the model one at a time—if the variable is not significant, it gets dropped. However, this method is very sensitive to the overall group of variables used, essentially just pushing decisions one step back, and then using an arbirtray non-theoertical standard for variable inclusion. There is no good theoretical reason to use this procedure.

Selecting Variables: RESET test

One question that comes up frequently is whether one or more variables ought to be expressed as quadratic or higher-order polynomials in the equation. The RESET test can help with this problem. Specifying the RESET test without any options means that Stata will fit the model with the second, third and fourth powers of $\widehat{y}$. Specifying the option `rhs` will use powers of the individual regressors.

In Stata, we would run:

```
.
. reg lwage hours age educ

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  3,   931) =   46.91
       Model |  21.7514568     3  7.25048559           Prob > F      =  0.0000
    Residual |  143.904838   931   .15457018           R-squared     =  0.1313
-------------+------------------------------           Adj R-squared =  0.1285
       Total |  165.656294   934  .177362199           Root MSE      =  .39315

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0047011   .0017887    -2.63   0.009    -.0082115   -.0011906
         age |   .0227339   .0041411     5.49   0.000     .0146069    .0308608
        educ |   .0616404   .0058814    10.48   0.000     .0500981    .0731827
       _cons |   5.403279   .1732026    31.20   0.000     5.063366    5.743191
------------------------------------------------------------------------------

.
. estat ovtest

Ramsey RESET test using powers of the fitted values of lwage
       Ho:  model has no omitted variables
                 F(3, 928) =      0.32
                  Prob > F =      0.8089

.
. estat ovtest, rhs

Ramsey RESET test using powers of the independent variables
       Ho:  model has no omitted variables
                 F(9, 922) =      2.12
                  Prob > F =      0.0255
```

The result of the first test is not significant, but the result of the second test is. This indicates that we might want to include some additional powers of the right hand variables. Let's begin by introducing a quadratic function of age:

```
. gen agesq=age^2

.
. label var agesq "Age squared"
```

```
.
. reg lwage hours educ age agesq

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  4,   930) =   35.15
       Model |  21.7551592     4  5.43878981           Prob > F      =  0.0000
    Residual |  143.901135   930  .154732403           R-squared     =  0.1313
-------------+------------------------------           Adj R-squared =  0.1276
       Total |  165.656294   934  .177362199           Root MSE      =  .39336

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |     -.0047   .0017897    -2.63   0.009    -.0082123   -.0011876
        educ |   .0615585   .0059083    10.42   0.000     .0499634    .0731536
         age |   .0388675   .1043805     0.37   0.710    -.1659811    .2437162
       agesq |  -.0002425   .0015678    -0.15   0.877    -.0033193    .0028343
       _cons |   5.138356   1.721371     2.99   0.003     1.760134    8.516578
------------------------------------------------------------------------------

.
. test age agesq

 ( 1)  age = 0
 ( 2)  agesq = 0

       F(  2,   930) =   15.07
            Prob > F =    0.0000
```

The two terms for age are jointly significant, but it looks like we could safely exclude age squared from the model without any loss of model fit.

Now let's try education squared:

```
. gen educsq=educ^2

.
. la var educsq "Education squared"

.
. reg lwage hours age educ educsq

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  4,   930) =   36.27
       Model |  22.3576551     4  5.58941378           Prob > F      =  0.0000
    Residual |  143.298639   930  .154084558           R-squared     =  0.1350
-------------+------------------------------           Adj R-squared =  0.1312
       Total |  165.656294   934  .177362199           Root MSE      =  .39254

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |   -.004608   .0017866    -2.58   0.010    -.0081141   -.0011018
         age |   .0243563   .0042147     5.78   0.000     .0160848    .0326278
        educ |   .2161619   .0781252     2.77   0.006     .0628397     .369484
      educsq |  -.0054912   .0027685    -1.98   0.048    -.0109245    -.000058
       _cons |   4.286926   .5887928     7.28   0.000      3.13141    5.442443
------------------------------------------------------------------------------
```

This does result in a statistically significant increase in model fit. The way I would prefer approaching this problem is to fully specify the model, then restrict it appropriately, like so:

```
.
. /*Preferred method */
```

```
.
. reg lwage hours age agesq educ educsq

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  5,   929) =   29.00
       Model |  22.3674226     5  4.47348452           Prob > F      =  0.0000
    Residual |  143.288872   929  .154239905           R-squared     =  0.1350
-------------+------------------------------           Adj R-squared =  0.1304
       Total |  165.656294   934  .177362199           Root MSE      =  .39273

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0046056   .0017875    -2.58   0.010    -.0081135   -.0010976
         age |    .050602   .1043806     0.48   0.628     -.154247     .255451
       agesq |  -.0003944   .0015672    -0.25   0.801    -.0034699    .0026812
        educ |   .2169837   .0782328     2.77   0.006     .0634502    .3705171
      educsq |  -.0055252   .0027732    -1.99   0.047    -.0109676   -.0000828
       _cons |   3.849225   1.836393     2.10   0.036     .2452658    7.453184
------------------------------------------------------------------------------

.
. test age agesq

 ( 1)  age = 0
 ( 2)  agesq = 0

       F(  2,   929) =   16.71
            Prob > F =    0.0000

.
. test educ educsq

 ( 1)  educ = 0
 ( 2)  educsq = 0

       F(  2,   929) =   56.44
            Prob > F =    0.0000
```

Selecting Variables: Non-Nested Models

In many situations, models are based on competing hypotheses, and so they don't nest within one another. Let's say we have one model that posits education as the key to wages, another that posits iq as the key to wages. To test whether one is better than the other, we use the Davidson-Mackinnon test:

```
. reg lwage hours iq

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  2,   932) =   54.14
       Model |  17.2420918     2  8.62104588           Prob > F      =  0.0000
    Residual |  148.414203   932  .159242707           R-squared     =  0.1041
-------------+------------------------------           Adj R-squared =  0.1022
       Total |  165.656294   934  .177362199           Root MSE      =  .39905

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0041302   .0018124    -2.28   0.023     -.007687   -.0005734
          iq |   .0089535   .0008698    10.29   0.000     .0072465    .0106606
       _cons |   6.053607   .1150416    52.62   0.000     5.827837    6.279378
------------------------------------------------------------------------------
```

```
.
. reg lwage hours educ

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  2,   932) =   53.62
       Model |  17.0930188     2  8.54650938           Prob > F      =  0.0000
    Residual |  148.563276   932  .159402656           R-squared     =  0.1032
-------------+------------------------------           Adj R-squared =  0.1013
       Total |  165.656294   934  .177362199           Root MSE      =  .39925

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0044454   .0018159    -2.45   0.015    -.0080091   -.0008817
        educ |   .0611697    .005972    10.24   0.000     .0494497    .0728898
       _cons |   6.150426    .108791    56.53   0.000     5.936923     6.36393
------------------------------------------------------------------------------

.
. nnest lwage hours iq (hours educ)

M1 : Y = a + Xb with X = [hours iq]
M2 : Y = a + Zg with Z = [hours educ]

J test for non-nested models

H0 : M1  t(931)      5.89476
H1 : M2  p-val       0.00000

H0 : M2  t(931)      5.97647
H1 : M1  p-val       0.00000

Cox-Pesaran test for non-nested models

H0 : M1  N(0,1)     -8.87744
H1 : M2  p-val       0.00000

H0 : M2  N(0,1)     -9.05449
H1 : M1  p-val       0.00000

.
```

The results of this test indicate that it would be better to include both of these models, in a sort of "super" model.

Interactions

Interactions can be difficult to understand, but they are key to getting a handle on sometimes important moderating variables in an analysis.

Interactions with two binary variables

Let's say we're interested in whether marriage affects wages differently for black and white men. The specification of an interaction between the two binary variables of white and married would look like this:

```
. reg lwage hours age educ i.black#i.married iq meduc south urban
```

```
      Source |       SS       df       MS              Number of obs =     857
-------------+------------------------------           F( 10,   846) =   29.83
       Model |  38.9381574    10  3.89381574           Prob > F      =  0.0000
    Residual |  110.422894   846  .130523515           R-squared     =  0.2607
-------------+------------------------------           Adj R-squared =  0.2520
       Total |  149.361051   856  .174487209           Root MSE      =  .36128

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0063301   .0017273    -3.66   0.000    -.0097204   -.0029398
         age |   .0220385   .0040456     5.45   0.000     .0140979    .0299792
        educ |   .0363765   .0068217     5.33   0.000     .0229871    .0497659
             |
      black#|
     married |
        0 1  |   .1817023   .0438419     4.14   0.000     .0956506     .267754
        1 0  |  -.2750052   .1036788    -2.65   0.008     -.478503   -.0715073
        1 1  |   .0597192   .0605138     0.99   0.324    -.0590557     .178494
             |
          iq |   .0039649   .0010438     3.80   0.000     .0019162    .0060137
       meduc |   .0104632   .0047967     2.18   0.029     .0010483    .0198781
       south |  -.0729291   .0276262    -2.64   0.008    -.1271531   -.0187051
       urban |    .179466   .0280946     6.39   0.000     .1243228    .2346092
       _cons |   5.084363   .1866763    27.24   0.000     4.717961    5.450766
------------------------------------------------------------------------------
```

Quick Exercise

Run a regression with an interaction between urban and south. Interpret the results.

Interactions with a binary and continous variable

Let's say we're interested in whether education affects wages differently for black and white men. If possible, we should start by plotting the data to see if these patterns are evident.

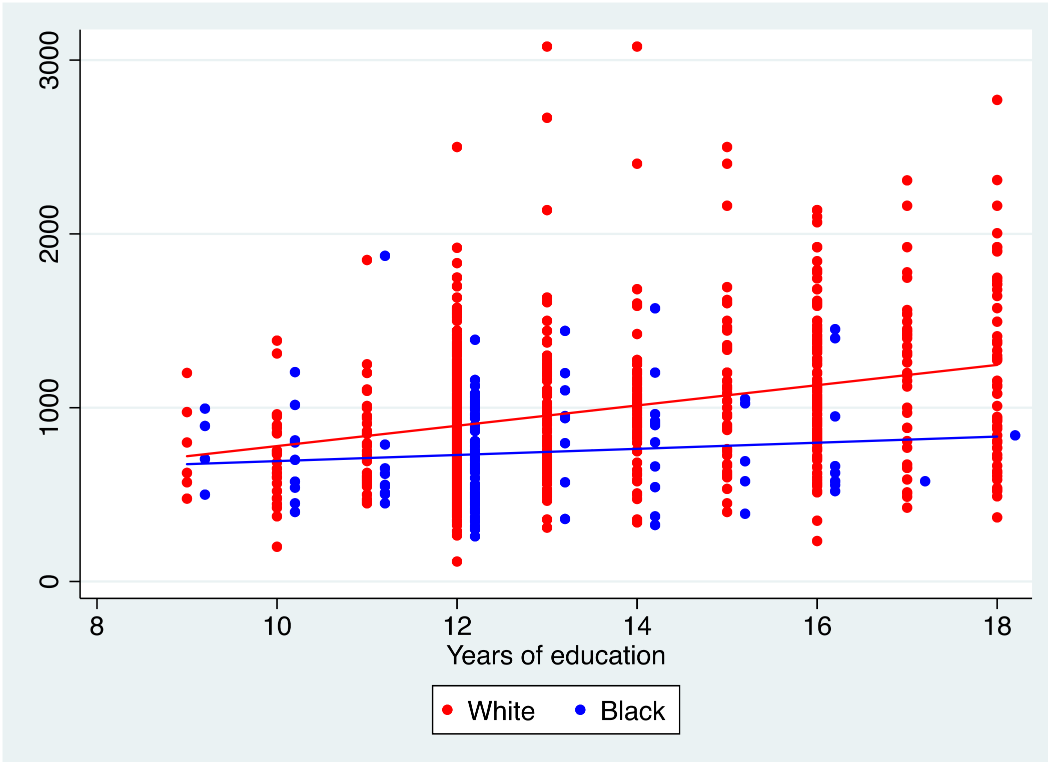
The specification of an interaction between a binary variable and a continous variable would look like this:

```
. reg lwage hours age educ black i.black#c.educ married  iq meduc south urban

      Source |       SS       df       MS              Number of obs =     857
-------------+------------------------------           F( 10,   846) =   29.69
       Model |  38.7974052    10  3.87974052           Prob > F      =  0.0000
    Residual |  110.563646   846  .130689889           R-squared     =  0.2598
-------------+------------------------------           Adj R-squared =  0.2510
       Total |  149.361051   856  .174487209           Root MSE      =  .36151

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0063045     .00173    -3.64   0.000    -.0097001    -.002909
         age |   .0215072   .0040455     5.32   0.000     .0135669    .0294476
        educ |   .0378404   .0069883     5.41   0.000     .0241239    .0515569
       black |   .1034702   .2766783     0.37   0.709    -.4395862    .6465265
             |
black#c.educ |
          1  |  -.0196117   .0215854    -0.91   0.364    -.0619789    .0227554
             |
     married |   .2051596   .0402851     5.09   0.000     .1260892      .28423
          iq |   .0040191   .0010442     3.85   0.000     .0019695    .0060687
```

Figure 1: Wages by education and race




```
       meduc |   .0102747   .0047964     2.14   0.032     .0008604    .0196889
       south |   -.069586   .0276172    -2.52   0.012    -.1237922   -.0153798
       urban |    .179133   .0281112     6.37   0.000     .1239572    .2343088
       _cons |    5.05532    .187864    26.91   0.000     4.686586    5.424055
------------------------------------------------------------------------------

.
```

Quick Exercise

Run a regression with an interaction between urban and education. Interpret the results.

Interactions with two continous variables

Finally let's say we think that eductation will affect your wages differently depending on your age. The specification of an interaction between two continous variables would look like this:

```
. reg lwage hours age educ c.age#c.educ black married iq meduc south urban

      Source |       SS       df       MS              Number of obs =     857
-------------+------------------------------           F( 10,   846) =   30.08
       Model |  39.1769054    10  3.91769054           Prob > F      =  0.0000
    Residual |  110.184146   846  .130241307           R-squared     =  0.2623
-------------+------------------------------           Adj R-squared =  0.2536
       Total |  149.361051   856  .174487209           Root MSE      =  .36089

------------------------------------------------------------------------------
```

```
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -.0066151   .0017294    -3.83   0.000    -.0100095   -.0032208
         age |  -.0280105   .0260076    -1.08   0.282    -.0790575    .0230364
        educ |  -.0876756   .0645406    -1.36   0.175    -.2143541    .0390029
             |
c.age#c.educ |   .0037019   .0019136     1.93   0.053    -.0000542    .0074579
             |
       black |  -.1466181   .0430343    -3.41   0.001    -.2310847   -.0621515
     married |   .2063299   .0402134     5.13   0.000     .1274003    .2852596
          iq |   .0040003   .0010423     3.84   0.000     .0019545    .0060461
       meduc |   .0100804   .0047844     2.11   0.035     .0006897     .019471
       south |    -.06698   .0276104    -2.43   0.015    -.121173    -.012787
       urban |    .182844     .02813     6.50   0.000     .1276312    .2380569
       _cons |   6.747921   .8848456     7.63   0.000     5.011171    8.484672
------------------------------------------------------------------------------

.
```

The do file has extensive examples of how to use margins to create nice plots of various types of interactions. We'll go over these in class.

Not-so-quick Exercise

In pairs, I would like you to estimate the best possible model using the wage2 dataset. Think about model specification and functional form, with an eye toward possible non-linearities and other issues. Generate a do file that walks through your process of identifying the best model. Generate a fancy graph that shows the predictions made by your model.