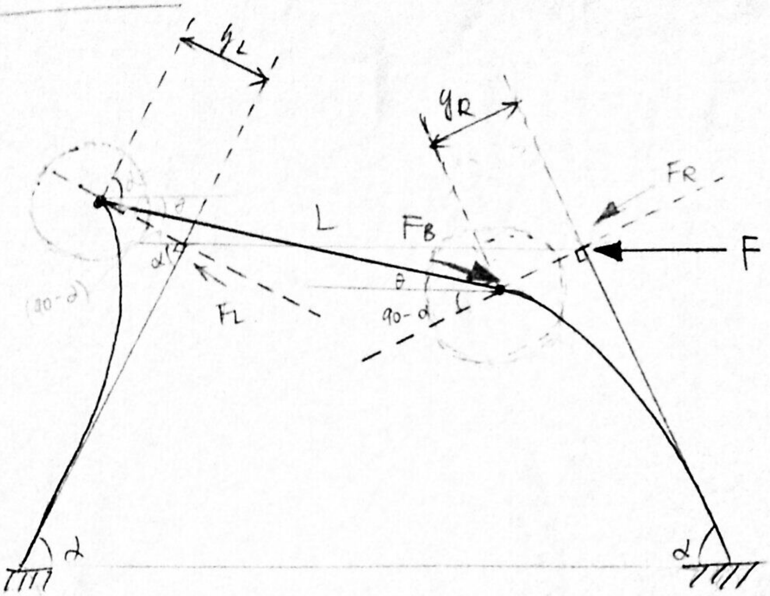
The 1^{st} Module

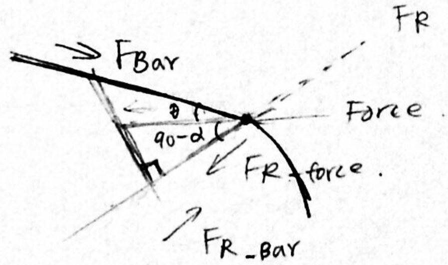
FBD :



Equation of Equlibrium :

- Force ① right hand side :

$$F_R = F \cdot \cos(90-\alpha) - F_B \cdot \cos(90-\alpha+\theta)$$



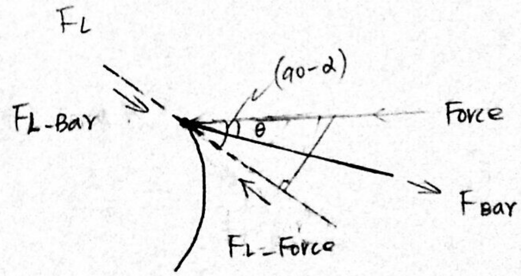
$$F_R = F \cdot \sin(\alpha) - F_B \cdot \sin(\alpha-\theta)$$

Displacement (y_R) :

$$y_R = \frac{F L^3}{3K} = \frac{F_R \cdot L^3}{3K}$$

- Force ② left hand side :

$$F_L = F_{L\text{-Force}} - F_{L\text{-Bar}}$$

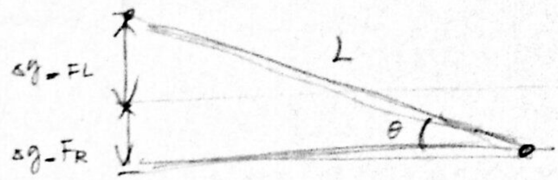


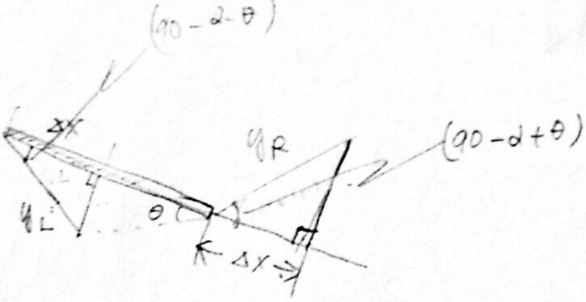
$$F_L = F \cdot \cos(90-\alpha) - F_B \cdot \cos(90-\alpha-\theta)$$

$$F_L = F \cdot \sin(\alpha) - F_B \sin(\alpha+\theta)$$

Displacement (y_L) :

$$y_L = \frac{F_L \cdot L^3}{3K}$$





$$\Delta y_{-F_L} = y_L \cdot \sin(90^\circ - \alpha) = y_R \cdot \cos\alpha$$

$$\Delta y_{-F_R} = y_R \cdot \sin(90^\circ - \alpha) = y_L \cdot \cos\alpha .$$

$$\sin\theta = \frac{\Delta y_{-F_L} + \Delta y_{-F_R}}{L}$$

$$\theta = \sin^{-1}\left[\frac{(y_R \cdot \cos\alpha + y_L \cdot \cos\alpha)}{L}\right]. \quad \text{—①}$$

$$F_R = f(F, \alpha, \overset{?}{F_B}, \overset{?}{\theta})$$

$$y_R = f(F_R) = f(F, \alpha, F_B, \theta)$$

$$\theta = f(y_R, y_L) = f(F, \alpha, F_B, \theta)$$

Force (α) =

The displacement on the bar :

$$\Delta x_{-L} = \Delta x_{-R}$$

$$y_L \cdot \cos(90-\alpha-\theta) = y_R \cdot \cos(90-\alpha+\theta)$$

$$y_L \cdot \sin(\alpha+\theta) = y_R \cdot \sin(\alpha-\theta) \quad \text{—— ②}$$

Arranging equations ① to ⑥, since we have known that external force (F), initial angle (α), and bar-length (L), But, we don't know the force acting on the right and left, which are F_R, F_L, and reacting force to the bar (F_B). Also, we don't know the angle theta showing in the figure, and displacement at both RHS and LHS (y_R and y_L)

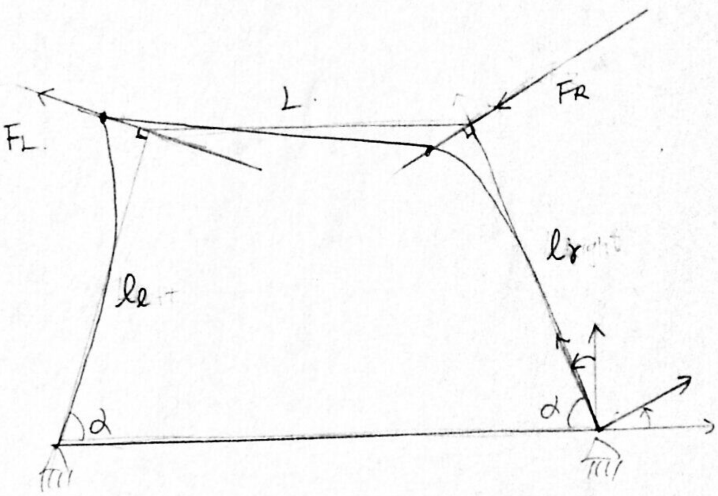
knows: F, α, L

unknows: $F_R, F_L, F_B, \theta, y_R, y_L$

$$
\begin{cases}
F_R = F \cdot \sin(\alpha) - F_B \cdot \sin(\alpha - \theta) & \text{——— ①} \\
y_R = \dfrac{F_R \cdot L^3}{3K} & \text{——— ②} \\
F_L = F \cdot \sin(\alpha) - F_B \cdot \sin(\alpha + \theta) & \text{——— ③} \\
y_L = \dfrac{F_L \cdot L^3}{3K} & \text{——— ④} \\
y_L \cdot \sin(\alpha + \theta) = y_R \cdot \sin(\alpha - \theta) & \text{——— ⑤} \\
\theta = \sin^{-1}\left[\dfrac{y_R \cdot \cos\alpha + y_L \cdot \cos\alpha}{L}\right] & \text{——— ⑥}
\end{cases}
$$

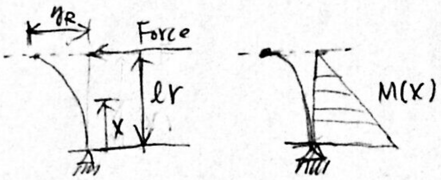
Since we have six unknows, we use six equations to solve the unknows.

Deflected Curve on Both Sides



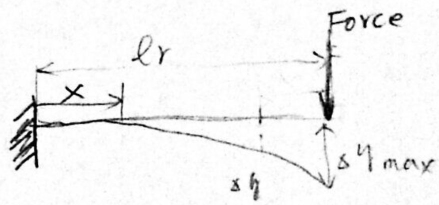
From elastic curve equation, we known that

$$\frac{d^2y}{dx^2} = \frac{M(X)}{EI}$$



$$\begin{cases} x=0, & y=0 \\ x=L, & y=y_{max} \end{cases}$$

For our cases here, on the RHS. Also, $EI = K$.



$$\frac{d^2y_R}{dx} = \frac{-F_R\cdot(l_r - x)}{K} = \frac{-F_R\cdot l_r}{K} + \frac{F_R\cdot X}{K}$$

Now we use double integration method to obtain elastic curve.

$$\frac{dy_R}{dx} = -\left(\frac{F_R\cdot l_r}{K}\right)X + \left(\frac{F_R}{2K}\right)X^2 + C_1$$

The initial B.Cs are $x=0$, $y=0$, then $0 = 0-0+C_1 \Rightarrow C_1 = 0$

The second time intergral:

$$y_R = -\left(\frac{F_R\cdot l_r}{2K}\right)x^2 + \left(\frac{F_R}{3\cdot 2K}\right)X^3 + C_2$$

Applying initial B.CS $(x=0,\ y=0)$, then $C_2 = 0$.

Therefore, we have displacement equation

$$y_R = -\left(\frac{F_R \cdot l_r}{2k}\right)x^2 + \left(\frac{F_R}{6K}\right)x^3 = \frac{F_R x^2}{6K}\left[x - 3l_r\right]. \quad 0 \le x \le L$$

Similarly, for the curvature on the LHS. we can obtain the displacement equation as well.

$$y_L = -\left(\frac{F_L \cdot l_e}{2k}\right)x^2 + \left(\frac{F_L}{6K}\right)x^3 = \frac{F_L x^2}{6K}\left[x - 3l_e\right]. \quad 0 \le x \le L.$$

Since we have rotated the cantilever beam, we need to multiply the rotation matrix to obtain our final position for displacement equation.

$$R = \begin{bmatrix} \cos(90-\alpha) & -\sin(90-\alpha) \\ \sin(90-\alpha) & \cos(90-\alpha) \end{bmatrix}$$

$$\boxed{\begin{aligned} y_R^* &= y_R \cdot R. \\ y_L^* &= y_L \cdot R. \end{aligned}}$$