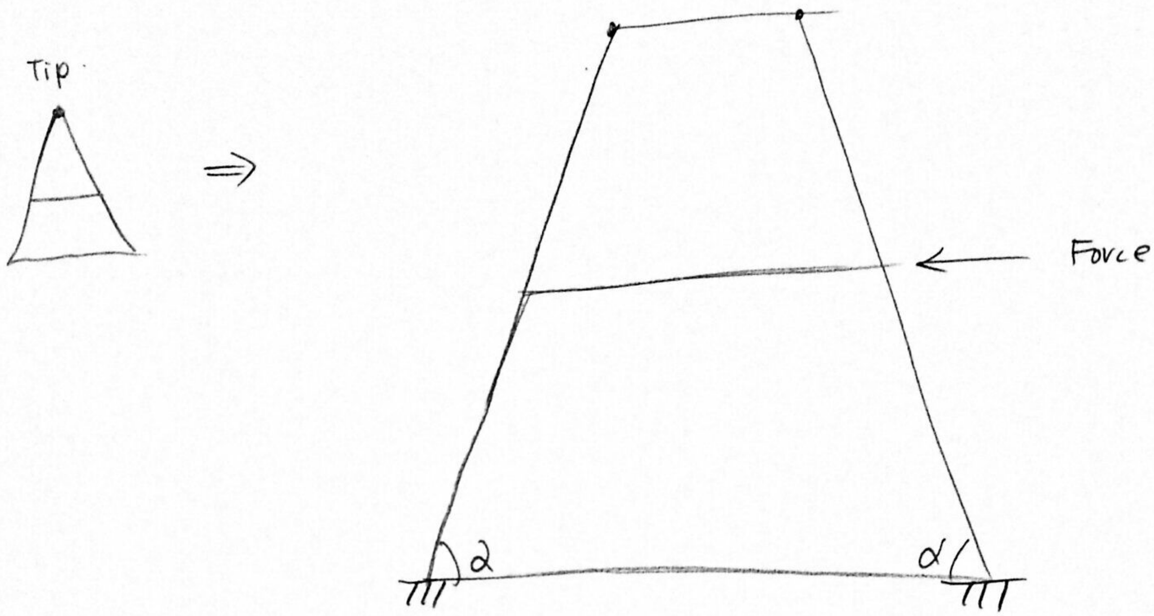
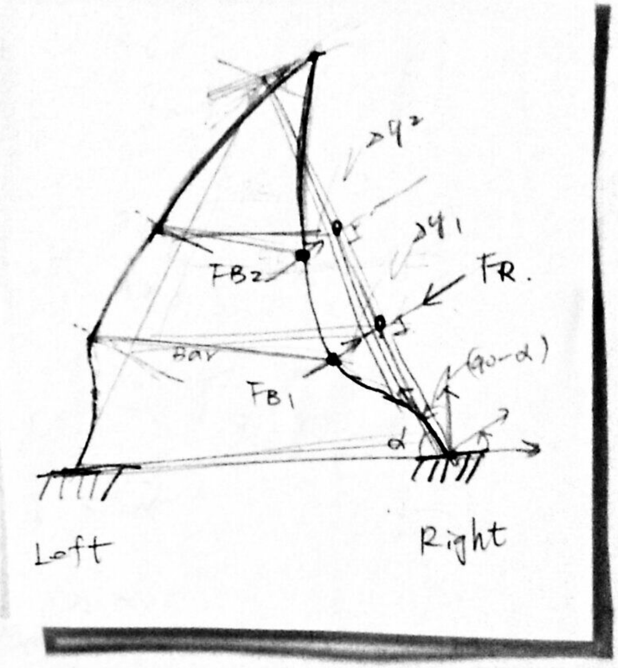
The z modules

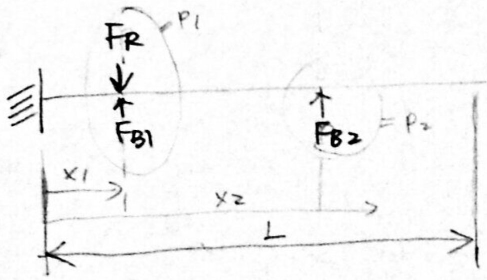


We assume the tip of the griper is not a point instead of a unlimited short bar connected two points.

Elastic Curve on both sides



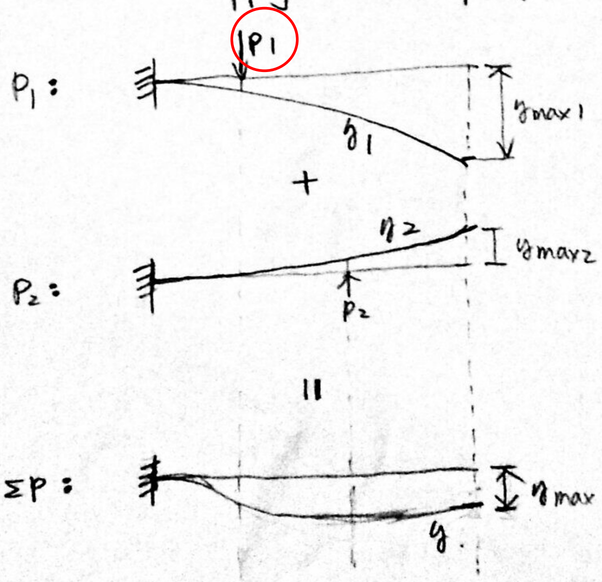
We start to analyze elastic curve on RHS first. The elastic curve here we regard it as a cantilever beam with concentrated load at any point.



F_R : force on the right

F_B : force from the bar.

Next we apply the superposition method to obtain final elastic curve. (y)

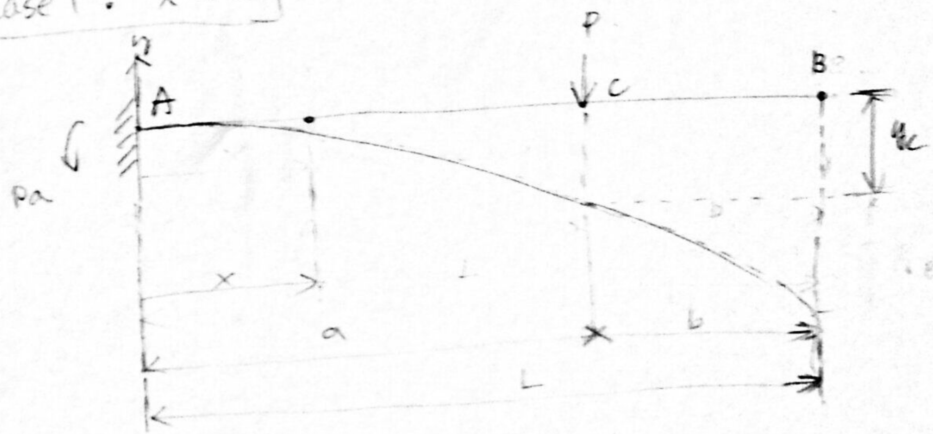


$$y = \Sigma(y_1 + y_2)$$

$$y = y_1(x_1) + y_2(x_1) + y_1(x_2) + y_2(x_2)$$

In order to obtain the final elastic curve (y), we need to know the equation when loading at any point. One equation is when $x < a$, the another one is when $x > a$.

Case 1 : $x < a$



Double Integral Method :

$$M = Px - Pa$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{dy}{dx} = \frac{Px^2}{2K} - \frac{Pax}{K} + C_1$$

$$y = \frac{Px^3}{6K} - \frac{Pax^2}{2K} + C_1x + C_2$$

Boundary Conditions :

$x = 0$, $\frac{dy}{dx} = 0$ $\Rightarrow$ $C_1 = 0$

$x = 0$, $y = 0$ $\Rightarrow$ $C_2 = 0$

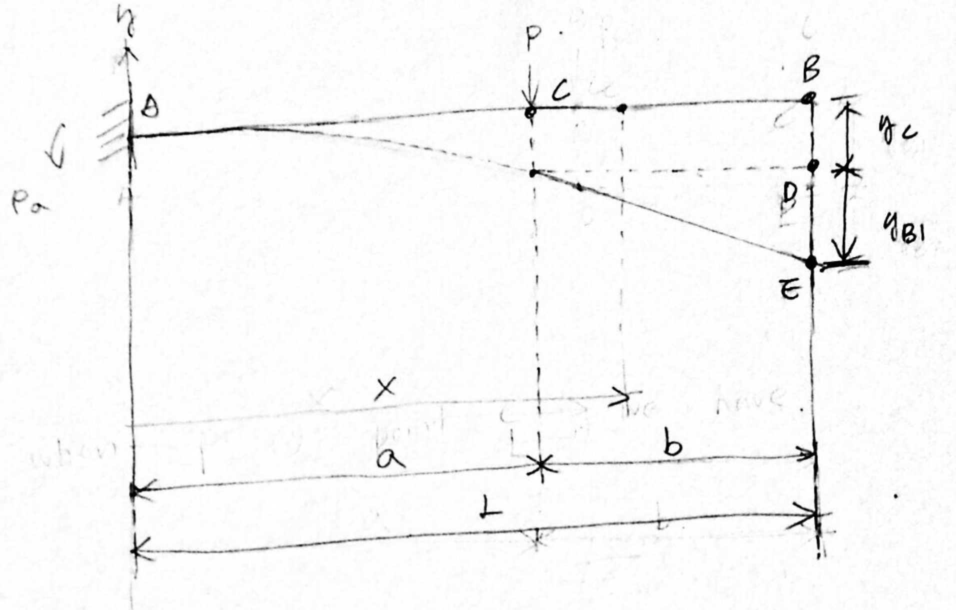
Arranging slope & defletion ,

$$\frac{dy}{dx} = \frac{Px}{2K}(x - 2a)$$

$$y = \frac{Px^2}{6K}(x - 3a) \ (\downarrow)$$

$$y_c = y\big|_{x=a} = \frac{-Pa^3}{3K}$$

Case 2: $x > a$



Double Intergal Method:

$$M = Px - Pa - P(x-a) = 0$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{dy}{dx} = C_1$$

$$y = C_1 x + C_2 = \frac{-Pa^2}{2K} x + \frac{Pa^3}{6K} = \frac{Pa^2}{6K}(a - 3x) \ (\downarrow)$$

Also,

$$\frac{dy}{dx} = \frac{\overline{DE}}{\overline{CB}} = \frac{-Pa^2}{2K}$$

$$\overline{DE} = \frac{-Pa^2}{2K}(\overline{CB})$$

$$y_{B1} = \frac{-Pa^2}{2K}(L-a)$$

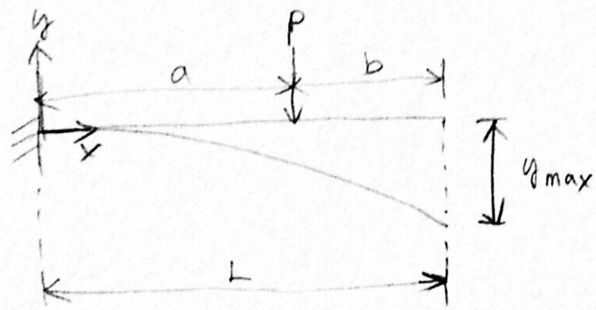
So,

$$y_{max} = y_c + y_{B1}$$

$$y_{max} = \frac{-Pa^3}{3K} - \frac{Pa^2}{2K}(L-a)$$

$$y_{max} = \frac{Pa^2}{6K}(a - 3L) \ (\downarrow)$$

Rearranging case 1 & case 2 :



$$y = \frac{Px^2}{6K}(x - 3a) \quad (\downarrow) \qquad \text{for } 0 < x < a \quad \text{— ①}$$

$$y = \frac{Pa^2}{6K}(a - 3x) \quad (\downarrow) \qquad \text{for } a < x < L. \quad \text{— ②}$$

Both cases on y_{max} are the same, which is

$$y_{max} = \frac{Pa^2}{6K}(a - 3L) \quad (\downarrow) \quad \text{— ③}$$

Similarly, we use equation ① to ③ to obtain elastic curve on LHS.

Since the cantilever beam is rotated by $(90 - \alpha)$, it is not vertical. We apply rotation matrix (R) to get the final position.

$$R = \begin{bmatrix} \cos(90-\alpha) & -\sin(90-\alpha) \\ \sin(90-\alpha) & \cos(90-\alpha) \end{bmatrix}$$

$$y^* = y \cdot R.$$