# Predicting the Lifespan of World Leaders

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8-30-20

## I. Introduction

World leaders are influential people that have the power to impact millions of people's lives. Some leaders are elected for terms while others serve for life, so their expected survival is of great interest. In this paper, we seek to examine Popes, US Presidents, Dalai Lamas, Chinese Emperors, and Japanese Emperors to see how their lifespans compare. Additionally, we want to analyze the impact of a leader's birth year on survival, and whether or not this impact might vary depending on the type of leadership. After we build a model to analyze the effects, we also wish to obtain predictions for the lifespans of leaders that are currently alive, as well as make comparative statements about the survival times of these leaders. We will accomplish this by using survival analysis. We use Bayesian Inference to estimate the model parameters with the help of the JAGS program.

The rest of the paper is structured as follows. First, we will describe the data used to carry out our analysis. Then, we will discuss the Weibull model and Bayesian framework used for analysis to ground our analysis plan. Then, we will show how we carried out our analysis plan and recount our results. After that, we discuss our conclusions and areas for further research. Finally, our code and some additional information can be found in the Appendix.

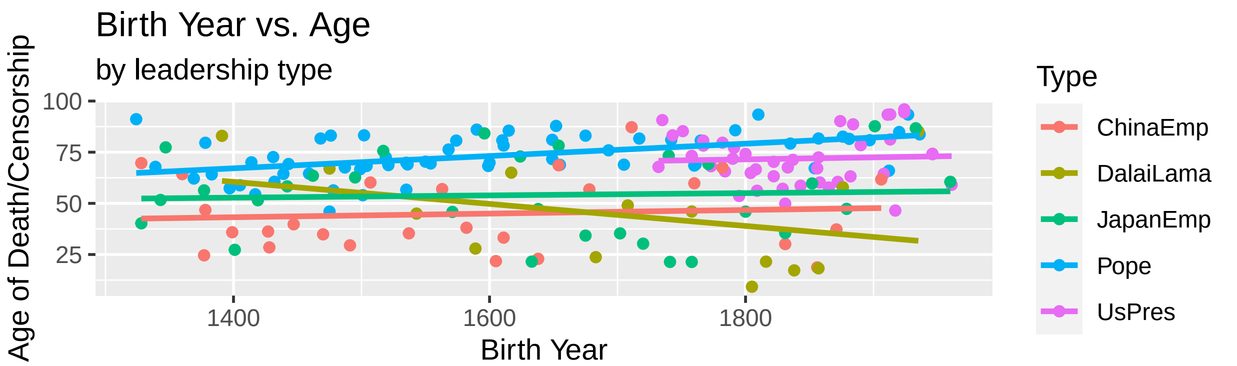
## II. Data

### II a. Description of the Data

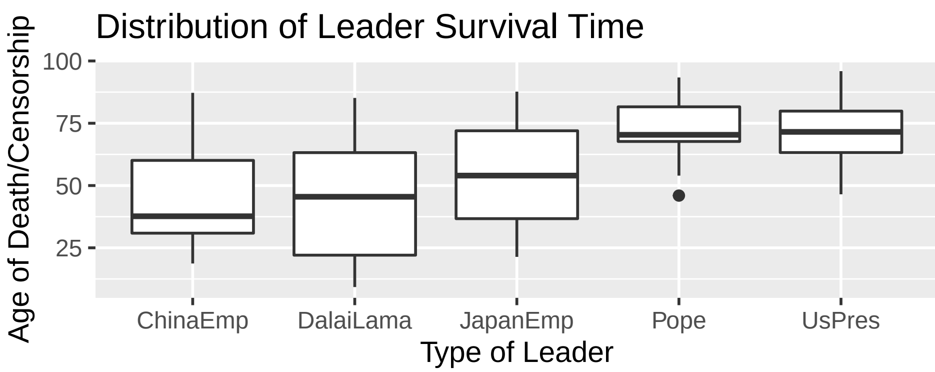
The data used in this analysis contains entries for 177 different world leaders. The types of world leaders present in the data include Popes, US Presidents, Dalai Lamas, Chinese Emperors, and Japanese Emperors. Each individual's birth date and their leadership position is recorded. We calculated each leader's birth year from the birth date column. For some of these groups, we have data dating all the way back to the 14th century. Additionally, leaders who have passed away also have their death date and age of death recorded. For leaders that are still living, these columns instead contain the date the dataset was created (July 31, 2020) and their current age on that date. We updated that date to the due date of the report, August 31, 2020. In order to further clarify who is dead or alive, there is a column titled "Censored," which takes a value of 0 if the person is dead and a value of 1 if that person is still alive. Related to this, there is another column called "Fail," which takes on a value of 1 if the person is dead and a value of 0 if the person is alive. The problem with censored data is that we don't know exactly when a person will die. Thus, we have to come up with a way to model a censored person's death date if we use them in building our model. In total, there are 10 living leaders in our dataset, 4 of whose age of death we are trying to predict.

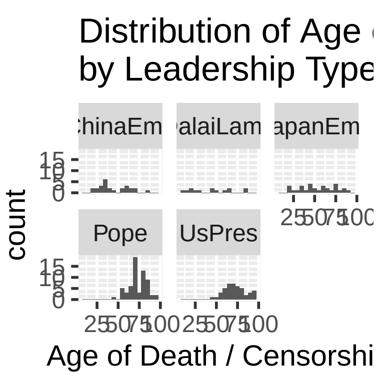
### II b. Exploratory Data Analysis

```
## `geom_smooth()` using formula 'y ~ x'
```




Based on this scatterplot, it appears that birth year's association with lifespan does depends on type of leadership. In particular, Dalai Lama's lifespans are negatively correlated with birth year, whereas the rest of the leader types have positive correlations with birth year.




From the boxplot, the distribution of the lifespans of Popes and US Presidents is centered higher, which makes intuitive sense given that these leaders are elected later in life, while Dalai Lamas and Emperors can be elected at much younger ages. For instance, if a person is a toddler and dies at age five, that person can never be president (and therefore would not have their age be part of the dataset). However, that person can become a Dalai Lama as a toddler and have their age at death influence the distribution plotted. These same patterns can also be observed in the histograms. Also, in the histograms, there is no extreme skewing, so it does not appear tha log transformations are needed.

# III. Methods

## III a. Motivating the Model

We use survival analysis as a way to model $T_i$, the lifespan of a given leader i depending on their year of birth and type of leadership. Survival analysis is useful in that it allows the consideration of "censored" data.

This means that we do not always observe the outcome for each data point. For example, we do not know the death date of leaders that are still alive. Thus, we do not know their lifespan, we just know that their survival time $T_i$ will be greater than their current age.

We model the lifespan $T_i$ of an individual i after the Weibull distribution specified below. We choose to use the Weibull distribution because it is often utilized in survival analysis and allows the user to specify a flexible shape parameter for the distribution. The first parameter $r$, also known as the shape parameter, is a positive scalar, and the second parameter $\mu$, the scale parameter, is a linear function of the covariates (in this case, the century of the birth year and types of leadership as well as their interactions).

$T_i \sim Weibull(r, \mu_i)$

$$\log(\mu_i) = \beta_0 + \sum_{j=1,\cdots,6} \beta_j I(Birth\ Century_i = j) + \sum_{k=7,\cdots,10} \beta_k I(Leadership_i = k)$$
$$+ \sum_{j=4,\cdots,6,\ k=7} \beta_{j,k} I(Birth\ Century_i = j) * I(Leadership_i = k)$$
$$+ \sum_{j=1,\cdots,6,\ k=8,\cdots,10} \beta_{j,k} I(Birth\ Century_i = j) * I(Leadership_i = k)$$

where the indicator functions $Birth\ Century_i = 1, \cdots, 6$ corresponds to a leader $i$ born in the 15th, 16th, 17th, 18th, 19th and 20th century respectively. The 14th century is the baseline for comparison. $Leadership_i = 7, \cdots, 10$ corresponds to a leader $i$ being a U.S. President, a Chinese Emperor, a Dalai Lamai and a Japanese Emperor respectively. $Leadership_i$ uses Pope the baseline for comparison. $\beta_{j,k}$ is the parameter for the interaction effect between brith year century $j$ and leadership type $k$.

For example, for Pope Francis who was born in 1963, his birth year century is the 20th century and his leadership type is Pope. $I(Birth\ Century_{Francis} = j)$ will all be 0 except for $j = 6$ (which indicates he was born in the 20th century) and $I(Leadership_i = k)$ will all be 0 since Pope is the baseline for comparison for leadership types. We can write $\log(\mu_{Francis}) = \beta_0 + \beta_6$ for Pope Francis.

Note that the first U.S. President, George Washington, was born in 1732 (18th century, $j = 4$) and hence the interactions between $Birth\ Century_i = 1, 2, 3$ (born in the 15th century, 16th century or 17th century) and $Leadership_7$ (being a U.S. President) are meaningless. As such, we didn't include those interaction terms in our model.

Prior to using this model, we considered a model that used leaders' birth years as a continuous variable instead of binning them into centuries. However, that model resulted in a less desirable model predictive ability as we ran model diagnostic. The residual plot showed a clear trend between residuals and predicted lifespans. Using a quadratic polynomial improved the residuals, but our current model is still superior. Model diagnostic of the current model can be found in section V.c and residual plots of previous models with continuous birth year variable can be found in the Appendix. *make sure we include them in the Appendix*

## III b. Addressing Censored Data

To handle censored observations, we specify their contribution to the likelihood function using the Poisson "zeros trick" and set the upper limit of age of death to 100. Details on this method and how to implement it can be found in Appendix A.1.

## III c. Prior Choice

We assume that our priors are independent because intuitively a given observation could not be two types of leaders. Additionally, we do not have sufficiently strong prior knowledge about the relationship between birth year and type of leadership to specify an informative prior. Therefore, we use uninformative priors for the betas in our model, in which we chose normal betas with mean 0 and variance 1.0E-3, this includes the coefficents and the intercept in our model.

For our prior for r, we chose a prior of $exp(1)$, as we believe the hazard of death increases with time. This is supported by the fact that, in the European Union in 2016, 82.9% of all deaths happened among people over 65. Within the Appendix A.3, different Weibull distributions are plotted with different values of r in order to showcase the fact that with $r = exp(1)$ gives an increasing rate of hazard, as the shape of the Weibull

Distribution starts to have an increasing level of density toward the upper x values for $r = exp(1)$, compared to smaller values of r.

# IV. Analysis

As Stander et al (2018) pointed out, following this Weibull distribution, $\log(T) = \frac{1}{r}(\mu_i) + \frac{1}{r}(\epsilon)$, or

$$\log(T) \sim -\frac{1}{r}(\beta_0+\beta_1x_1+\cdots+\beta_6x_6+\beta_7x_7+\cdots\beta_{10}x_{10}+\beta_{1,8}x_1x_8+\cdots+\beta_{6,10}x_6x_{10})+\frac{1}{r}\log(\epsilon) \qquad =\alpha_0+\alpha_1x_1+\cdots+\alpha_6x_6+\alpha_7x$$

where $\epsilon \sim exp(1)$ and $\epsilon > 0$. $\alpha = -\beta/r$ is thus a monotone transformation of $\beta$. $x_1, \cdots, x_6$ are indicator values (i.e., 0s and 1s) corresponding to birth year's century, and $x_7, \cdots, x_{10}$ are indicator values corresponding to types of leadership.

The interpretation of coefficients depends on the interaction terms (i.e. both birth year century and the types of leadership). For example, while keeping all others constant, if a Pope was born in the 20th century, he is expected to live longer by a multiplicative factor of $exp(\alpha_6)$ compared to a Pope born in the 14th century, or his lifespan is expected to increase by a percentage of $100 * exp(\alpha_6 - 1)$. If a U.S. President was born in the 18th century, he is expected to live $exp(\alpha_0 + \alpha_4 + \alpha_7 + \alpha_{4,7})$ years; if the president were born in the 20th century instead, he is expected to live $exp(\alpha_0 + \alpha_6 + \alpha_7 + \alpha_{6,7})$. In other words, his lifespan is expected to increase by a multiplicative factor of $exp(\alpha_6 - \alpha_4 + \alpha_{6,7} - \alpha_{4,7})$. If a Chinese Emperor and a Dalai Lama were both born in the 19th century, the Chinese emperor is expected to live longer by a multiplicative factor of $exp(\alpha_8 - \alpha_9 + \alpha_{5,8} - \alpha_{5,9})$ and so on.

# V. Results

## V a. Output and Interpretation of the Model

The table below shows the model output of the multiplicative factors, $\alpha$s. Note that since $\alpha = -\beta/r$ is a monotone transformation, if $\beta$ is statistically significant (i.e. 95% credible interval doesn't contain 0), it should be the same for the corresponding $\alpha$.

| Coefficient | Coefficient Name | Mean | Standard Deviation | 2.5% Quantile | Median | 97.5% Quantile |
|---|---|---|---|---|---|---|
| alpha_0 | Intercept | 4.319 | 0.079 | 4.181 | 4.313 | 4.490 |
| alpha_1 | Born in 15th Century | -0.087 | 0.096 | -0.284 | -0.085 | 0.095 |
| alpha_2 | Born in 16th Century | -0.029 | 0.092 | -0.219 | -0.026 | 0.150 |
| alpha_3 | Born in 17th Century | 0.076 | 0.102 | -0.130 | 0.077 | 0.271 |
| alpha_4 | Born in 18th Century | 0.049 | 0.109 | -0.166 | 0.050 | 0.266 |
| alpha_5 | Born in 19th Century | 0.103 | 0.109 | -0.121 | 0.104 | 0.315 |
| alpha_6 | Born in 20th Century | 0.305 | 0.176 | 0.008 | 0.289 | 0.699 |
| alpha_7 | U.S. President | -0.372 | 3.597 | -5.944 | 0.627 | 5.829 |
| alpha_8 | Chinese Emperor | -0.246 | 0.122 | -0.483 | -0.250 | 0.000 |
| alpha_9 | Dalai Lama | 0.221 | 0.257 | -0.181 | 0.181 | 0.836 |
| alpha_10 | Japanese Emperor | -0.153 | 0.130 | -0.402 | -0.155 | 0.117 |
| alpha_1_8 | 15th Century*Chinese Emperor | -0.415 | 0.161 | -0.740 | -0.415 | -0.102 |

| Coefficient | Coefficient Name | Mean | Standard Deviation | 2.5% Quantile | Median | 97.5% Quantile |
|---|---|---|---|---|---|---|
| alpha_1_9 | 15th Century*Dalai Lama | -0.141 | 0.361 | -0.847 | -0.141 | 0.604 |
| alpha_1_10 | 15th Century*Japanese Emperor | -0.012 | 0.168 | -0.348 | -0.009 | 0.323 |
| alpha_2_8 | 16th Century*Chinese Emperor | -0.063 | 0.169 | -0.390 | -0.062 | 0.274 |
| alpha_2_9 | 16th Century*Dalai Lama | -0.768 | 0.300 | -1.419 | -0.751 | -0.225 |
| alpha_2_10 | 16th Century*Japanese Emperor | 0.211 | 0.180 | -0.139 | 0.211 | 0.583 |
| alpha_3_8 | 17th Century*Chinese Emperor | -0.143 | 0.166 | -0.469 | -0.147 | 0.186 |
| alpha_3_9 | 17th Century*Dalai Lama | -0.526 | 0.308 | -1.226 | -0.500 | 0.023 |
| alpha_3_10 | 17th Century*Japanese Emperor | -0.061 | 0.169 | -0.400 | -0.063 | 0.275 |
| alpha_4_7 | 18th Century*U.S. President | 0.354 | 3.597 | -5.901 | -0.629 | 5.907 |
| alpha_4_8 | 18th Century*Chinese Emperor | 0.236 | 0.187 | -0.125 | 0.232 | 0.603 |
| alpha_4_9 | 18th Century*Dalai Lama | -0.677 | 0.309 | -1.354 | -0.651 | -0.120 |
| alpha_4_10 | 18th Century*Japanese Emperor | -0.161 | 0.169 | -0.495 | -0.155 | 0.155 |
| alpha_5_7 | 19th Century*U.S. President | 0.203 | 3.597 | -6.011 | -0.769 | 5.758 |
| alpha_5_8 | 19th Century*Chinese Emperor | -0.677 | 0.186 | -1.029 | -0.680 | -0.312 |
| alpha_5_9 | 19th Century*Dalai Lama | -0.873 | 0.282 | -1.539 | -0.837 | -0.408 |
| alpha_5_10 | 19th Century*Japanese Emperor | -0.295 | 0.191 | -0.668 | -0.298 | 0.093 |
| alpha_6_7 | 20th Century*U.S. President | 0.294 | 3.595 | -5.994 | -0.660 | 5.876 |
| alpha_6_8 | 20th Century*Chinese Emperor | -0.141 | 0.317 | -0.729 | -0.156 | 0.561 |
| alpha_6_9 | 20th Century*Dalai Lama | 4.585 | 3.799 | -0.148 | 3.726 | 13.877 |
| alpha_6_10 | 20th Century*Japanese Emperor | 0.254 | 0.316 | -0.338 | 0.232 | 0.935 |

*double check variable significance after knitting the whole document* Since the baseline for comparison is a Pope born in the 14th century, it's meaningful to interpret the intercept in this case. The intercept shows that we expect a Pope born in the 14th century to live $\exp(\alpha_0) = \exp(4.3193694) = 75.1412292$ years. $\alpha_{6,9}$ which correspond to the interaction between born in the 20th century and being a Dalai Lama has an abnormally large point estimate, largely because there is only one Dalai Lamai born the last century and he is a censored data point. *should double check why alpha_6_9 is so large*

While $\alpha_1, \cdots, \alpha_5$ have 95% credible intervals containing 0 (an indication that these coefficients are statistically insignificant), $\alpha_6$ has a positive credible interval. Hence birth year overall is an significant predictor in this model. If we only look at the main effect of leadership types ($\alpha_7, \cdots, \alpha_10$), these parameters' 95% credible intervals all contain 0. However, some of their interactions with birth year are statistically significant, for example, $\alpha_{1,8}$'s CI is always negative. Due to model hierachy, we need to keep main effects of leadership types in the model as well.

The specific interpretation of these parameters depends on inputs from birth year and type of leadership. Below are some examples.

- A Japanese emperor born in different centuries
  - 14th century: $\log(T_{14th,JapanEmp}) = \alpha_0 + \alpha_{10} + \frac{1}{r}\log(\epsilon)$
  - 15th century: $\log(T_{15th,JapanEmp}) = \alpha_0 + \alpha_1 + \alpha_{10} + \alpha_{1,10} + \frac{1}{r}\log(\epsilon)$
  - 16th century: $\log(T_{16th,JapanEmp}) = \alpha_0 + \alpha_2 + \alpha_{10} + \alpha_{2,10} + \frac{1}{r}\log(\epsilon)$
  - 17th century: $\log(T_{17th,JapanEmp}) = \alpha_0 + \alpha_3 + \alpha_{10} + \alpha_{3,10} + \frac{1}{r}\log(\epsilon)$
  - 18th century: $\log(T_{18th,JapanEmp}) = \alpha_0 + \alpha_4 + \alpha_{10} + \alpha_{4,10} + \frac{1}{r}\log(\epsilon)$
  - 19th century: $\log(T_{19th,JapanEmp}) = \alpha_0 + \alpha_5 + \alpha_{10} + \alpha_{5,10} + \frac{1}{r}\log(\epsilon)$
  - 20th century: $\log(T_{20th,JapanEmp}) = \alpha_0 + \alpha_6 + \alpha_{10} + \alpha_{6,10} + \frac{1}{r}\log(\epsilon)$

The summary table is shown below if we comapre the expected lifespans of later Japanese emperors with that of a Japanese emperor born in the 14th century.

| Japanese Emperor | Multiplicative Factor | % Increase / Decrease in Lifespan |
|---|---|---|
| Born in 14th Century | 1.00 | 0 |
| Born in 15th Century | 0.91 | -9 |
| Born in 16th Century | 1.20 | 20 |
| Born in 17th Century | 1.02 | 2 |
| Born in 18th Century | 0.89 | -11 |
| Born in 19th Century | 0.83 | -17 |
| Born in 20th Century | 1.75 | 75 |

Surprisingly, the model suggests that longevity doesn't always increase with calendar time. The biggest increase in lifespan is in the 20th century, where the expected lifespan of a Japanese emperor increases by -17% compared to a Japanese emperor born 6 centuries ago. This might make sense, given the last century saw the biggest number of technological breakthroughs and improvement in healthcare.

- Dalai Lamai born in different centuries
  - 14th century: $\log(T_{14th,Dalai}) = \alpha_0 + \alpha_9 + \frac{1}{r}\log(\epsilon)$
  - 15th century: $\log(T_{15th,Dalai}) = \alpha_0 + \alpha_1 + \alpha_9 + \alpha_{1,9} + \frac{1}{r}\log(\epsilon)$
  - 16th century: $\log(T_{16th,Dalai}) = \alpha_0 + \alpha_2 + \alpha_9 + \alpha_{2,9} + \frac{1}{r}\log(\epsilon)$
  - 17th century: $\log(T_{17th,Dalai}) = \alpha_0 + \alpha_3 + \alpha_9 + \alpha_{3,9} + \frac{1}{r}\log(\epsilon)$
  - 18th century: $\log(T_{18th,Dalai}) = \alpha_0 + \alpha_4 + \alpha_9 + \alpha_{4,9} + \frac{1}{r}\log(\epsilon)$
  - 19th century: $\log(T_{19th,Dalai}) = \alpha_0 + \alpha_5 + \alpha_9 + \alpha_{5,9} + \frac{1}{r}\log(\epsilon)$
  - 20th century: $\log(T_{20th,Dalai}) = \alpha_0 + \alpha_6 + \alpha_9 + \alpha_{6,9} + \frac{1}{r}\log(\epsilon)$

| Dalai Lama | Multiplicative Factor | % Increase / Decrease in Lifespan |
|---|---|---|
| Born in 14th Century | 1.00 | 0 |
| Born in 15th Century | 0.80 | -20 |
| Born in 16th Century | 0.45 | -55 |
| Born in 17th Century | 0.64 | -36 |
| Born in 18th Century | 0.53 | -47 |
| Born in 19th Century | 0.46 | -54 |
| Born in 20th Century | 132.96 | 13196 |

For Dalai Lamas, percentage increase in lifespan becomes generally more negative, which means later born Dalai Lamas are expected to live shorter, confirming the negative correlation between lifespan and birth year exihibited in Figure 1 of section II.b EDA. *double check the Figure number.*The Dalai Lama born in the 20th century, however, is expected to live abnormally longer than the 14th Dalai Lama, due the abnormal point estimate of $\alpha_{6,9}$ that corresponds to the interaction between born in the 20th century and being a Dalai Lama.

- Leaders born in the 18th century
  - Pope: $\log(T_{18th,Pope}) = \alpha_0 + \alpha_4 + \frac{1}{r}\log(\epsilon)$
  - U.S. President: $\log(T_{18th,UsPres}) = \alpha_0 + \alpha_4 + \alpha_7 + \alpha_{4,7} + \frac{1}{r}\log(\epsilon)$
  - Chinese Emperor: $\log(T_{18th,ChinaEmp}) = \alpha_0 + \alpha_4 + \alpha_8 + \alpha_{4,8} + \frac{1}{r}\log(\epsilon)$
  - Dalai Lama: $\log(T_{18th,Dalai}) = \alpha_0 + \alpha_4 + \alpha_9 + \alpha_{4,9} + \frac{1}{r}\log(\epsilon)$
  - Japanese Emperor: $\log(T_{18th,JapanEmp}) = \alpha_0 + \alpha_4 + \alpha_{10} + \alpha_{4,10} + \frac{1}{r}\log(\epsilon)$

| Leaders born in 18th Century | Multiplicative Factor | % Increase / Decrease in Lifespan |
|---|---:|---:|
| Pope | 1.00 | 0 |
| U.S. President | 0.98 | -2 |
| Chinese Emperor | 0.99 | -1 |
| Dalai Lama | 0.63 | -37 |
| Japanese Emperor | 0.73 | -27 |

- Leaders born in the 19th century
  - Pope: $\log(T_{20th,Pope}) = \alpha_0 + \alpha_5 + \frac{1}{r}\log(\epsilon)$
  - U.S. President: $\log(T_{20th,UsPres}) = \alpha_0 + \alpha_5 + \alpha_7 + \alpha_{5,7} + \frac{1}{r}\log(\epsilon)$
  - Chinese Emperor: $\log(T_{20th,ChinaEmp}) = \alpha_0 + \alpha_5 + \alpha_8 + \alpha_{5,8} + \frac{1}{r}\log(\epsilon)$
  - Dalai Lama: $\log(T_{20th,Dalai}) = \alpha_0 + \alpha_5 + \alpha_9 + \alpha_{5,9} + \frac{1}{r}\log(\epsilon)$
  - Japanese Emperor: $\log(T_{20th,JapanEmp}) = \alpha_0 + \alpha_5 + \alpha_{10} + \alpha_{5,10} + \frac{1}{r}\log(\epsilon)$

| Leaders born in 20th Century | Multiplicative Factor | % Increase / Decrease in Lifespan |
|---|---:|---:|
| Pope | 1.00 | 0 |
| U.S. President | 0.84 | -16 |
| Chinese Emperor | 0.40 | -60 |
| Dalai Lama | 0.52 | -48 |
| Japanese Emperor | 0.64 | -36 |

Popes born in the 18th and 19th century are expected to higher lifespan than other leaders born in the same century. If born in the 18th century, a U.S. president and a Chinese emperor are expected to enjoy the comparable lifespans as a Pope. However, if born in the 19th century instead, the expected lifespans of a U.S. president and a Chinese emperor are significantly shorter than that of a Pope (16% and 60% shorter respectively).

*do we need more interpretation examples?*

The table below shows the estimate and 95% credible interval (the bounds are the 2.5% quantile and 97.5% quantile) for all 10 living leaders in the dataset.

| Variable | Mean | Standard Deviation | 2.5% Quantile | Median | 97.5% Quantile |
|---|---|---:|---:|---|---:|
| age_Francis_predictive | 94.072 | 3.804 | 87.137 | 94.340 | 99.780 |
| age_Obama_predictive | 96.640 | 1.914 | 93.580 | 96.589 | 99.846 |
| age_Dalai_predictive | 87.349 | 7.308 | 74.935 | 87.305 | 99.320 |
| age_Naruhito_predictive | 97.934 | 1.172 | 96.013 | 97.929 | 99.888 |
| age_Benedict_predictive | 87.441 | 7.309 | 74.809 | 87.736 | 99.227 |
| age_Carter_predictive | 92.665 | 4.309 | 85.544 | 92.776 | 99.610 |
| age_Clinton_predictive | 91.910 | 4.646 | 84.135 | 91.874 | 99.517 |

| Variable | Mean | Standard Deviation | 2.5% Quantile | Median | 97.5% Quantile |
|---|---|---|---|---|---|
| age_Bush_predictive | 86.410 | 10.357 | 63.258 | 88.768 | 99.569 |
| age_Trump_predictive | 81.897 | 10.996 | 61.262 | 82.831 | 99.113 |
| age_Akihito_predictive | 87.311 | 7.289 | 74.883 | 87.317 | 99.390 |

## V b. Posterior Inference for Selected Leaders

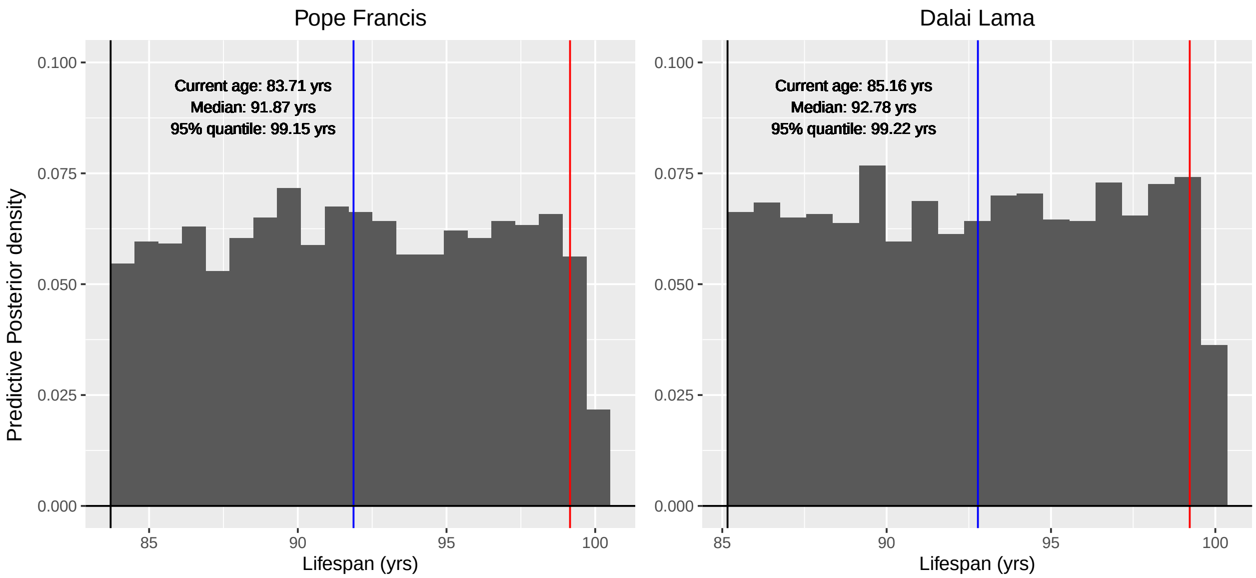



Fig. 4a The posterior predictive probability density function of the lifespan for hypothetical leaders with the same attributes as Pope Francis (right) and the 14th Dalai Lama (left). The black vertical line marks current age the blue marks posterior median, and the red marks the posterior 95% quantile.

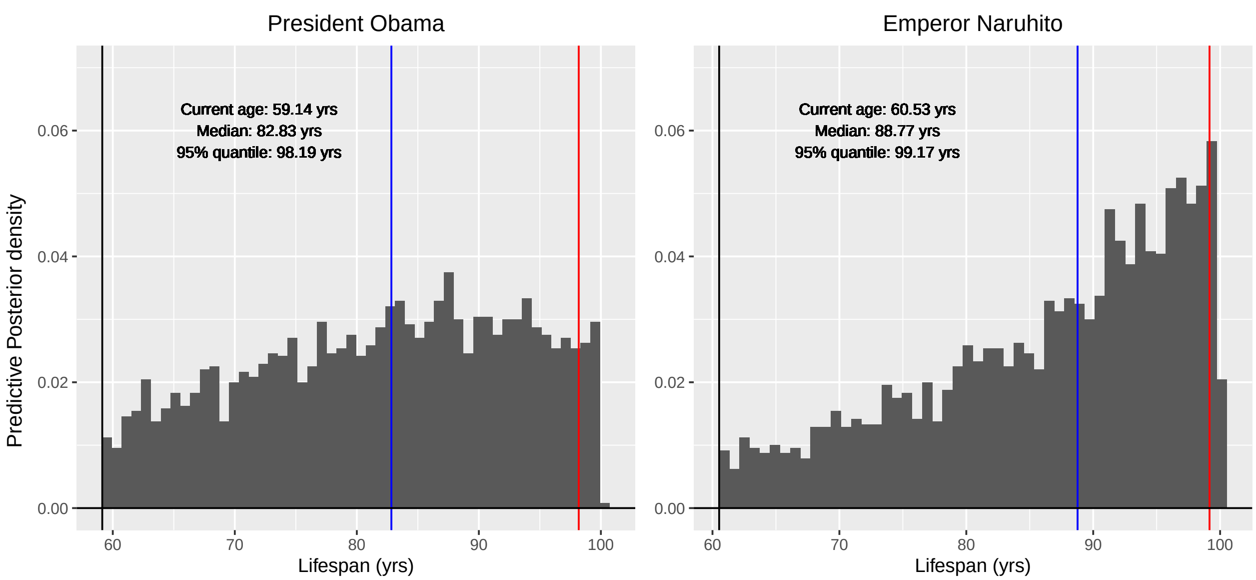



Fig. 4b The posterior predictive probability density function of the lifespan for hypothetical leaders with the same attributes as Obama (right) and Emperor Naruhito (left). The black vertical line marks current age, the blue marks posterior median, and the red marks the posterior 95% quantile.

The histogram of the posterior predictive distributions of the lifespans for a leader with the same birth year and leadership type as Pope Francis and that of a leader with the same above attributes as the 14th Dalai Lama are more uniform. The histogram for a leader with the same birth year and leadership type as President Obama and that of a leader with the same above attributes as Emperor Naruhito are both left skewed, with modes around the 90s.

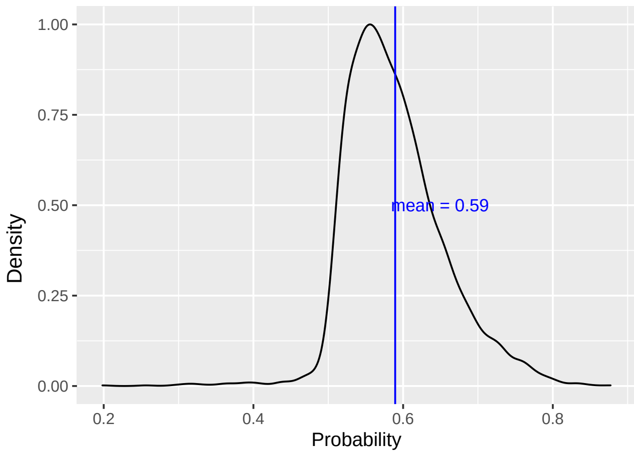



Figure 5a. This density plot shows the posterior distribution of the probability that the 14th Dalai Lama outlives Pope Francis.

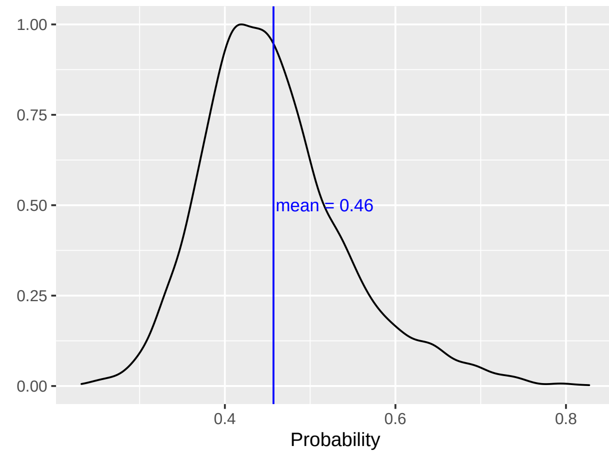
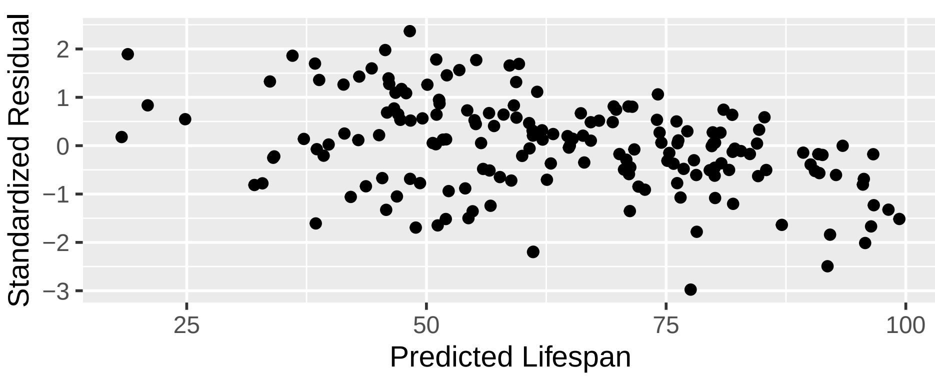


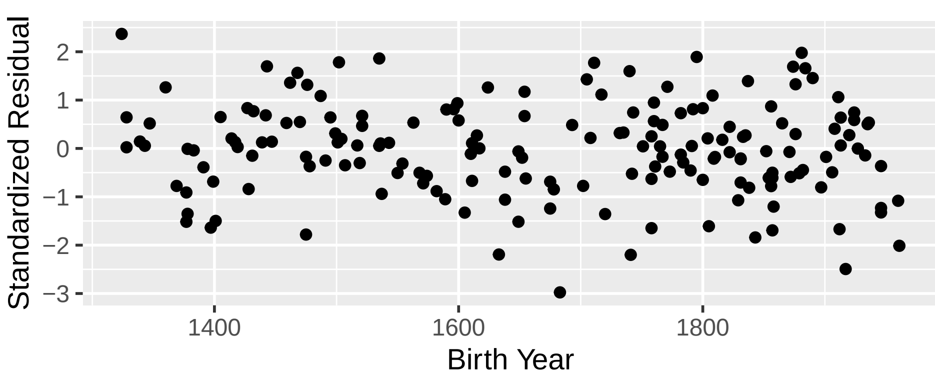

Figure 5b. This density plot shows the posterior distribution of the probability that Obama outlives Emperor Naruhito.

Figure 5 depicts the posterior distribution of the probability that Dalai Lama outlives Pope Francis, and that Obama outlives Emperor Naruhito. The probability that the 14th Dalai Lama will have a longer lifespan than Pope Francis is 0.589 with a 95% confidence interval of (0.503, 0.741). The probability that President Obama will have a longer lifespan than Emperor Naruhito is 0.457 with a 95% confidence interval of (0.323, 0.66).

## V c. MCMC and Model Diagnostics

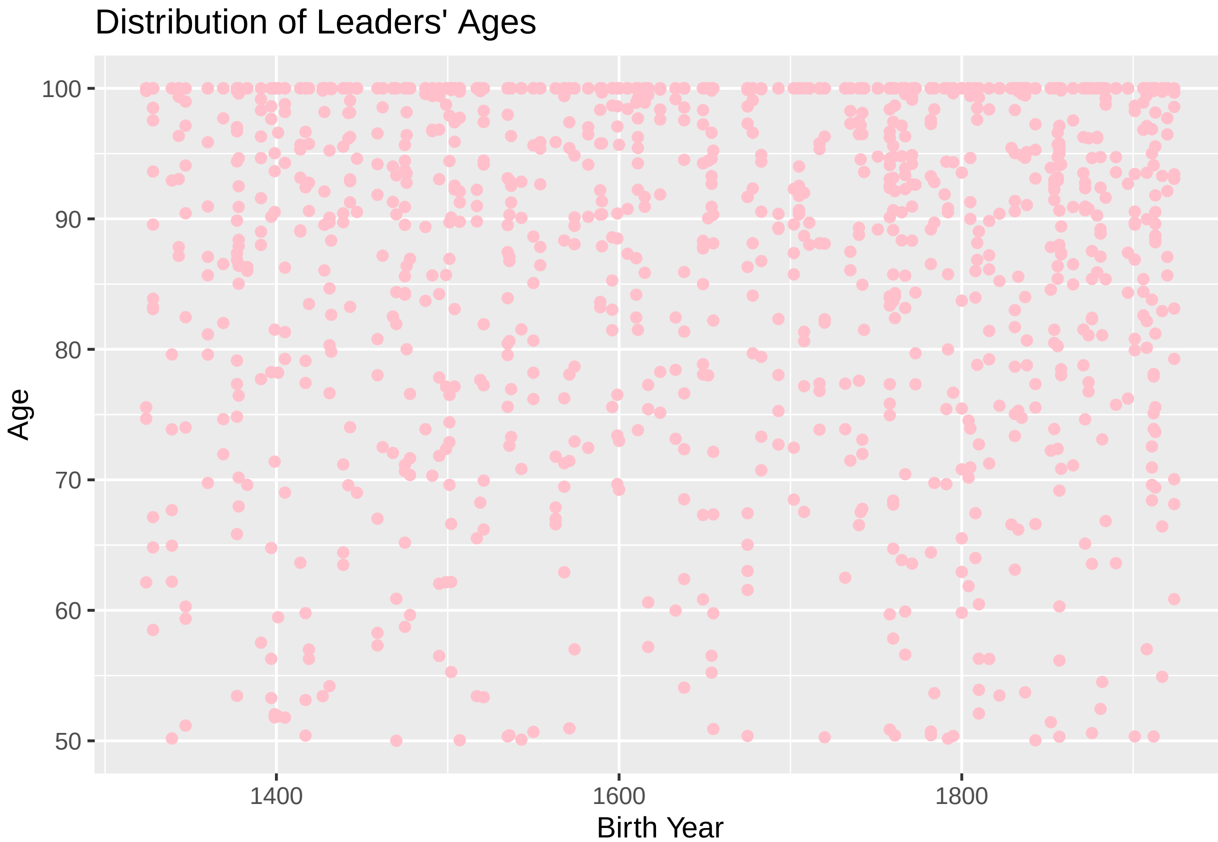
The combination of traceplots, lag-1 scatterplots, and acf plots suggest the chain for each parameters converges. Code for those plots are included in Appendix A.2. The Rhat's are all close to 1, which is another indicator of convergence. Most of the effective sample sizes are greater than 1000 (the effective sample size of 400 for r is lower, but the number of iterations was kept at 50,000 for computational efficiency).

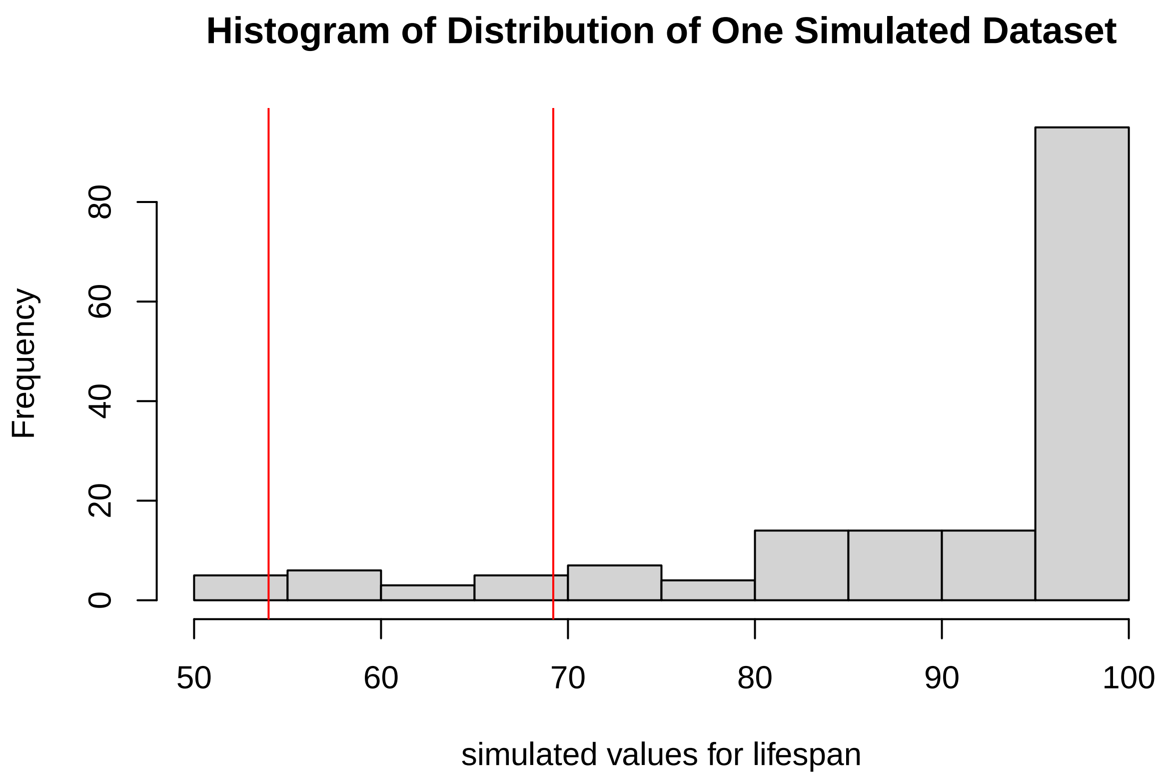



The standardized residual plot for predicted lifespan shows patterning. It appears that our model tends to overpredict lifespan compared to what was actually observed in many cases. This might be due to the fact that we only have two predictors, Birth Year and Type of Leader. Therefore, we are missing information that might help predict whether or not a leader will die earlier, for example if they have some underlying health condition, etc.

The standardized residuals when plotted against Birth Year show random scattering, however, and this means that our model is not over or underpredicting lifespan according to the year a leader was born.






In order to perform posterior predictive checks, a simulated dataset was created by first extracting the leaders that were non-censored. There was ten different iterations that performed the same task: every single row of the non-censored leaders put into our defined Weibull posterior model alongside the random sample from the JAGS model output. These simulations were put into a matrix, and one of the simulated datasets (of which ten were created total) is displayed in the Histogram of Distribution of One Simulated Dataset.

The Distribution of Leaders' Age and the Birth Year showcases the relationship of the actual birth year and the predicted age from the performed posterior predictive checks.

For the posterior predictive checks, it appears that our model returns values that are more concentrated around the upper truncation limit, which is why the dataframe's Inf values are encoded as 100 (the upper truncation limit defined in our function). This follows what the standardized residuals found, where most of the values from the model are left-skewed, and the posterior predictive checks is showing that a lot of the leaders are being predicted to having a larger lifespan. On the Histogram of Distribution of One Simulated dataset, the red lines on our model are defined as the actual ages from two randomly sampled leaders that are non-censored. This emphasizes the previous point that the model is predicting leaders to have larger lifespans than in actuality.

## VI. Sensitivity Analysis

```
## Note: Using an external vector in selections is ambiguous.
## i Use `all_of(parameter)` instead of `parameter` to silence this message.
## i See <https://tidyselect.r-lib.org/reference/faq-external-vector.html>.
## This message is displayed once per session.
```

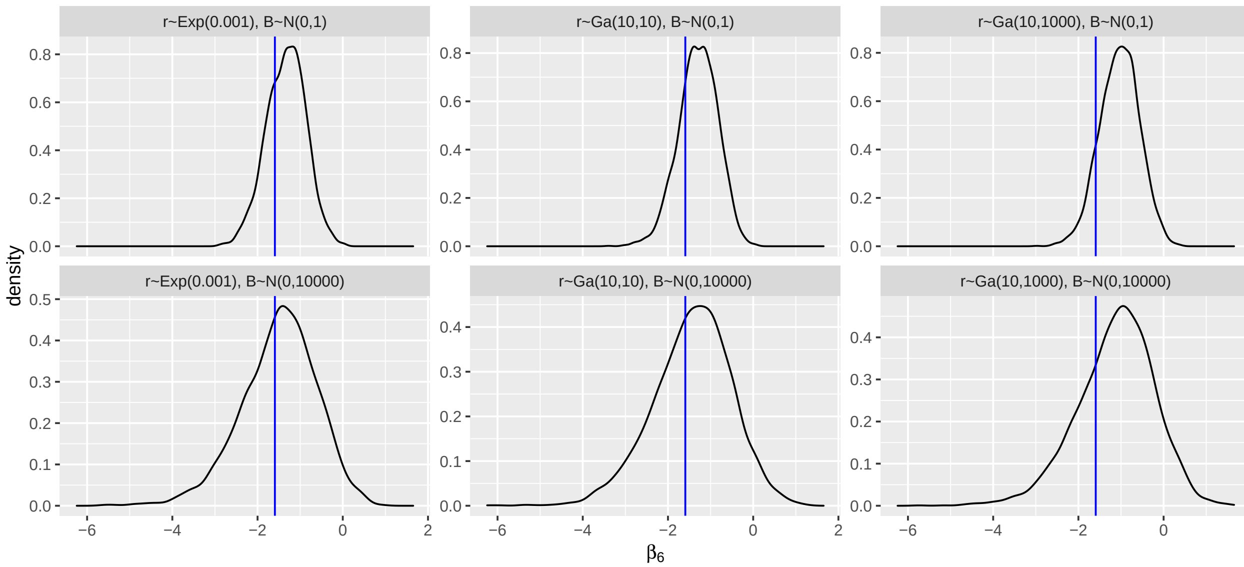



Fig. 9 The above plots show sensitivity analysis for different priors on r and the coefficients. The blue line indicates the estimate of the coefficient from our model. It appears that the analysis is generally not sensitive to different priors.

We tested $N(0,1)$ and $N(0,10000)$ priors for the $\beta$s and $Exp(0.001)$ (Stander et al prior), $Gamma(10,10)$ (mean $=1$), and $Gamma(10,100)$ (mean $=0.1$) priors for $r$. Distribution of $\beta_6$ was approximated using those different priors and plotted above. The blue line, which is the estimate for $\beta_6$ from our model, crosses (or is close to) the mode of all but one of the distributions, so our analysis is mostly robust against changes in prior. However, it is somewhat sensitive to the prior combination of $r \sim Gamma(10,1000)$ and $\beta_6 \sim N(0,1)$ Code to conduct sensitivity analysis for the remaining betas can be found in Appendix A.3.

# VII. Cross Validation

```
#read data
monarchs <- read.csv("english_monarchs.csv", header = FALSE)
monarchs$Age <- as.numeric(monarchs$V3) - as.numeric(monarchs$V2)
monarchs <- monarchs %>%
  filter(V2 >= 1300)
```

```
monarchs$Age[monarchs$V1 == "Elizabeth II"] <- 94
```

```
monarchs$Censored <- 0
monarchs$Censored[monarchs$V1 == "Elizabeth II"] <- 1
```

```
monarchs$survival <- monarchs$Age
monarchs$survival[monarchs$V1 == "Elizabeth II"] <- NA
```

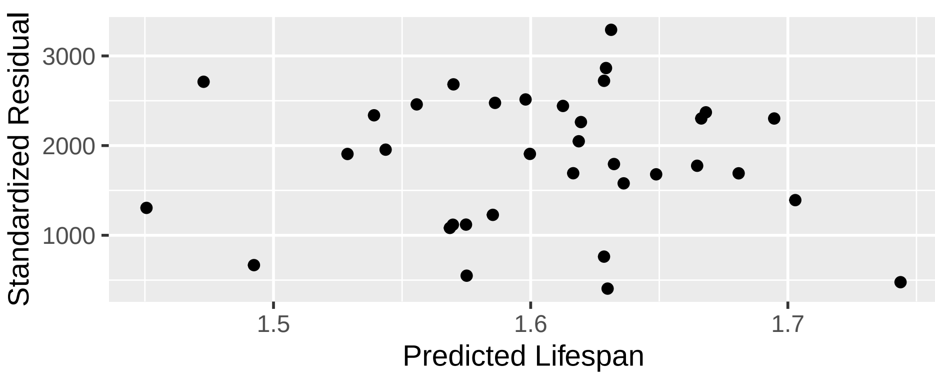
```
monarchs$TypeJapanEmp <- 1
monarchs <- monarchs %>%
  mutate(Yr14 = as.factor(case_when(V2 <= 1400 ~1,
                                      TRUE ~0)),
         Yr15 = as.factor(case_when(V2 >= 1401 & V2 <= 1500 ~ 1,
                                      TRUE ~0)),
         Yr16 = as.factor(case_when(V2 >= 1501 & V2 <= 1600 ~ 1,
                                      TRUE ~0)),
         Yr17 = as.factor(case_when(V2 >= 1601 & V2 <= 1700 ~ 1,
                                      TRUE ~0)),
         Yr18 = as.factor(case_when(V2 >= 1701 & V2 <= 1800 ~ 1,
```

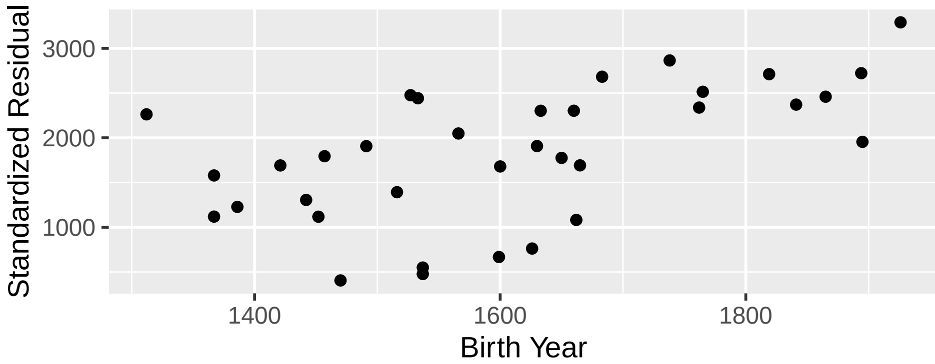
```
                                        TRUE ~0)),
         Yr19 = as.factor(case_when(V2 >= 1801 & V2 <= 1900 ~ 1,
                                        TRUE ~0)),
         Yr20 = as.factor(case_when(V2 >= 1901 & V2 <= 2000 ~ 1,
                                        TRUE ~0))
         )

z_1 <- as.numeric(as.character(monarchs$Yr15))
z_2 <- as.numeric(as.character(monarchs$Yr16))
z_3 <- as.numeric(as.character(monarchs$Yr17))
z_4 <-  as.numeric(as.character(monarchs$Yr18))
z_5 <-  as.numeric(as.character(monarchs$Yr19))
z_6 <-  as.numeric(as.character(monarchs$Yr20))
z_7 <- 0
z_8 <- 0
z_9 <- 0
z_10 <- as.numeric(as.character(monarchs$TypeJapanEmp))

# add interaction b/w Yr__ and Type___
z_1_8 <- z_1*z_8; z_1_9 <- z_1*z_9; z_1_10 <- z_1*z_10


z_2_8 <- z_2*z_8
z_2_9 <- z_2*z_9
z_2_10 <- z_2*z_10


z_3_8 <- z_3*z_8
z_3_9 <- z_3*z_9
z_3_10 <- z_3*z_10

z_4_7 <- z_4*z_7
z_4_8 <- z_4*z_8
z_4_9 <- z_4*z_9
z_4_10 <- z_4*z_10

z_5_7 <- z_5*z_7
z_5_8 <- z_5*z_8
z_5_9 <- z_5*z_9
z_5_10 <- z_5*z_10

z_6_7 <- z_6*z_7
z_6_8 <- z_6*z_8
z_6_9 <- z_6*z_9
z_6_10 <- z_6*z_10

cv_data.mat = as.matrix(cbind(z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_10,

                           z_1_8,
                           z_1_9,
                           z_1_10,

                           z_2_8,
```

```
                            z_2_9,
                            z_2_10,

                            z_3_8,
                            z_3_9,
                            z_3_10,
                              z_4_7,
                            z_4_8,
                            z_4_9,
                            z_4_10,
                              z_5_7,
                            z_5_8,
                            z_5_9,
                            z_5_10,
                              z_6_7,
                            z_6_8,
                            z_6_9,
                            z_6_10))

beta.mat = as.matrix(betas[2:length(betas)], nrow = p, byrow = TRUE)
temp = cv_data.mat %*% t(beta.mat)
logmus = as.numeric(betas["beta_0"]) + temp

# draw predicted values based on mus calculated above
set.seed(123)
cv_n = nrow(monarchs)
Ts = rtweibull(cv_n, r, (1/exp(logmus))^(1/r), 0, 100)
```

```
## Warning in (1/exp(logmus))^(1/r): longer object length is not a multiple of
## shorter object length
```

```
#calculate residuals
resid = monarchs$Age - Ts

#calculate standard deviations
meanTs = mean(Ts)
diff = Ts - meanTs
sd = sqrt((1/n)*(sum((diff)^2)) * (1+ (1/n) + (diff)^2/sum((diff)^2)))
```






We sought to perform external cross-validation on our model. The data that we used for the cross-validation was from all of England's monarchs, accessed from Wikipedia. This dataset only included one person that was still alive. As shown in the standardized residuals plot, there is an upward trend, as the birth year of the monarchs increases. Additionally, in the plot showing Predicted Lifespan and the standardized residuals, there is a downward trend, underpredicting a lot of the leaders' lifespans.

# VI. Conclusion and Further Discussion

We sought to compare Popes, US Presidents, Dalai Lamas, Chinese Emperors, and Japanese Emperors to see how their lifespans compare. Our model has found that the impact of birth year on lifespan does not change based on leadership type. Additionally, our model has found that lifespan does depend on year of birth, but the association is not strong.

One limitation worth noting is that our data does not include election year, so we have no way to control for the fact that Presidents and Popes must have a minimum lifespan. On the other hand, Dalai Lamas can be chosen close to birth and thus have a younger death date.

Additionally, there is a lack of previous existing work in the field to inform stronger prior choices, especially for the $\beta_i$ coefficients.

This model could be improved by including more predictors; for example, health varies widely among people, and could lead to a better model for prediction. For further implementations of this research question, it would be valuable to interpret economic or quality of life data as it concerns the different countries that the leaders grew up in, as the conditions that a person live in lead to changes in a person's lifespan.

# VII. References

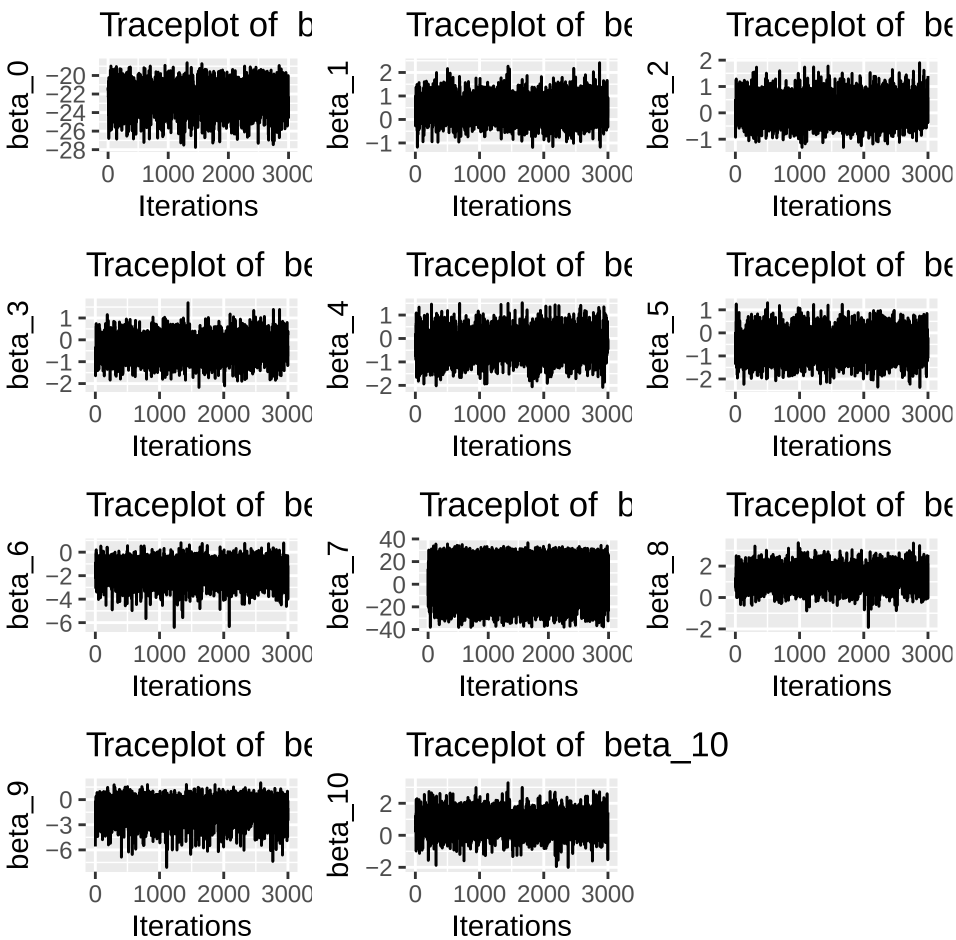
Stander, J., Dalla Valle, L., and Cortina-Borja, M. (2018). A Bayesian Survival Analysis of a Historical Dataset: How Long Do Popes Live? The American Statistician 72(4):368-375.

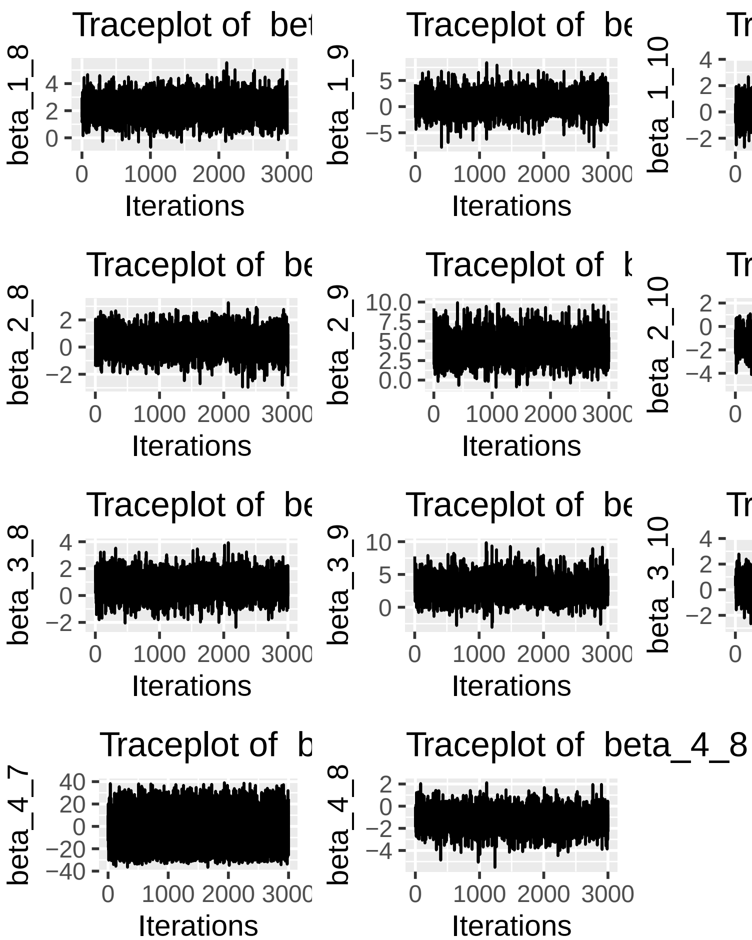
# VIII. Appendix

## A.1 Details on the Poisson "Zero-Trick" Method

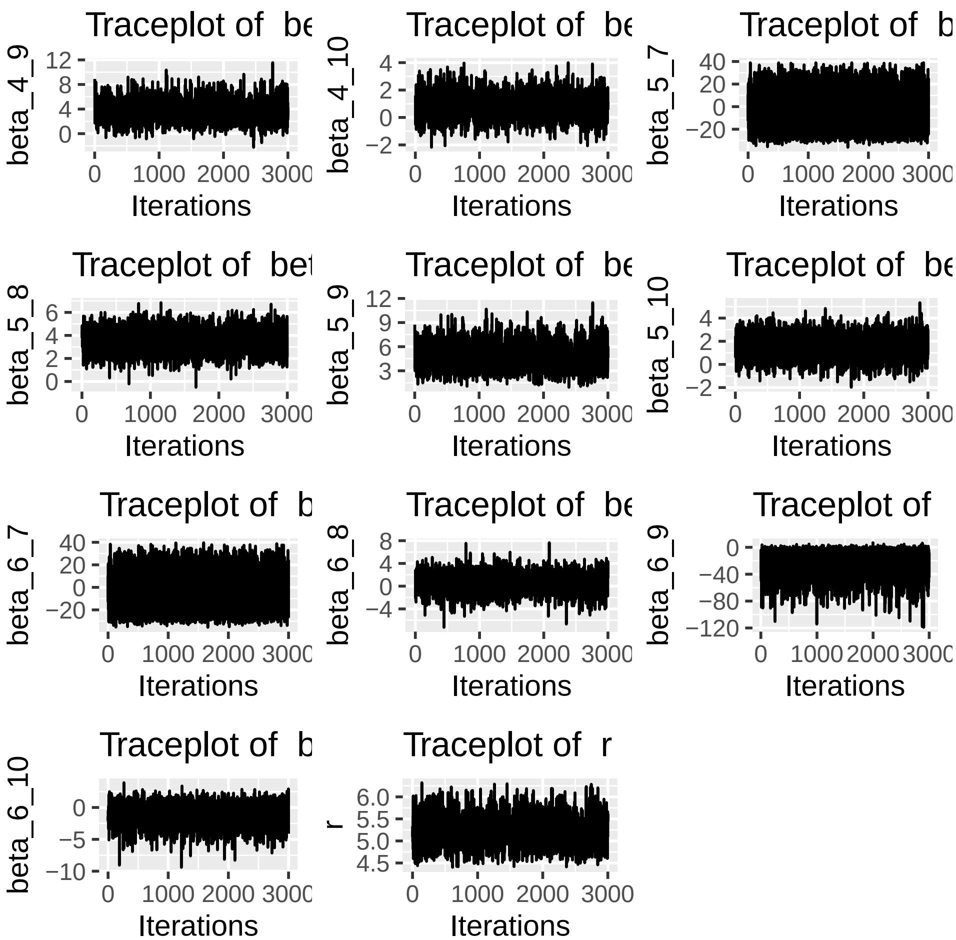
We use the Poisson "zeros trick," where a set of 0's are created and the likelihood of observing an 0 follows a $Poisson(\phi_i)$ distribution. The likelihood $L_i$ is $Pr(z_i = 0) = exp(-\phi)$. In other words, $\phi_i$ equals $-log(L_i)$. This mechanism is necessary because we don't know the lifespan of currently living people, so we need a way to model their expected survival times. The censored vector has 0 for dead leaders and 1 for alive leaders. The vector censoring_limits contains 100 (chosen as an upper bound on age) for dead leaders and current age for alive leaders. The non-100 observations in censoring_limits are saved in t_censored, and t_censored is taken to the power of r then multiplied by mu_censored to calculate phi_censored, which is the mean parameter of the Poisson likelihood described above.

### A.2 Code for Traceplots, Lag-1 Scatterplots, and Acf Plots

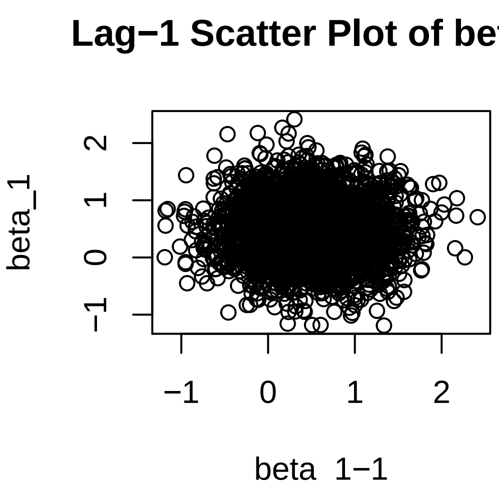


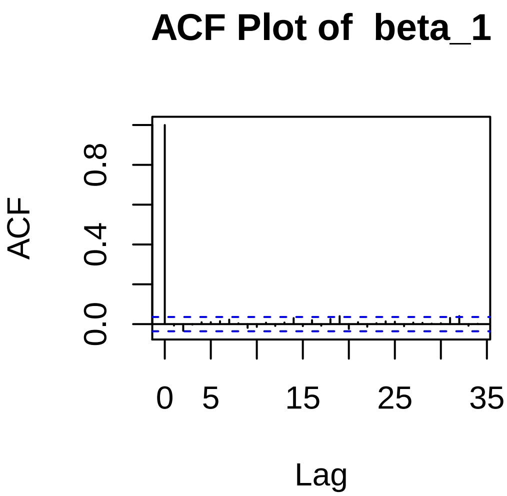


Traceplot of bet
Traceplot of be
Traceplot of be
Traceplot of be
Traceplot of b
Traceplot of be
Traceplot of be
Traceplot of be
Traceplot of be
Traceplot of b
Traceplot of beta_4_8
beta_1_8
beta_1_9
beta_1_10
beta_2_8
beta_2_9
beta_2_10
beta_3_8
beta_3_9
beta_3_10
beta_4_7
beta_4_8
Iterations




All of the traceplots and code for plotting is above.





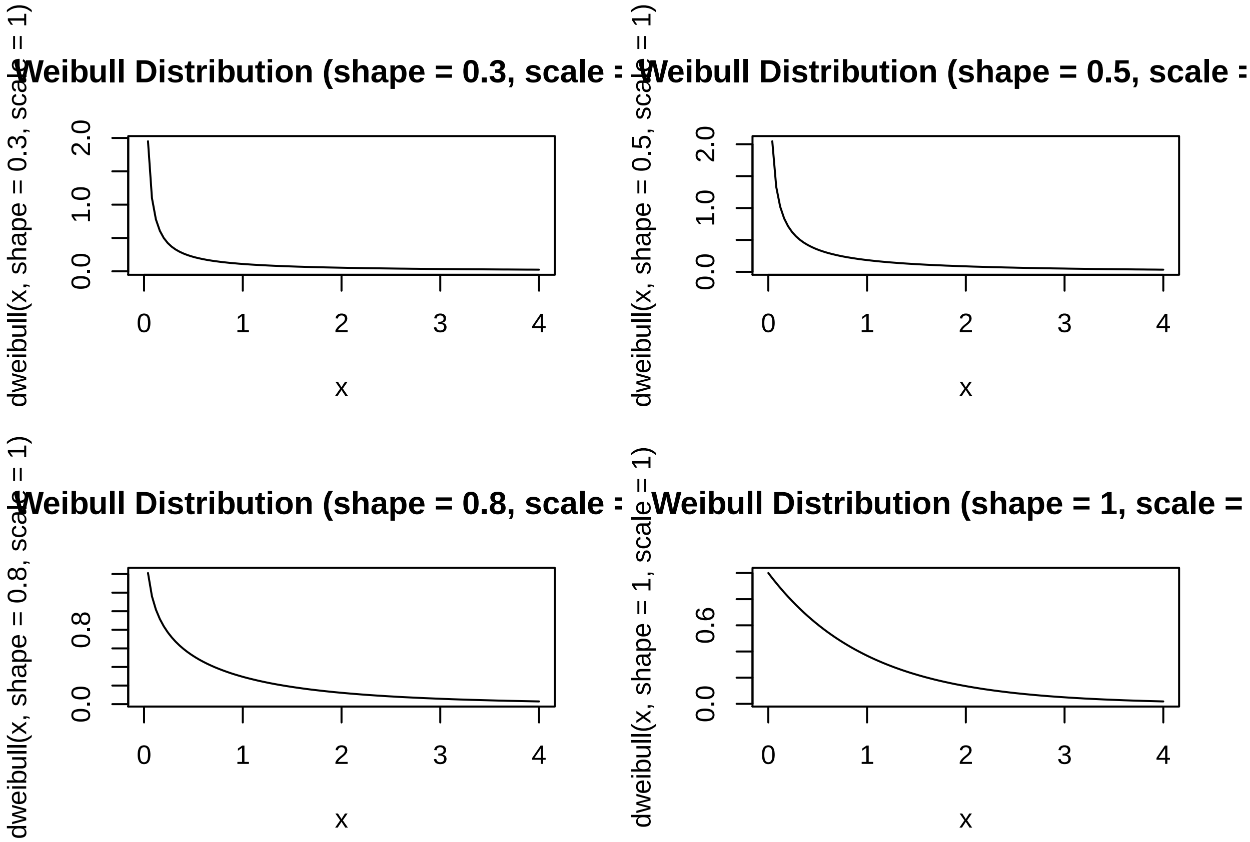

Display of lag-1 scatterplot and acf plot for $\beta_1$ is shown above, code provided allows generations of plots for all parameters.

*add old model and resid code here*

### A.3 Code for Sensitivity Analysis

### A.4 Weibull Distributions with Differing r Values

```
par(mfrow=c(2,2))
curve(dweibull(x, shape=0.3, scale = 1), from=0, to=4, main = 'Weibull Distribution (shape = 0.3, scale
curve(dweibull(x, shape=0.5, scale = 1), from=0, to=4, main = 'Weibull Distribution (shape = 0.5, scale
curve(dweibull(x, shape=0.8, scale = 1), from=0, to=4, main = 'Weibull Distribution (shape = 0.8, scale
curve(dweibull(x, shape=1, scale = 1), from=0, to=4, main = 'Weibull Distribution (shape = 1, scale = 1)
```


Weibull Distribution (shape = 0.3, scale =
dweibull(x, shape = 0.3, scale = 1)
0.0
1.0
2.0
0
1
2
3
4
x
Weibull Distribution (shape = 0.5, scale =
dweibull(x, shape = 0.5, scale = 1)
0.0
1.0
2.0
0
1
2
3
4
x
Weibull Distribution (shape = 0.8, scale =
dweibull(x, shape = 0.8, scale = 1)
0.0
0.8
0
1
2
3
4
x
Weibull Distribution (shape = 1, scale =
dweibull(x, shape = 1, scale = 1)
0.0
0.6
0
1
2
3
4
x