

3D Medical Image Segmentation Using Level Set Models and Anisotropic Diffusion

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Abstract— This paper is concerned with the segmentation of structures from 3D medical datasets where an important challenge is the need to overcome problems associated with intensity inhomogeneities, noise and proximity to other tissue of similar intensity levels resulting in weak delineating boundaries. Level sets are part of an important class of methods that utilize partial differential equations (PDEs) and have been extensively applied in image segmentation. During the evolution of the curve, anisotropic diffusion is adaptively applied to the image to remove noise while preserving boundary information. The speed of curve evolution and the segmentation result has been significantly improved compared with traditional level set methods and region-based implementation of level sets. The method is tested on 3D CTA datasets containing abdominal and thoracic aneurysm (AAA and TAA). The results demonstrate the capability of the methodology to segment an AAA and TAA from these datasets.

Keywords- level sets, anisotropic diffusion, intensity inhomogeneity, abdominal and thoracic aortic aneurysm.

I. INTRODUCTION

The 3D segmentation of anatomical structures is essential for many medical applications, both for diagnosis and visualization. Due to the massive amount of data associated with 3D datasets and the complexity of the anatomical structures, 3D segmentation methods are required to fully utilize 3D data. It is a challenging task because tissue is not homogeneous and the boundaries between structures are not clearly defined in the image data. It is often necessary to use prior knowledge about the shape or particular image properties of the structure of interest.

Image segmentation commonly utilises one of two principal approaches to classify pixels belonging to a particular object or region, either edge-based or region-based. Edge-based segmentation looks for discontinuities in image intensity [4, 7, 9], whilst region-based methods look

for uniformity within an image sub-region, based on some consistent property such as intensity, colour or texture [2, 6, 10]. In this paper we have used an edge-based approach because region-based methods do not cope well with segmentation of the aortic thrombus due to intensity inhomogeneities and weak boundaries. Region-based methods also require more parameters to be tuned and the complex structure of medical images makes the setting of these parameters more difficult.

Active contours methods, also referred to as geometric deformable models, evolve an image contour from an initial contour which can either be a curve in 2-D space or a surface in 3-D space using image forces derived from region properties that drive the search to locate the boundaries of the desired objects. Level sets provide an implementation of an active contour method based on regions or edges using partial differential equations (PDEs). Energy functional, defined according to edge or region properties, drives the zero level curve towards the object boundary. For example, a closed 3D surface propagates from an initial surface such as a sphere towards the desired boundaries through the iterative evolution of a 4D implicit function.

For stability, at each mesh point, the domain of dependence of the PDE must lie within the domain of dependence of the numerical scheme [11, 15]. We have used the Courant Friedrichs Lewy (CFL) condition as a necessary condition for stability of explicit Finite Difference Methods (FDM) applied to the level set PDEs which also satisfies a criterion for algorithmic convergence; we also assume the Neumann Boundary condition at the borders of the image domain.

We have used an edge-based level set segmentation [9] for tracking curves, localised and regularized with a Dirac delta function around the zero level set; a diffusion function has been used for smoothing and reduce connectivity at weak boundaries. The approach uses anisotropic diffusion to

aid noise suppression whilst preserving edges and then guides the search motion of the evolving contour, particularly for the detection of weak boundaries. Segmentation time can be significantly reduced by improving the convergence criteria and the diffusion equation, for which we have applied the CFL condition.

We have applied 3D level sets and anisotropic diffusion for the detection of thrombus in the cardiovascular system imaged by computed tomography angiography (CTA), which uses a contrast dye to enhance detection of the vasculature. We have segmented abdominal and thoracic aortic aneurysm on 2D CTA slices in our previous work [16] using a region-based approach. In this paper we apply it to 3D CTA image data. Segmentation of the thrombus volume is a challenge, in part because of the lack of delineating contrast at anatomical tissue boundaries, and also artifacts associated with stents and calcium deposits.

The rest of the paper is organized as follows. Section II reviews the level set formulation for edge-based active contours, anisotropic diffusion and introduces the proposed computational methods. Section III describes the numerical implementation; section IV presents the experimental results and conclusions are presented in section V.

II. LEVEL SETS AND ANISOTROPIC DIFFUSION

The level set methodology proposed by Osher and Sethian [4] has been used to track the evolution of 2D curve or 3D surface. It has been applied to the formulation of both region and edge-based approaches for image segmentation. It offers a robust and accurate technique for tracking boundaries moving under complex motions. Level set segmentation involves solving the energy-based active contour minimization problem by computation of geodesics or minimal distance curves [6, 7]. The main idea of the level set method is to represent a closed curve on the plane as the zero level set of a higher dimensional function and can efficiently handle the topological change by splitting and merging. The motion of the curve is then embedded within the motion of the higher dimensional surface. This means that the closed curves on a two or three -dimensional surface are regarded as a continuous surface of a three or four -dimensional space.

The definition of a smoothing function $\phi(x, y, z, t)$ represents the four-dimensional space while the set of definitions $\phi(x, y, z, t) = 0$ define the surface. Thus the evolution of a surface can be transformed into the evolution of a four-dimensional level set function. Given a level set function ($\phi(x, y, z, t) = 0$) whose zero level set corresponds to a surface, with the curve as the boundary, the entire surface

can be divided into two regions, either internal or external to the curve.

The evolution (motion) formula of the level set is:

$$\begin{aligned}\phi_t(x, y, z, t) + F|\nabla \phi(x, y, z, t)| &= 0 \\ \phi(x, y, z, t = 0) &= \phi_0(x, y, z)\end{aligned}$$

where F denotes a constant speed term that moves the curve either outwards or inwards in search of the contour. A common edge indicator function can be defined by a positive and decreasing function of the image gradient given by:

$$g = \frac{1}{1 + |\nabla G_\sigma * I(x, y, z)|^2}$$

where G_σ is a Gaussian kernel with standard deviation σ and $G_\sigma * I$, the convolution of the image with a Gaussian, is a smoothed version of I . We suggest $F(\phi)$ as an external energy that drives the zero level set toward object boundaries and g is the edge stopping function. We indicate

by $\frac{\partial F}{\partial \phi}$ the Gateaux derivative [11] of the functional F and

the following evolution equation:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial F}{\partial \phi}$$

is the gradient flow [11] that minimizes the functional F .

During the evolution of ϕ according to the gradient flow that minimizes the functional $F(\phi)$, the zero level curves will be moved by external energy.

F is defined as

$$F(\phi) = \lambda L_g(\phi) + \nu A_g(\phi)$$

where $\lambda > 0$ and ν are constants and

$$\begin{aligned}L_g(\phi) &= \int_{\Omega} g \delta(\phi) |\nabla \phi| dx dy dz \\ A_g(\phi) &= \int_{\Omega} g H(-\phi) dx dy dz\end{aligned}$$

where δ is the Dirac function and H the Heaviside function. By the calculus of variations [9], function F can be written

$$\frac{\partial F}{\partial \phi} = -\nu g \delta(\phi) - \lambda \delta(\phi) \text{div} \left(g \frac{\nabla \phi}{|\nabla \phi|} \right)$$

The gradient descent process for minimization of the functional F has been used and we have applied the following equation to segment the image:

$$\frac{\partial \phi}{\partial t} = \nu g \delta(\phi) + \lambda \delta(\phi) \operatorname{div} \left(g \frac{\nabla \phi}{|\nabla \phi|} \right) \quad (1)$$

To preserve sharp and / or flat shapes during evolution of the level set function, we have used a re-initialize function.

Re-initialization has been widely applied as a numerical remedy for maintaining a stable curve evolution in level set methods [4, 7]. The standard re-initialization method is to solve the following re-initialization equation:

$$\frac{\partial \phi}{\partial t} = \operatorname{sign}(\phi_0)(1 - |\nabla \phi|) \quad (2)$$

where ϕ_0 is the function to be re-initialized and $\operatorname{sign}(\phi_0)$ is the sign function.

Anisotropic diffusion is a method for image de-noising first introduced by Perona and Malik [12] that can preserve edges. During evolution of the curve, anisotropic diffusion is adaptively applied to the image to remove noise. By appropriate selection of the edge detection function, boundary information can be preserved, whilst also sharpening brightness edges. This is an important property when applied in medical imaging, where the similarity of image intensities of different tissue in adjacent regions results in weak boundaries that typically lead to poor segmentation (see fig. 1).

We use the following partial differential equation for smoothing image and denoising:

$$\frac{\partial I}{\partial t} = C(x, y, z, t) \Delta I + \nabla C(x, y, z, t) \cdot \nabla I \quad (3)$$

The diffusion strength is controlled by $C(x, y, z, t)$; the function $I(x, y, z, t)$ represents the image intensity, and C is a function of I : $C(x, y, z, t) = g(|\nabla I(x, y, z, t)|)$; if $C(x, y, z, t)$ edge attraction is a constant the equation reduces to the isotropic diffusion equation. Differences between continuous and discrete anisotropic diffusion has been examined by Weickert [13] and You [14]. They show that discrete anisotropic diffusion is well-posed even if its continuous counterpart is ill-posed.

III. IMPLEMENTATION

The Dirac function $\delta(x)$ in (1) is a smoothed variant of the following function $\delta_\varepsilon(x)$ defined by:

$$\delta_\varepsilon(x) = \begin{cases} 0 & |x| > \varepsilon \\ \frac{1}{2\varepsilon} [1 + \cos(\frac{\pi x}{\varepsilon})] & |x| \leq \varepsilon \end{cases}$$

We use the regularized Dirac with $\varepsilon = 2$ for all the experiments in this paper and all partial derivatives can be discretized as central finite differences; also the temporal derivative is discretized as a forward difference [15]. Let h

be the space step, Δt the time step and $(x_i, y_j, z_k) = (ih, jh, kh)$ the grid points. Let $\phi_{i,j,k}^m$ and $\phi_{i,j,k}^{m+1}$ denote the embedding function ϕ in the i^{th} and $(i+1)^{\text{th}}$ iterations respectively, the approximation of (1) by the above difference scheme can be simply written as:

$$\frac{\phi_{i,j,k}^{m+1} - \phi_{i,j,k}^m}{\Delta t} = L(\phi_{i,j,k}^m)$$

where $L(\phi_{i,j,k}^m)$ is the approximation of the right hand side in (1) by the above spatial difference scheme and can be expressed as the following iteration:

$$\phi_{i,j,k}^{m+1} = \phi_{i,j,k}^m + \Delta t L(\phi_{i,j,k}^m)$$

The Courant Friedrichs Lewy (CFL) condition has been applied to ensure a stability criterion for our algorithm. CFL is a necessary condition for stability of explicit Finite difference Methods (FDM) applied to Hyperbolic PDEs. We used the CFL condition [8]:

$$\frac{F \cdot \Delta t}{\Delta x} \leq 1 \quad (4)$$

Where Δt is the time step, $\Delta x = x_{i+1} - x_i$ is the grid spacing in the x -direction and F is the maximum absolute speed of all the points on the grid. This means that the evolving contour cannot cross more than one grid at each time step.

The main steps of the algorithm can be expressed as follows:

1. Initialize the level set function ϕ_0 , with an initial curve or surface inside the object.
2. Compute the time-step according to (4) that contains the CFL condition.
3. Calculate anisotropic diffusion from (3)
4. Evolve the level set function ϕ according to (1)
5. Reinitialize ϕ to the signed distant function from (2) and repeat step 4.
6. Stop when the solution is stationary:

$$|\phi^{m+1} - \phi^m| \leq \xi$$

IV. EXPERIMENTAL RESULTS

To validate and assess the robustness of the proposed method we have applied the algorithm to 2D and 3D image slices of computed tomography angiography (CTA) datasets. The task was to segment an aortic thrombus from abdominal and thoracic scan sections. Eight CTA datasets have been used in the evaluation, each containing thrombus and the segmentation is applied to approximately 20 slices

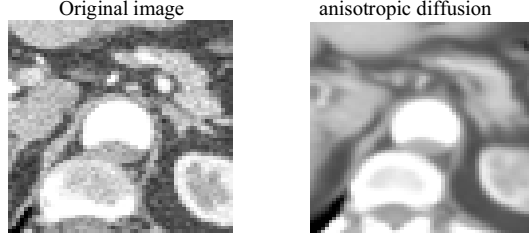


Fig.1: Example showing the effect of smoothing by anisotropic diffusion on a CTA slice section.

from each CTA dataset that contain the thrombus. These datasets were collected at Lausanne University and by MedIAR Ltd. They have noisy and inhomogeneous intensities, and exhibit weak boundaries between the thrombus and surrounding tissue. A signed distance function is used as the initial level set function so that new contours can readily emerge and curve evolution is considerably faster than the evolution from an initial function such as a binary step function. We have utilised anisotropic diffusion as a pre-processing step to initially smooth the CTA dataset (see figure 1). We also apply anisotropic diffusion of the level set algorithm 30th iteration every. This ensures that boundary information is preserved, whilst sharpening brightness edges which allow the curve to evolve more easily.

The methodology has been tested with the following set of parameters which were empirically determined:

$$\sigma = 2, \lambda = 3, \Delta t = 3, \nu = 3$$

Using a larger time step (Δt) can speed up the evolution but this may lead to errors in boundary location. A smaller scale σ can produce a more accurate location of the object boundaries, whilst a larger value of σ is more independent of the location of the initial contour. The coefficient ν can be positive or negative, depending on the relative position of the initial contour to the object of interest so that the contours can shrink or expand. The algorithm requires a number of iterations to generate a stable boundary and to ignore calcium deposits that may be found near the thrombus boundary. The initial contour (a sphere) is manually placed on the thrombus to enable the aorta and thrombus to be segmented either separately or together.

Traditional and region-based level set methods [2, 3] generally fail to segment images with significant intensity inhomogeneity as shown in figures 2, 4. In these circumstances some part of the background/foreground is incorrectly identified as foreground/background, halting the curve evolution. The results presented in figure 3 show the

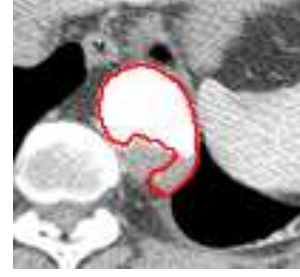


Fig. 2: Result of unsuccessful 2D segmentation of an abdominal aortal thrombus showing the final contour (in red) computed using the region-based function [16].

2D delineation of the boundary of both the aorta (the bright region) and thrombus (the small darker region adjacent and below the bright region) using the edge indicator function. The weak boundary on the lower edge of the thrombus region has been reliably detected. Figure 3 shows an example where the thrombus encircles the entire vessel wall. As shown in figures 4 and 5 our method using anisotropic diffusion results in more reliable detection of the thrombus and vessel. As can be seen some areas of the thrombus are missed and also some irrelevant areas are segmented in the result. Also our method takes shorter time in comparison with the method without anisotropic diffusion.

The number of iterations required using anisotropic diffusion was 450 whilst 800 were required with conventional (Gaussian) smoothing. The results of the segmentation with and without diffusion are depicted in Fig.4 and 5. Because of the different colors and weak boundaries associated with aortal thrombus, region-based methods are not suitable for segmenting them and the edge-based method and anisotropic diffusion show promising results. The algorithm can reliably segment the aorta and thrombus, either separately or together by choosing a suitable initial contour and curvature. Finally, figure 5 shows the operation of the edge-based approach, 3D detecting the aortic thrombus in several slices and 3D view of aortic, thrombus and aortic thrombus region is shown in figure 6. Similar results were achieved with the other six datasets.

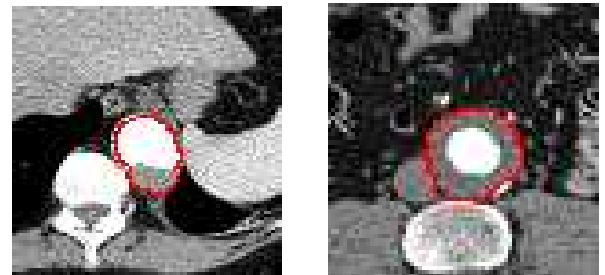


Fig.3: Result of 2D segmentation for two examples of an abdominal aortal thrombus showing the final contour (in red) computed using the edge detection function.

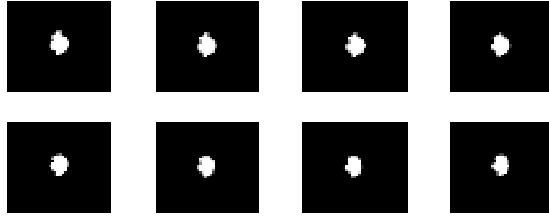


Fig.4: Result (consecutive 2D sections) of unsuccessful 3D segmentation of an abdominal aortal thrombus showing the final contour computed without anisotropic diffusion.

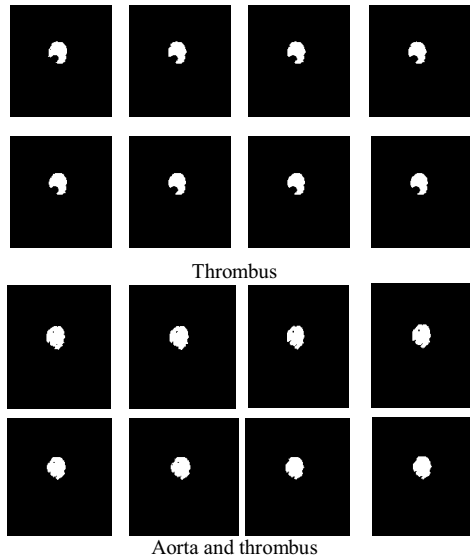
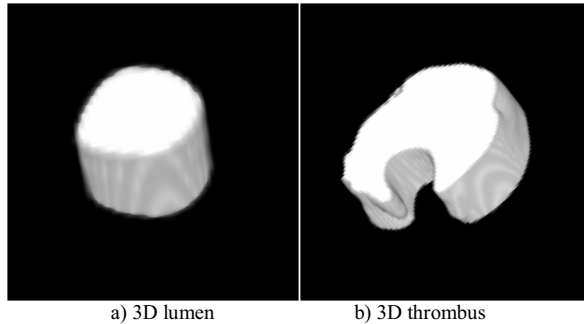
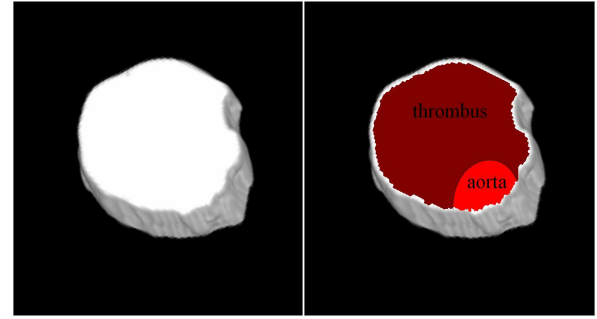


Fig.5. Result (consecutive 2D sections) of 3D segmentation for an abdominal aorta thrombus without aorta (first) and with aorta (second) computed in CTA data using the edge detection function with anisotropic diffusion.



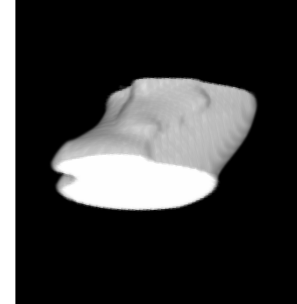
a) 3D lumen

b) 3D thrombus



c) 3D aortal thrombus

d) labeling of final segmentation



e) Second example of final segmentation of aorta + thrombus.

Fig.6. (a - d) Result of reconstruction of 3D segmentation for abdominal aorta thrombus computed using the edge detection function with 3D view.
e) Example from a second dataset

V.CONCLUSIONS

We have presented an active contour model based on an edge indicator that uses anisotropic diffusion which is better adapted to the problem of intensity inhomogeneities and weak boundaries in the image. The method was demonstrated by segmenting the ascending and descending thoracic aorta thrombus and the abdominal aorta thrombus. The effectiveness of the algorithm has been validated on a 3D CTA dataset to assess its performance in terms of accuracy.

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