CS 4830/6830
Project 3: Continuous System Simulation

The purpose of this programming assignment is to introduce the concept of continuous system simulation. In this assignment, you will simulate a simple biological model. This model is derived from the well-known logistic growth equation:

$$\frac{dN}{dt} = \lambda \cdot N \cdot \left(1 - \frac{N}{N_{max}}\right)$$

where N is the mass of some biological species (e.g. biomass of a deer population) and λ is a constant controlling the growth of the biomass. For example, if N_{max} =1000 and λ is 2.0, this equation says the change of the biomass is determined by a growth factor 2N and a suppression term -(N^2 / 500). These terms roughly correspond to an average birth rate and death rate for a population.

We can extend this idea to model two populations, a prey species and predator species, by introducing two equations of the form:

$$\frac{dN_1}{dt} = \lambda_1 \cdot N_1 \cdot \big(1 - (\alpha \cdot N_2)\big)$$

$$\frac{dN_2}{dt} = -\lambda_2 \cdot N_2 \cdot (1 - (\beta \cdot N_1))$$

These equations are the famous Lotka-Volterra equations describing predator-prey dynamics. Examining the first equation, we see the change in the amount of prey (N_1) is controlled by a growth term ($\lambda_1 \cdot N_1$) that depends on the current amount of prey and a suppression term ($\lambda_1 \cdot \alpha \cdot N_1 \cdot N_2$) that depends on the product of the amount of predator and prey. As the amount of predator increase, the rate of change in the amount of prey will become negative and the amount of prey is reduced. Similarly, the second equation is governed by two terms. The growth term ($\lambda_2 \cdot \beta \cdot N_1 \cdot N_2$) says that the rate of increase in the amount of predator grows as the amount of prey increases. The suppression term ($-\lambda_2 \cdot N_2$) adjusts the rate of change in the amount of predator in proportion to the current amount of predator. This system is in equilibrium when $dN_1/dt = 0$ and $dN_2/dt = 0$. If the system is not in equilibrium (e.g. $dN_1/dt < 0$ and $dN_2/dt > 0$), the biomass of one population will drop and the other will rise. At some point, one of the suppression terms will start to dominate its equation and the sign of the derivate will change causing the mass of the populations to move in the opposite direction. The net result is the system oscillates. There is no closed form solution for the Lotka-Volterra equation so the only way to study them is through simulation experiments.

To make this problem concrete, imagine the owner of golf course has asked you to model of the relationship between the growth of grass and the insects that destroy the grass. There are many variables that affect the growth of grass, but after careful study you determine that the total mass of grass (G) is influenced by two constant parameters, the birth rate (β_g) of new grass and the death rate of old grass (δ_g). In addition, grass dies in proportion to the mass of insects (I) that feed on the grass. The insect population has a constant birth rate (β_i) and death rate (δ_i), but the birth rate is also proportional to the mass of grass. Given this information, you quickly formulate the following relationship (looks familiar doesn't it).

$$\frac{dG}{dt} = \beta_g \cdot G(t) - \delta_g \cdot G(t) \cdot I(t)$$

$$\frac{dI}{dt} = \beta_i \cdot G(t) \cdot I(t) - \delta_i \cdot I(t)$$

TASK 1: Your first task is to simulate the Grass/Insect (Lotka-Volterra) model of predator-prey dynamics by deriving a simulation program from the base class Dynamics discussed in class. The source code for the Dynamics class is provided in the Pilot dropbox associated with this assignment.

Try the following parameter values as a starting point for your simulation experiments:

$$\beta_g = 0.05$$

$$\beta_i = 0.0005$$

$$\delta_g = 0.001$$

$$\delta_i = 0.01$$

with the initial conditions set to:

$$G(0) = 150$$

$$I(0) = 50$$

This should produce an oscillating system for both grass and insects. Now vary the parameters $\beta_g, \beta_i, \delta_g, \delta_i, G(0)$, and $I(0)$ Try a Euler integration step size of dt=0.005 and a print out the state after every 1000 steps (dt).

TASK 2: After you have gained a fundamental understand of the behavior of the system, modify the system to incorporate a third variable that reflects the golf course owner interest in applying pesticide to control the insects. Assume that once applied, pesticide simply decays at a constant rate. Further assume that the application of pesticide kills some amount of grass and insects over time and the effect is proportional to the mass of pesticide remaining at any time instant. Modify the original equations to reflect the influence of pesticide on the growth/death of grass and insects. Please the define the meaning of each parameter and variable introduced in your solution. Pick an initial setting of parameters that maintain the oscillations of the grass/insect system. Using the same parameter setting for birth and death of grass, introduce the pesticide equation and modification to the original system to incorporate the effect of pesticide, finally examine the behavior of the new system for a variety of different levels of initial pesticide and parameter settings for the effect of pesticide.

The goal of this assignment to come to some basic understanding of the qualitative nature of this system through the use of simulation experiments. When you have gathered enough information to explain the behavior and relationship among the variables and initial conditions, submit a short (3-5) page report discussing the qualitative behavior of this system along with supporting graphs and tables.