CHALMERS

SSY130 Applied Signal Processing

Project 1A: Acoustic Communication System

Group 9

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I. Introduction

In this study, we analyze an OFDM (Orthogonal Frequency Division Multiplexing) system's performance with several parameters: Cyclic Prefix length(N_{cp}), Signal-to-Noise Ratio(snr), and Synchronization error($sync\_err$) under two distinct scenarios: known and unknown channel conditions. The focus is on assessing the system's effectiveness through the metrics of Bit Error Rate (BER) and Error Vector Magnitude (EVM). This comparison aims to highlight the impact of channel knowledge on OFDM system performance, providing insights crucial for optimizing communication systems in varying channel environments.

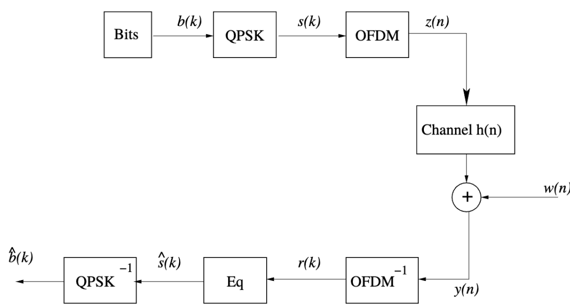

Figure 1: Block diagram for part A.

FIG. 1: Block diagram from project specification

II. Question 1: Signal encoding/decoding from the bit stream to symbol

Start with the default case: The ideal (trivial) channel h_1 is used, no noise is added ($snr = inf$, i.e. $w(n) = 0$), the cyclic prefix(N_{cp}) is set to zero, there is no synchronization error, and the channel is known. We will repeatedly return to this setup, so make a note as to these settings. (The choice of a channel without any dynamics and no noise implies that $y(n) = z(n)$ in Eq.10)

A. Investigate and consider how this setup affects the received symbols, both before $r(k)$ and after $\hat{s}(k)$ equalization. (See the plots that are generated in the proj1a.m script). How are the transmitted symbols related to the received symbols? What is the EVM and the BER? Why?

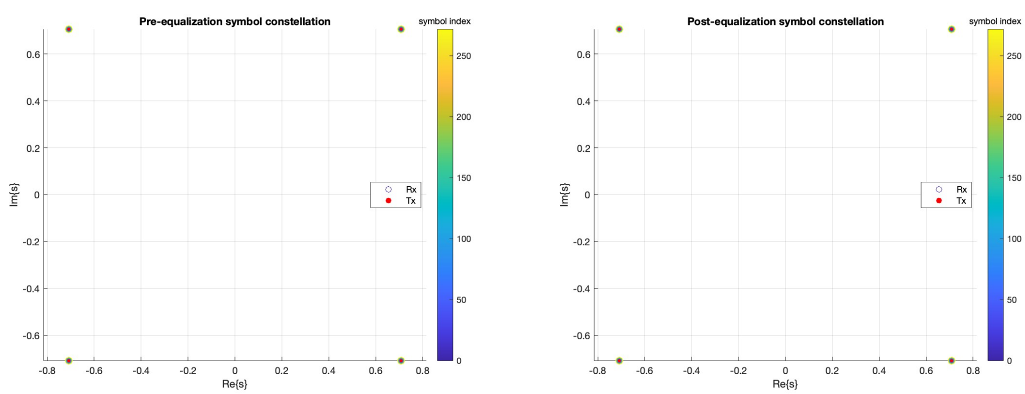

FIG. 2: pre- and post-equalization symbol constellation of ideal channel

Transmitted: `'Alice:  Would you tell me, please, which way I ought to go from here?'`
Received: `'Alice:  Would you tell me, please, which way I ought to go from here?'`
EVM: 3e-16, BER: 0

- With the ideal setup (Figure 2), the transmitted and received symbols are identical, indicated by an EVM of 3×10^{-16} and a BER of 0. This perfect alignment in the constellation diagram implies that the channel is relatively free from impairments and effective equalization techniques are employed to compensate for channel distortions.

B. What influence does the cyclic prefix have on the ability of our communication scheme to work? How long does the cyclic prefix need to be to give a near-zero EVM? Why?

When $N_{cp} = 0$ (i.e. the cyclic prefix), EVM: 3e-16, BER: 0
$N_{cp} = 10$, EVM: 3e-16, BER: 0
$N_{cp} = 20$, EVM: 3e-16, BER: 0
$N_{cp} = 30$, EVM: 3e-16, BER: 0
$N_{cp} = 50$, EVM: 3e-16, BER: 0
$N_{cp} = 100$, EVM: 3e-16, BER: 0

- The cyclic prefix shows remarkable consistency in this communication scheme. Varying lengths (0, 10, 20, 30, 50, 100) consistently yield an EVM of 3×10^{-16} and a BER of 0, indicating its non-impact under ideal channel conditions.
- Theoretically, to achieve a near-zero EVM, the cyclic prefix should be at least as long as the maximum delay spread of the channel, ensuring that the received symbols are free from inter-symbol interference(ISI) and distortion.

C. Now, back to zero cyclic prefix, change the channel to h_2 (magnitude-scaling) and h_3 (constant phase-shift). Notably, with these channels we are ensured that $y(n) = \alpha \cdot z(n)$, for a scalar, possibly complex, value α. How does this choice of channel affect the pre- and post-equalization constellation diagrams, the EVM, and the BER? What is α in both of these cases?

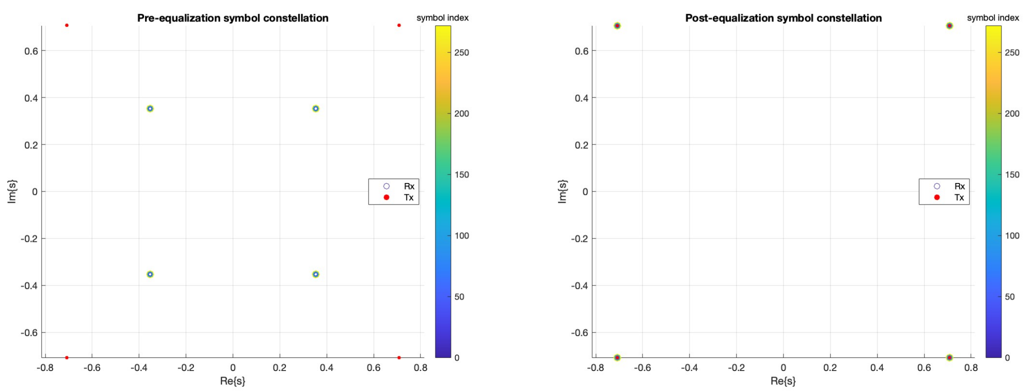

FIG. 3: h_2 channel, EVM: 3e-16, BER: 0

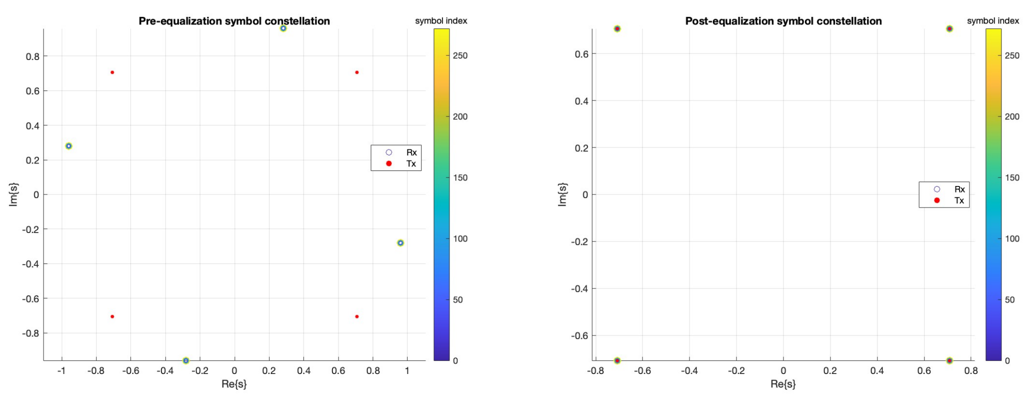

FIG. 4: h_3 channel, EVM: 3.4e-16, BER: 0

- The two channels maintain the fidelity of the transmission.

- Channels h_2 (magnitude-scaling) yields the EVM value of 3×10^{-16} and BER value at 0, with the scaling factor $\alpha = 0.5$. The magnitude scaling factor would affect the distances between the constellation points.
  1. For the pre-equalization diagram: since the scaling factor is less than 1, the constellation points will be compressed towards the origin.
  2. For the post-equalization diagram: the equalization algorithm will adjust the received symbol amplitudes to counteract the magnitude scaling introduced by the channel, and restore the original spacing between the constellation points.
- Channels h_3 (constant phase-shift) yields the EVM value of 3.4×10^{-16} and BER value at 0, with the scaling factor $\alpha = exp(1j * 1/2)$, which is a phase shift by 1/2 radian. The constant phase shift would cause a uniform rotation of the constellation points by the same angle.
  1. For the pre-equalization diagram: we can see that all the constellation points rotate by the same angle and form a circle.
  2. For the post-equalization diagram: the equalization process will also apply a phase correction to counteract the constant phase shift, aligning the symbols back to their original phase positions.

D. Now we will consider the effect of synchronization error. Return back to the initial setup. Initially test a synchronization error of ±1. What happens when it is increased? Why does this have such a large influence on the EVM and BER?

For $sync\_err = 1$, EVM: 1.4, BER: 0.496
Transmitted: `'Alice:  Would you tell me, please, which way I ought to go from here?'`
Recieved: `W_t62_5_066_40_E_E_E_E_E_m here?`

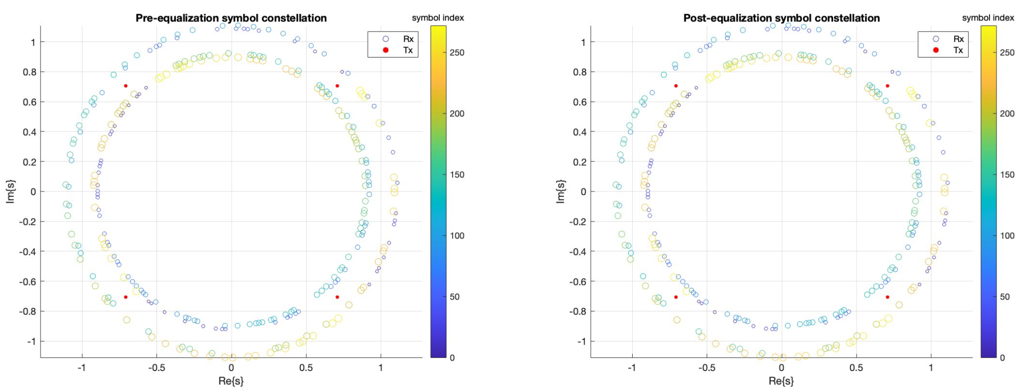

FIG. 5: $sync\_err = 1$

For $sync\_err = -1$, EVM: 1.41, BER: 0.5
Transmitted: `'Alice:  Would you tell me, please, which way I ought to go from here?'`
Recieved: Alice: `WJ_E_E_E_IE_,_5_1>_5_15_3_om here?`

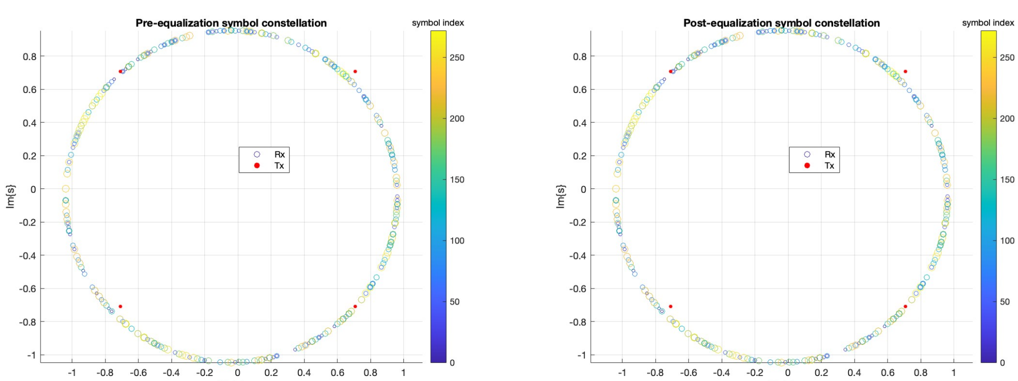

FIG. 6: $sync\_err = -1$

- Compared with the ideal case($sycn\_err = 0$), which EVM: 3e-16, BER: 0. Synchronization errors (± 1) significantly deteriorate the signal quality. EVM jumps to 1.4 and 1.41, while BER increases to 0.496 and 0.5, respectively (Figures 5 and 6). This demonstrates the system's high sensitivity to timing misalignment while *sycn_err* controls the starting point when the receiver starts to 'listen', if the timing is wrong, the misalignment will lead to symbol misalignment and cause interference between adjacent symbols.

E. Finally return to the initial setup and start adding some noise to the system, e.g. set $snr = 30$. What happens, and why? What about for $snr = 5$?

For $snr = 30$, EVM: 0.0281, BER: 0
Transmitted: `'Alice:  Would you tell me, please, which way I ought to go from here?'`
Received: `'Alice:  Would you tell me, please, which way I ought to go from here?'`

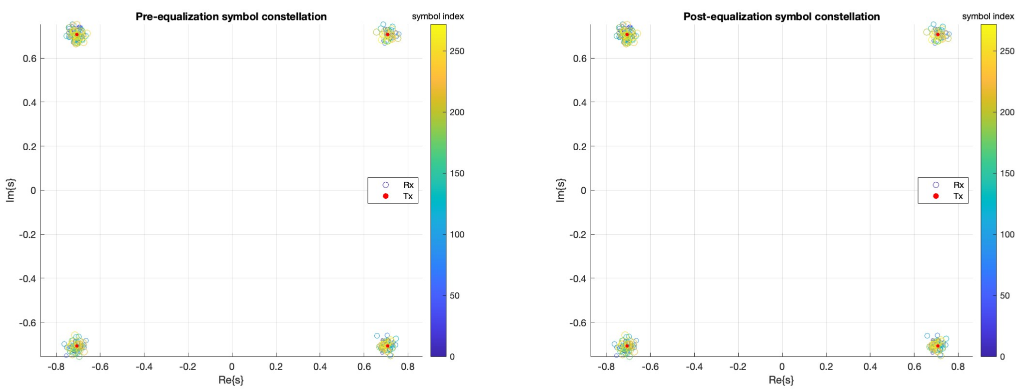

FIG. 7: $snr = 30$

For $snr = 5$, EVM: 0.499, BER: 0.0221
Transmitted: `'Alice:  Would you tell me, please, which way I ought to go from here?'`
Received: `'Alice:  Would you tull(md,_pleese, _hich say I oueht to co froo$here?`

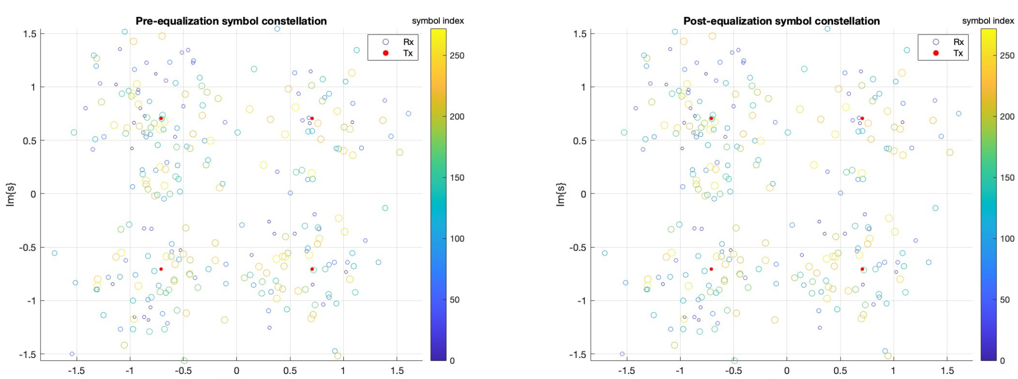

FIG. 8: $snr = 5$

- Introducing noise (Figures 7 and 8) shows a clear degradation of signal quality. A higher SNR value in decibels represents a better signal quality, with a larger separation between the signal power and noise power.

- At an SNR of 30, the EVM is 0.0281 with a BER of 0, indicating manageable noise impact. However, at an SNR of 5, the EVM escalates to 0.499 and BER to 0.0221, revealing significant degradation.

III. Question 2: Conversion to an analog coding/decoding technique; orthogonal frequency division multiplexing (OFDM)

Let us now progress to (slightly) more realistic scenarios. Return to the initial setup, but change the channel model to h4 (the low-pass system) and set the cyclic prefix to 60. Notably, this system does have dynamics, so generally, $y(n) \neq \alpha \cdot z(n)$.

A. How are $H(k)$ and the pre-equalization constellation diagram related? Your answer to this question can be fairly brief and based only on visual observation of the plots of $H(k)$ and the pre-equalization constellation diagram

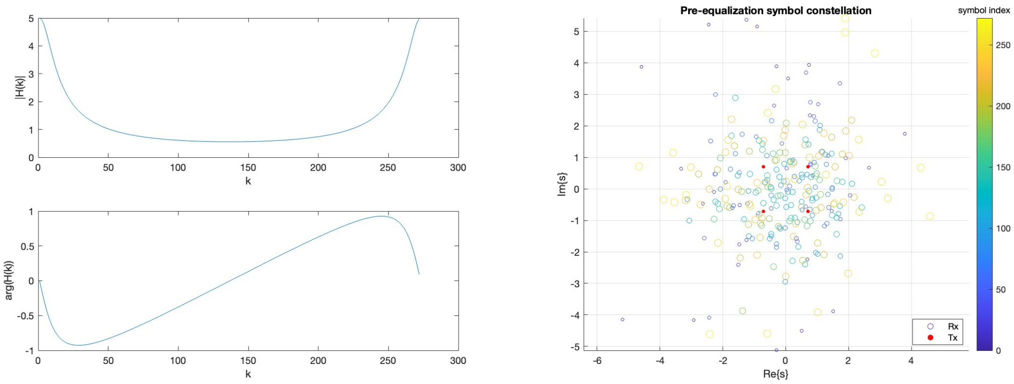

FIG. 9: The channel gain H(k) and the pre-equalization constellation diagram

- The above 2 plots are related in the context of equalization.
- H(k) represents the frequency response of the channel at each subcarrier, indicating how the channel affects transmitted symbols at different frequencies and estimating the channel distortion.
- The pre-equalization constellation diagram shows the desired positions of symbols in the complex plane before transmission through the channel.
- During the equalization process, which adjusts the received symbols based on H(k) to align them with their intended positions in the pre-equalization constellation diagram, reducing channel-induced distortions and improving symbol recovery.

B. What is the BER and EVM of the system with this setup? What happens when N_{cp} is increased/decreased? Is there some "magic" number that makes EVM = 0 (or at least $< 10^{-15}$, which is within the machine's accuracy of zero)? Why is it this particular value? How is the BER influenced by the choice of N_{cp}?

Current setup:

$N_{cp} = 60$: EVM: 5.99e-16, BER: 0

When N_{cp} decrease:

$N_{cp} = 55$: EVM: 2.81e-07, BER: 0
$N_{cp} = 50$: EVM: 3.5e-07, BER: 0
$N_{cp} = 40$: EVM: 1.18e-05, BER: 0
$N_{cp} = 20$: EVM: 0.000611, BER: 0

When N_{cp} increase:

$N_{cp} = 80$: EVM: 5.99e-16, BER: 0
$N_{cp} = 100$: EVM: 5.99e-16, BER: 0

The magic number is $N_{cp} = 60$, when $N_{cp} >= 60$, the EVM value will stably equal to 5.99e-16.

- The OFDM system's performance varies with N_{cp}. The EVM and BER are lowest (5.99e-16 and 0, respectively) at $N_{cp} = 60$, suggesting this as the optimal cyclic prefix length for this setup. To demonstrate the correlation of N_{cp} and the other values, when N_{cp} varies from 20 to 60, the EVM drops significantly while it remains stable when the N_{cp} exceeded 60.
- As for why the "magic number" has to be 60. Our assumption is that since we are transmitting signals under a lowpass filter, due to the transition band effect, 60 could be the threshold that well-preserved most of the useful part of our message.

C. Modify the channel definition to $h_4' = 0.99n$. How important is the choice of N_{cp} with this channel, both with respect to the BER and EVM

$N_{cp} = 0$: EVM: 0.699, BER: 0.068
$N_{cp} = 20$: EVM: 0.471, BER: 0.0129
$N_{cp} = 40$: EVM: 0.285, BER: 0
$N_{cp} = 50$: EVM: 0.102, BER: 0
$N_{cp} = 55$: EVM: 0.0837, BER: 0
$N_{cp} = 58$: EVM: 0.0064, BER: 0
$N_{cp} = 59$: EVM: 3.45e-15, BER: 0
Current setup $N_{cp} = 60$: EVM: 3.26e-15, BER: 0
$N_{cp} = 61$: EVM: 3.84e-15, BER: 0
$N_{cp} = 70$: EVM: 3.58e-15, BER: 0
$N_{cp} = 80$: EVM: 3.26e-15, BER: 0
$N_{cp} = 90$: EVM: 3.58e-15, BER: 0
$N_{cp} = 100$: EVM: 3.26e-15, BER: 0

- Modifying to h_4' while keeping $N_{cp} = 60$ results in increased EVM and BER, as depicted in the data. This change underscores the significant influence of channel characteristics on OFDM system performance. But $N_{cp} = 60$ is still the 'magic number' which performs the smallest BER and EVM value.

IV. Question 3: Transmission channel equalization

Now, let us move to the fairly realistic case: return to the initial setup between each of these subquestions, but keep channel_known = false

A. Select the ideal channel h_1, set N_{cp} to zero and investigate the effect of nonzero sync error. How and why is this different compared to the knownchannel scenario?

Known Channel with $N_{cp} = 0$:
sync error = 1: EVM: 1.4, BER: 0.496
sync error = -1: EVM: 1.41, BER: 0.5
Unknown Channel with $N_{cp} = 0$:
sync error = 1: EVM: 0.132, BER: 0
sync error = -1: EVM: 0.206, BER: 0

- Exploring the effect of nonzero synchronization error on the ideal channel h_1 with no cyclic prefix ($N_{cp} = 0$), we observe significant changes in the EVM and BER. Under unknown channel conditions, with a sync error of ± 1, the EVM increases to 0.132 and 0.206, respectively, but the BER remains at 0. This indicates that while the synchronization error affects the signal's alignment, leading to increased EVM, it does not necessarily result in bit errors under these specific conditions.
- In the unknown channel, since the channel-impulse-response is unknown, some method must be employed to establish sufficient information about the channel. The reason why it performs better is because it allows for accurate channel estimation using pilot symbols. By comparing the received and transmitted pilot symbols, enables effective equalization and reduces distortion, resulting in lower EVM and BER.

B. Select the lowpass channel h_4. How large does the cyclic prefix need to be to give a nearzero EVM? Is this the same value as before? Why? Now, keep the cyclic prefix at a large enough value and change the SNR. How does the noise level influence the BER and EVM? How sensitive is this setup to noise compared to the known-channel scenario? Why

To test the cyclic prefix number in order to give a near-zero EVM:
lowpass h_4 with cyclic prefix=0: EVM: 0.254, BER: 0
lowpass h_4 with cyclic prefix=20: EVM: 0.00152, BER: 0
lowpass h_4 with cyclic prefix=40: EVM: 1.48e-05, BER: 0
lowpass h_4 with cyclic prefix=50: EVM: 2.27e-06, BER: 0
lowpass h_4 with cyclic prefix=58: EVM: 2.62e-07, BER: 0
lowpass h_4 with cyclic prefix=59: EVM: 7.85e-16, BER: 0
lowpass h_4 with cyclic prelix=60: EVM: 7.15e-16, BER: 0
Keep the cyclic prefix at a this value and change the SNR (unknown — known):
lowpass h_4 with cyclic prefix=60, snr=inf: EVM: 7.15e-16, BER: 0 — EVM: 5.99e-16, BER: 0
lowpass h_4 with cyclic prefix=60, snr=0: EVM: 2.81, BER: 0.364 — EVM: 1.98, BER: 0.278

lowpass h_4 with cyclic prefix=60, snr=10: EVM: 2.28, BER: 0.125 — EVM: 0.625, BER: 0.068
lowpass h_4 with cyclic prefix=60, snr=20: EVM: 0.311, BER: 0 —EVM: 0.198, BER: 0
lowpass h_4 with cyclic prefix=60, snr=40: EVM: 0.0302, BER: 0 — EVM: 0.0198, BER: 0

- For the lowpass channel h_4, varying cyclic prefix lengths reveal its critical role in achieving near-zero EVM. With no cyclic prefix, the EVM is substantially high, but increasing the cyclic prefix to 59 results in an EVM of $7.15e-16$. This dramatic reduction in EVM underscores the importance of the cyclic prefix in mitigating intersymbol interference in lowpass channels. Further, adjusting the SNR levels (inf, 20, 0, ...) while maintaining a cyclic prefix of 60 shows varying degrees of EVM and BER, highlighting the system's sensitivity to noise. The higher the SNR, the lower the EVM and BER, demonstrating the system's ability to maintain signal integrity in less noisy environments.
- Compared to the known channel setup, while the cyclic prefix increases, the unknown channel's EVM and BER drop slower than the known channel. Since in the known channel, the receiver can leverage additional information about the channel to enhance equalization, instead of the unknown channel which affects the cyclic prefix alone. This optimized equalization can result in a faster reduction of EVM and BER compared to the unknown channel setup,

C. Select the multi-path channel h5. Set N_{cp} to a large enough value (which value is this, and why?). Why do we always have some bit errors despite the absence of any noise and sync error

Initial setup with $N_{cp} = 0$: EVM: 0.659, BER: 0.0404
$N_{cp} = 1$: EVM: 0.756, BER: 0.0294
$N_{cp} = 7$: EVM: 0.36, BER: 0.0184
$N_{cp} = 8$: EVM: 0.239, BER: 0.0184
$N_{cp} = 10$: EVM: 0.239, BER: 0.0184
$N_{cp} = 20$: EVM: 0.239, BER: 0.0184
BER decrease from 0.0294 to 0.0184 from $N_{cp} = 0$ to $N_{cp} = 8$, with the value $N_{cp} > 8$, the BER value will only stay at 0.0184.
EVM decrease from 0.756 to 0.239 from $N_{cp} = 0$ to $N_{cp} = 8$, with the value $N_{cp} > 8$, the EVM value will only stay at 0.239.

- On the multi-path channel h_5, setting N_{cp} to different values provides insight into the behavior of the system in a multi-path environment. With $N_{cp} = 0$, the EVM is 0.659, and the BER is 0.0404. Increasing N_{cp} to 8 reduces the BER to 0.239, indicating some mitigation of multi-path effects. However, further increases in N_{cp} do not significantly decrease the BER, which stabilizes at 0.0184. This suggests that while a sufficient cyclic prefix length is necessary to reduce multi-path effects to a certain extent, the length of the cyclic prefix should be chosen to be larger than the maximum delay spread of the channel, but it cannot completely eliminate bit errors. These persistent errors are likely due to the inherent characteristics of the multi-path environment, such as Inter-Symbol Interference (ISI) or nonlinearities and distortions.