AGTA notes

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1 Nash equilibrium

Nash equilibrium occurs when two players are simultaneously playing their best strategy and no player can benefit by deviating from this strategy. Every game has a mixed strategy Nash equilibrium but not all games have a pure strategy Nash.

1.1 Methods

1.2 Informal methods

The following is a quick, informal method of finding Nash equilibrium although it is not advised to use definitively.

Example

Consider the following matrix:

$$\begin{array}{c} \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 5,1 & 2,0 & 2,2 \\ 0,4 & 1,5 & 4,5 \\ 2,3 & 3,6 & 1,0 \end{pmatrix} \end{array}$$

In order to find the Nash Equilibrium, we need to find each players best responses to each of the other players best responses. Starting with player 1 (the rows), we highlight the best response that player 1 can make if player 2 plays A:

$$\begin{array}{c} \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} \mathbf{5},1 & 2,0 & 2,2 \\ 0,4 & 1,5 & 4,5 \\ 2,3 & 3,6 & 1,0 \end{pmatrix} \end{array}$$

And then player 1's best response if player 2 plays B or C:

$$\begin{array}{c} \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} \mathbf{5}, 1 & 2, 0 & 2, 2 \\ 0, 4 & 1, 5 & \mathbf{4}, 5 \\ 2, 3 & \mathbf{3}, 6 & 1, 0 \end{pmatrix} \end{array}$$

We then do the same for player 2 for responses to player 1, obtaining.

$$\begin{array}{c} \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} \mathbf{5}, 1 & 2, 0 & 2, \mathbf{2} \\ 0, 4 & 1, \mathbf{5} & \mathbf{4}, \mathbf{5} \\ 2, 3 & \mathbf{3}, \mathbf{6} & 1, 0 \end{pmatrix} \end{array}$$

As (4, 5) and (3, 6) are both underlined, these are the Nash equilibriums of the game.

1.3 Proofs and properties

2 Iterative Dominance

We can intuitively see that if one strategy is better than another(i.e it is a dominant strategy) than the dominated strategy will never be played. The order that iterative dominance is conducted does not effect the result.

There are two types types of dominance - strict and weak dominance.

If a game can be reduced to one strategy by strict dominance then we can say that there is only one Nash Equilibrium of the game.

Strictly dominated strategy

A strictly dominated strategy can never be a best reply.

Thus, we should remove it as it will never be played.

3 Notation and Definitions

$u_i(a_i, a_{-i})$: the pay off for playing a_i, regardless of all other strategies
$u_i(a_i, a_{-i}) < u_i(a`_i, a_{-i})$: utility of $a`_i$ is better than a_i

4 Examples

4.1 Nash Equilibrium