AGTA notes

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Contents

Chapter 1

Mixed and pure strategies

Chapter 2

Nash equilibrium

Nash equilibrium occurs when two players are simultaneously playing their best strategy and no player can benefit by deviating from this strategy. Every game has a mixed strategy Nash equilibrium but not all games have a pure strategy Nash.

2.1 Methods

2.2 Informal methods

The following is a quick, informal method of finding Nash equilibrium although it is not advised to use definitively.

Example

Consider the following matrix:

$$
\begin{array}{c c}
 & \begin{array}{ccc} A & B & C \end{array} \\
\begin{array}{c} D \\ E \\ F \end{array} & \begin{pmatrix} 5,1 & 2,0 & 2,2 \\ 0,4 & 1,5 & 4,5 \\ 2,3 & 3,6 & 1,0 \end{pmatrix}
\end{array}
$$

In order to find the Nash Equilibrium, we need to find each players best responses to each of the other players best responses. Starting with player 1 (the rows), we highlight the best response that player 1 can make if player 2 plays A:

$$\begin{array}{c|c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} D \\ E \\ F \end{array} & \begin{pmatrix} \mathbf{5},1 & 2,0 & 2,2 \\ 0,4 & 1,5 & 4,5 \\ 2,3 & 3,6 & 1,0 \end{pmatrix} \end{array}$$

And then player 1's best response if player 2 plays B or C:

$$\begin{array}{c|c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} D \\ E \\ F \end{array} & \begin{pmatrix} \mathbf{5},1 & 2,0 & 2,2 \\ 0,4 & 1,5 & \mathbf{4},5 \\ 2,3 & \mathbf{3},6 & 1,0 \end{pmatrix} \end{array}$$

We then do the same for player 2 for responses to player 1, obtaining.

$$\begin{array}{c|c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} D \\ E \\ F \end{array} & \begin{pmatrix} \mathbf{5},1 & 2,0 & 2,\mathbf{2} \\ 0,4 & 1,\mathbf{5} & \mathbf{4},\mathbf{5} \\ 2,3 & \mathbf{3},\mathbf{6} & 1,0 \end{pmatrix} \end{array}$$

As (4, 5) and (3, 6) are both underlined, these are the Nash equilibriums of the game.

2.3 Proofs and properties

Chapter 3

Iterative Dominance

We can intuitively see that if one strategy is better than another(i.e it is a dominant strategy) than the dominated strategy will never be played. The order that iterative dominance is conducted does not effect the result.

There are two types types of dominance - strict and weak dominance.

If both player are playing strictly dominated strategies, it must be a unique Nash Equilibrium.

Strictly dominated strategy

> A strictly dominated strategy can never be a best reply.

Thus, we should remove it as it will never be played.

If both player are playing strictly dominated strategies, it must be a unique Nash Equilibrium.

Chapter 4

Notation and Definitions

4.1 Basic definitions

Definition: Mixed strategy profile

A set of all the possible combinations of mixed strategies usually denoted by x_i. A mixed strategy is pure if $x_i = 1$ and this is denoted $pi_{i,j}$

Definition: Pure strategy profile

A set of all the possible combinations of mixed strategies usually denoted by S_i

Definition: Mixed strategy

A mixed strategy is a randomised strategy x_i with a probability distribution over S_i(**pure strategies**). A player will choose a random strategy based on the probabilities of x_i. In other words x_i is a vector of probabilities that sum:
$x_i(1) + x_i(2) + x_i(3) + ... + x_i(m) = 1$

Definition: Zero-sum game

In a zero sum game the utilities/pay-offs of the players must sum to zero. In other words $u_1 = -u_2$

Definition: $u_i(a_i, a_{-i})$

the pay off for playing a_i, regardless of all other strategies

Definition: $u_i(a_i, a_{-i}) < u_i(a`_i, a_{-i})$

utility of $a`_i$ is better than a_i

Definition: Pareto efficient

To do

Definition: ESS: Evolutionary Stable Solution

todo