BIOSTAT701 Homework 1

Note: Only use R (or other software) if it is indicated in the question. Full credit will not be given to answers without work shown.

$2 \times 2 \times 2$.

1. (5 pts) A coin is to be flipped three times. List the possible outcomes in the form: (result on toss 1, result in toss 2, result in toss 3). Assume that each one of the outcomes has probability 1/8 of occurring. Find the probability of

a. A: Observing exactly 1 head $P(A) = 3/8$
b. B: Observing 1 or more heads $P(B) = (3+3+1)/8 = 7/8$
c. C: Observing exactly 2 heads $P(C) = 3/8$

2. (5 pts) For the events given in the above problem, compute:

a. P (A | B) $P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A)/P(B) = 3/7$
b. P (A | C) $P(A|C) = 0$
c. P (B | C) $P(B|C) = P(B \cap C)/P(C) = P(C)/P(C) = 1$
d. Are A, B independent? Are A, B mutually exclusive? $P(A \cap B) = P(A) \neq P(A) \cdot P(B)$ not independent $A \subseteq B$, not mutually exclusive
e. Are B, C independent? Are B, C mutually exclusive?

$P(B \cap C) = 1 \neq P(B) \cdot P(C) \neq 0$ not independent, not mutually exclusive

3. (10 pts) Consider the following outcomes for an experiment:

Outcome	1	2	3	4	5

a. $P(A) = 0.20 + 0.25 + 0.15 = 0.6$

$P(B) = 0.15 + 0.10 + 0.30 = 0.55$

c. $P(A \cup B) = 1$

Probability	.20	.25	.15	.10	.30

b. $P(A \cap B) = P(3) = 0.15$

Let A consists of outcomes 1, 2, 3 and B consist of outcomes 3, 4 and 5

a. Find P(A) and P(B)
b. Find P(both A and B occur)
c. Find P(either A or B occur)
d. Are A, B independent?
e. Are A, B mutually exclusive.

d. $P(A \cap B) = 0.15$

$P(A) \cdot P(B) \neq P(A \cap B)$

not independent

e. not mutually exclusive

$P(A \cap B) \neq 0$

4. (10 pts)The utility company in a large metropolitan area finds that 80% of its customers pay a given monthly bill in full.

$P(a) = 0.8 \times 0.8 = 0.64$

a. Suppose two customers are chosen at random from the list of all customers. What is the probability that both customers will pay their monthly bill in full?

b. What is the probability that at least one of them will pay in full? $P(b) = 0.8 \times 0.2 \times 2 + 0.64 = 0.96$

5. (10 pts) Refer to the above problem. A more detailed examination of the company records indicates that 95% of the customers who pay one monthly bill in full will also pay the next monthly bill in full; only 10% of those who pay less than the full amount one month will pay in full the next month.

a. Find the probability that a customer selected at random will pay two consecutive months in full.

A: pay one month.

B: pay the next month

a. $P(A \cap B) = P(B|A) \cdot P(A) = 0.95 \times 0.8$
$= 0.76$

$P(B|\bar{A})=0.1$

b. $P(\bar{A}\cap\bar{B}) = P(\bar{A}) \cap P(\bar{B}|\bar{A}) = 0.2 \times (1-0.1) = 0.18.$

b. Find the probability that a customer selected at random will pay neither of two consecutive months in full.

c. Find the probability that a customer chosen at random chosen at random will pay exactly one month in full.

c. exactly one month paid $= P(A\cap\bar{B}) + P(B\cap\bar{A})$ $= P(A) \times P(\bar{B}|A) + P(\bar{A}) \times P(B|\bar{A}) = 0.8 \times 0.05 + 0.2 \times 0.1 = 0.06$

6. (10 pts) The number of daily requests for emergency assistance at a fire station in a medium-sized city has the probability distribution shown here:

y	0	1	2	3	4	5	6	7	8	9	10
P(y)	0.06	.14	.16	.14	.12	.10	.08	.07	.06	.04	.03

a. What is the probability that four or more requests will be made in a particular day?

$P(a) = 0.12 + 0.10 + 0.08 + 0.07 + 0.06 + 0.04 + 0.03 = 0.5$

b. What is the probability that the requests for assistance will be at least four but no more than 6?

$P(b) = 0.12 + 0.10 + 0.08 = 0.30$

c. Suppose the fire station must call for additional equipment from a neighboring city whenever the number of requests for assistance exceeds eight in a given day. The neighboring city then charges for its equipment. What is the probability the city will call for additional equipment on a given day?

$P(C) = (0.4 + 0.3) / 0.06 + 0.14 + 0.16 + 0.14 + 0.5 = 0.7$

7. (15 pts) Health status, denoted by Y, can be classified as terrible, poor, fair, good or excellent.

Part a: What type of random variable is Y: discrete with a finite number of possible values, discrete with a countably infinite number of possible values, continuous. ✓

Part b: Is Y ordinal? Yes.

Part c: Suppose the probability that Y falls into various categories is given by the table below:

Category	Probability
Terrible	.05
Poor	.10
Fair	.15
Good	.25
excellent	.45

Calculate the probability for these events:

- Y is excellent 0.45
- Y is good 0.25
- Y is good or better 0.25 + 0.45 = 0.7
- Y is good or worse 1 − 0.45 = 0.55
- Y is poor or worse 0.1 + 0.05 = 0.15

8. (20 pts) Consider an urn with 10,000 balls, 5,000 of which are red and 5,000 of which are green. Draw a ball from the urn, and let C1 denote its color. What is the probability that C1 is red? Return the ball to the urn (which continues to contain 5,000 red balls and 5,000 green balls). Draw another ball from the urn, and let C2 denote its color. What is the probability that C2 is red?

$P(C_1 = \text{red}) = 1/2$

$P(C_2 = \text{red}) = 1/2$

$Pr(C_2) = 4999/9999$

not similar

$Pr\{C_1=red\} = \frac{1}{2}$

$Pr\{C_2=red \mid C_1=red\} = 4999/9999$

Repeat this experiment, with C1 being red, but do not return the ball to the urn (which now contains 4,999 red balls and 5,000 green balls). Draw another ball from the urn, and let C2 denote its color. What is the probability that C2 is red? Are Pr{C1=red} and Pr{C2=red | C1=red} similar? What about Pr{C3=red | C1=red and C2=red)? What is Pr{C6=red | C1-C5 are red)?

$Pr\{C_3=red \mid C_1=red \text{ and } C_2=red\} = 4998/9998$

Now repeat this experiment with an urn containing 10 balls, 5 of which are red and 5 of which are green. Are Pr{C1=red} and Pr{C2=red | C1=red} similar? What about Pr{C3=red | C1=red and C2=red)? What is Pr{C6=red | C1-C5 are red)?

not similar

$Pr\{C_1=red\} = 1/2$

$Pr\{C_2=red \mid C_1=red\} = 4/9$

$Pr\{C_6=red \mid C_1-C_5 \text{ are red}\} = 4995/9995$

9 (15 pts) A deck of 52 cards has 13 denominations (i.e., A, 2, 3, … , Q, K). A "pair +" consists of pairs, three of a kind, and 4 of a kind. An example of a pair is {3,3,5, J,K}. An example of 3 of a kind is {4,6,6,6,J}. An example of 4 of a kind is {2,A,A,A,A}. What is the probability of drawing a pair +?

$Pr\{C_3=red \mid C_1=red \text{ and } C_2=red)$

$= 3/8$

$Pr\{C_6=red \mid C_1-C_5 \text{ are red}\}$

$= 0$

DDDDD

$P\{\text{not pairs+}\} = (C_{13}^5 \cdot 4^5)/C_{52}^5$

$\therefore P\{\text{pairs+}\} = 1 - \frac{C_{13}^5 \cdot 4^5}{C_{52}^5}$

≈ 0.493