Appendix III

Common probability distributions

In this appendix, we present some common probability distributions that are useful in statistical methods that we have used in this book. There is a much greater variety of distributions that are very important in particular areas of applications. A good reference can be found at http://www.causascientia.org/math_stat/Dists/Compendium.pdf. We give the density function, mean, variance, and moment-generating function (mgf). For some distribution functions, if the mgf is complicated, we just leave it out and refer the reader to one of the references in the book.

Name	Probability density function	Mean	Variance	Moment-generating function
Bernoulli distribution	$f(x,p) = \begin{cases} p, & x = 1 \\ 1-p, & x = 0 \quad 0 \le p \le 1 \\ 0, & \text{otherwise.} \end{cases}$	p	$p(1-p)$	$q + pe^t$, $q = 1 - p$
Binomial	$f(x,n,p) = \binom{n}{x} p^x q^{n-x}, x = 0,1,\ldots,n$	np	npq	$(q+pe^t)^n$
Geometric	$f(x,p) = q^{x-1}p, x = 1,2,\ldots, 0 \le p \le 1.$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1-qe^t}$
Hypergeometric	$f(x,N,m,n) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$, $N = 0,1,2,\ldots, m = 0,1,\ldots,N,$ $n = 0,1,\ldots,N, x = 0,1,\ldots,n$	$\frac{mn}{N}$	$\frac{n\left(\frac{m}{N}\right)\left(1-\frac{m}{N}\right)\left(1-\frac{n}{N}\right)}{N-1}$	No closed form
Negative binomial	$f(x,N,m,n) = \binom{x+r-1}{x} p^r q^x$, $x = 0,1,2,\ldots$	$r\frac{q}{p}$	$r\frac{q}{p^2}$	$\left(\frac{p}{1-qe^t}\right)^r$
Poisson	$f(x,\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$, $x = 0,1,2,\ldots$	λ	λ	$\exp(\lambda(e^t - 1))$

Name	Probability density function	Mean	Variance	Moment-generating function
Beta	$f(x,\alpha,\beta) = \left(\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right)x^{\alpha-1}(1-x)^{\beta-1}$, $0<x<1$	$\dfrac{\alpha}{\alpha+\beta}$	$\dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Chi-square	$f(x,n) = \dfrac{x^{(n/2)-1}e^{-x/2}}{\Gamma(n/2)2^{n/2}}$, $x\geq 0, n>0$(*degrees of freedom*)	n	$2n$	$\dfrac{1}{(1-2t)^{n/2}}, t<\dfrac{1}{2}$
Exponential	$f(x,\beta) = \begin{cases}\dfrac{1}{\beta}e^{-x/\beta}, & \beta>0, 0\leq x<\infty\\ 0, & otherwise\end{cases}$	β	β^2	$\dfrac{1}{(1-\beta t)},\ t<\dfrac{1}{\beta}$
Gamma	$f(x,\alpha,\beta) = \begin{cases}\dfrac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta}, & x>0,\alpha,\beta>0\\ 0, & otherwise\end{cases}$	$\alpha\beta$	$\alpha\beta^2$	$\dfrac{1}{(1-\beta t)^\alpha}, t<\dfrac{1}{\beta}$
Laplace	$f(x,\mu,\sigma) = \dfrac{1}{2\sigma}\exp\left(-\dfrac{\lvert x-\mu\rvert}{\sigma}\right)$, $x,\mu>-\infty$	μ	$2\sigma^2$	
Normal	$f(x,\mu,\sigma^2) = \dfrac{1}{\sigma\sqrt{2\pi}}\exp\left(-\dfrac{(x-\mu)^2}{2\sigma^2}\right)$, $-\infty<x,\mu<\infty,\sigma>0$	μ	σ^2	$e^{t\mu+\frac{1}{2}t^2\sigma^2}$
Uniform	$f(x,a,b) = \frac{1}{b-a}$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$	$\dfrac{e^{tb}-e^{ta}}{t(b-a)}$
Three-parameter gamma	$f(x:\alpha,\beta,\gamma) = \begin{cases}\dfrac{1}{\beta^\alpha\Gamma(\alpha)}(x-\gamma)^{\alpha-1}\exp-\dfrac{(x-\gamma)}{\beta}, & \gamma<x<\infty,\alpha,\beta>0\\ 0, & \text{otherwise.}\end{cases}$	$\gamma+\alpha\beta$	$\alpha\beta^2$	$\dfrac{\exp(\gamma t)}{(1-\beta t)^\alpha}$.
Two-parameter Weibull	$f(x,\alpha,\beta) = \dfrac{\alpha}{\beta}\left(\dfrac{x}{\beta}\right)^{\alpha-1}e^{-\left(\frac{x}{\beta}\right)^\alpha}, x\geq 0,\alpha,\beta>0$	$\beta\Gamma\left(1+\frac{1}{\alpha}\right)$	$\beta^2\left[\Gamma\left(1+\frac{2}{\alpha}\right)-\left(\Gamma\left(1+\frac{1}{\alpha}\right)\right)^2\right]$	$\sum_{n=0}^{\infty}\dfrac{t^n\beta^n}{n!}\Gamma\left(1+\frac{n}{\alpha}\right)$
Three-parameter Weibull	$f(x,\alpha,\beta,\gamma) = \dfrac{\beta}{\alpha}(x-\gamma)^{\beta-1}e^{-\left((x-\gamma)^{\frac{\beta}{\alpha}}\right)}, x>\gamma,\alpha,\beta,\gamma>0.$	$\alpha^{\frac{1}{\beta}}\Gamma\left(1+\frac{1}{\beta}\right)+\gamma$	$\alpha^{\frac{2}{\beta}}\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left(\Gamma\left(1+\frac{1}{\beta}\right)\right)^2\right\}$	$e^{t\gamma}\int_0^\infty e^{-y+t(\alpha y)^{1/\beta}}dy$