Optimal Monetary Policy and the Welfare Cost of Inflation of a Currency Union

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Abstract

This paper studies the welfare cost of inflation and optimal monetary policy of a currency union between two countries using a search-theoretic framework with endogenous composition of buyers and sellers. The model includes three features of a currency union that are key to welfare and policy analysis: heterogeneous market structure (characterized by buyers' bargaining power), imperfect market integration, and immigration policy. The model yields optimal monetary policy that deviates from the Friedman rule, with the magnitude of the optimal inflation rate and welfare cost of inflation determined by different policy regimes. The Friedman rule is suboptimal, because of a matching congestion externality in the labor market arising from the endogenous composition of buyers and sellers. Higher labor mobility reduces the cost of inflation by alleviating congestion, regardless of buyers' bargaining power. Market integration may also reduce congestion, lowering the cost of inflation, but only when sellers are relatively scarce.

Keywords: currency union; optimal monetary policy; welfare cost of inflation; search models

1 Introduction

Currency unions play a central role in the international monetary system. It is widely thought that a currency union eliminates extra costs and risks involved in cross-border transactions, facilitating trade and investment. The euro area, as of 2017, has become one of the largest currency unions. In the euro area, the European Commission has been promoting a single market strategy[1] for years, because free movement of goods and labor benefits both people and enterprises. Numerous studies on the effect of EA on trade. Baldwin, DiNino, Fontagné, De Santis, and Taglioni (2008) confirms that the introduction of the euro has increased trade by 5%, in-to-in FDI ranging by 15% to 200%, and out-to-in FDI ranging by 7.5% to 100%[2], signifies the significant effects of mobility in factors and goods as a result of forming a currency union. A literature represented by Mundell (1961) emphasizes free mobility of factors of production and free mobility of goods as ideal characteristics for a currency union[3]. The monetary literature has also long recognized the benefits of introducing a single currency to improve transaction efficiency, such as Krugman (1984) and Black (1991).** More lit: Shapley and Shubik (1977), Hartmann (1998), Starr (2000), and Howitt (2005), Devereux and Shi (2013).

In this paper, we analyze a situation where countries are already in a currency union, and investigate the optimal monetary policy in the presence of immigration policy and imperfect market integration. I am curious whether the promotion of free mobility of goods and labor indeed benefits a currency union if member countries are vastly different in terms of market structure. And are the gains of some countries at the cost of others? For these reasons, we develop an open-economy model to capture the difference in market structure across countries, and the mobility of goods and labor across borders. We want to look at optimal monetary policy and the welfare cost of inflation under the effects of these factors.

How to determine optimal policy? How does mobility/immigration and trade policy affect optimal policy? How to see the impact of the policy on different countries – different market structure? In order to answer these questions, I develop an open-economy monetary search model that considers immigration policy, imperfect market integration, and heterogeneous market structure. It follows Lagos and Wright (2005), a now-standard workhorse search-theoretic monetary model, coming from a long standing literature that allows an essential role of medium of exchange to arise from an environment with mi-

[1] "The single market is at the heart of the European project, enabling people, services, goods and capital to move more freely, offering opportunities for European businesses and greater choice and lower prices for consumers. It enables citizens to travel, live, work or study wherever they wish."

[2] pg 132 out-to-in: non euro to eruo area countries

[3] See Dellas and Tavas (2009) for a review of the optimal-currency-union literature.

crofounded frictions, developed by Kiyotaki and Wright (1989). The model combines an over-the-counter goods market characterized by bilateral meetings and bargaining, and a centralized Walrasian market for settlement. Within this framework, the lack of commitment and enforcement makes a medium of exchange socially useful. There are three key features that are different from LW. First, an open economy allows for analyses regarding labor and goods mobility across multiple countries. Second, a congestion externality from an endogenous composition of producers (sellers) and consumers (buyers), whose magnitude is affected by immigration and market integration. This congestion externality affects optimal monetary policy because positive interest rate could improve welfare by alleviating the congestion externality[4]. Third, a difference in market structure across countries (characterized by the heterogeneity in buyers' bargaining power) allows the monetary policy to have different effects on different countries. The mobility of goods and labor shifts the effects to different directions based on buyers' bargaining power.

In the model, monetary policy manifests as the growth rate of the money supply through lump-sum transfers. With endogenous composition of buyers and sellers, monetary policy has real effects on the economy due to two effects of inflation on the level of economic activity: the intensive effect and extensive effect. The intensive marginal effect captures the reduced quantity per trade between a buyer and a seller due to inflation. When monetary policy is such that the nominal interest rate (or cost of holding money) is positive, trade quantities suffer as agents are less willing to hold real balances. The extensive marginal effect captures the potential increase in the number of trades between sellers and buyers due to positive interest rate. An increase in the nominal interest rate, which translates to a higher inflation, reduces the value of being a buyer, who holds real balances in equilibrium. Since agents are able to switch roles between buyers and sellers, a higher inflation incentivizes a buyer to become a seller. When buyers' bargaining power is too high, there are too many buyers in the market; an inflationary monetary policy serves to improve matching by "rebalancing" the number of buyers and sellers. In the case where the positive extensive marginal effect dominates the negative intensive marginal effect, the Friedman rule is suboptimal. The optimal interest rate increases with buyers' bargaining power.

While there are numerous benefits of forming a currency union, one of the main challenges is to determine a single monetary policy taking into account the differences between all the member states. Along with the many benefits, a single currency carries a cost: that monetary policy can not be individualized for different nations. There is a growing literature discussing the macroeconomic implications of market structure. De Loecker,

[4]Rocheteau and Wright (2006): optimal monetary policy deviates from FR, because inflation improves matching by reducing congestion.

Eeckhout, and Unger (2020) find a negative net effect of market power on welfare, because despite the higher productivity of the high markup firms, they also extract more rents from the customer and affect the labor market adversely. Market structures are different across states due to economic institutions, regulations and policies. How do we determine buyers' bargaining power? In a currency union, the member countries differ in market structures. In the model, market structure is characterized by buyers' bargaining power. We can think of buyers as households, and sellers as firms. In a monopoly market, the firm retains all the trade surplus, and households have zero bargaining power. In a monopolistic market, households retain all the trade surplus; therefore, they have a high bargaining power. Since an agent is free to choose his role as a buyer or a seller, more agents choose to be buyers if the market is more competitive; in other words, a higher buyers' bargaining power results in a higher proportion of buyers in the market. In a country with a more competitive market, an inflationary monetary policy acts as a counterforce to the high bargaining power to reduce matching congestion and thus improves welfare. In the literature on market structure, Roeger's (1995) method or its variations of estimating firm markups is widely applied in the trade literature but rarely in the new monetarist models, especially open-economy ones. Part of the problem might be how to connect the bargaining methods commonly used to market structure and data. Recent paper Choi and Rocheteau (2021) provides a thorough consideration of market power and its implications in New Monetarist models.

The mobility of goods is characterized by the degree of market integration - the probability of a buyer having access to the goods produced in a foreign country, or the ease of import. If the probability of a buyer meeting a foreign seller is high, international trade occurs often, and the goods market is highly integrated. As shown in Table 1, a higher degree of market integration increases the cost of inflation for the country with lower buyers' bargaining power, and reduces the cost of inflation for the country with higher buyers' bargaining power. In other words, all else equal, free mobility of goods reduces the cost of inflation for the country with a less competitive market. The mobility of labor is characterized by whether agents are allowed to relocate to another country, or whether there is free immigration. An agent wants to move to the country with higher bargaining power and become a buyer there for higher surplus until the congestion effect kicks in. All else equal, free immigration reduces the cost of inflation for both countries. To combine, the effect of free mobility of goods is beneficial for the country with a higher buyers' bargaining power, and is ambiguous for the one with a lower bargaining power. The model is calibrated using data of the euro area countries. The optimal interest rate is the highest when there is free mobility of goods and labor. The model yields an optimal interest rate lower than the interest rate implemented by the European Central Bank; the discrepancy translates to .02% of the Euro Area gross output, or 2.59 billion euros.

Table 1: Effects on the Welfare Cost of Positive Inflation

	Country with Low Bargaining Power	Country with High Bargaining Power
Market Integration	> 0	< 0
Free Immigration	< 0	< 0
Combined	Ambiguous	< 0

1.1 Related Literature

This paper is related to a broad search-theoretic monetary literature on optimal monetary policy, with Lagos and Wright (2005) as a now standard workhorse model, coming from a long standing literature that allows an essential role of medium of exchange to arise from an environment with microfounded frictions, developed by Kiyotaki and Wright (1989). A monetary search model typically yields FR as the optimal monetary policy, as fiat money tends to lose value over time. In practice, however, FR is rarely implemented: central banks usually target a positive nominal interest rate or even a positive inflation rate. Models are developed to account for the nonobservance of the Friedman rule in practice and to study optimal monetary policy. In the literature, reasons for suboptimality of the Friedman rule are mostly based on the idea that positive interest rates correct for the externalities arise from agents' choices. These include but are not limited to: Papers that endogenize the composition of buyers and sellers to account for congestion externalities include Hosios (1990), Rocheteau and Wright (2005, 2006), Berentsen, Rocheteau and Shi (2007). Papers such as Head and Kumar (2005), and Berentsen, Rocheteau and Shi (2007) include endogenous search effort. Zhang (2014) includes endogenous currency acceptance.

Congestion externality is particularly relevant in this paper in an open-economy market because it is affected by both labor and goods mobility. This model includes immigration as a policy consideration to evaluate the effect of labor mobility, and a metric to measure the effect of goods mobility – degree of market integration. For a currency union, it is interesting to study the optimal monetary in an environment with an inefficiency directly affected by immigration/export/import policy. Recent literature that is most relevant to this framework: Gomis-Porqueras and Zhang (2018, 2021) study effects of frictional goods markets and migration in a currency union, respectively. Difference: different frictions. Theirs: search frictions arise from search intensity in labor or goods market. Mine: congestion arises from agents' endogenous choice of being buyers or sellers by; magnitude of congestion is affected by labor mobility (immigration) and degree of market integration.

More new monetarist lit: Herrenbrueck (2015): studies optimal monetary policy in an open economy model with price posting and shows inflation can have non-monotonic

effects due consumers' search intensity. Bignon, Brenton and Rojas-Breu (2019): symmetric two-country monetary model with credit to study the interplay between currency integration and credit markets integration. capture credit markets integration by the extra cost incurred to obtain credit for cross-border transactions Duc and Rojas-Breu (2021): In this paper we construct a two-country model of monetary exchange and bank credit to study resource allocation and risk sharing within a currency union. Fragmented credit markets: prevents banks from engaging in cross-border lending activities. We use this framework to study the implications of two supra-national policies when credit markets are fragmented: a common deposit insurance and a central-bank digital currency.

NK lit:

The main contribution of this paper is extending a closed economy model to an open economy model that enables policy and welfare analysis within an environment that incorporates heterogeneity in market structure, immigration policy and market integration.

2 Environment

The model modifies and extends the closed-economy model featured by endogenous seller-buyer choice in Rocheteau and Wright (2006) to an open-economy setting similar to Zhang (2014). In addition, we allow for heterogeneous buyers' bargaining power and immigration across countries.

Time is infinite and discrete. There are two countries $i \in \{1, 2\}$, populated with a continuum of measure 1 and $n \in [0, 1]$ of agents, respectively. All agents are infinitely lived and have a discount factor $\beta \in (0, 1)$. As shown in Figure 1, each period is divided into two subperiods: the first for over-the-counter trade of domestic and foreign goods, and the second for settlement.

In the first subperiod, a decentralized market (DM) opens in each country. Agents in each country are divides between buyers and sellers based on their choices from the last period. An agent decides whether to become a buyer or a seller and in which country to live in the following period based on the continuation value of each option. After choosing a role and a location, a seller produces a special good that he cannot consume, while a buyer wants to consume the special good but cannot produce.

Taking meeting probabilities as given, a There are two divisible and non-storable consumption goods being produced and consumed: a general numeraire good x produced and consumed by all agents, and a special good q produced only by sellers and consumed

only by buyers.

At the beginning of the second subperiod, the government makes a lump-sum transfer of T to all agents. A single centralized market (CM) opens after the transfer; all agents produce and consume the general good x. All agents have a linear production function that allows them to transfer one unit of labor into one unit of general good. The supply of labor hours is h, which implies that the real wage is normalized to 1. All agents readjust their real balances to carry forward to the decentralized market (DM) of the next period.

For simplicity, the instantaneous utility of each agent is additively separable and quasi-linear in labor supply h. $u(.)$ and $c(.)$ denote the utility function and cost function of the special good respectively, and $U(.)$ denotes the utility function of the general good. U^B and U^S are instantaneous utility of a buyer and a seller, respectively:

$$U^B = u(q) + U(x) - h,$$
$$U^S = -c(q) + U(x) - h,$$

where $u(.)$ and $c(.)$ are C^2, with $u' > 0, u'' < 0, c' > 0, c'' \geq 0, u(0) = c(0) = c'(0) = 0$, and $u'(0) = \infty$.

Sellers in country i have immobile factors of production and can produce in country i only; buyers in i, on the other hand, can purchase the special good in either country. As shown in Figure 1, in DM_i, only sellers i are selling the special goods, while buyers can be from both countries. Buyers and sellers meet pairwise and at random in the two distinct DMs. When goods market is perfectly integrated, international trade happens no more often than local trade; otherwise, search frictions make international trade more difficult than local trade. The degree of market integration can be measured by how easy it is for a buyer to access the foreign DM and purchase the foreign goods; or it can be thought of as how easy it is to import foreign goods. Let the buyer stays in the domestic DM with a probability $\alpha \in [\frac{1}{2}, 1]$, and visits the foreign DM with a probability $1-\alpha$. $\alpha = \frac{1}{2}$ indicates a perfectly integrated economy, since a buyer has equal probability of staying in his own DM and visiting the foreign DM. Similarly, $\alpha = 1$ indicates a disintegrated economy.

Let b_1, b_2 be the proportion of agents who choose to by buyers in countries 1 and 2, respectively; the number of buyers are b_1, nb_2, and the number of sellers are $(1-b_1), n(1-b_2)$ in the two countries, respectively. Meeting probabilities between sellers and buyers in the two DMs are determined by the matching technology. Let the matching function $\mathcal{M}_i = \mathcal{M}(\mathcal{B}_i, \mathcal{S}_i)$, where $\mathcal{B}_i, \mathcal{S}_i, \mathcal{M}_i$ denote the number of buyers, sellers, and matches present in country i's DM, respectively. $\mathcal{M}(.,.)$ is increasing in both arguments, concave,

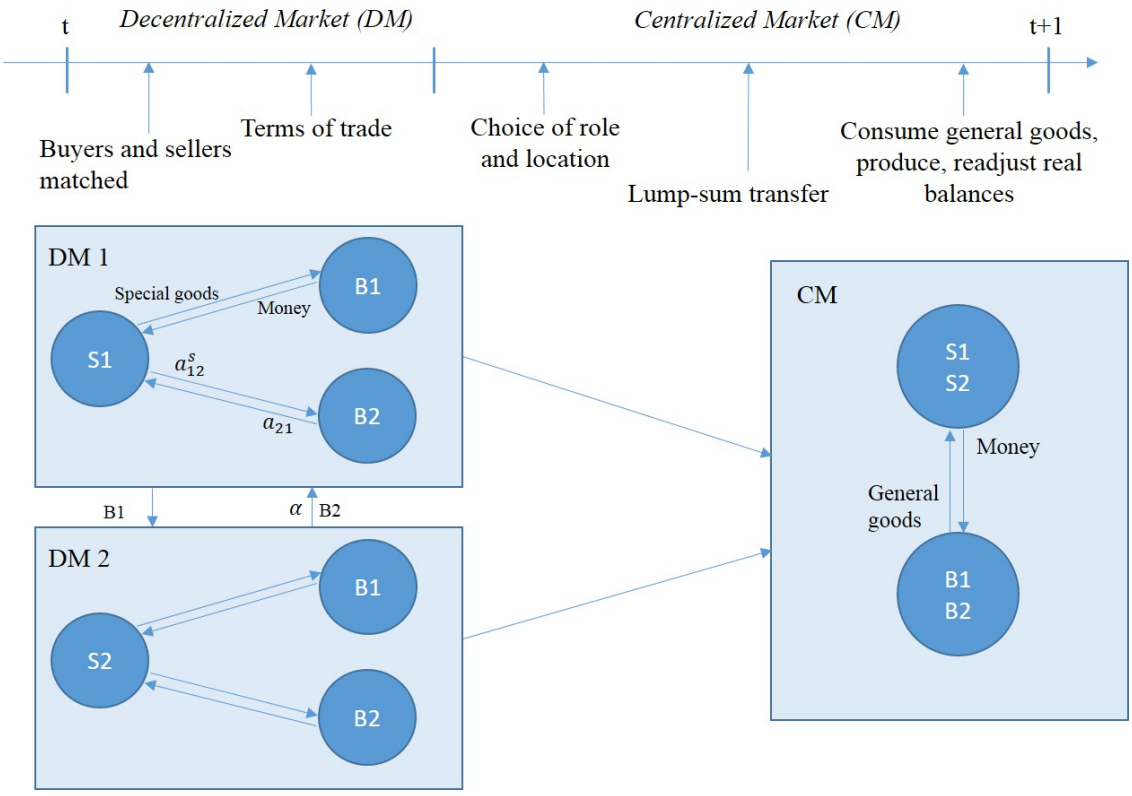

Figure 1: Timing of representative period

and homogeneous of degree 1. In equilibrium, denote the population of the two countries n_1, n_2. In country 1, $\mathcal{B}_1 = \alpha n_1 b_1 + (1-\alpha) n_2 b_2$, $\mathcal{S}_1 = n_1(1-b_1)$, and in country 2, $\mathcal{B}_2 = (1-\alpha) n_1 b_1 + \alpha n_2 b_2$, $\mathcal{S}_2 = n_2(1-b_2)$. The ratio of sellers to buyers present in country i, or the market tightness, is $\tau_i \equiv \mathcal{S}_i/\mathcal{B}_i$. In country j, denote the probability of a buyer from country i but present in country j meeting a seller a_{ij}, and the probability of a seller in i meeting a buyer originally from j a_{ij}^s. Assume a telegraph matching function $\mathcal{M}_i = \mathcal{M}(\mathcal{B}_i, \mathcal{S}_i) = \mathcal{B}_i \mathcal{S}_i/(\mathcal{B}_i + \mathcal{S}_i)$, the buyers' probabilities are shown in Table 2. Sellers' meeting probabilities are calculated in a similar way.[5] The elasticity of the matching function with respect to the number of buyers in i is denoted $\epsilon(\tau_i) \equiv \tau_i/(1+\tau_i)$.

Table 2: Buyer i's Meeting Probability a_{ij}

	Seller from 1	Seller from 2
Buyer from 1	$a_{11} = \alpha \mathcal{M}_1/\mathcal{B}_1$	$a_{12} = (1-\alpha)\mathcal{M}_2/\mathcal{B}_2$
Buyer from 2	$a_{21} = (1-\alpha)\mathcal{M}_1/\mathcal{B}_1$	$a_{22} = \alpha \mathcal{M}_2/\mathcal{B}_2$

Note:
$\mathcal{B}_1 = \alpha n_1 b_1 + (1-\alpha) n_2 b_2$, $\mathcal{S}_1 = n_1(1-b_1)$
$\mathcal{B}_2 = (1-\alpha) n_1 b_1 + \alpha n_2 b_2$, $\mathcal{S}_2 = n_2(1-b_2)$
$\mathcal{M}_i = \mathcal{M}(\mathcal{B}_i, \mathcal{S}_i) = \mathcal{B}_i \mathcal{S}_i/(\mathcal{B}_i + \mathcal{S}_i)$

[5] Appendix A3

After buyers and sellers are matched, they negotiate terms of trade according to Kalai's proportional bargaining rule. Denote buyers' bargaining power in country i $\theta_i \in (0, 1]$, indicating the proportion of the DM trade surplus that goes to the buyers present in i. In the DM, trade is bilateral and anonymous, and buyers cannot commit to repaying their debt. As a result of this friction, in equilibrium, all trade has to be *quid pro quo*. This lack of commitment and record keeping gives an essential role to a medium of exchange.

The two countries are part of a currency union, where a single monetary authority sets monetary policy for the union. Monetary policy consists of changes in the supply of a perfectly divisible and storable fiat currency issued by the monetary authority denoted by $M \in \mathbb{R}_+$. The currency is valued at ϕ, which is the price of the currency in terms of the numeraire, which we take to be labor in the CM. Money supply M grows at a constant rate $(\gamma - 1)$, where $\gamma \equiv \frac{M'}{M}$, M' being the next period money supply.

3 Monetary Equilibrium

In each period, an agent makes a decision of trade quantities and country of residency in the DM given the terms of trade in the DM and the choice of real balances in the CM. Equilibrium trade quantities, agent composition, population, and real balances are solved using backwards induction; therefore, we start with the description of a representative period with the second subperiod and then proceed to the first subperiod.

3.1 Centralized Market (CM)

In the CM, an agent $k \in \{b, s\}$ from country $i \in \{1, 2\}$ chooses consumption of the general good x, labor supply h, and money holding m' carried forward to the next DM, where b denotes a role as a buyer and s denotes a seller. The value of currency is ϕ in terms of CM labor, and the current real balances are $z = \phi m$. An agent k in country i has a value function $W_i^k(.)$ in the CM and $V_i^k(.)$ in the following DM. Assume $U(x) = x$. In the CM, an agent solves the following maximization problem

$$\begin{aligned} W_i^k(z) &= \max_{x,h,m'} [x - h + \beta V_i^k(z')] \\ s.t. \quad & x + \phi m' = h + z + T \\ & z = \phi m, \; T = (1 - \gamma)M, \end{aligned}$$

where T is the lump-sum transfer from the government to the agents. Since the government cannot distinguish buyers from sellers, the transfer is made to both agents.

Substituting the budget constraint into the value function yields

$$W_i^k(z) = \max_{x,h,m'} \{-\phi m' + z + T + \beta V_i^k(z')\}. \tag{1}$$

Note that the choice of the next period real balances m' is independent of the current real balances z. This is a result of the quasi-linearity in CM labor supply, and it is a common feature of the money-search literature following Lagos-Wright.

3.2 DM - Terms of Trade

Entering the DM, a seller and a buyer meets and negotiate terms of trade. The terms of trade are determined by Kalai's (1977) proportional bargaining rule. A buyer chooses the quantity purchased q and amount of money paid d to maximize his surplus, subject to a fixed share of the total DM surplus specified by the buyer's bargaining power $\theta \in (0, 1]$ and the buyer's real balances z. Let (q_1, q_2) be buyer 1's quantities of purchase in DM_1 and DM_2, and $(\tilde{q}_1, \tilde{q}_2)$ be buyer 2's quantities of purchase in DM_1 and DM_2, where DM_1 and SM_2 are the DM in countries 1 and 2. When buyer 1 meets a seller $j \in \{1, 2\}$, he solves the following problem:

$$\begin{aligned} q_j^* &\in argmax\ \theta_j[u(q_j) - c(q_j)] \\ s.t. \quad d(q_j) &= (1 - \theta_j)u(q_j) + \theta_j c(q_j) \leq z_1, \end{aligned} \tag{2}$$

where θ_j is buyers' bargaining power in country j, because the buyer's bargaining power depends on the market structure of the country where the trade occurs. If $d(q_j^*) \leq z_1$, where q_j^* solves $u'(q_j) = c'(q_j)$, buyer 1 holds sufficient real balances to purchase the efficient level of the special good q_j^* in either DM. If $d(q_j^*) > z_1$, a buyer holds real balances less than the payment for the efficient quantity of goods. Buyer 2 solves a similar problem when meeting a seller from country j

$$\begin{aligned} \tilde{q}_j^* &\in argmax\ \theta_j[u(\tilde{q}_j) - c(\tilde{q}_j)] \\ s.t. \quad d(\tilde{q}_j) &= (1 - \theta_j)u(\tilde{q}_j) + \theta_j c(\tilde{q}_j) \leq z_2. \end{aligned} \tag{3}$$

3.3 DM - Value Functions

Buyer i's DM value is determined by his expected surplus from trade and the continuation value of his choice of role and location for the following CM and DM. A buyer chooses quantity of trade $q_i \in [0, q_i^*]$, taking the probability of meeting a seller as given. Buyer i has value functions as follows:

$$
\begin{aligned}
V_1^b(z) &= \max_{q_1\in[0,q_1^*],q_2\in[0,q_2^*]} \big[a_{11}\theta_1 S(q_1) + a_{12}\theta_2 S(q_2) + R\big], \\
V_2^b(z) &= \max_{\tilde{q}_1\in[0,\tilde{q}_1^*],\tilde{q}_2\in[0,\tilde{q}_2^*]} \big[a_{21}\theta_1 S(\tilde{q}_1) + a_{22}\theta_2 S(\tilde{q}_2) + R\big],
\end{aligned} \tag{4}
$$

where $R = \max[W_1^b(z'), W_1^s(z'), W_2^b(z'), W_2^s(z')]$ denotes the buyer's choice of role as a seller or a buyer, and residence in country 1 or 2. Let $S(q) \equiv u(q) - c(q)$ be the total surplus of trade between a buyer and a seller in the DM. For buyer 1, $a_{11}\theta_1 S(q_1)$ is the probability of buyer 1 meeting a seller in country 1 multiplied by his proportion of the DM_1 surplus. The second term $a_{12}\theta_2 S(q_2)$ is the probability of buyer 1 meeting a seller in country 2 multiplied by his proportion of DM_2 surplus.

A seller j makes a decision on his role for the following CM and DM, taking the terms of trade as given. Value functions of a seller are as follows:

$$
\begin{aligned}
V_1^s(z) &= (1-\theta_1)\big[a_{11}^s S(q_1) + a_{12}^s S(\tilde{q}_1)\big] + R, \\
V_2^s(z) &= (1-\theta_2)\big[a_{21}^s S(q_1) + a_{22}^s S(\tilde{q}_1)\big] + R.
\end{aligned} \tag{5}
$$

The first term on the right-hand side is the seller's share of the DM surplus, which comes from trade with buyers from both countries. For seller 1, $a_{11}^s S(q_1)$ is probability of seller 1 meeting buyer 1 multiplied by the surplus in DM_1, and $a_{12}^s S(\tilde{q}_1)$ is probability of seller 1 meeting buyer 2 multiplied by the surplus in DM_1. Substituting buyers' DM value functions (4) into the CM value function (1) yields (6)

$$
\begin{aligned}
W_1^b(z) = \max_{q_1\in[0,q_1^*],q_2\in[0,q_2^*]} \big\{ & z + T - \phi m' + \beta \\
& \big[a_{11}\theta_1 S(q_1) + a_{12}\theta_2 S(q_2) + \max[W_1^b(z'), W_1^s(z'), W_2^b(z'), W_2^s(z')]\big]\big\}.
\end{aligned} \tag{6}
$$

Due to linearity $W(z) = z + W(0)$, we can transform the (6) into the following buyer's value functions (7) and (8)

$$
\begin{aligned}
(q_1, q_2) \in \underset{q_1\in[0,q_1^*],q_2\in[0,q_2^*]}{argmax} & \big\{ -\iota z_1 + a_{11}\theta_1 S(q_1) + a_{12}\theta_2 S(q_2)\big\} \\
s.t. \quad & (1-\theta_1)u(q_1) + \theta_1 c(q_1) \le z_1 \\
& (1-\theta_2)u(q_2) + \theta_2 c(q_2) \le z_1,
\end{aligned} \tag{7}
$$

where $\iota = \frac{\phi}{\phi'\beta} - 1$ is the cost of holding money, or nominal interest rate. Since holding money is costly for buyers, in equilibrium they will not hold any more money than necessary in the DM. Therefore, a buyer holds real balances of $z_1 = d_1$ in equilibrium, where $d_1 = (1-\theta_1)u(q_1) + \theta_1 c(q_1) = (1-\theta_2)u(q_2) + \theta_2 c(q_2)$ is the payment made by buyer 1, depending on quantities of trade in either DM. A buyer faces a trade-off between the

cost and benefit of holding positive real balances and chooses DM quantities to maximize utility. In the special case when a buyer does not have access to the foreign DM, $\alpha = 1$, $d_1 = (1-\theta_1)u(q_1)$ and $q_2 = 0$. Similarly,

$$(\tilde{q}_1, \tilde{q}_2) \in \underset{\tilde{q}_1 \in [0,\tilde{q}_1^*], \tilde{q}_2 \in [0,\tilde{q}_2^*]}{argmax} \big\{ -\iota z_2 + a_{22}\theta_2 S(\tilde{q}_2) + a_{21}\theta_1 S(\tilde{q}_1) \big\}, \tag{8}$$

where buyer 2's real balances in equilibrium z_2 satisfies $z_2 = (1-\theta_1)u(\tilde{q}_1) + \theta_1 c(\tilde{q}_1) = (1-\theta_2)u(\tilde{q}_2) + \theta_2 c(\tilde{q}_2)$ when $\alpha \in [\frac{1}{2}, 1)$; $z_2 = (1-\theta_2)u(\tilde{q}_2) + \theta_2 c(\tilde{q}_2)$ and $\tilde{q}_1 = 0$ when $\alpha = 1$.

A seller, without the need to make any payment in the DM, holds zero real balances in equilibrium, $z' = 0$. Given the buyers' quantity choices, substituting (5) into (1), we obtain expressions for sellers' value functions

$$\begin{aligned} \frac{1}{\beta} W_1^s(z) &= (1-\theta_1)\big[a_{11}^s S(q_1) + a_{12}^s S(\tilde{q}_1)\big] + (R - z') + \frac{1}{\beta}(z + T), \\ \frac{1}{\beta} W_2^s(z) &= (1-\theta_2)\big[a_{21}^s S(q_2) + a_{22}^s S(\tilde{q}_2)\big] + (R - z') + \frac{1}{\beta}(z + T). \end{aligned} \tag{9}$$

In equilibrium, an agent in each country must be indifferent between being a buyer or a seller, i.e. $W_1^s(z) = W_1^b(z)$ and $W_2^s(z) = W_2^b(z)$. Combining (7), (8), and (9), we have

$$\begin{aligned} (1-\theta_1)\big[a_{11}^s S(q_1) + a_{12}^s S(\tilde{q}_1)\big] &= -\iota z_1 + a_{11}\theta_1 S(q_1) + a_{12}\theta_2 S(q_2), \\ (1-\theta_2)\big[a_{21}^s S(q_2) + a_{22}^s S(\tilde{q}_2)\big] &= -\iota z_2 + a_{22}\theta_2 S(\tilde{q}_2) + a_{21}\theta_1 S(\tilde{q}_1). \end{aligned} \tag{10}$$

If immigration is not allowed, the values of staying in the two countries are not necessarily equal; otherwise, all agents must be indifferent between living in either country in equilibrium, i.e. $W_1^s(z) = W_1^b(z) = W_2^s(z) = W_2^b(z)$, which yields

$$(1-\theta_1)\big[a_{11}^s S(q_1) + a_{12}^s S(\tilde{q}_1)\big] = (1-\theta_2)\big[a_{21}^s S(q_2) + a_{22}^s S(\tilde{q}_2)\big]. \tag{11}$$

Equations (10) and (11) determines the meeting probabilities, population, and proportion of buyers given trade quantities. Going back to (7) and (8), we take first-order conditions with respect to the quantities, so the quantities can be solved given meeting probabilities

$$\begin{aligned} \frac{a_{11}\theta_1 S'(q_1)}{(1-\theta_1)u'(q_1) + \theta_1 c'(q_1)} + \frac{a_{12}\theta_2 S'(q_2)}{(1-\theta_2)u'(q_2) + \theta_2 c'(q_2)} &= \iota, \\ \frac{a_{21}\theta_1 S'(\tilde{q}_1)}{(1-\theta_1)u'(\tilde{q}_1) + \theta_1 c'(\tilde{q}_1)} + \frac{a_{22}\theta_2 S'(\tilde{q}_2)}{(1-\theta_2)u'(\tilde{q}_2) + \theta_2 c'(\tilde{q}_2)} &= \iota. \end{aligned} \tag{12}$$

The left-hand side of (12) is the marginal benefit of holding real balances, and the right-hand side is the marginal cost. When $\iota > 0$, quantities decline as the marginal cost

increases given meeting probabilities, capturing the negative intensive marginal effect. (10), (11) and (12) together characterize a system in which meeting probabilities and quantities are jointly determined.

Definition 1 A stationary equilibrium is the number of population $n_i \in [0, 1+n]$, proportion of buyers $b_i \in (0,1)$, and quantities of trade $q_i \in [0, q_i^*]$, $\tilde{q}_i \in [0, \tilde{q}_i^*]$, for $i \in \{1,2\}$, such that

1. $(n_1, n_2, b_1, b_2, q_1, q_2, \tilde{q}_1, \tilde{q}_2)$ solve
   Free immigration:
   $$W_1^s(z) = W_1^b(z) = W_2^s(z) = W_2^b(z),$$ [6]
   No immigration:
   $$W_1^s(z) = W_1^b(z),\ W_2^s(z) = W_2^b(z);$$ [7]
2. $(n_1, n_2, b_1, b_2, q_1, q_2, \tilde{q}_1, \tilde{q}_2)$ solve $W_i^b(z)$ [8];
3. Money market clears: $\phi M = \mathcal{B}_1 d_1 + \mathcal{B}_2 d_2$;
   Labor market clears: $n_1 + n_2 = 1 + n$.

Social optimum is achieved when total welfare of the monetary union is maximized. Total welfare is measure by $\mathcal{W} = \mathcal{W}_1 + \mathcal{W}_2$, where $\mathcal{W}_i$ is the welfare of country i. $\mathcal{W}_1 = \mathcal{M}_{11} S(q_1) + \mathcal{M}_{21} S(\tilde{q}_1)$, and $\mathcal{W}_2 = \mathcal{M}_{12} S(q_2) + \mathcal{M}_{22} S(\tilde{q}_2)$, where $\mathcal{M}_{ij}$ is the number of matches between buyers from country i and sellers from country j.

Lemma 1 shows that social optimum can be achieved at the Friedman rule if and only if Hosios (1990) condition is satisfied. The Friedman rule requires that money supply grows at the discount rate such that the cost of holding real balances is zero, i.e. $\gamma = \beta$. With zero cost of holding real balances, trade quantities achieve the first-best q^*, and are not affected by the change in the buyers' proportion, or their meeting probabilities. Hosios condition is satisfied when buyers' bargaining power is equal to their contribution to the matching process; it ensures a composition of the agents in each country to maximize the number of matches, or the number of trades. Although the Friedman rule can be imposed by the central bank, the Hosios condition is often violated. Therefore, monetary equilibrium can be inefficient even under the Friedman rule because of the inefficiency on the extensive margin.

Lemma 1 Equilibrium achieves the social optimum if and only if

[6] Equations (10): $(1-\theta_1)\big[a_{11}^s S(q_1) + a_{12}^s S(\tilde{q}_1)\big] = -\iota z_1 + a_{11}\theta_1 S(q_1) + a_{12}\theta_2 S(q_2), (1-\theta_2)\big[a_{21}^s S(q_2) + a_{22}^s S(\tilde{q}_2)\big] = -\iota z_2 + a_{22}\theta_2 S(\tilde{q}_2) + a_{21}\theta_1 S(\tilde{q}_1)$, and Equation (11): $(1-\theta_1)\big[a_{11}^s S(q_1) + a_{12}^s S(\tilde{q}_1)\big] = (1-\theta_2)\big[a_{21}^s S(q_2) + a_{22}^s S(\tilde{q}_2)\big]$.

[7] Equations (10).

[8] Equations (12): $\frac{a_{11}\theta_1 S'(q_1)}{(1-\theta_1)u'(q_1)+\theta_1 c'(q_1)} + \frac{a_{12}\theta_2 S'(q_2)}{(1-\theta_2)u'(q_2)+\theta_2 c'(q_2)} = \iota$,
$\frac{a_{21}\theta_1 S'(\tilde{q}_1)}{(1-\theta_1)u'(\tilde{q}_1)+\theta_1 c'(\tilde{q}_1)} + \frac{a_{22}\theta_2 S'(\tilde{q}_2)}{(1-\theta_2)u'(\tilde{q}_2)+\theta_2 c'(\tilde{q}_2)} = \iota$.

Friedman rule is implemented:

$$\iota = 0$$

and Hosios condition is satisfied:

$$\theta_1 = \epsilon_1(\tau_1), \quad \theta_2 = \epsilon_2(\tau_2),$$

where $\epsilon(\tau_i) = \tau_i/(1+\tau_i)$.[9]

*** proof Define $\bar{\iota}$ as optimal monetary policy. As stated in Proposition 1, $\bar{\iota}$ is determined such that the total welfare of the currency union is maximized. When $\bar{\iota}$ deviates from the Friedman rule, it increases with buyers' bargaining power, because a higher interest rate is needed to reduce the matching congestion. When α decreases, the goods market becomes more integrated; this increases the optimal interest rate. A free immigration policy also has a positive impact on optimal policy. With a higher mobility of goods and labor, the power of a high buyers bargaining power is magnified; therefore, a higher interest rate is needed to obtain social optimum.

Proposition 1 If $\theta_2 \geq \theta_1 = 1/2$, there exist an $\bar{\iota} > 0$ such that when $0 \leq \iota < \bar{\iota}$, total welfare of the currency union increases in ι, and when $\iota \geq \bar{\iota}$, total welfare decreases in ι. $\bar{\iota}$ increases with θ_2, decreases with α, and increases with a change from no immigration to free immigration.

The market tightness τ, τ_i, and trade quantities $q_i, \tilde{q}_i$ decrease monotonically with nominal interest rate ι, which is intuitive, since a higher interest rate motivates agents to become buyers and hold less money. The total number of matches or trades in the currency union always increases at a lower level of interest rate, and starts to decrease after a certain threshold. This results from a combined effect of decreases in the matches in country 1 and increases in the matches in country 2.[10]

4 Calibration

In this section, we want to use the pre 2008 financial crisis data of the Euro Area (EA) to calibrate the model. In doing so, we will 1) learn the optimal interest rate of the EA and the welfare gain/loss at the optimal interest rate, 2) compare the implemented monetary policy to the theoretical optimal, and 3) have a better understanding of the market structure of the EA member states and the degree of goods mobility between states.

[9] Appendix A4

[10] Figure in A6.

Six parameters are jointly calibrated using Simulated Method of Moments: market integration parameter α, utility function parameter σ, bargaining power θ_1, θ_2, and the CM outputs A_1, A_2. We assume a free immigration policy for calibration. Targets are calculated using 1996-2007 data of the EA countries from OECD Statistics, and the European Central Bank.[11] Calibrated results are presented in Table 4. All the EA member countries before 2007 are divided into two groups roughly based on their economic influences : the core countries and the periphery countries.[12] The core includes Austria, Belgium, Finland, France, Germany, and Luxembourg, and the periphery includes Greece, Ireland, Italy, Portugal, Slovenia, and Spain as of 2007. These two groups correspond to the two countries in the model, with the core group corresponding to country 1, and the periphery group corresponding to country 2.

4.1 Firm Markups

How do we account for the differences between member states in a currency union. Asymmetric effects of policy could come from any aspect of the modeling. I will focus on the difference in market structure. Market structure is characterized by buyers' bargaining power in this model, which measures the market competitiveness. It is relevant in this model because congestion externality is heavily affected by buyers' bargaining power. The asymmetry applies to the effect of both immigration and trade policy.

Buyers' bargaining power is closely related to firm markups. Markup is defined as the market price over the marginal cost of the special good.[13] Using variables in the model, the markup μ_i of DM_i can be expressed as follows

$$\begin{aligned} \mu_1 &= \frac{\alpha z_1/q_1 + (1-\alpha) z_2/\tilde{q}_1}{c'(\alpha q_1 + (1-\alpha)\tilde{q}_1)}, \\ \mu_2 &= \frac{\alpha z_2/\tilde{q}_2 + (1-\alpha) z_1/q_2}{c'(\alpha \tilde{q}_2 + (1-\alpha) q_2)}. \end{aligned} \tag{13}$$

Take the first equation for instance, the numerator is the price in DM_1, a weighted average of home and foreign buyers' real balances over trade quantities, with the weight determined by the degree of market integration α. The higher the market integration, the more weight is put on the price of imported goods. The denominator is the marginal cost of seller 1 at the quantity of trade with buyers from both home and foreign countries. If firm markups are higher in a country, the market is less competitive on average; it translates into a lower bargaining power of the buyers, because buyers in this market extract a smaller proportion of the total trade surplus.

[11] Tables 4-5 in the appendix A4 provides the source and description of the parameters used.
[12] De Santis and Cesaroni (2016), Bartlett and Prica (2016)
[13] The special goods are tradable, and the general goods are non-tradable.

The estimation of firm markups follows Roeger (1995), whose method has been used frequently in the literature. For each country c, the markup μ_c is assumed to be constant over time, and can be consistently estimated using an OLS regression. For each country,

$$y_{ct} = \zeta_c x_{ct} + \epsilon_{ct},$$

where $y_{ct} = \Delta p_{ct} + \Delta Q_{ct} - (\Delta w_{ct} + \Delta N_{ct})$, $x_{ct} = \Delta p_{ct} + \Delta Q_{ct}$. $\Delta p_{ct}, \Delta Q_{ct}, \Delta w_{ct}, \Delta N_{ct}$ are the log difference of the price of output, production quantities, labor cost, and labor hours, respectively, between year t and year $t-1$. The coefficient is $\zeta_c = (1 - 1/\mu_c)$, so we can derive the markup $\mu_c = 1/(1-\zeta_c)$.[14]. Data used include gross output at current prices and compensation of employees. Country-level markups are estimated for each EA country over the time period 1996-2007 in Table 3. Core and periphery markups are weighted average of country-level markups using the gross outputs as the weight.[15] The periphery, on average, has a higher markup than the core, meaning that on average, the goods market in the core region is more competitive.

Table 3: Markups μ_c

Core (1)		Periphery (2)	
Austria	1.73	Greece	2.33
Belgium	1.66	Ireland	2.15
Finland	1.73	Italy	2.15
France	1.59	Portugal	1.61
Germany	1.67	Slovenia	1.61
Luxembourg	1.78	Spain	1.71
Netherlands	1.68		
Core average	1.67	Periphery average	2.00

4.2 Real Money Demand

The real demand for money is defined as the aggregate real balances divided by aggregate nominal output

$$L(\iota) = \frac{Z_1 + Z_2}{y_1 + y_2 + A_1 + A_2}. \tag{14}$$

Z_1, Z_2 are the aggregate money balances, y_1, y_2 are the aggregate DM outputs, and A_1, A_2 are the aggregate CM outputs, where

$$Z_1 = \mathcal{B}_1 z_1, \quad Z_2 = \mathcal{B}_2 z_2,$$
$$y_1 = \mathcal{M}_{11}\alpha[(1-\theta_1)u(q_1) + \theta_1 c(q_1)] + \mathcal{M}_{21}(1-\alpha)[(1-\theta_1)u(\tilde{q}_1) + \theta_1 c(\tilde{q}_1)],$$
$$y_2 = \mathcal{M}_{22}\alpha[(1-\theta_2)u(\tilde{q}_2) + \theta_2 c(\tilde{q}_2)] + \mathcal{M}_{12}(1-\alpha)[(1-\theta_2)u(q_2) + \theta_2 c(q_2)].$$

[14] For any variable X_t, $\Delta X_t = ln(X_t) - ln(X_{t-1})$.

[15] Gross output here excludes the trade with non-EA countries.

The sum of aggregate real balances corresponds to the monetary aggregate M1 in the economy.[16] $(y_i + A_i)$ corresponds to the nominal gross output of country i. ι is the overnight facility deposit rate implemented by the ECB.

4.3 Import and Population

The model yields quantitative results for trade flows between regions, which are also observable in data. Using the bilateral trade flow data of the EA member countries, we are able to calculate the value of imports from core to periphery countries, and that from periphery to core countries.

$$\begin{aligned} \frac{\mathcal{M}_{12} z_1}{y_1 + A_1} &= \frac{\text{Core import from periphery}}{\text{Core gross output}}, \\ \frac{\mathcal{M}_{21} z_2}{y_2 + A_2} &= \frac{\text{Periphery import from core}}{\text{Periphery gross output}}. \end{aligned} \tag{15}$$

Predictions from the model are on the left-hand side and data is on the right-hand side. In the first equation of (15) for instance, the numerator of the term on the left-hand side is buyer 1's payment to seller 2 for goods purchased in country 2; this is equivalent to the nominal value of the core's import of the periphery's goods. We use the import share in output of the two regions as two calibration targets. Assuming free mobility of labor, the model also yields predictions for steady state relative population, which can be observed in data

$$\frac{n_2}{n_1} = \frac{\text{Periphery population}}{\text{Core population}}. \tag{16}$$

Table 4: Calibration

Parameters	Equations	Sources	Estimates	Targets
$\alpha = 0.7674$	(13)	OECD	1.4715	1.5015
$\sigma = 0.5302$	(13)	OECD	2.0308	1.9960
$A_1 = 4.7325$	(14)	ECB	0.2060	0.2159
$A_2 = 0.9238$	(15)	OECD	0.0301	0.0331
$\theta_1 = 0.5823$	(15)	OECD	0.0268	0.0222
$\theta_2 = 0.3240$	(16)	OECD	0.7671	0.7746

4.4 EA Estimates

Parameter estimates are presented in Table 4[17]. $\alpha = 0.7674 < 1$ means the euro area does not have a fully integrated goods market. $\theta_1 > \theta_2$ means the core countries on average

[16] M1 here is the sum of currency in circulation and overnight deposits.

[17] Appendix A4

has a higher buyers' bargaining power/more competitive goods market than the periphery countries. With the calibrated parameter values in Table 4, we can determine the optimal monetary policy of the monetary union and estimate the welfare cost of inflation at the optimal rate. The optimal nominal interest rate predicted by the model is .52% per annum, which is lower than the rate implemented by the ECB .93%, meaning that the EA could have benefited more as a whole with a lower interest rate. The welfare cost of inflation is -.71% at interest rate .52%, meaning that there is a welfare benefit of .71% at the optimal rate. The welfare cost of inflation is -.69% at interest rate .93%. Using the gross output data of the EA countries, the difference is equivalent to approximately 2.59 billion euros.

5 Optimal Policy and the Welfare Cost of Inflation

Using data and the model, we have learned the basic facts about the EA, but the EA is only one form of a currency union that takes shape. What about other forms of currency union? We want to do numerical analysis as policy experiment to study the interaction between monetary policy and currency union structure details. The goal is to gain a deeper and more general understanding of the impact of these details. This quantitative analysis aims to answer these questions: What is the impact of market structure on the welfare of member states? Is free labor and goods mobility necessarily beneficial for a currency union and the member states? Can member states be negatively affected by free mobility?

In order to do so, we design four regimes for quantitative analysis. The four regimes are specified in Table 5, with differences in immigration and market integration policy. Regime I is a closed economy, where there is no mobility in goods ($\alpha = 1$) and no immigration (the equilibrium population of each country is equal to the initial population). Regime IV is an open economy with full mobility in goods an labor. Regime II and III are somewhere in between, with no immigration and disintegrated market, respectively.

Table 5: Market Integration and Immigration

	No Immigration	Free Immigration
Disintegrated Market	I. $\alpha = 1, n_1 = 1, n_2 = n$	III. $\alpha = 1, n_1 + n_2 = 1 + n$
Integrated Market	II. $\alpha = 0.5, n_1 = 1, n_2 = n$	IV. $\alpha = 0.5, n_1 + n_2 = 1 + n$

For each regime, the optimal interest rate is determined by maximizing the total welfare of the currency union. The welfare of each country can thus be calculated at the optimal interest rate. We adopt the functional form of agents' utility and cost as follows: $u(q) = q^{1-\sigma}/(1-\sigma)$, where $\sigma \geq 0$ and $c(q) = q$. The choice of parameter values are

shown in Table 6. β and σ are standard values from the literature for easy comparison.[18] α dictates whether the goods market is integrated ($\alpha = 0.5$) or disintegrated ($\alpha = 1$). $n = 1$, meaning we assume two countries have identical size of initial population. For simplicity, we choose $\theta_1 = 0.5$ as the bargaining power in country 1, because it satisfies the Hosios condition in a closed economy.[19] The bargaining power in country 2 may vary between 0 and 1. When $\theta_2 \in (0, 0.5]$ – lower or equal to θ_1, the optimal policy of the currency union always follows the Friedman rule, regardless of regime.[20] When $\theta_2 \in (0.5, 1]$, however, more agents are tempted to become buyers, causing a congestion externality in the goods market that can only be alleviated by positive interest rate – costly money holding reduces the number of buyers and improves matching. The optimal interest rate thus deviates from FR, and the benefit of improved matching outweighs the loss from reduced trade. To illustrate, we choose $\theta_2 = 0.8$ and show in Figure 2 the welfare of the currency union and each member country in each regime. Take the open-economy regime for example (regime IV, purple line), the optimal interest rate of the currency union is 3.5%. The welfare level of the two countries at 3.5% is 4.4 and 1.6, respectively.

Table 6: Parameter Values

	Values	Descriptions
β	0.97	Discount factor
σ	0.14	Parameter in utility function
α	0.5,1	Integrated, Disintegrated market
n	1	Country 2 initial population
θ_1	0.5	Country 1 bargaining power
θ_2	0.6-0.8	Country 2 bargaining power

One obvious disadvantage of comparing the level of welfare is the difficulty to map it to the level of output. Instead of the level of welfare, a better way to measure the impact of monetary policy (change in interest rate) is the welfare cost of inflation, as we are comparing the percentage change of welfare, and it can be easily mapped to the percentage change of output. We calculate the welfare cost of inflation of each regime following Craig and Rocheteau (2008), where the cost of inflation is defined as the fraction of total consumption that individuals are willing to give up to reduce inflation to zero (compensated welfare). The compensated welfare approach captures the change in social benefits of real

[18] σ is chosen from Table 2 in Craig and Rocheteau (2008) for numerical examples.

[19] When the Hosios condition is satisfied in a closed economy, the proportion of buyers, market tightness, and matching elasticity in country 1 is $b_1 = 0.5$, $\tau_1 = 1$, $\epsilon(\tau_1) = \tau_1/(1 + \tau_1) = 0.5$, respectively.

[20] If $\theta_2 < 0.5$, there are too few buyers in the market. Since costly money holding reduces the number of buyers, optimal policy follows the Friedman rule.

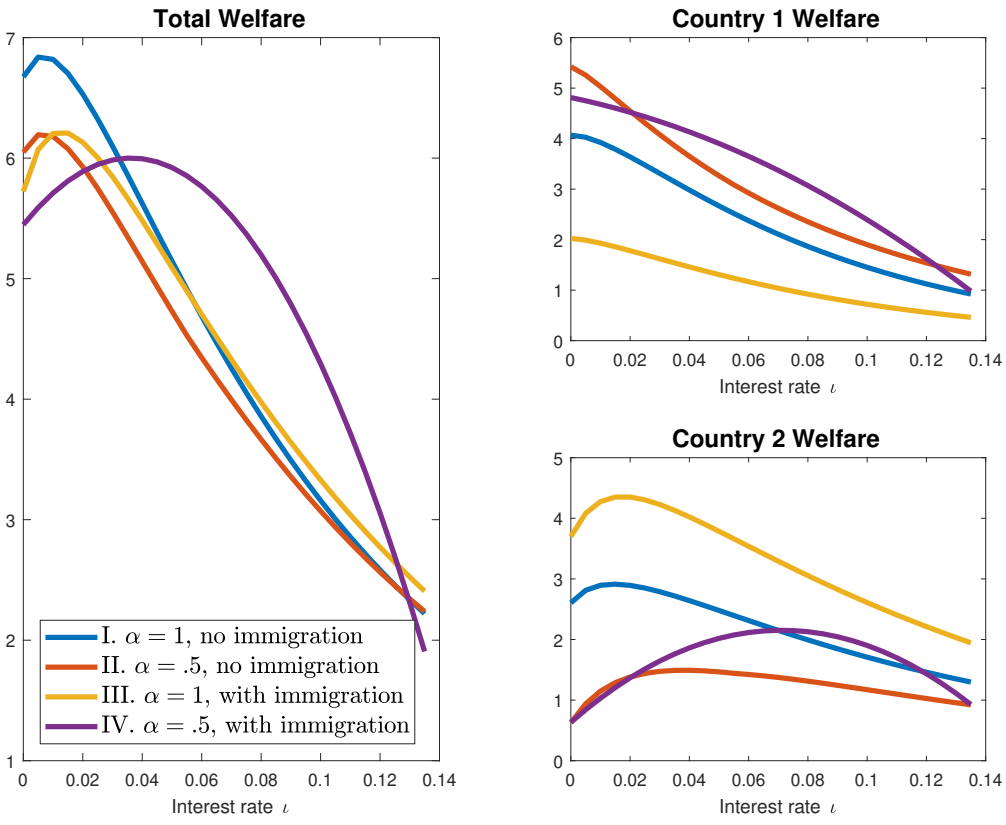

Figure 2: Welfare and optimal policy of regimes I-IV, $\theta_1 = 0.5, \theta_2 = 0.8$

balances. It is different from the traditional "welfare triangle" approach[21], which captures the change in buyers' private benefits of holding real balances. The welfare triangle approach underestimates the true welfare cost of inflation in a search framework with proportional bargaining because of a discrepancy between the marginal social benefits and buyers' marginal private benefits of real balances arises from a rent-sharing externality.[22] Under proportional bargaining, buyers bear the marginal cost of holding real balances but only extract a proportion of the marginal social benefits, reducing real balances below the socially desirable level. With the discrepancy between marginal social and private benefits, compensated welfare is a more accurate measure of the true welfare cost of inflation.

5.1 Closed Economy

Regime I - a closed economy, can be used as the benchmark case. In a closed economy, buyers' in a country do not have access to the foreign DM and immigration is closed. With $\alpha = 1$, buyers never visit the foreign DM.

To measure the welfare cost of inflation, we define $\Delta, \Delta_1, \Delta_2 \in (0, 1)$ as the welfare cost of inflation of the monetary union and individual countries, respectively. (****optimal policy of currency union is the same when only total welfare if considered. optimal policy for each country if it was not part of a currency union and only cared about themselves.) Optimal monetary policy is determined such that the welfare cost of inflation of the monetary union Δ is minimized. Let $X^{0.03}$ be the value of variable X evaluated at $\iota = 0.03$ (zero inflation), and X^{ι} be the value of X evaluated at nominal interest rate ι. Country i's cost of inflation Δ_i must satisfy

$$\mathcal{M}_i^{0.03}[u(q_i^{0.03}(1-\Delta_i)) - c(q_i^{0.03})] - A_i\Delta_i = \mathcal{M}_i[u(q_i^{\iota}) - c(q_i^{\iota})]. \tag{17}$$

(17) shows that in order to achieve the welfare at zero inflation, country i is willing to give up a proportion of Δ_i of its DM output $q_i^{0.03}$ and CM output A_i at nominal interest rate $\iota = \gamma/\beta - 1$, where $\gamma = M'/M$. The CM output cannot be pinned down by the model, but we calibrated this quantity in Section 4. Similarly, Δ is the proportion of the gross output that the monetary union is willing to give up to achieve zero inflation. Δ satisfies

[21] Baily (1956) and Lucas (2000). This is also called Bailey-Lucas approach. Welfare cost of inflation is measured by the area of the "welfare triangle," or the area underneath the money demand curve.

[22] This rent-sharing externality is generated by pricing mechanisms that do not allow buyers to extract the full trade surplus.

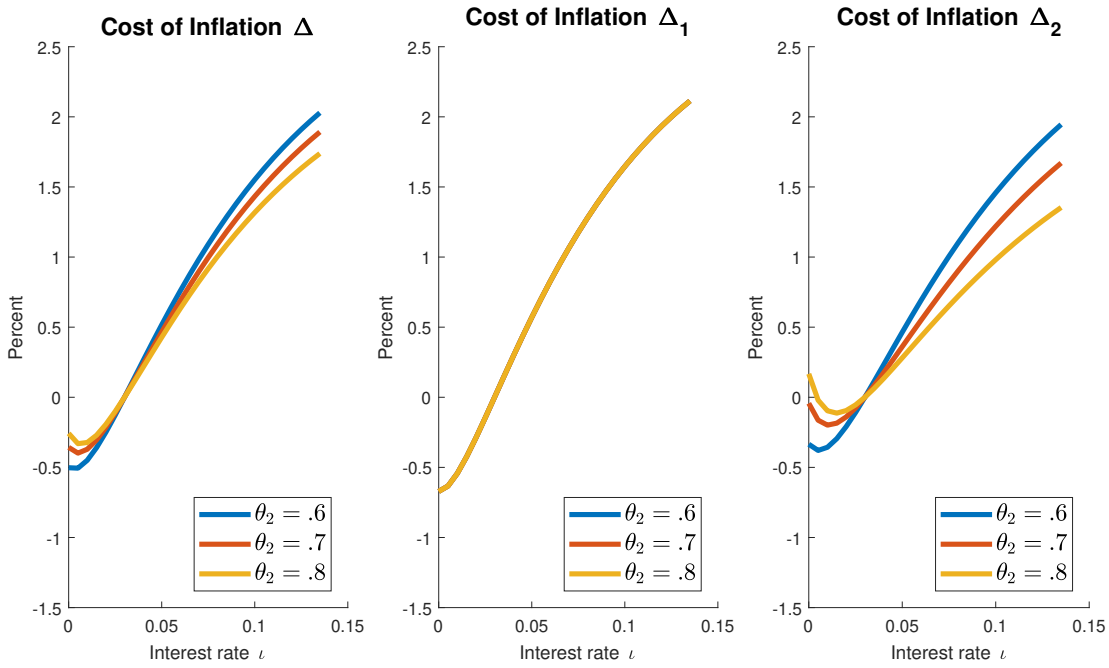

Figure 3: I. $\alpha = 1$, no immigration

$$\begin{aligned}\mathcal{M}_1^{0.03}[u(q_1^{0.03}(1-\Delta)) - c(q_1^{0.03})] + \mathcal{M}_2^{0.03}[u(q_2^{0.03}(1-\Delta)) - c(q_2^{0.03})] - (A_1+A_2)\Delta \\ = \mathcal{M}_1[u(q_1^{\iota}) - c(q_1^{\iota})] + \mathcal{M}_2[u(q_2^{\iota}) - c(q_2^{\iota})].\end{aligned} \tag{18}$$

The terms on the left-hand side of (18) are the welfare of the two DM's and the two CM's after giving up Δ outputs.

Regime I: disintegrated market and no immigration

Figure 3 plots the welfare cost of inflation of individual countries and the monetary union as the nominal interest rate ι increases from 0 to 13.5% (equivalent to appoximately 10% of inflation), confirming the analytical results. When $\theta_2 > 0.5$, the cost of inflation is negative for small positive ι, meaning that a country with bargaining power greater than 0.5 benefits from a small positive ι. Note that when $\iota < 0.03$, monetary policy is still deflationary.

5.2 Open Economy

Each of the other three regimes has either integrated market, free immigration, or both. The levels of welfare are illustrated in Figure 2 and optimal monetary policy can be observed. We can see that in all cases, the optimal interest rate deviates from the Friedman rule. Free immigration and a higher degree of market integration pushes the optimal interest rate to a higher level.

The equations to compute the welfare cost of inflation change because of buyers and goods from a foreign country. For country 1, Δ_1 satisfies

$$\begin{aligned}\mathcal{M}_{11}^{0.03}[u(q_1^{0.03}(1-\Delta_1))-c(q_1^{0.03})]+\mathcal{M}_{21}^{0.03}[u(\tilde{q}_1^{0.03}(1-\Delta_1))-c(\tilde{q}_1^{0.03})]-A_1\Delta_1 \\ =\mathcal{M}_{11}[u(q_1^{\iota})-c(q_1^{\iota})]+\mathcal{M}_{21}[u(\tilde{q}_1^{\iota})-c(\tilde{q}_1^{\iota})].\end{aligned} \tag{19}$$

$\mathcal{M}_{ij}$ denotes the number of matches between buyers from i and sellers in j. $\mathcal{M}_{ij}=\frac{B_i}{\mathcal{B}_j}\mathcal{M}(\mathcal{B}_j,\mathcal{S}_j)$, where B_i is the number of buyers visiting from country i, and $\mathcal{B}_j$ is the total number of buyers present in the DM in country j. Given that in country j, $\frac{B_i}{\mathcal{B}_j}$ of the total matches is attributed to matches between buyers from country i and sellers in j. Country 1 needs to give up a proportion of Δ_1 of the domestic DM goods and imported foreign DM goods. Similarly, Δ_2 satisfies

$$\begin{aligned}\mathcal{M}_{12}^{0.03}[u(q_2^{0.03}(1-\Delta_2))-c(q_2^{0.03})]+\mathcal{M}_{22}^{0.03}[u(\tilde{q}_2^{0.03}(1-\Delta_2))-c(\tilde{q}_2^{0.03})]-A_2\Delta_2 \\ =\mathcal{M}_{12}[u(q_2^{\iota})-c(q_2^{\iota})]+\mathcal{M}_{22}[u(\tilde{q}_2^{\iota})-c(\tilde{q}_2^{\iota})],\end{aligned}$$

and Δ satisfies

$$\begin{aligned}&\mathcal{M}_{11}^{0.03}[u(q_1^{0.03}(1-\Delta))-c(q_1^{0.03})]+\mathcal{M}_{21}^{0.03}[u(\tilde{q}_1^{0.03}(1-\Delta))-c(\tilde{q}_1^{0.03})] \\ &+\mathcal{M}_{12}^{0.03}[u(q_2^{0.03}(1-\Delta))-c(q_2^{0.03})]+\mathcal{M}_{22}^{0.03}[u(\tilde{q}_2^{0.03}(1-\Delta))-c(\tilde{q}_2^{0.03})]-(A_1+A_2)\Delta \\ &=\mathcal{M}_{11}[u(q_1^{\iota})-c(q_1^{\iota})]+\mathcal{M}_{21}[u(\tilde{q}_1^{\iota})-c(\tilde{q}_1^{\iota})] \\ &+\mathcal{M}_{12}[u(q_2^{\iota})-c(q_2^{\iota})]+\mathcal{M}_{22}[u(\tilde{q}_2^{\iota})-c(\tilde{q}_2^{\iota})].\end{aligned}$$

Regime II: integrated market and no immigration

In regime II, market is completely integrated ($\alpha=0.5$), but immigration is not allowed. Optimal monetary policy deviates from the Friedman rule slightly more than the closed economy, putting the optimal interest rate at 0.5% when $\theta_2=0.8$. In Figure 4, the optimal interest rate benefits country 1 (lower bargaining power) more than country 2 (higher bargaining power). The higher country 2's bargaining power, the more optimal policy benefits country 1, and the more it hurts country 2.

Regime III: disintegrated market and immigration

In regime III, market is disintegrated ($\alpha=1$), but immigration is allowed. Optimal policy deviates from the Friedman rule more than regime II, putting the optimal interest rate at 1.5% when $\theta_2=0.8$. In Figure 5, the optimal interest rate benefits country 2 more at a lower bargaining power, and benefits country 1 more at a higher bargaining power.

Regime IV: integrated market and immigration

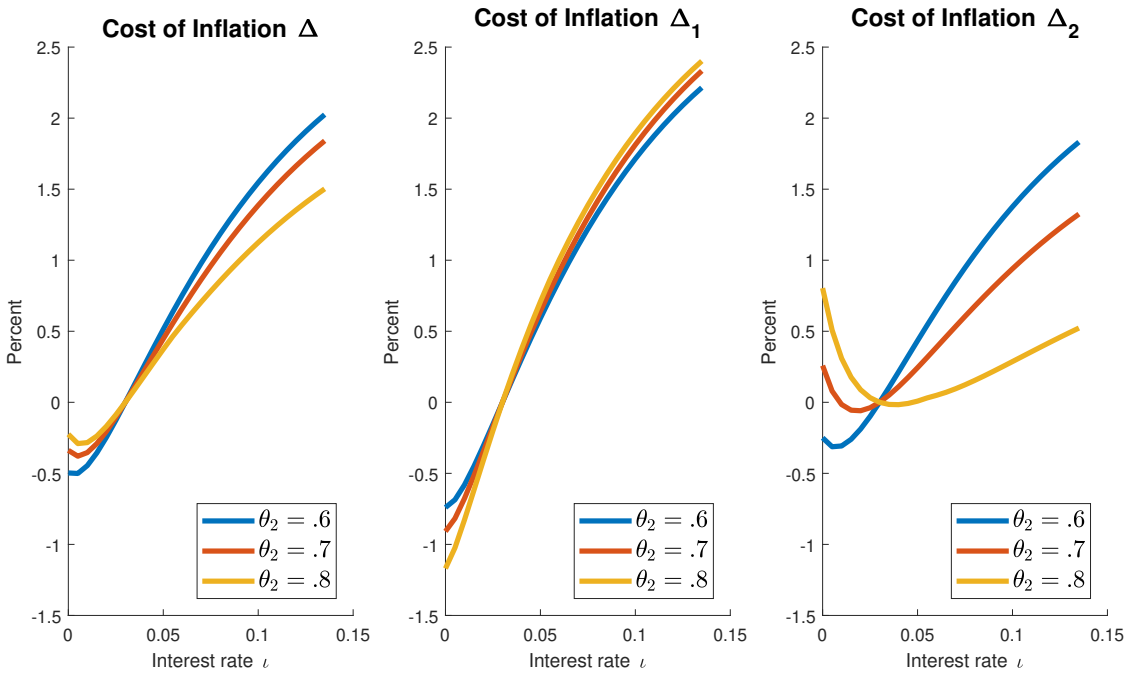

Figure 4: II. $\alpha = 0.5$, immigration not allowed

In regimes IV, market is completely integrated and immigration is allowed. In Figure 6, Optimal policy deviates from the Friedman rule the furthest compared to the other three regimes. The optimal interest rate is 3.5%, which benefits country 2 more in general.

5.2.1 Bargaining power

In each of the four regimes, the optimal interest rate increases with buyers' bargaining power, because a higher interest rate is required to reduce the congestion externality generated by a higher bargaining power. The welfare cost of inflation of the currency union decreases with buyers' bargaining power for a positive inflation in each regime. The higher country 2's bargaining power, the higher the proportion of the trade surplus buyers in country 2 extract, reducing the rent-sharing externality, and therefore resulting in a lower welfare cost of inflation.[23]

We plot the welfare cost of inflation in Figure 7 with $\theta_2 = 0.8$ to illustrate the effect of immigration policy and market integration on the welfare cost of inflation.[24] The left panel is the cost of inflation of the currency union. The welfare gain of the monetary

[23]Rent-sharing externality is discussed extensively in Craig and Rocheteau (2008). When a buyer does not get the full marginal benefit of his real balances, discrepancy between the private and social benefits of real balances arises from a rent-sharing externality. In other words, the social welfare cost of inflation is reduced when buyers' bargaining power is increased.

[24]The figure does not change substantially with θ_2 as long as θ_2 is greater than 0.5.

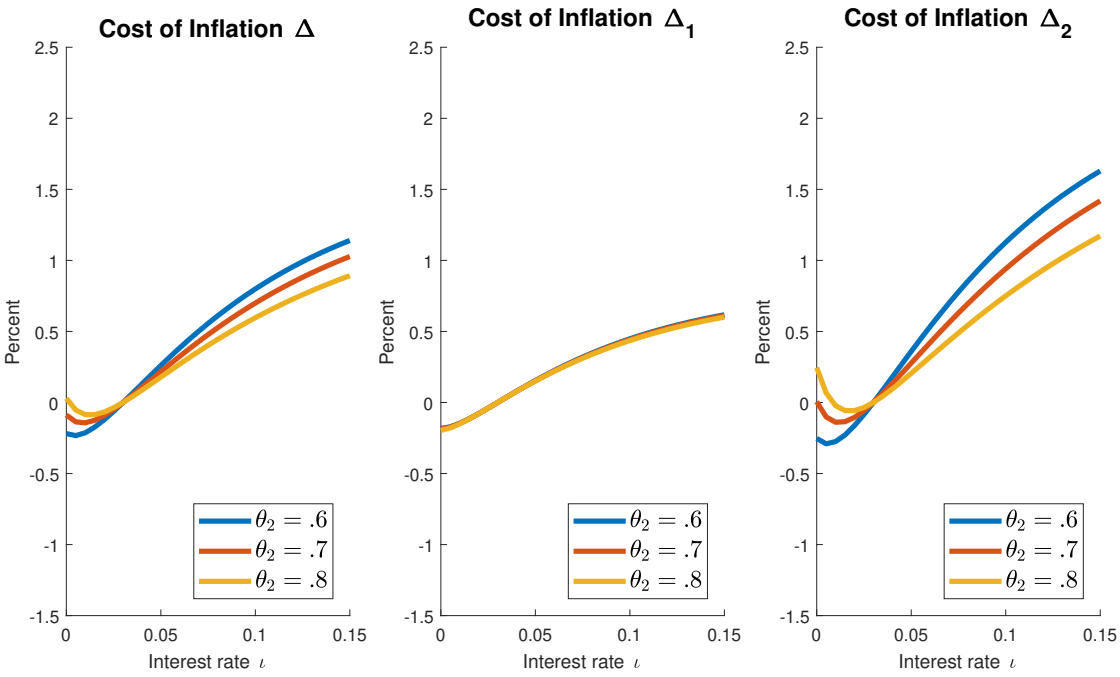

Figure 5: III. $\alpha = 1$, immigration allowed

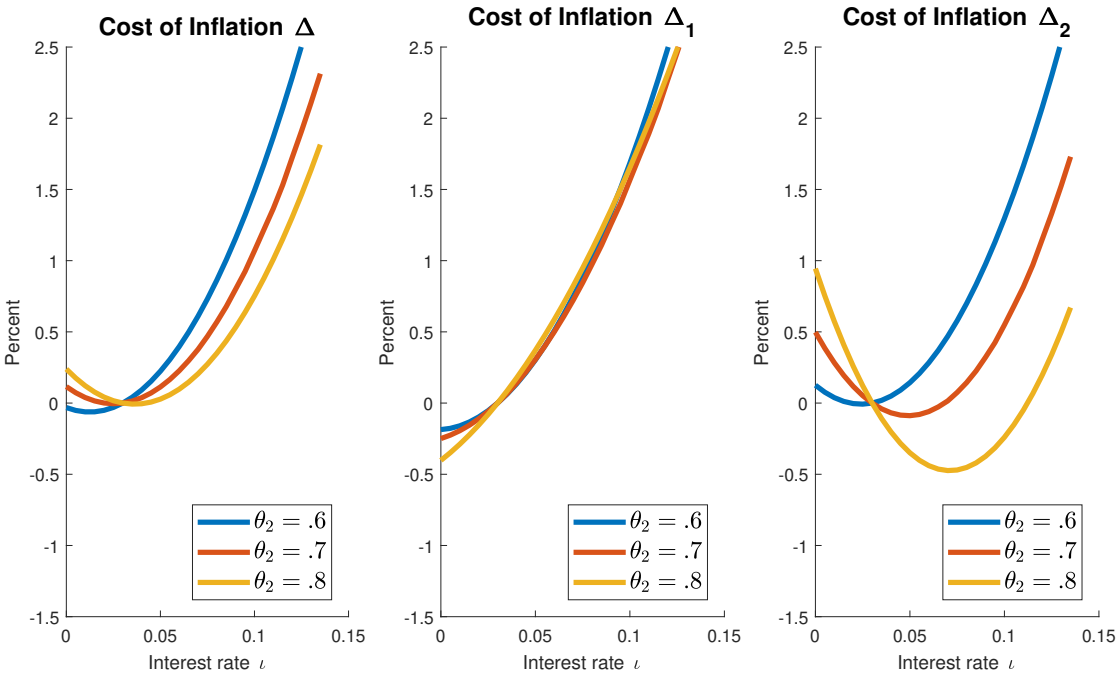

Figure 6: IV. $\alpha = 0.5$, immigration allowed

union associated with a reduction of the interest rate from 0 to 3% (corresponding to an inflation between from -3% to 0) is the lowest with disintegrated market and no immigration; the gain is as high as 0.33% of output. For a positive inflation, it is always true that $\Delta_I > \Delta_{II} > \Delta_{III}$. The lack of mobility in goods and labor limits the positive effect of inflation on matching, leading to the highest cost of inflation in regime I. Free immigration policy alone has a larger effect in reducing the welfare cost of inflation compared to having a perfectly integrated market alone, because free immigration allows monetary policy to better rebalance the agent composition in the entire monetary union.

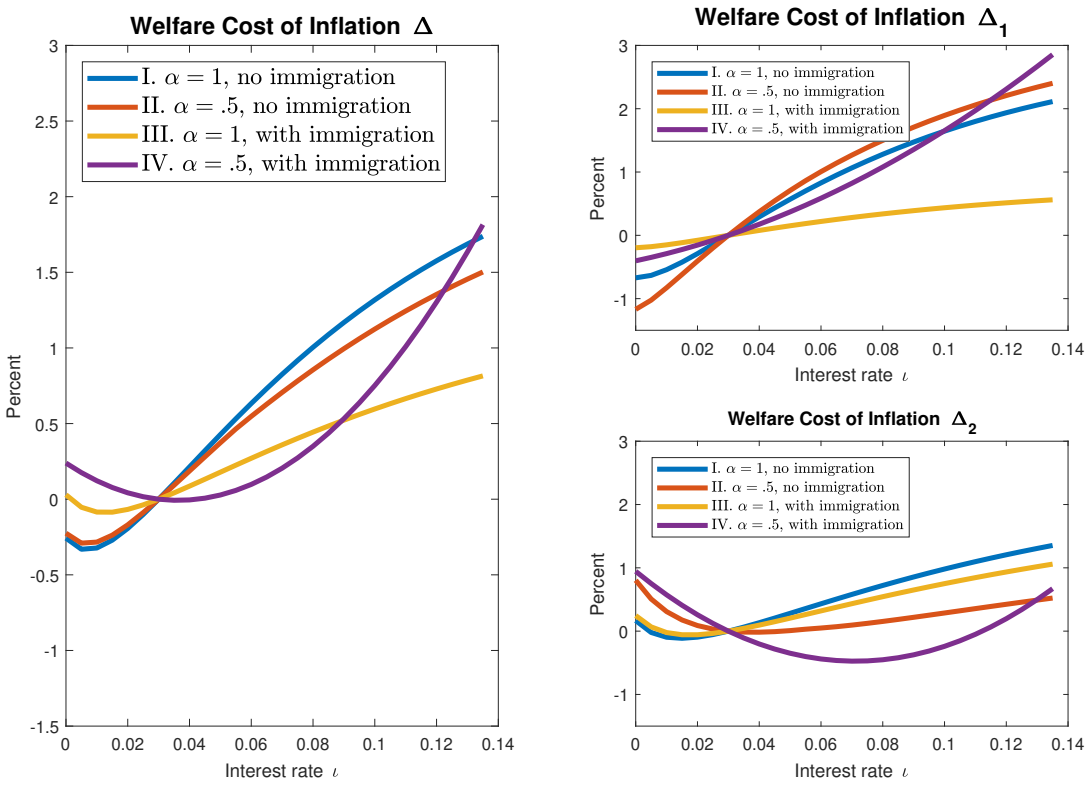

Figure 7: Welfare cost of inflation of regimes I-IV, $\theta_1 = 0.5, \theta_2 = 0.8$

We also observe a nonmonotonic relationship between interest rate and welfare cost in regime IV. When the interest rate is moderate, $\iota \in [3\%, 8.5\%)$, corresponding to an inflation between [0, 5.3%), the cost of inflation is the lowest with regime IV, but when interest rate is high, $\iota \in [8.5\%, 13.5\%]$, corresponding to an inflation between [5.3%, 10%], the welfare cost of inflation with regime IV increases rapidly and eventually exceeds the other three regimes.

On the right panel of Figure 7, we plot the cost of inflation of the two countries separately, where country 1 has a lower bargaining power, and country 2 has a higher bargaining power. we look at how bargaining power induces different effect of each policy

for positive inflation.

5.2.2 Immigration policy

The effect of immigration policy on the two countries are to the same direction; allowing for free mobility of labor decreases the cost of inflation in general given degree of market integration. A nonmonotonic relationship between inflation and cost of inflation occurs only in country 2 (where bargaining power is higher) when the market is fully integrated. For a sufficiently high inflation in regime IV, free immigration policy increases cost of inflation in country 2.

5.2.3 Market integration

Market integration has the opposite effect on the cost of inflation in the two countries. Given immigration policy, a higher degree of market integration increases the cost of inflation of the country with lower bargaining power, and decreases the cost of inflation of the country with higher bargaining power. In other words, free mobility of goods reduces the cost of inflation for the country with a less competitive market.

When monetary policy is inflationary, free mobility of goods and labor is unambiguously beneficial for country 2. For country 1 in this experiment, free immigration is the dominating factor, and therefore, country 1 would prefer not to have an integrated market although it is beneficial to the union. The situation is reversed when monetary policy is deflationary. For instance, country 1 favors an environment with no immigration but fully integrated market. In either case, country 1's interest does not align with the interest of the currency union.

5.3 Welfare cost of inflation

Table 7 presents the welfare cost of inflation of the four regimes corresponding to 2% and 10% of inflation.[25] We are interested in the welfare cost of inflation at 2% inflation because this is the inflation target set by the European Central Bank. ***what would be the most beneficial regime for the union? At 2% inflation, regime IV has the lowest cost of inflation; at 10% inflation, regime III has the lowest cost of inflation. *** what if it was up to individual countries to decide?

[25] 2% and 10% inflation corresponds to interest rates 5.06% and 13.3%, respectively.

Table 7: Welfare Cost of Inflation Δ (%)

Regimes	Description	2% Inflation	10% Inflation
I	Disintegrated market, no immigration	0.43	1.74
II	Integrated market, no immigration	0.37	1.50
III	Disintegrated market, free immigration	0.18	0.82
IV	Integrated market, free immigration	0.03	1.81

6 Conclusion

This paper studies optimal monetary policy and the welfare cost of inflation of a currency union using an open-economy search model with endogenous composition of buyers and sellers, and heterogeneous bargaining power. When buyers' bargaining power is too high, the Friedman rule is suboptimal due to matching congestion; a positive interest rate motivates agents to switch roles from buyers to sellers, alleviating the congestion. A decrease in buyers' bargaining power, or an increase in market integration or labor mobility limits the ability of monetary policy to correct for the matching congestion, thus deviating optimal monetary policy from the Friedman rule even further. A higher labor mobility decreases the cost of inflation for both countries regardless of buyers' bargaining power. A higher market integration, however, only decreases the cost of inflation in the country with higher bargaining power and has the opposite effect on the other. When one country's bargaining power is sufficiently higher than the other country, the interest of the country with lower bargaining power does not align with the interest of the currency union.

For the four regimes in the quantitative analysis, a higher degree of market integration and an open immigration policy both raise the optimal interest rate. Comparing the cost of inflation at optimal policy in each regime, a closed economy gains the highest benefit (lowest cost). Which regime generates the lowest cost of inflation depends on the level of nominal interest rate. At a low interest rate, closed economy has the highest benefit; at a moderate interest rate, integrated market with free immigration policy has the lowest cost of inflation; at a high interest rate, disintegrated market with free immigration policy has the lowest cost of inflation.

References

Aghion, Philippe, Peter Howitt, and David Mayer-Foulkes. "The effect of financial development on convergence: Theory and evidence". In: *The quarterly journal of economics* 120.1 (2005), pp. 173–222.

Alvarez, Fernando, Robert E Lucas, and Warren E Weber. "Interest rates and inflation". In: *American Economic Review* 91.2 (2001), pp. 219–225.

Araujo, Luis and Leo Ferraris. "Societal Benefit of Multiple Currencies". In: *Journal of Money, Credit and Banking* 53.8 (2021), pp. 2201–2214.

Aruoba, S Borağan, Guillaume Rocheteau, and Christopher Waller. "Bargaining and the Value of Money". In: *Journal of Monetary Economics* 54.8 (2007), pp. 2636–2655.

Baldwin, Richard E et al. "Study on the impact of the euro on trade and foreign direct investment". In: *European Economic and Monetary Union Working Paper* 321 (2008).

Bartlett, William and Ivana Prica. "Interdependence between core and peripheries of the European economy: secular stagnation and growth in the Western Balkans". In: *LEQS Paper* 104 (2016).

Becker, Sascha S and Dieter Nautz. "Inflation, price dispersion and market integration through the lens of a monetary search model". In: *European Economic Review* 56.3 (2012), pp. 624–634.

Benigno, Pierpaolo. "Optimal monetary policy in a currency area". In: *Journal of international economics* 63.2 (2004), pp. 293–320.

Berentsen, Aleksander, Guillaume Rocheteau, and Shouyong Shi. "Friedman meets Hosios: Efficiency in search models of money". In: *The Economic Journal* 117.516 (2007), pp. 174–195.

Bignon, Vincent, Régis Breton, and Mariana Rojas Breu. "Currency union with or without banking union". In: *International Economic Review* 60.2 (2019), pp. 965–1003.

Black, Stanley W. "Transactions costs and vehicle currencies". In: *Journal of International Money and Finance* 10.4 (1991), pp. 512–526.

Cacciatore, Matteo, Giuseppe Fiori, and Fabio Ghironi. "Market deregulation and optimal monetary policy in a monetary union". In: *Journal of International Economics* 99 (2016), pp. 120–137.

Choi, Michael and Guillaume Rocheteau. "Foundations of market power in monetary economies". In: *Available at SSRN 3763587* (2021).

Christopoulou, Rebekka and Philip Vermeulen. "Markups in the Euro area and the US over the period 1981–2004: a comparison of 50 sectors". In: *Empirical Economics* 42.1 (2012), pp. 53–77.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger. "The rise of market power and the macroeconomic implications". In: *The Quarterly Journal of Economics* 135.2 (2020), pp. 561–644.

De Santis, Roberta and Tatiana Cesaroni. "Current account 'core–periphery dualism'in the EMU". In: *The World Economy* 39.10 (2016), pp. 1514–1538.

Dellas, Harris and George S Tavlas. "An optimum-currency-area odyssey". In: *Journal of International Money and Finance* 28.7 (2009), pp. 1117–1137.

Devereux, Michael B and Shouyong Shi. "Vehicle currency". In: *International Economic Review* 54.1 (2013), pp. 97–133.

Duc, Julien and Mariana Rojas-Breu. "Central-Bank Digital Currency and Risk Sharing in a Currency Union". In: (2021).

Eichengreen, Barry. "European monetary unification". In: *Journal of Economic literature* 31.3 (1993), pp. 1321–1357.

Faig, Miquel. "Endogenous buyer–seller choice and divisible money in search equilibrium". In: *Journal of Economic Theory* 141.1 (2008), pp. 184–199.

Forni, Lorenzo, Andrea Gerali, and Massimiliano Pisani. "Macroeconomic effects of greater competition in the service sector: the case of Italy". In: *Macroeconomic Dynamics* 14.5 (2010), pp. 677–708.

Frieden, Jeffry A. "The impact of goods and capital market integration on European monetary politics". In: *Comparative Political Studies* 29.2 (1996), pp. 193–222.

Fuerst, Timothy S. "Liquidity, loanable funds, and real activity". In: *Journal of monetary economics* 29.1 (1992), pp. 3–24.

Gomes, Sandra et al. "Structural Reforms and Macroeconomic Performance in the Euro Area Countries: A Model-B ased Assessment". In: *International Finance* 16.1 (2013), pp. 23–44.

Gomis-Porqueras, Pedro and Cathy Zhang. "Optimal monetary and fiscal policy in a currency union with frictional goods markets". In: *Macroeconomic Dynamics* 25.7 (2021), pp. 1726–1754.

— *Optimal Monetary and Fiscal Policy with Migration in a Currency Union*. Tech. rep. University Library of Munich, Germany, 2018.

Hall, Robert E and Dale W Jorgenson. "Tax policy and investment behavior: Reply and further results". In: *The American Economic Review* 59.3 (1969), pp. 388–401.

Hartmann, Philipp. "The currency denomination of world trade after European Monetary Union". In: *Journal of the Japanese and International Economies* 12.4 (1998), pp. 424–454.

Head, Allen and Alok Kumar. "Price dispersion, inflation, and welfare". In: *International Economic Review* 46.2 (2005), pp. 533–572.

Head, Allen, Beverly Lapham, et al. "Search, market power, and inflation dynamics". In: *2006 Meeting Papers*. Vol. 559. 2005.

Head, Allen and Shouyong Shi. "A fundamental theory of exchange rates and direct currency trades". In: *Journal of Monetary Economics* 50.7 (2003), pp. 1555–1591.

Herrenbreuck, L. *Optimal Monetary Policy, Currency Unions, and the Eurozone Divergence*. Tech. rep. working paper, 2015.

Hosios, Arthur J. "On the efficiency of matching and related models of search and unemployment". In: *The Review of Economic Studies* 57.2 (1990), pp. 279–298.

Kalai, Ehud. "Proportional solutions to bargaining situations: interpersonal utility comparisons". In: *Econometrica: Journal of the Econometric Society* (1977), pp. 1623–1630.

Kiyotaki, Nobuhiro and Randall Wright. "A contribution to the pure theory of money". In: *Journal of economic Theory* 53.2 (1991), pp. 215–235.

— "A search-theoretic approach to monetary economics." In: *The American Economic Review* (1993), pp. 63–77.

Krugman, Paul et al. "The international role of the dollar: theory and prospect". In: *Exchange rate theory and practice* 1 (1984), pp. 261–278.

Lagos, Ricardo. "Some results on the optimality and implementation of the Friedman rule in the search theory of money". In: *Journal of Economic Theory* 145.4 (2010), pp. 1508–1524.

Lagos, Ricardo and Randall Wright. "A unified framework for monetary theory and policy analysis". In: *Journal of political Economy* 113.3 (2005), pp. 463–484.

Liu, Qing and Shouyong Shi. "Currency areas and monetary coordination". In: *International Economic Review* 51.3 (2010), pp. 813–836.

Lucas Jr, Robert E. "Inflation and welfare". In: *Monetary Theory as a Basis for Monetary Policy*. Springer, 2001, pp. 96–142.

— "Nobel lecture: Monetary neutrality". In: *Journal of political economy* 104.4 (1996), pp. 661–682.

Lucas Jr, Robert E and Juan Pablo Nicolini. "On the stability of money demand". In: *Journal of Monetary Economics* 73 (2015), pp. 48–65.

Matsuyama, Kiminori, Nobuhiro Kiyotaki, and Akihiko Matsui. "Toward a theory of international currency". In: *The Review of Economic Studies* 60.2 (1993), pp. 283–307.

Mundell, Robert A. "A theory of optimum currency areas". In: *The American economic review* 51.4 (1961), pp. 657–665.

Pappa, Evi. "Do the ECB and the Fed really need to cooperate? Optimal monetary policy in a two-country world". In: *Journal of Monetary Economics* 51.4 (2004), pp. 753–779.

Rocheteau, Guillaume and Ed Nosal. *Money, payments, and liquidity*. MIT Press, 2017.

Rocheteau, Guillaume and Randall Wright. "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium". In: *Econometrica* 73.1 (2005), pp. 175–202.

Roeger, Werner. "Can imperfect competition explain the difference between primal and dual productivity measures? Estimates for US manufacturing". In: *Journal of political Economy* 103.2 (1995), pp. 316–330.

Shapley, Lloyd and Martin Shubik. "Trade using one commodity as a means of payment". In: *Journal of political economy* 85.5 (1977), pp. 937–968.

Shi, Shouyong. "A divisible search model of fiat money". In: *Econometrica: Journal of the Econometric Society* (1997), pp. 75–102.

— "Liquidity, bargaining, and multiple equilibria in a search monetary model". In: *Annals of Economics and Finance* 2.2 (2001), pp. 325–351.

Silva, Mario. "New monetarism with endogenous product variety and monopolistic competition". In: *Journal of Economic Dynamics and Control* 75 (2017), pp. 158–181.

Starr, Martha A. "One world, one currency: exploring the issues". In: *Contemporary Economic Policy* 24.4 (2006), pp. 618–633.

Thum-Thysen, Anna, Erik Canton, et al. *Estimation of service sector mark-ups determined by structural reform indicators*. Tech. rep. Directorate General Economic and Financial Affairs (DG ECFIN), European Commission, 2015.

Wallace, Neil. "Whither monetary economics?" In: *International Economic Review* 42.4 (2001), pp. 847–869.

Waller, Christopher J. "Microfoundations of Money: Why They Matter". In: *Federal Reserve Bank of St. Louis Review* 97.4 (2015), pp. 289–301.

Weber, Warren E et al. "International Evidence on Long Run Money Demand". In: *2017 Meeting Papers*. 1154. Society for Economic Dynamics. 2017.

Wright, Randall and Alberto Trejos. "International currency". In: *The BE Journal of Macroeconomics* 1.1 (2001).

Zhang, Cathy. "An information-based theory of international currency". In: *Journal of International Economics* 93.2 (2014), pp. 286–301.