School of Information Technology

Module	:	Business Statistics
Topic	:	Discrete Probability Distribution

Learning Outcomes:

By the end of this lesson, you should be able to

1. Identify characteristics of a probability distribution.
2. Calculate the mean, variance and expected value of a discrete probability distribution.
3. Identify characteristics of a Binomial and Poisson distribution through the assumptions for each distribution.
4. Calculate probabilities, mean and standard deviation of the Binomial and Poisson distribution using relevant formulas.
5. Solve real-life business problems by applying concepts of discrete, Binomial and Poisson distribution.

Topic: Discrete Probability Distribution

Part I: Discrete Probability Distribution

QUESTION 1

Randomly selected households from a particular estate were asked on the number of children they have, and the following frequency distribution shows the result of the survey:

Number of children	0	1	2	3
Households	300	280	95	20

(a) Construct a probability distribution table.

(b) Let X denotes the number of children from a particular estate. Compute the following probabilities:

(i) $P(X = 1)$ (ii) $P(X \geq 2)$

(iii) $P(X < 1)$ (iv) $P(1 \leq X \leq 3)$

QUESTION 2

The following table shows the distribution of household sizes k in a small town.

k	1	2	3	4	5	6
$P(X = k)$	0.266	0.330	0.166	0.140	0.064	0.034

(a) Show that the distribution is a probability distribution.

(b) What is the expected size of a household in the town?

QUESTION 3

Let the random variable X be the number of errors that a randomly selected page of a book contains. The following table lists the probability distribution of X.

X	0	1	2	3	4
$P(X=x)$	0.73	0.16	k	0.04	0.01

(a) Find the value of k

(b) Find the mean and standard deviation of X.

QUESTION 4

John sells new cars for Toyota. He usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

x, No. cars sold	0	1	2	3	4
P(X=x)	0.1	0.2	0.3	0.3	0.1

(a) Compute the expected value.

(b) What is the variance of the distribution?

Part 2: Binomial Distribution

QUESTION 5

Given that X ~ B(10,0.3), calculate the following:

(a) mean (b) variance

(c) $P(X \leq 2)$ (d) $P(X > 8)$

QUESTION 6

A recent study by Singapore Police revealed that only 60 percent of the Singaporean drivers wear seat belts. In order to confirm this, a sample of 10 drivers on the expressway is selected.

(a) Explain if this situation fits the assumptions of Binomial distribution.

(b) Compute the probability that exactly 7 are wearing seat belts.

(c) Compute the probability that 7 or fewer of the drivers are wearing seat belts.

QUESTION 7

A company manufactures toy robots. 1 out of 100 toy robot is damaged during the manufacturing process. You purchased 35 toy robots.

(a) Compute the probability that exactly 4 robots were damaged.

(b) Compute the probability at least 1 toy robot was damaged.

(c) Compute the mean and standard deviation of robots that were damaged.

QUESTION 8

A firm owns cameras. Each camera has an 8% probability of not working. 20 cameras were randomly selected for inspection. What is the probability that:

(a) 5 cameras will not work.?

(b) all the cameras will work?

QUESTION 9

A recent survey found that 32% of all the Small and Medium Enterprises [SMEs] in Singapore are ready to adopt digital technologies in their business. Out of 10 randomly selected firms, what is the probability that

(a) less than two SMEs are ready to adopt digital technologies in their business?

(b) at least two SMEs are ready to adopt digital technologies in their business?

(c) What is the expected number and standard deviation of SMEs which are ready to adopt digital technologies in their business?

Part 3: Poisson Distribution

QUESTION 10

Given that $X \sim P_0(3)$, calculate the following:

(a) mean (b) variance

(c) $P(X \leq 2)$ (d) $P(X \geq 3)$

QUESTION 11

On average, a household receives 1.8 junk mails per day. Using the Poisson formula, find the probability that a randomly selected household receives

(a) exactly 3 junk mails on a certain day,

(b) at most 2 junk mails on a certain day.

QUESTION 12

A budget airline receives an average of 9.7 complaints per day from its passengers. Using the Poisson formula, find the probability that on a certain day this airline will receive

(a) exactly 5 complaints.

(b) at least 3 complaints.

QUESTION 13

A customer service department receives an average of 1.6 telephone calls in any 10-minute interval. Find the probability that the department receives

(a) no calls in any 10-minute interval.

(b) at most 1 calls in any 5-minute interval.

(c) more than 2 calls in any 15-minute interval.

QUESTION 14

During a graduation ceremony in NYP, guests arrive at an average of 30 per hour. What is the probability that

(a) at least 2 guests will arrive in any particular minute?

(b) 3 guests will arrive in any particular minute?

SUPPLEMENTARY QUESTIONS

QUESTION 15

Find the mean, variance and standard deviation of the random variable in the following probability distribution:

k	0	1	2	3	4
P (X = k)	0.54	0.16	0.06	0.04	0.20

QUESTION 16

A recent survey conducted by the police showed that at 35% of Singaporean drivers exceeded the speed limit allowed on the expressway. Given that the speed of cars follow a binomial distribution, a random sample of 7 drivers was surveyed.

(a) Compute the probability that fewer than 3 drivers exceeded the speed limit.

(b) Compute the probability that exactly 4 drivers exceeded the speed limit.

QUESTION 17

On average, six students per hour use a printer in the computer lab. Compute the probability that:

(a) No one will use the printer during a ten-minute interval.
(b) Exactly one student will use the printer during a five-minute interval
(c) Compute the mean and standard deviation of printer used for half an hour.

Answers:

Q1(a)

Number of children	0	1	2	3
$P(X = x)$	60/139	56/139	19/139	4/139

(b) (i) 0.403 (ii) 0.165 (iii) 0.432 (iv) 0.568
Q2 (b) 2.508
Q3 (a) 0.06 (b) mean = 0.44, standard deviation = 0.852
Q4 (a) 2.1 (b) 1.29
Q5 (a) 3 (b) 2.1 (c) 0.383 (d) 0.000144
Q6 (b) 0.215 (c) 0.8327
Q7 (a) 0.0004 (b) 0.2966 (c) 0.5886
Q8 (a) 0.0145 (b) 0.1887
Q0 (a) 0.1206 (b) 0.8794 (c) Expected number = 3.2, standard deviation = 1.4751
Q10 (a) 3 (b) 3 (c) 0.423 (d) 0.577
Q11 (a) 0.161 (b) 0.731
Q12 (a) 0.0439 (b) 0.996
Q13 (a) 0.202 (b) 0.809 (c) 0.430
Q14 (a) 0.0902 (b) 0.0126
Q15 mean = 1.2, variance = 3.06, standard deviation = 1.7493
Q16 (a) 0.5323 (b) 0.1442
Q17 (a) 0.3679 (b) 0.3033 (c) mean = 3, standard deviation = 1.7321