ECE 403
Assignment #1

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All MATLAB code used to generate answers for this assignment can be found at

github.com/ashlynns/ECE403/tree/master/A1

1.1

a) $X = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$ $\quad X^T = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$ $\quad C = XX^T = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$$\left.\begin{aligned} C_{11} &= 1\cdot1 + 1\cdot1 + 2\cdot2 + 2\cdot2 = 10 \\ C_{12} &= 1\cdot1 + 1\cdot2 + 2\cdot3 + 2\cdot2 = 13 \\ C_{21} &= 1\cdot1 + 2\cdot1 + 3\cdot2 + 2\cdot2 = 13 \\ C_{22} &= 1\cdot1 + 2\cdot2 + 3\cdot3 + 2\cdot2 = 18 \end{aligned}\right\} \quad C = \begin{bmatrix} 10 & 13 \\ 13 & 18 \end{bmatrix}$$

b) $\det(\lambda I - C) = \begin{bmatrix} \lambda-10 & -13 \\ -13 & \lambda-18 \end{bmatrix}$

$$= (\lambda-10)(\lambda-18) - (-13)(-13)$$

$$= \lambda^2 - 28\lambda + 180 - 169$$

$$= \lambda^2 - 28\lambda + 11$$

$$\lambda_{1,2} = \frac{28 \pm \sqrt{(-28)^2 - 4(1)(11)}}{2(1)} = 14 \pm \frac{\sqrt{784-44}}{2} = 14 \pm \frac{\sqrt{740}}{2}$$

$$\lambda_1 = 27.6$$
$$\lambda_2 = 0.398$$

c)

Q1_1.m

```
1 -   X = [1 1 2 2; 1 2 3 2];
2 -   XT = X';
3 -   C = X*XT;
4 -   E = eig(C)
```

Command Window

```
>> Q1_1

E =

    0.3985
   27.6015
```

1.2 - See attached MATLAB live script

1.3 - $\left(\frac{v^T u}{u^T \cdot u}\right) u = \left(\frac{v^T u}{\|u\| \|u\| \cos(0)}\right) u = \left(\frac{v^T u}{\|u\|}\right) \cdot \frac{u}{\|u\|}$

$= \left(v^T \left(\frac{u}{\|u\|}\right)\right) \cdot \frac{u}{\|u\|}$

$= (v^T \tilde{u}) \tilde{u}$