```
load('building256.mat')
X = building 256/255;
% a
C = X*X'
C = 256 \times 256
 242.8823 242.6921 242.5849 242.2224 242.3723 242.4943 242.3410 242.6505 ...
  242.6921 242.5772 242.4313 242.0689 242.2204 242.3420 242.1862 242.4918
  242.5849 242.4313 242.3654 241.9604 242.1112 242.2369 242.0824 242.3853
  242.2224 242.0689 241.9604 241.6424 241.7473 241.8747 241.7177 242.0258
  242.3723 242.2204 242.1112 241.7473 241.9348 242.0253 241.8715 242.1717
  242.4943 242.3420 242.2369 241.8747 242.0253 242.1966 241.9904 242.2975
  242.3410 242.1862 242.0824 241.7177 241.8715 241.9904 241.8777 242.1454
  242.6505 242.4918 242.3853 242.0258 242.1717 242.2975 242.1454 242.4840
 242.0516 241.8996 241.7886 241.4254 241.5780 241.7005 241.5482 241.8506
  241.9173 241.7598 241.6530 241.2898 241.4390 241.5684 241.4087 241.7192
% b
[V, D] = eigs(C);
[d,ind] = sort(diag(D), 'descend');
Ds = D(ind, ind);
Vs = V(:,ind);
sigma = diag(Ds);
sigma = sigma(1:5)
sigma = 5x1
10^{4} \times
   3.8236
   0.0679
   0.0247
   0.0070
   0.0035
u = Vs(:, 1:5)
u = 256 \times 5
   0.0796
          0.0217 -0.0096 0.0394 -0.0825
          0.0207 -0.0094 0.0380 -0.0781
   0.0795
          0.0211 -0.0077
                            0.0380 -0.0782
   0.0795
          0.0212 -0.0076
                            0.0360 -0.0738
   0.0794
            0.0218 -0.0068
                              0.0371 -0.0734
   0.0794
            0.0228
   0.0795
                    -0.0081
                              0.0382
                                      -0.0746
   0.0794
            0.0215
                    -0.0083
                                       -0.0775
                              0.0404
   0.0795
            0.0216 -0.0089
                              0.0362 - 0.0775
                            0.0373 -0.0782
   0.0793
          0.0212 -0.0083
                            0.0379
   0.0793
            0.0214 -0.0085
                                     -0.0755
% C
V = zeros(size(u));
for i = 1:length(sigma)
```

```
V(:,i) = sigma(i)^{(-1/2)*X'*u(:,i)};
end
% Each column in V represents vi where i is the column index
V
V = 256 \times 5
   0.0607
           -0.0760
                    -0.0344
                             -0.0090
                                       -0.0956
                             -0.0047
                                       -0.0819
   0.0618
           -0.0802 -0.0367
           -0.0751
   0.0615
                    -0.0409
                              0.0058
                                       -0.0749
           -0.0688
   0.0611
                    -0.0475
                              0.0245
                                       -0.0459
   0.0606
           -0.0650
                    -0.0467
                              0.0430
                                       -0.0407
           -0.0640 -0.0511
   0.0606
                            0.0456
                                      -0.0379
   0.0605
          -0.0586 -0.0497 0.0456 -0.0499
   0.0603
          -0.0583 -0.0519
                            0.0478 -0.0407
          -0.0576 -0.0522 0.0526
   0.0605
                                      -0.0506
   0.0604
          -0.0581 -0.0493 0.0539 -0.0463
% d
X_{i1} = sigma(1)^{(1/2)}u(:,1)*V(:,1)';
X_{i2} = sigma(2)^{(1/2)}u(:,2)*V(:,2)';
X i3 = sigma(3)^{(1/2)}u(:,3)*V(:,3)';
X i4 = sigma(4)^{(1/2)}u(:,4)*V(:,4)';
X_{i5} = sigma(5)^{(1/2)}u(:,5)*V(:,5)';
X k1 = X i1
X_k1 = 256 \times 256
   0.9441
          0.9619
                   0.9569 0.9504 0.9428 0.9431 0.9418
                                                                   0.9381 •••
   0.9436
          0.9614 0.9565 0.9499
                                      0.9424 0.9426 0.9413
                                                                   0.9377
   0.9432
          0.9610 0.9560 0.9495
                                       0.9419
                                               0.9422 0.9409
                                                                   0.9372
   0.9419
           0.9596 0.9547
                             0.9482
                                        0.9406
                                               0.9408
                                                          0.9396
                                                                   0.9359
                   0.9552
   0.9424
          0.9602
                             0.9487
                                        0.9411
                                                 0.9414
                                                          0.9401
                                                                   0.9364
   0.9428
          0.9606
                   0.9557
                             0.9491
                                        0.9416
                                                 0.9418
                                                          0.9406
                                                                   0.9369
            0.9599
                   0.9550
                             0.9485
                                        0.9409
                                                 0.9412
                                                          0.9399
                                                                   0.9362
   0.9422
   0.9434
           0.9612 0.9563
                             0.9497
                                        0.9421 0.9424
                                                          0.9411
                                                                   0.9375
   0.9411
            0.9588
                   0.9539
                              0.9474
                                        0.9399
                                               0.9401
                                                          0.9389
                                                                   0.9352
   0.9406
            0.9583 0.9534
                             0.9469
                                        0.9393
                                                 0.9396
                                                          0.9383
                                                                   0.9347
X_k2 = X_{i1}+X_{i2}
X_k2 = 256 \times 256
   0.9011
            0.9165
                      0.9144
                               0.9115
                                        0.9060
                                                 0.9068
                                                          0.9087
                                                                   0.9051 ...
   0.9026
            0.9180
                      0.9159
                               0.9128
                                        0.9072
                                                 0.9080
                                                          0.9097
                                                                   0.9062
   0.9014
            0.9168
                      0.9147
                              0.9116
                                        0.9062
                                                 0.9070
                                                          0.9087
                                                                   0.9052
            0.9153
                               0.9102
                                        0.9047
                                                          0.9073
                                                                   0.9037
   0.8999
                      0.9132
                                                 0.9055
   0.8992
            0.9145
                      0.9125
                              0.9096
                                        0.9041
                                                 0.9050
                                                          0.9068
                                                                   0.9033
   0.8977
            0.9129
                      0.9110
                               0.9082
                                        0.9029
                                                 0.9038
                                                          0.9058
                                                                   0.9022
                                                 0.9053
   0.8996
            0.9150
                      0.9129
                               0.9099
                                        0.9045
                                                          0.9071
                                                                   0.9036
   0.9006
            0.9160
                      0.9139
                               0.9110
                                        0.9055
                                                 0.9063
                                                          0.9081
                                                                   0.9046
   0.8992
                     0.9125
                               0.9095
                                        0.9040
                                                 0.9048
                                                          0.9065
                                                                   0.9030
            0.9146
   0.8983
            0.9136
                     0.9115
                               0.9085
                                        0.9031
                                                 0.9039
                                                          0.9057
                                                                   0.9022
```

```
X_k3 = 256 \times 256
   0.9063
         0.9220 0.9206 0.9186
                                  0.9131 0.9146 0.9162
                                                            0.9130 •••
   0.9076
         0.9235 0.9219 0.9198
                                  0.9141 0.9156 0.9170
                                                            0.9138
         0.9213 0.9197
                         0.9174
                                  0.9118 0.9132
                                                  0.9147
   0.9056
                                                            0.9115
         0.9197 0.9181
                                  0.9103 0.9116
                         0.9159
                                                  0.9132
   0.9040
                                                            0.9100
         0.9184 0.9168
                         0.9146
                                  0.9091 0.9104
                                                  0.9121
                                                            0.9088
   0.9028
                                          0.9103
         0.9176 0.9162
                         0.9143
                                                  0.9121
   0.9020
                                   0.9089
                                                            0.9088
          0.9197 0.9182
                                          0.9120
                                                  0.9136
   0.9041
                           0.9161
                                   0.9106
                                                            0.9103
                  0.9196
   0.9054
          0.9211
                           0.9176
                                   0.9120
                                           0.9134
                                                   0.9151
                                                            0.9118
   0.9037
          0.9193
                 0.9178
                           0.9156
                                   0.9100
                                           0.9114
                                                   0.9130
                                                            0.9098
   0.9029
          0.9185
                  0.9170
                          0.9149
                                   0.9094
                                           0.9108
                                                   0.9124
                                                            0.9092
X k4 = X i1+X i2+X i3+X i4
X_k4 = 256 \times 256
   0.9033
          0.9205
                  0.9225
                           0.9267
                                   0.9273
                                           0.9296
                                                    0.9312
                                                            0.9287 •••
   0.9048
          0.9220
                  0.9237
                           0.9275
                                   0.9278
                                           0.9301
                                                   0.9315
                                                            0.9290
   0.9027
          0.9198
                  0.9215
                           0.9252
                                           0.9276
                                                    0.9292
                                   0.9255
                                                            0.9266
                 0.9198
   0.9013
          0.9183
                           0.9232
                                   0.9232
                                           0.9254
                                                    0.9269
                                                            0.9243
   0.9000
          0.9169
                  0.9186
                           0.9222
                                   0.9224
                                           0.9245
                                                   0.9262
                                                            0.9236
                         0.9221
                                                  0.9266
   0.8992
          0.9161 0.9181
                                   0.9226
                                           0.9248
                                                            0.9241
                                   0.9251
                                           0.9273 0.9290
   0.9011
         0.9182 0.9202 0.9244
                                                            0.9265
   0.9027
         0.9197 0.9214 0.9250 0.9250 0.9272 0.9288
                                                            0.9263
   0.9009
         0.9179 0.9196 0.9232 0.9234 0.9256 0.9272
                                                            0.9247
   0.9000
         0.9170 0.9189 0.9227
                                   0.9230 0.9252 0.9268
                                                            0.9243
X k5 = X i1+X i2+X i3+X i4+X
X_k5 = 256 \times 256
   1.8759
          1.9087
                  1.9107
                         1.8993
                                   1.9155
                                           1.9178
                                                  1.9038
                                                            1.9013 ...
          1.9102 1.8963 1.9001 1.9003 1.9026 1.9198
                                                            1.9016
   1.8773
   1.8753
         1.8923 1.9097 1.8977
                                  1.8980 1.9159 1.9174
                                                            1.8992
   1.8739
          1.9065 1.8924 1.9115
                                   1.8958 1.8979 1.8995
                                                            1.9126
   1.8412 1.8895 1.9069 1.8947 1.8950 1.8971 1.8988
                                                            1.8962
   1.8403 1.8886 1.8906 1.8946 1.8638 1.8974 1.8992
                                                            1.8966
   1.8736
          1.9064 1.8927 1.8969 1.8976 1.9156 1.9015
                                                            1.8990
          1.8922 1.9096 1.8975 1.8976 1.9155 1.9014
   1.8752
                                                            1.8675
          1.9061
                  1.8608 1.8644 1.8960 1.8982 1.8684
                                                            1.8658
   1.8420
   1.8412
          1.8896 1.9071 1.8952 1.8642 1.8664 1.8994
                                                            1.8655
% e
fro_X = norm(X,'fro');
e_1 = norm((X_k1-X), 'fro')/fro_X
e_1 = 0.1742
e 2 = norm((X k2-X), 'fro')/fro X
```

X k3 = X i1+X i2+X i3

 $e_2 = 0.1145$

```
e_3 = norm((X_k3-X), 'fro')/fro_X

e_3 = 0.0828

e_4 = norm((X_k4-X), 'fro')/fro_X

e_4 = 0.0713

e_5 = norm((X_k5-X), 'fro')/fro_X

e_5 = 0.9975
```

```
f figure(1)
subplot(221)
imshow(X)
title("original building256")
subplot(222)
imshow(X_k1)
title("rank-1 approximation")
subplot(223)
imshow(X_k3)
title("rank-3 approximation")
subplot(224)
imshow(X_k5)
title("rank-5 approximation")
```

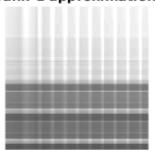
original building 256



rank-3 approximation



rank-1 approximation



rank-5 approximation



```
% g
V_values = prod(size(V));
sigma_values = prod(size(sigma));
u_values = prod(size(u));

n5 = V_values + sigma_values + u_values;
compression_ratio = (256^2)/n5
```

compression_ratio = 25.5501