ECE 403
Assignment #1

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All MATLAB code used to generate answers for this assignment can be found at

github.com/ashlynns/ECE403/tree/master/A1

1.1

a) $X = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$ $\quad X^T = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$ $\quad C = XX^T = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$$\left.\begin{aligned} C_{11} &= 1\cdot1 + 1\cdot1 + 2\cdot2 + 2\cdot2 = 10 \\ C_{12} &= 1\cdot1 + 1\cdot2 + 2\cdot3 + 2\cdot2 = 13 \\ C_{21} &= 1\cdot1 + 2\cdot1 + 3\cdot2 + 2\cdot2 = 13 \\ C_{22} &= 1\cdot1 + 2\cdot2 + 3\cdot3 + 2\cdot2 = 18 \end{aligned}\right\} \quad C = \begin{bmatrix} 10 & 13 \\ 13 & 18 \end{bmatrix}$$

b) $\det(\lambda I - C) = \begin{bmatrix} \lambda-10 & -13 \\ -13 & \lambda-18 \end{bmatrix}$

$$= (\lambda-10)(\lambda-18) - (-13)(-13)$$

$$= \lambda^2 - 28\lambda + 180 - 169$$

$$= \lambda^2 - 28\lambda + 11$$

$$\lambda_{1,2} = \frac{28 \pm \sqrt{(-28)^2 - 4(1)(11)}}{2(1)} = 14 \pm \frac{\sqrt{784-44}}{2} = 14 \pm \frac{\sqrt{740}}{2}$$

$$\lambda_1 = 27.6$$
$$\lambda_2 = 0.398$$

c)

Q1_1.m

```
1 -   X = [1 1 2 2; 1 2 3 2];
2 -   XT = X';
3 -   C = X*XT;
4 -   E = eig(C)
```

Command Window

```
>> Q1_1

E =

    0.3985
   27.6015
```

1.2 - See attached MATLAB live script

1.3 - $\left(\frac{v^T u}{u^T \cdot u}\right) u = \left(\frac{v^T u}{\|u\| \|u\| \cos(0)}\right) u = \left(\frac{v^T u}{\|u\|}\right) \cdot \frac{u}{\|u\|}$

$= \left(v^T \left(\frac{u}{\|u\|}\right)\right) \cdot \frac{u}{\|u\|}$

$= (v^T \tilde{u}) \tilde{u}$

```
load('building256.mat')
X = building256/255;
```

```
% a
C = X*X'
```

```
C = 256×256
  242.8823  242.6921  242.5849  242.2224  242.3723  242.4943  242.3410  242.6505 ···
  242.6921  242.5772  242.4313  242.0689  242.2204  242.3420  242.1862  242.4918
  242.5849  242.4313  242.3654  241.9604  242.1112  242.2369  242.0824  242.3853
  242.2224  242.0689  241.9604  241.6424  241.7473  241.8747  241.7177  242.0258
  242.3723  242.2204  242.1112  241.7473  241.9348  242.0253  241.8715  242.1717
  242.4943  242.3420  242.2369  241.8747  242.0253  242.1966  241.9904  242.2975
  242.3410  242.1862  242.0824  241.7177  241.8715  241.9904  241.8777  242.1454
  242.6505  242.4918  242.3853  242.0258  242.1717  242.2975  242.1454  242.4840
  242.0516  241.8996  241.7886  241.4254  241.5780  241.7005  241.5482  241.8506
  241.9173  241.7598  241.6530  241.2898  241.4390  241.5684  241.4087  241.7192
     ⋮
```

```
% b
[V, D] = eigs(C);
[d,ind] = sort(diag(D),'descend');
Ds = D(ind,ind);
Vs = V(:,ind);

sigma = diag(Ds);
sigma = sigma(1:5)
```

```
sigma = 5×1
10^4 ×
    3.8236
    0.0679
    0.0247
    0.0070
    0.0035
```

```
u = Vs(:, 1:5)
```

```
u = 256×5
    0.0796    0.0217   -0.0096    0.0394   -0.0825
    0.0795    0.0207   -0.0094    0.0380   -0.0781
    0.0795    0.0211   -0.0077    0.0380   -0.0782
    0.0794    0.0212   -0.0076    0.0360   -0.0738
    0.0794    0.0218   -0.0068    0.0371   -0.0734
    0.0795    0.0228   -0.0081    0.0382   -0.0746
    0.0794    0.0215   -0.0083    0.0404   -0.0775
    0.0795    0.0216   -0.0089    0.0362   -0.0775
    0.0793    0.0212   -0.0083    0.0373   -0.0782
    0.0793    0.0214   -0.0085    0.0379   -0.0755
     ⋮
```

```
% c
V = zeros(size(u));
for i = 1:length(sigma)
```

```
    V(:,i)= sigma(i)^(-1/2)*X'*u(:,i);
end

% Each column in V represents vi where i is the column index
V
```

```
V = 256×5
    0.0607   -0.0760   -0.0344   -0.0090   -0.0956
    0.0618   -0.0802   -0.0367   -0.0047   -0.0819
    0.0615   -0.0751   -0.0409    0.0058   -0.0749
    0.0611   -0.0688   -0.0475    0.0245   -0.0459
    0.0606   -0.0650   -0.0467    0.0430   -0.0407
    0.0606   -0.0640   -0.0511    0.0456   -0.0379
    0.0605   -0.0586   -0.0497    0.0456   -0.0499
    0.0603   -0.0583   -0.0519    0.0478   -0.0407
    0.0605   -0.0576   -0.0522    0.0526   -0.0506
    0.0604   -0.0581   -0.0493    0.0539   -0.0463
     ⋮
```

```
% d
X_i1 = sigma(1)^(1/2)*u(:,1)*V(:,1)';
X_i2 = sigma(2)^(1/2)*u(:,2)*V(:,2)';
X_i3 = sigma(3)^(1/2)*u(:,3)*V(:,3)';
X_i4 = sigma(4)^(1/2)*u(:,4)*V(:,4)';
X_i5 = sigma(5)^(1/2)*u(:,5)*V(:,5)';

X_k1 = X_i1
```

```
X_k1 = 256×256
    0.9441    0.9619    0.9569    0.9504    0.9428    0.9431    0.9418    0.9381 ···
    0.9436    0.9614    0.9565    0.9499    0.9424    0.9426    0.9413    0.9377
    0.9432    0.9610    0.9560    0.9495    0.9419    0.9422    0.9409    0.9372
    0.9419    0.9596    0.9547    0.9482    0.9406    0.9408    0.9396    0.9359
    0.9424    0.9602    0.9552    0.9487    0.9411    0.9414    0.9401    0.9364
    0.9428    0.9606    0.9557    0.9491    0.9416    0.9418    0.9406    0.9369
    0.9422    0.9599    0.9550    0.9485    0.9409    0.9412    0.9399    0.9362
    0.9434    0.9612    0.9563    0.9497    0.9421    0.9424    0.9411    0.9375
    0.9411    0.9588    0.9539    0.9474    0.9399    0.9401    0.9389    0.9352
    0.9406    0.9583    0.9534    0.9469    0.9393    0.9396    0.9383    0.9347
     ⋮
```

```
X_k2 = X_i1+X_i2
```

```
X_k2 = 256×256
    0.9011    0.9165    0.9144    0.9115    0.9060    0.9068    0.9087    0.9051 ···
    0.9026    0.9180    0.9159    0.9128    0.9072    0.9080    0.9097    0.9062
    0.9014    0.9168    0.9147    0.9116    0.9062    0.9070    0.9087    0.9052
    0.8999    0.9153    0.9132    0.9102    0.9047    0.9055    0.9073    0.9037
    0.8992    0.9145    0.9125    0.9096    0.9041    0.9050    0.9068    0.9033
    0.8977    0.9129    0.9110    0.9082    0.9029    0.9038    0.9058    0.9022
    0.8996    0.9150    0.9129    0.9099    0.9045    0.9053    0.9071    0.9036
    0.9006    0.9160    0.9139    0.9110    0.9055    0.9063    0.9081    0.9046
    0.8992    0.9146    0.9125    0.9095    0.9040    0.9048    0.9065    0.9030
    0.8983    0.9136    0.9115    0.9085    0.9031    0.9039    0.9057    0.9022
     ⋮
```

```
X_k3 = X_i1+X_i2+X_i3
```

```
X_k3 = 256×256
    0.9063    0.9220    0.9206    0.9186    0.9131    0.9146    0.9162    0.9130 ···
    0.9076    0.9235    0.9219    0.9198    0.9141    0.9156    0.9170    0.9138
    0.9056    0.9213    0.9197    0.9174    0.9118    0.9132    0.9147    0.9115
    0.9040    0.9197    0.9181    0.9159    0.9103    0.9116    0.9132    0.9100
    0.9028    0.9184    0.9168    0.9146    0.9091    0.9104    0.9121    0.9088
    0.9020    0.9176    0.9162    0.9143    0.9089    0.9103    0.9121    0.9088
    0.9041    0.9197    0.9182    0.9161    0.9106    0.9120    0.9136    0.9103
    0.9054    0.9211    0.9196    0.9176    0.9120    0.9134    0.9151    0.9118
    0.9037    0.9193    0.9178    0.9156    0.9100    0.9114    0.9130    0.9098
    0.9029    0.9185    0.9170    0.9149    0.9094    0.9108    0.9124    0.9092
     ⋮
```

```
X_k4 = X_i1+X_i2+X_i3+X_i4
```

```
X_k4 = 256×256
    0.9033    0.9205    0.9225    0.9267    0.9273    0.9296    0.9312    0.9287 ···
    0.9048    0.9220    0.9237    0.9275    0.9278    0.9301    0.9315    0.9290
    0.9027    0.9198    0.9215    0.9252    0.9255    0.9276    0.9292    0.9266
    0.9013    0.9183    0.9198    0.9232    0.9232    0.9254    0.9269    0.9243
    0.9000    0.9169    0.9186    0.9222    0.9224    0.9245    0.9262    0.9236
    0.8992    0.9161    0.9181    0.9221    0.9226    0.9248    0.9266    0.9241
    0.9011    0.9182    0.9202    0.9244    0.9251    0.9273    0.9290    0.9265
    0.9027    0.9197    0.9214    0.9250    0.9250    0.9272    0.9288    0.9263
    0.9009    0.9179    0.9196    0.9232    0.9234    0.9256    0.9272    0.9247
    0.9000    0.9170    0.9189    0.9227    0.9230    0.9252    0.9268    0.9243
     ⋮
```

```
X_k5 = X_i1+X_i2+X_i3+X_i4+X
```

```
X_k5 = 256×256
    1.8759    1.9087    1.9107    1.8993    1.9155    1.9178    1.9038    1.9013 ···
    1.8773    1.9102    1.8963    1.9001    1.9003    1.9026    1.9198    1.9016
    1.8753    1.8923    1.9097    1.8977    1.8980    1.9159    1.9174    1.8992
    1.8739    1.9065    1.8924    1.9115    1.8958    1.8979    1.8995    1.9126
    1.8412    1.8895    1.9069    1.8947    1.8950    1.8971    1.8988    1.8962
    1.8403    1.8886    1.8906    1.8946    1.8638    1.8974    1.8992    1.8966
    1.8736    1.9064    1.8927    1.8969    1.8976    1.9156    1.9015    1.8990
    1.8752    1.8922    1.9096    1.8975    1.8976    1.9155    1.9014    1.8675
    1.8420    1.9061    1.8608    1.8644    1.8960    1.8982    1.8684    1.8658
    1.8412    1.8896    1.9071    1.8952    1.8642    1.8664    1.8994    1.8655
     ⋮
```

```
% e
fro_X = norm(X,'fro');

e_1 = norm((X_k1-X), 'fro')/fro_X
```

```
e_1 = 0.1742
```

```
e_2 = norm((X_k2-X), 'fro')/fro_X
```

```
e_2 = 0.1145
```

```
e_3 = norm((X_k3-X), 'fro')/fro_X
```

```
e_3 = 0.0828
```

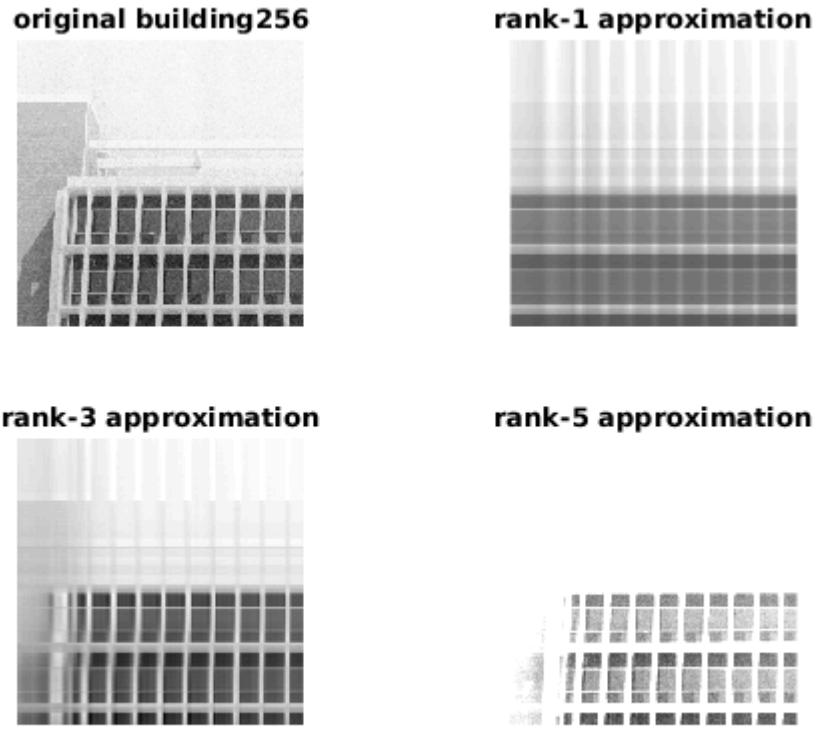
```
e_4 = norm((X_k4-X), 'fro')/fro_X
```

```
e_4 = 0.0713
```

```
e_5 = norm((X_k5-X), 'fro')/fro_X
```

```
e_5 = 0.9975
```

```
% f
 figure(1)
 subplot(221)
 imshow(X)
 title("original building256")
 subplot(222)
 imshow(X_k1)
 title("rank-1 approximation")
 subplot(223)
 imshow(X_k3)
 title("rank-3 approximation")
 subplot(224)
 imshow(X_k5)
 title("rank-5 approximation")
```



```
% g
V_values = prod(size(V));
sigma_values = prod(size(sigma));
u_values = prod(size(u));

n5 = V_values + sigma_values + u_values;
compression_ratio = (256^2)/n5
```

```
compression_ratio = 25.5501
```