21BCE10695
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MASTER ASSIGNMENT.

Q1)

$y'' + 5y' - 6y = 0$

Let $y' = x_1$, and $y = x_2$

$\Rightarrow \frac{dy'}{dt} = \frac{dx_1}{dt}$. $\frac{dy}{dt} = \frac{dx_2}{dt}$

$$\boxed{y'' = \frac{dx_1}{dt}}$$

$\therefore$ The eq^n from second order eq^n are

$\frac{dx_1}{dt} + 5x_1 - 6x_2 = 0$

$\frac{dx_2}{dt} - x_1 = 0$

$\frac{dx_1}{dt} = -5x_1 + 6x_2$; and $\frac{dx_2}{dt} = x_1 + 0x_2$

The matrix will be $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

for. Charac Eq^n

$A = \begin{bmatrix} -5 & 6 \\ 1 & 0 \end{bmatrix}$

$|A - \lambda I| = 0$

$\lambda(5+\lambda) - 6 = 0 \Rightarrow \lambda^2 + 5\lambda - 6 = 0$ — ①

B) $y = e^{\lambda x}$

$\therefore y' = \lambda e^{\lambda x}$ and $y'' = \lambda^2 e^{\lambda x}$

$\Rightarrow \lambda^2 e^{\lambda x} + 5\lambda e^{\lambda x} - 6 e^{\lambda x} = 0$

i.e. $\lambda^2 + 5\lambda - 6 = 0$ — ②

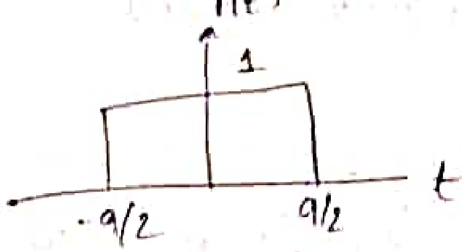
From ① and ② The characteristics eq^n of given of Eq^n is same as of derived Eq^n.

Q3)

$$F\{f(t)\} = \int_{-a/2}^{a/2} e^{-ist}\cdot dt$$

$$= -\frac{1}{s}\left[e^{-isa/2} - e^{isa/2}\right]$$

$$= \frac{2}{s}\cdot \sin\left(\frac{sa}{2}\right) \quad \text{Ans}$$



Q2)

As per given question, $f(x)$ is odd f^n

So $[a_0 = a_n = 0]$

$$f(x) = \sum b_n \sin nx$$

$$b_n = \frac{2}{\pi}\int f(x)\cdot \sin nx \cdot dx$$

$$b_n = \frac{2}{\pi}\left[\int_0^{\pi/2} x\cdot \sin nx\cdot dx + \int_{\pi/2}^{\pi} \pi/2 \sin nx \cdot dx\right.$$

$$= \frac{1}{\pi}\left[\frac{2\sin(n\pi/2)}{n^2} - \frac{\pi}{n}(-1)^n\right]$$

$$b_1 = \frac{2+\pi}{\pi}, \quad b_2 = \frac{-1}{2}, \quad b_3 = \frac{-2+3\pi}{3\pi}$$

So
$$f(x) = \frac{1}{\pi}\left[(2+\pi)\sin x + \frac{1}{9}(-2+3\pi)\sin 3x + \frac{1}{25}(2+5\pi)\sin 5x\right].$$

Q4).

$$f(t) = \begin{cases} 0 ; & -4 \leq t < 0 \\ 5 ; & 0 \leq t < 4. \end{cases}$$

$$F(t) = f(t+0).$$

Here $(C=4)$

$$F(n) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{4} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{4}.$$

$$a_0 = \frac{1}{4}\int_{-4}^{4} f(t)\cdot dt = \frac{1}{4}\left[(5)[t]_0^4\right]$$

$$a_n = \frac{1}{4}\int_{-4}^{4} F(t) \cos\frac{n\pi t}{4} \cdot dt$$

$$= \frac{1}{4}\left[\int_{-4}^{0} 0 + \int_{0}^{4} 5 \left(\cos\left(\frac{n\pi}{4}\right)t\right)\cdot dt\right.$$

$$a_n = 0.$$

$$b_n = \frac{1}{4}\int_{-4}^{4} F(t)\cdot \sin\left(\frac{n\pi}{4}\right)t \cdot dt = \frac{1}{4}\int_{0}^{4}(5)\sin\left(\frac{n\pi}{4}\right)t\,dt.$$

$$b_n = \frac{5}{4}\left[-\frac{\left(\cos\frac{n\pi t}{4}\right)}{n\pi/4}\right]_0^4 = \frac{-5}{4}\times\frac{4}{n\pi}\left(\cos n\pi - 1\right).$$

$$b_n = \frac{5}{n\pi}\left[1-(-1)^n\right]$$

Fourier Series.

$$F(n) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \cos\frac{n\pi t}{4}\, a_n + \sum_{n=1}^{\infty} \sin\frac{n\pi t}{4}\cdot b_n.$$

$$F(n) = \frac{5}{2} + \sum_{n=1}^{\infty} \frac{5}{n\pi}\left[1-(-1)^n \sin\frac{n\pi t}{4}\right]$$ Ans.

Q5).

$y(n+2) - 3y(n+1) + 2z(n) = \delta(n)$.

Take z-transform on both sides.

$z[z_{n+2}] - 3z[z_{n+1}] + 2z(z_n] = z[\delta(n)]$

$\{z^2($

$\Rightarrow y_n = z^{-1}\{z(z)\} = z^{-1}\left\{\frac{1}{(z-2)(z-1)}\right.$

$y_n = z^{-1}\left[\frac{1}{z-2}\right] \times z^{-1}\left[\frac{1}{z-1}\right]$.

$= 1 \times 2^{n-1} = 1 \times \frac{2^n}{2}$

$= \frac{1}{2} \sum^{n} 2^r \cdot (1)^{n-r}$.

$= \frac{1}{2}\left[1 + 2 + 2^2 + \cdots 2^n\right]$

$= \frac{1}{2}\left[\frac{2^{n+1} - 1}{1}\right] = \left\{\frac{2^{n+1} - 1}{2}\right\}$ Ans.