P441/P442 - Open Lab Experiment

Antenna Simulation for 21cm Hydrogen Line

Submitted By

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Contents

Page

Abstract

21cm Hydrogen line is one of the most useful tool of Radio Astronomy in current times. It can give us information about galaxy structures and rotation curves. It also finds applications in cosmology to probe the period from recombination to reionisation.

In this report, an attempt has been made to design an antenna for detecting this 21cm line. First, the theory of waveguides and antennas has been discussed. The optimum parameters have been calculated using ewa library in Matlab/Octave. The final design has been implemented using the software FEKO.

I. Introduction

Antennas are one of the most widely used instrumental tool in physics. They are devices which either convert voltage from a transmitter to radio waves, or pick up radio waves from the atmosphere and convert it into voltage which can be detected by a receiver.

In this this report, we aim to :

i.) discuss the importance of the 21cm line in modern astronomy and cosmology

ii.) discuss the theory of waveguides

iii.) discuss properties of antennas

iv.) design the antenna using FEKO

II. 21cm Hydrogen Line

2.1 Theory

Neutral hydrogen is the most abundant element of the Universe. It is made up of one proton and one electron. Both the proton and electron are spin-1/2 particles. They can either be in up-spin or down-spin orientation.

At any given point of time, the electron and proton in the neutral hydrogen atom, can either be aligned parallel to each other, or anti-parallel to each other.

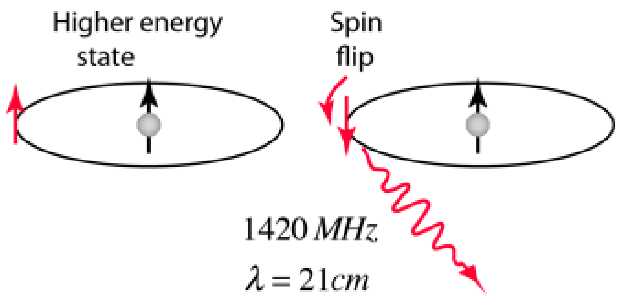

Fig 1. Two orientations of Neutral Hydrogen

Source : http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/h21.html

The parallel spin state is the higher energy state compared to the anti-parallel spin state. The energy difference between the states is around 5.874eV.

Whenever a neutral hydrogen atom goes from the parallel spin state to the anti-parallel spin state, it releases this energy in the form of a photon. This transition is called the the Spin-flip transition.

From the Einstein-Planck equation :

$$E = h\nu \tag{1}$$

We can find the frequency of the emitted photon as 1420MHz. This corresponds to a wavelength of $\lambda = 21$cm.

2.2 Importance

2.2.1 In Radio Astronomy

The 21cm line falls in the radio frequency range. It can easily penetrate the interstellar clouds and the Earth's atmosphere, thus can be observed without much interference.

If we assume hydrogen atoms to be uniformly distributed throughout the galaxies, we should observe the 21cm line from all directions. The waves we receive will either be redshifted or blueshifted to different extents, depending on the location of their emission in the galaxy.

We can then use this information to measure the rotation curve of our galaxy and the relative speed of each spiral arm. This information could in turn be used to indirectly calculate the mass of the galaxy.

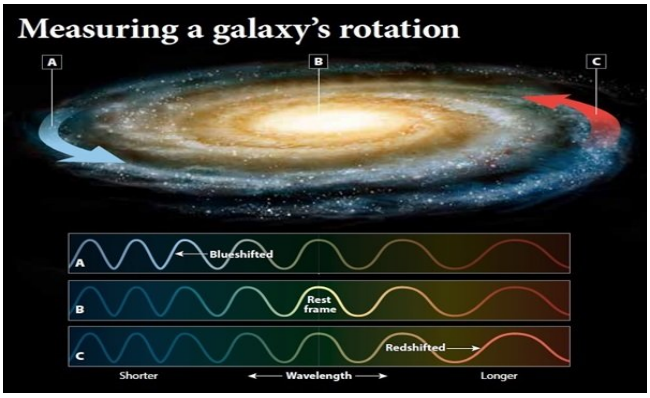

Fig 2. Rotation Curve of a Galaxy

Source : https://physicsopenlab.org/2020/09/08/measurement-of-the-milky-way-rotation/

2.2.2 In Cosmology

The 21cm hydrogen line could be used in cosmology to probe the "dark ages" of the Universe, i.e., the period from recombination to reionisation.
The hydrogen line from that epoch is highly red-shifted and obtained in the frequency range of 200MHz to 9MHz on Earth.

We can obtain a picture of how the reionisation occured via this information, as we should obtain holes in the 21cm background spectrum from that epoch which correspond to hydrogen atoms which got ionised by the radiation from stars or quasars.

III. Waveguides

Before building an antenna, we need to first talk about waveguiding structures.

3.1 What Are Waveguides?

Waveguides are structures which can guide waves, like sound waves or electromagnetic waves in a particular direction with minimal loss in energies.

Waveguides for electromagnetic waves are made up of hollow metallic (conducting) tubes. They can carry high frequency radio waves.

They can be of several types, namely circular, rectangular, elliptical, ridged etc.

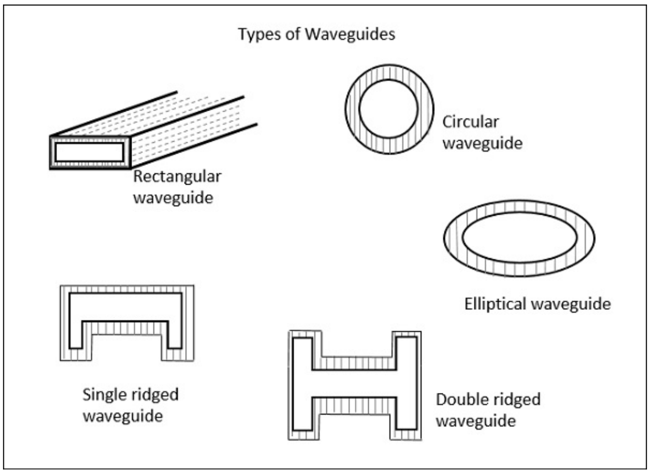

Fig 3. Types of Waveguides

Source : https://www.tutorialspoint.com/microwave_engineering/microwave_engineering_waveguides.htm

3.2 Arbitrary Waveguide

Let us consider a waveguide with some arbitrary cross section.

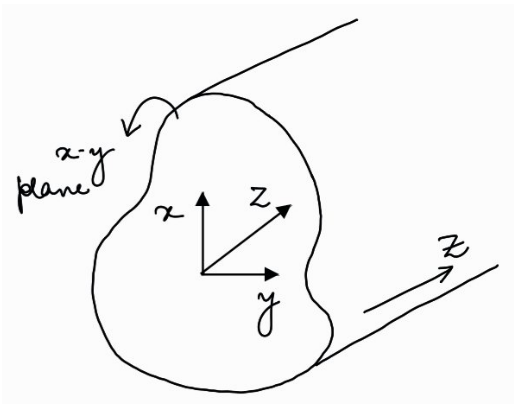

Fig 4. Arbitrary Waveguide Structure

Let's consider z to be the longitudinal direction, and x-y to be the transverse directions. Writing the EM waves :

$$\vec{E} = \vec{E}_\perp + E_z\hat{z}\,, \qquad \vec{E}_\perp = E_x\hat{x} + E_y\hat{y}$$
$$\vec{H} = \vec{H}_\perp + H_z\hat{z}\,, \qquad \vec{H}_\perp = H_x\hat{x} + H_y\hat{y}$$

We know from Maxwell's equation :

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \tag{2}$$

where, μ is the permeability of the space.

Using the forms of EM waves written above :

$$\left(\nabla_\perp + \frac{\partial}{\partial z}\hat{z}\right) \times \left(\vec{E}_\perp + E_z\hat{z}\right) = -j\omega\mu\left(\vec{H}_\perp + H_z\hat{z}\right)$$

$$\implies \nabla_\perp \times E_\perp + \nabla_\perp \times (E_z\hat{z}) + \frac{\partial}{\partial z}\hat{z} \times \vec{E}_\perp + \cancelto{0}{\frac{\partial}{\partial z}\hat{z} \times (E_z\hat{z})} = -j\omega\mu\left(\vec{H}_\perp + H_z\hat{z}\right)$$

The last term on LHS goes to zero as it involves the cross product of two parallel vectors. We can also see that the first term on LHS is along z direction, and the second and third terms are transverse terms. Equating the transverse components we obtain :

$$\vec{H}_\perp = -\frac{1}{j\omega\mu}\left\{\nabla_\perp \times (E_z\hat{z}) + \frac{\partial}{\partial z}\hat{z} \times \vec{E}_\perp\right\} \tag{3}$$

Similarly, using the other Maxwell's equation :

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} \tag{4}$$

where, ϵ is the permittivity of the space.
We obtain the transverse component of electric field as :

$$\vec{E}_\perp = \frac{1}{j\omega\varepsilon}\left\{\nabla_\perp \times (H_z\hat{z}) + \frac{\partial}{\partial z}\hat{z} \times \vec{H}_\perp\right\} \tag{5}$$

Now substituting for $\vec{H}_\perp$ from (3) in (5) :

$$\omega^2\mu\varepsilon - \frac{\partial}{\partial z}\hat{z} \times \frac{\partial}{\partial z}\hat{z} \times \vec{E}_\perp = -j\omega\mu\nabla_\perp \times (H_z\hat{z}) + \left(\frac{\partial}{\partial z}\hat{z}\right) \times \nabla_\perp \times (E_z\hat{z})$$

Using the triple product identity :

$$\vec{A} \times \vec{B} \times \vec{C} = \left(\vec{A}.\vec{C}\right)\vec{B} - \left(\vec{A}.\vec{B}\right)\vec{C}$$

We simplify the triple product terms in the equation above :

$$\frac{\partial}{\partial z}\hat{z} \times \frac{\partial}{\partial z}\hat{z} \times \vec{E}_\perp = -\frac{\partial^2}{\partial z^2}\vec{E}_\perp$$
$$\frac{\partial}{\partial z}\hat{z} \times \nabla_\perp \times E_z\hat{z} = \nabla_\perp\left(\frac{\partial E_z}{\partial z}\right)$$

The final equation now becomes :

$$\omega^2\mu\varepsilon\vec{E}_\perp + \frac{\partial^2}{\partial z^2}\vec{E}_\perp = -j\omega\mu\nabla_\perp \times (H_z\hat{z}) + \nabla_\perp\left(\frac{\partial E_z}{\partial z}\right) \tag{6}$$

The Maxwell's equations have only two independent components. Thus, we can choose the z-component of the electric and magnetic fields as the two independent components. So, (6) gives us how the transverse electric field depend on the independent components E_z and H_z.

Taking a travelling wave antasz along the z direction :

$$E\,,H \sim \exp(-\gamma z) \tag{7}$$

γ is called the absorption coefficient.

We ignore solutions for waves travelling in negative direction after reflection by assuming the waveguide is of infinite length.
Given the ansatz in (7), we can write :

$$\frac{\partial}{\partial z} \equiv -\gamma\,, \quad \frac{\partial^2}{\partial z^2} \equiv \gamma^2$$

Using this in (6) :

$$\left(\omega^2\mu\varepsilon + \gamma^2\right)\vec{E}_\perp = -j\omega\mu\nabla_\perp \times (H_z\hat{z}) - \gamma\nabla_\perp E_z$$

We can define :

$$\omega^2\mu\varepsilon + \gamma^2 = h^2 \tag{8}$$

h is the propagation constant for the transverse wave.

Using this, we get the final form for the transverse electric and magnetic fields as :

$$\vec{E}_\perp = -\frac{j\omega\mu}{h^2}\nabla_\perp \times (H_z\hat{z}) - \frac{\gamma}{h^2}\nabla_\perp E_z \tag{9}$$

$$\vec{H}_\perp = \frac{j\omega\varepsilon}{h^2}\nabla_\perp \times (E_z\hat{z}) - \frac{\gamma}{h^2}\nabla_\perp H_z \tag{10}$$

Since we have chosen the z-components of electric and magnetic fields as the independent components, we can find the complete solutions for the wave motion if we can determine E_z and H_z.

For a medium that is lossless,

$$\gamma = j\beta \tag{11}$$

We can conclude that :

i.) E_z and H_z both cannot be zero, unless h is zero. Converse is also true. This implies that if both the longitudinal components are zero, there is no transverse wave propagation in the waveguide. EM wave travels as if it is in free space.

ii.) The above mode is called the TEM mode, i.e., Transverse Electric and Magnetic field mode. It is a non-dispersive mode.

iii.) When $E_z = 0$ and $H_z \neq 0$, it is called the TE mode, i.e., the Transverse Electric mode.

It is dispersive mode.

iv.) When $E_z \neq 0$ and $H_z = 0$, it is called the TM mode, i.e., the Transverse Magnetic mode. It is a dispersive mode.

3.3 Rectangular Waveguides

A rectangular waveguide is one of the most common types of waveguides used.

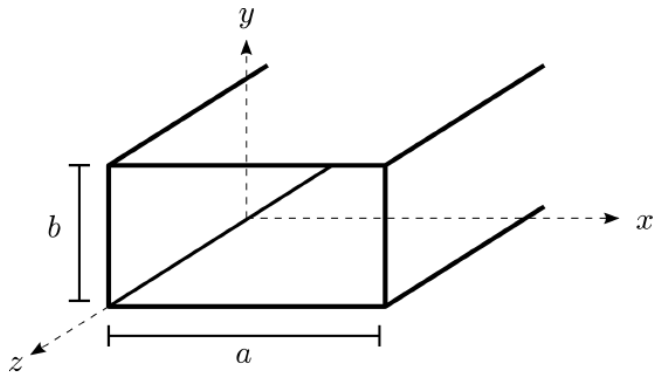

Fig 5. Rectangular Waveguide

Source : https://www.tutorialspoint.com/microwave_engineering/microwave_engineering_waveguides.htm

Let us consider the electromagnetic waves moving along z-direction.
We will also assume that for this structure $a \geq b$.

3.3.1 TM Mode

We will first consider the solutions of TM Mode, i.e.,

$$E_z \neq 0\,, \quad H_z = 0$$

Writing the wave equation for electric field travelling in z-direction :

$$\begin{aligned} &\nabla^2 E_z + \omega^2 \mu\varepsilon E_z = 0 \\ \Longrightarrow\ &\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2 \mu\varepsilon E_z = 0 \end{aligned}$$

Applying separation of variables :

$$E_z(x, y, z) = X(x)Y(y)Z(z)$$

Putting this in the wave equation, we obtain the final solution as :

$$\begin{aligned}
X(x) &= c_1 \cos(Ax) + c_2 \sin(Ax) \\
Y(y) &= c_3 \cos(By) + c_4 \sin(By) \\
Z(z) &= c_5 \exp(-j\beta z) + c_6 \exp(j\beta z)
\end{aligned}$$

The solutions $X(x)$ and $Y(y)$ are standing wave solutions. The solution $Z(z)$ is the travelling wave solution.

As we had assumed earlier that the waveguide we are using is of infinite length, and there is no reflected wave (wave travelling in -z direction), we can write $c_6 = 0$.

Now using the four boundary conditions due to the four walls upon which E_z is tangential, i.e.,

$$E_z = 0 \quad \text{for} \quad x = 0\,, \quad x = a\,, \quad y = 0\,, \quad y = a$$

we obtain the constants A and B as :

$$A = \frac{m\pi}{a}\,, \quad B = \frac{n\pi}{b} \tag{12}$$

We obtain the final solution for E_z as :

$$E_z = C \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \exp\left(-j\beta z\right) \tag{13}$$

Now using the expression for E_z (13) in (9) and (10), we obtain the transverse wave equations as :

$$E_x = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial x} = -\frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right) C \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \exp(-j\beta z) \tag{14}$$

$$E_y = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial y} = -\frac{j\beta}{h^2}\left(\frac{n\pi}{a}\right) C \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \exp(-j\beta z) \tag{15}$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} = \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) C \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \exp(-j\beta z) \tag{16}$$

$$H_y = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} = -\frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) C \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \exp(-j\beta z) \tag{17}$$

The conclusions we can immediately draw from the above equations are as follows :

i.) The TM_{00} mode does not exist. All the waves become zero.

ii.) If either $m = 0$ or $n = 0$, then also none of the components exist. Thus, $\mathrm{TM}_{\mathrm{m0}}$ and $\mathrm{TM}_{\mathrm{0n}}$ modes also do not exist.

iii.) TM_{11} is the lowest TM mode that can exist in the rectangular waveguide.

Also, from (8), (11) and the separable solutions, we can write :

$$h^2 = \omega^2\mu\varepsilon - \beta^2 = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \tag{18}$$

3.3.2 TE Mode

Now considering the TE Mode, i.e.,

$$E_z = 0\,, \quad H_z \neq 0$$

We need to first solve for H_z, as we did for the case of TM mode.
After following the similar procedure, we obtain the form of H_z as :

$$H_z = C\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{a}\right)\exp(-j\beta z) \tag{19}$$

We can analyse the above equation directly to look for possible lowest order modes.

i.) For $m = n = 0$, H_z is a constant. It does not vary with time. So, all transverse components of the field is zero, i.e., $\vec{E}_\perp = \vec{H}_\perp = 0$. This is because $\vec{E}_\perp$ and $\vec{H}_\perp$ contain space derivatives of H_z. From this we can immediately as H_z must be zero too, as magnetic field cannot exist in the absence of a time-varying electric field. Thus, the TE_{00} mode does not exist.

ii.) The $\mathrm{TE}_{\mathrm{m}0}$ and $\mathrm{TE}_{0\mathrm{n}}$ modes exist. Thus the lowest order mode could be either TE_{10} or TE_{01} mode.

3.3.3 Cut-off Frequency, Dominant Mode and Field Pattern

We know from the preceding analysis that :

$$\begin{aligned} h^2 &= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \\ \beta^2 &= \omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 \end{aligned}$$

Now for a propagating wave, the frequency should be greater than some critical value for the term β to be a real number. If β is a complex number, it would mean the EM wave decays exponentially inside the waveguide.

At the cut-off frequency of a mode, we have $\beta = 0$.
With this, we can find the equation for the cut-off frequency as follows :

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right\}^{1/2} \tag{20}$$

From the above equation (20), we can calulate the cutoff frequencies of TE_{10}, TE_{02} and TM_{11} modes. We observe that :

$$f^c_{TE_{10}} < f^c_{TE_{01}} < f^c_{TM_{11}}$$

Thus, TE_{10} mode is the lowest order mode of a rectangular waveguide. It has the minimum cutoff frequency. Consequently, it is also the dominant mode of the rectangular waveguide as the probability of a wave being in TE_{10} mode is the greatest. It depends only on the 'a' side of the waveguide.

In the dominant TE_{10} mode :

$$\begin{aligned}
H_z &= C\cos\left(\frac{\pi x}{a}\right)\exp(-j\beta z) \\
E_y &= -\frac{j\omega\mu}{(\pi/a)^2}C\left(\frac{\pi}{a}\right)\sin\left(\frac{\pi x}{a}\right)\exp(-j\beta z) \\
H_x &= -\frac{j\omega\varepsilon}{(\pi/a)^2}C\left(\frac{\pi}{a}\right)\sin\left(\frac{\pi x}{a}\right)\exp(-j\beta z) \\
E_x &= 0\,, \quad H_y = 0
\end{aligned}$$

The field pattern of electric and magnetic field inside the waveguide depends on which mode is it being operated on.
In the TE_{10} mode, magnetic flux lines appear as continuous loops, and electric field lines occur as sinusodial waves on the x-y plane, which become zero on the $y = 0$ and $y = b$ walls.

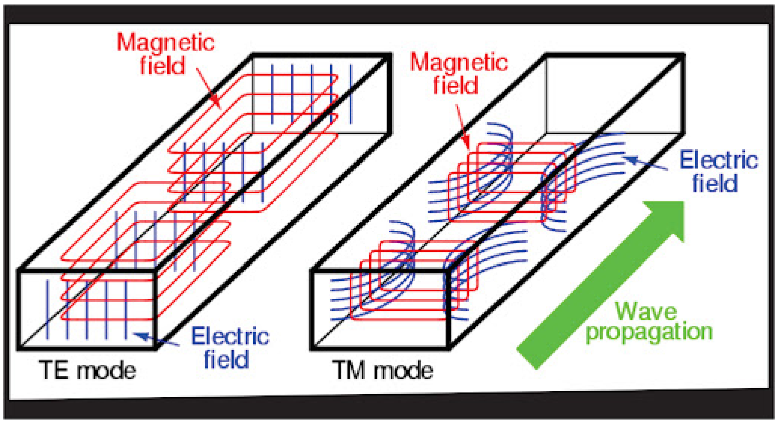

Fig 6. Field Patterns inside Rectangular Waveguide

Source : http://www.engineeringdone.com/te-tm-modes/te-tm-modes/

IV. Antenna Parameters

All kinds of antennas can be characterised or studied using certain parameters which have been summarised in this section.

4.1 Bandwidth

The bandwidth gives us the frequency range over which an antenna operates. All waveguides have a lower cutoff frequency, ω_c. So for an antenna to function properly, the frequency of the wave must be greater than the cutoff frequency. For optimum transport, the frequency of the wave must in the range of $1.25\omega_c$ to $1.9\omega_c$. If the frequency is greater than $1.25\omega_c$ then dispersion down the waveguide gets minimised, and if the frequency is lesser than $1.9\omega_c$, then the higher order modes gets suppressed.

4.2 Radiation Pattern

The radiation pattern of an antenna gives us the power radiated by it as a function of θ and ϕ, i.e., as a function of direction.
We can classify antennas into three types based on their radiation patterns :

i.) **Isotropic** : Antenna radiates same amount of radiation in all directions.

ii.) **Omni-directional** :