P441/P442 - Open Lab Experiment

NON-LINEAR DYNAMICS CIRCUIT

Submitted By

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Abstract

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I. Introduction

Chua circuit is the simplest electronic circuit which exhibits the phenomenon of chaos. It was invented by Leon Chua in 1983.

A dynamical system is said to have chaotic behaviour when despite its deterministic nature, it is not predictable. The apparent random behaviour of the system is usually governed by deterministic laws that are highly sensitive to initial conditions. A small change in initial conditions can result in widely varying results.

To exhibit chaos, a circuit must have been :

i.) at least one locally active resistor

ii.) at least one non-linear element

iii.) at least three energy storage units

II. Theoretical Design

2.1 Circuit Elements and Constraints

In order to physically exhibit the phenomenon of chaos Chua decided to design a physical circuit with 3 unstable equilibrium points with further contraints that number of passive elements should be as few as possible and there should be only one non-linear resistor with two terminals which has piecewise linear characteristic.

There must be 3 energy storage elements as the dynamical system must have at least order 3 to be chaotic. He also decided to have only one passive element in the circuit - a linear resistor.
Passive elements are the circuit elements which donot generate power but instead store or dissipate it.

Also since we want to observe oscillations, we cannot have only capacitors or only inductors as all 3 energy storage elements. There must be some combination of both. Chua preferred the combination of two capacitors and one inductor to make the circuit more cost-efficient.

2.2 Possible Configurations

With these constraints in place, there can be 8 possible configurations.

Fig 1. Possible configurations for the circuit

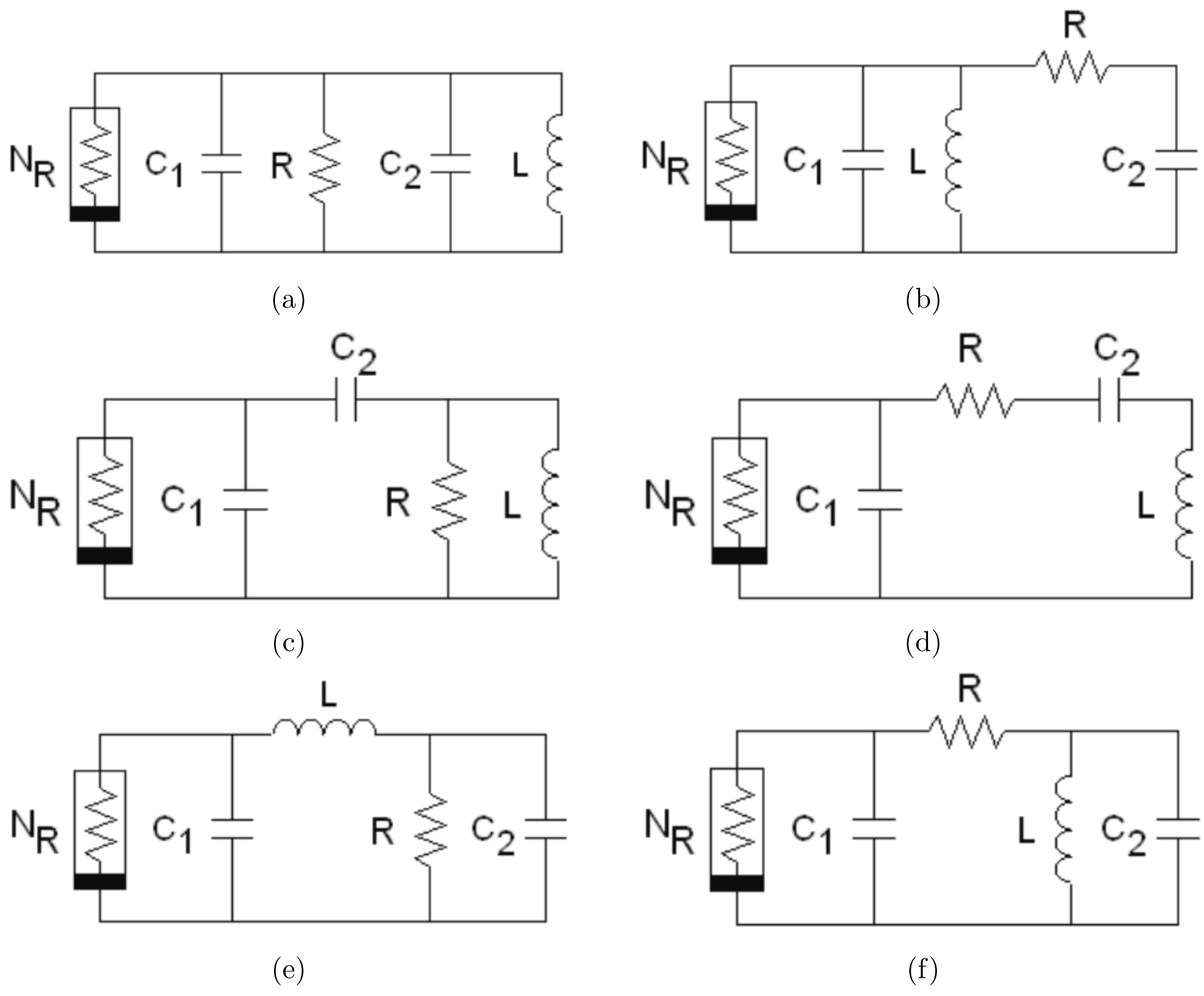
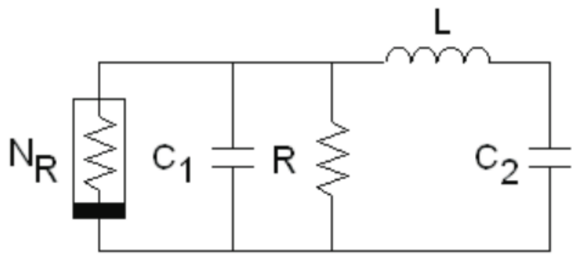
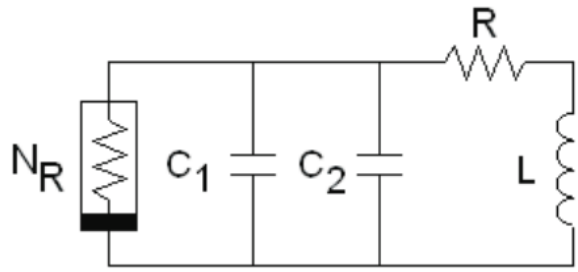


Fig 1. Possible configurations for circuit



(g)



(h)

Configuration (g) and (h) can be immediately rejected.
In (g) the characteristic of resistance R can be absorbed in the characteristics of non-linear resistor N_R. In (h) the C_1 and C_2 capacitances can be replaced by a single effective capacitor $C = C_1 + C_2$. So in both of these configurations all circuit elements donot give unique contribution. Thus they can be rejected.

For (a) and (b), the DC equilibrium calculations show that non-linear resistor gets short-circuited by the inductor. For (c) and (d), the DC equilibrium calculations show that non-linear resistor terminals are open. So all the four configurations can be rejected.

The remaining configuration (e) and (f) are both valid, but Chua selected configuration (f) because the RLC subcircuit generates oscillations.

2.3 Final Circuit

The final Chua circuit is given as follows :

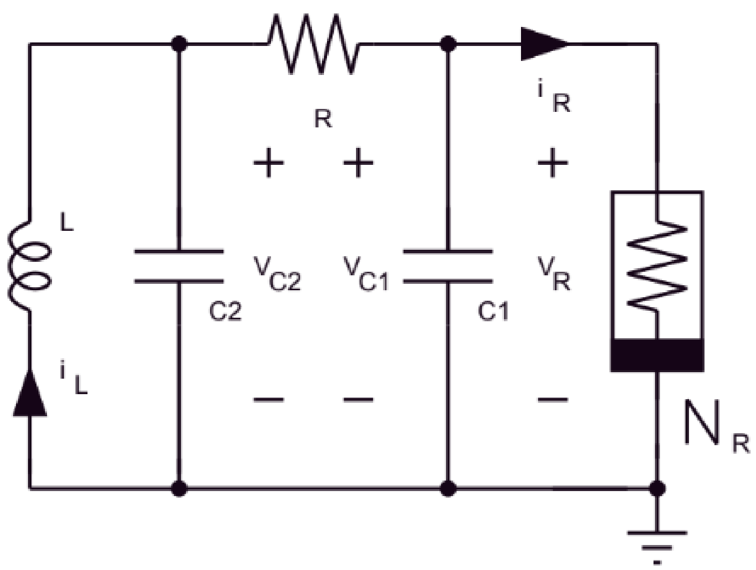


Fig 2. Chua's Circuit

III. State Equations and Simulations

3.1 State Equations

The equations of Chua's circuit are given as a system of three coupled differential equations :

$$C_1 \frac{dv_{C_1}}{dt} = G\left(v_{C_2} - v_{C_1}\right) - g(v_{C_1}) \tag{1}$$

$$C_2 \frac{dv_{C_2}}{dt} = G\left(v_{C_1} - v_{C_2}\right) - i_L \tag{2}$$

$$L \frac{i_L}{dt} = -v_{C_2} \tag{3}$$

where, $G = \dfrac{1}{R}$ is the conductance, and $g(x)$ is a piece-wise linear function. It is given as :

$$g(v) = m_0 v + \frac{1}{2}(m_1 - m_0)\left[|v + B_p| - |v - B_p|\right] \tag{4}$$

where,

$$m_0 \implies \text{slope of outer region}$$
$$m_1 \implies \text{slope of inner region}$$
$$B_P \implies \text{breakpoints (both positive and negative values)}$$

3.2 Simulation

The variables were redefined and all constants were taken to right hand side to make handling the equations easier.

$$\frac{dx}{dt} = \frac{1}{C_1}\left\{G\left(y - x\right) - g(x)\right\} \tag{5}$$

$$\frac{dy}{dt} = \frac{1}{C_2}\left\{G\left(x - y\right) - z\right\} \tag{6}$$

$$\frac{dz}{dt} = -\frac{y}{L} \tag{7}$$

where,

$$x \equiv v_{C_1} \quad y \equiv v_{C_2} \quad z \equiv i_L$$

The equation $g(x)$ remains the same as in (4).
The equations are solved numerically using Runge Kutta 4 method in Python. All plots are made using Gnuplot.

3.2.1 Python Codes

The code for RK4 is as follows :

```python
#Chua circuit simulations

import math
import handling_files
import numpy as np

#RK4 to solve Chua circuit equations (a system of 3-ODEs)
def RK4_chua(F,b,t,h,N,name):
    handling_files.append_file(name, f'{t} {b[0]} {b[1]} {b[2]}\n')
    for i in range(N):
        K1=F(t,b)
        #
        K2=F(t+h/2, b+np.multiply(K1,h/2))
        #
        K3=F(t+h/2, b+np.multiply(K2,h/2))
        #
        K4=F(t+h, b+np.multiply(K3,h))
        #
        b=b+np.multiply((K1+np.multiply(K2,2)+np.multiply(K3,2)+K4), h/6)
        t=t+h
        handling_files.append_file(name, f'{t} {b[0]} {b[1]} {b[2]}\n')
        #
    return(1)
```

The code inputs the three differential equations as a column vector F which is a function of x, y, z and t (time). x, y and z are arranged as column vector b. For the first iterations, it has the initial values. h is the increment factor. N is the number of iterations. t_0 is the initial time value.

The 'append.file()' function saves the data points (t, x, y and z) after each iteration in a file (filename provided to function as variable 'name'). All codes for manipulation with files is as follows :

```python
#Library for handling files and their contents

# READ FILE
def read_matrix(x): #more than one column #parameter = name of file
    f=open(x,'r') #'r' ==> read only
    X=[[float(num) for num in line.split('\t')] for line in f]
    f.close()
    return(X)

#
    ###############################################################################
```

```python

# APPEND FILE
def append_file(x, str): #arguments = name of file, string to append
    f=open(x, 'a') #'a' ==> append file
    f.write(str)
    f.close()
    #

# ############################################################################

#WRITE AT BEGINNING (hopefully)
def write_beginning(x, str):
    f=open(x, 'r+')
    old=f.read()
    f.seek(0)
    f.write(str + old)
    f.close()
    # it works :D

# PRINT CONTENTS OF A TEXT FILE
def print_file(x): #argument = name of file
    f=open(x, 'r')
    contents=f.read()
    print(contents)

# ##############################################################################
```

3.2.2 Plots with Dimensionless Constants

The following values were used for the constants :

$$G = 0.7 \quad C_1 = 1/9 \quad C_2 = 1 \quad L = 1/7 \quad B_p = 1 \quad m_0 = -0.5 \quad m_1 = -0.8$$

The function g(v) in (4) can be plotted using the constants.

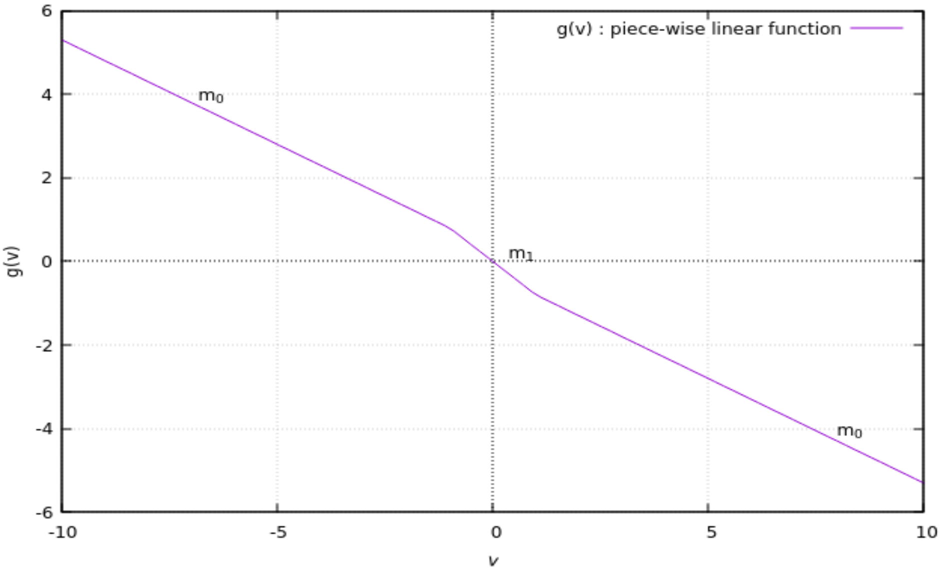


Fig 3. Three Segment Linear Function : g(v)

The code for defining the function, initial values and calling the function is :

```
# Dimensionless Chua Circuit

import math
import sys
import numpy as np

sys.path.append('/home/ashmita/Desktop/ASHMITA/APanda_Lib')
# importing all files at once, now we just need to write function name to
    access it
from APanda_Lib import *

import chua_circuit_simulations

#dimensionless chua
R=1.4285 # R corresponding to G=0.7
C1=1/9
C2=1
L=1/7
Bp=1
m0=-0.5
m1=-0.8

x0=0.1
y0=0.0
z0=0.0
t0=0.0

b0=[x0, y0, z0]

def Yfunc(t,b):
```

```
    x,y,z=b
    gx=m0*x+0.5*(m1-m0)*(abs(x+1)-abs(x-1))
    Y=[(1/C1)*((1/R)*(y-x)-gx), (1/C2)*((1/R)*(x-y)+z), (-1/L)*y]
    return Y
    #

h=0.1
N=5000
path="/home/ashmita/Desktop/ASHMITA/NISER Study/7th Semester/Open Lab/Non-Linear Circuit/Dimensionless/"
name=f'dimensionless'
n=path+name
f=open(n, "w")
f.close()
out2=chua_circuit_simulations.RK4_chua(Yfunc,b0,t0,h,N,n)
#
```

The data file obtained is plotted using Gnuplot.

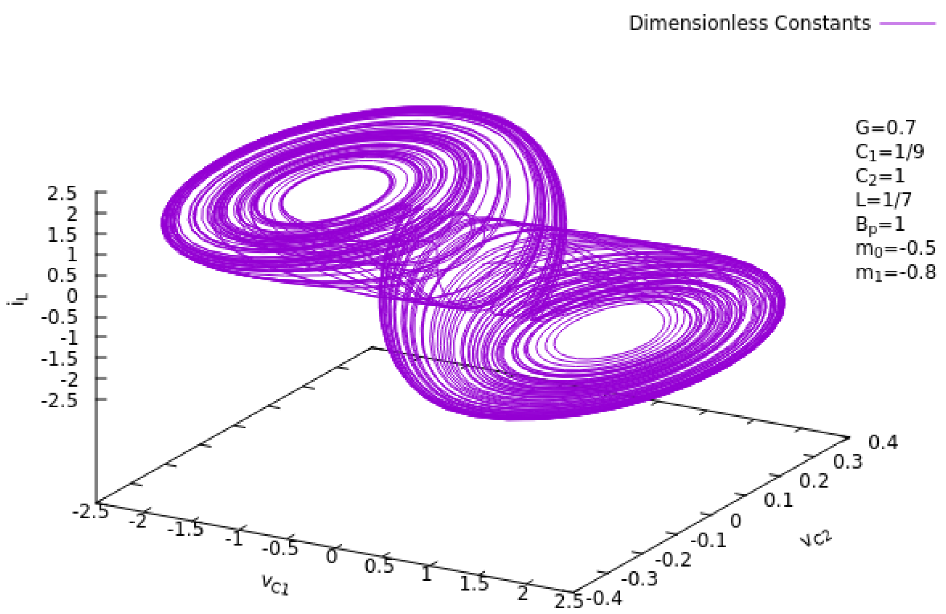


Fig 4. 3D plot of v_{C_1} vs v_{C_2} for dimensionless constants

Thus, we do obtain a double scroll attractor for the Chua Circuit. In principle, the Chua Ciruit does exhibit chaotic behaviour.

3.2.3 Varying R with Dimensionful Constants

Now, we will attempt to use constants which represent actual dimensionful values and try to observe how the graph changes when we change the value of resistance R.

We will define conversion factors to relate the value of our constants to values of actual electronic circuit components. Current will be measured in Amperes(A), potential differences in Volts(V), capacitances in Farads(F), inductance in Henry(H) and resistance in Ohm(Ω). Resistivity is expressed in Siemens(S)

If we want currents of milliamperes to be in the circuit, we will adjust all current values by 1000. It will thus increase resistances and inductance by 1000, while decreasing capacitances by the same factor. Also, we can also rescale the values of time by some factor k in (3). This will leave all resistances unaffected, and all capacitors and inductors will be scaled by same factor k. For ease of using values in the code, I have chosen k to be 10^{-4}, i.e., I rescale all capacitances and inductances by 10^{-4}. This gives us the final conversion factors as :

$$
\begin{aligned}
R : 1 &\equiv 1000\Omega = 1k\Omega \\
C_1, C_2 : 1 &\equiv 10^{-7}F = 100nF \\
L : 1 &\equiv 10^{-1}H = 100mH \\
m_0, m_1 : 1 &\equiv 10^{-3}S = 1mS \\
B_p : 1 &\equiv 1V
\end{aligned}
$$

So, the constants used in the previous part correspond to :

$$R = 1.43k\Omega \,; C_1 = 11.11nF \,; C_2 = 100nF \,; L = 14.29mH \,; B_p = 1V \,; m_0 = -0.5mS \,; m_1 = -0.8mS$$

We will now attempt to vary R and observe how the output changes.

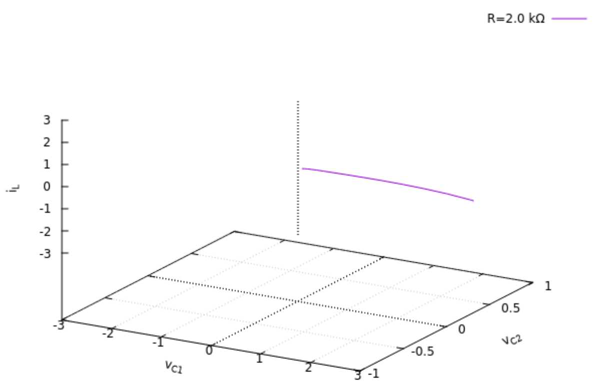
The code for defining the function, initial values, varying R values and calling the function is :

```
# Varying R

import math
import sys
import numpy as np

sys.path.append('/home/ashmita/Desktop/ASHMITA/APanda_Lib')# importing all
    files at once
from APanda_Lib import *

import chua_circuit_simulations

#varying R
R=float(input('Please enter the value of R.\n'))
C1=1/9
C2=1
L=1/7
Bp=1
m0=-0.5
```

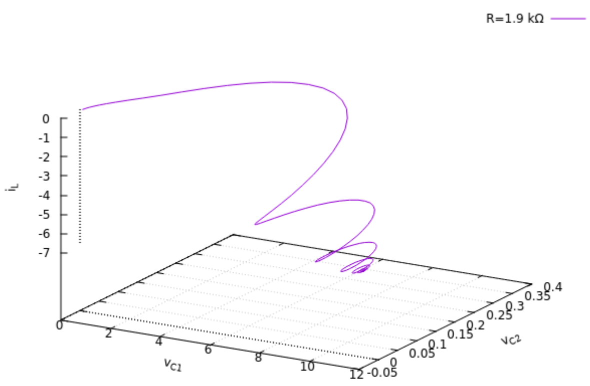
```python
m1=-0.8

x0=0.1
y0=0.0
z0=0.0
t0=0.0

b0=[x0, y0, z0]

def Yfunc(t,b):
    x,y,z=b
    gx=m0*x+0.5*(m1-m0)*(abs(x+1)-abs(x-1))
    Y=[(1/C1)*((1/R)*(y-x)-gx), (1/C2)*((1/R)*(x-y)+z), (-1/L)*y]
    return Y
    #

h=0.1
N=5000
path="/home/ashmita/Desktop/ASHMITA/NISER Study/7th Semester/Open Lab/Non-Linear Circuit/Varying R/"
name=f'R={R}'
n=path+name
f=open(n, "w")
f.close()
out2=chua_circuit_simulations.RK4_chua(Yfunc,b0,t0,h,N,n)
#
```

Plotting the data files obtained in Gnuplot.

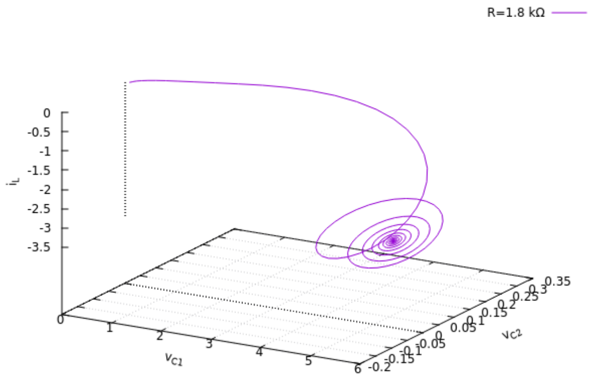
Fig 5. R Bifurcation in Theoretical Chua Circuit using a Three Segment Non-Linear Resistance



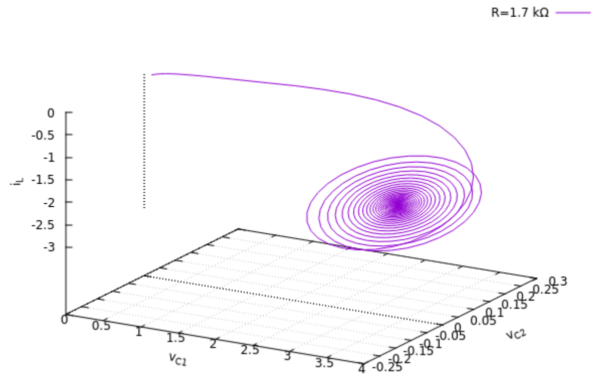
(a) $R = 2.0k\Omega$



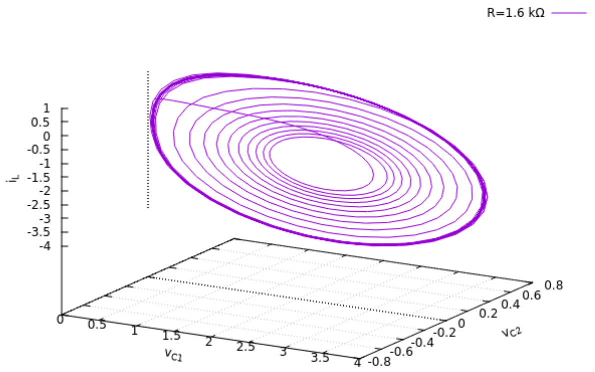
(b) $R = 1.9k\Omega$



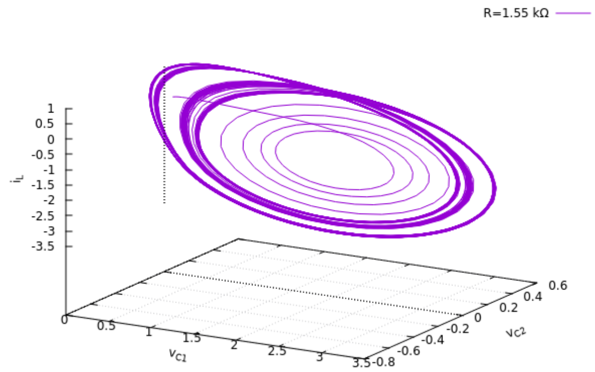
(c) $R = 1.8k\Omega$



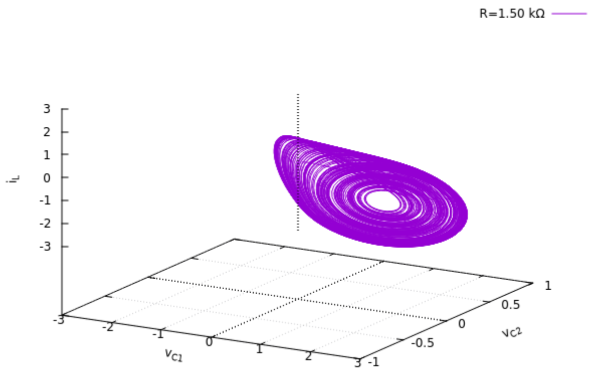
(d) $R = 1.7k\Omega$



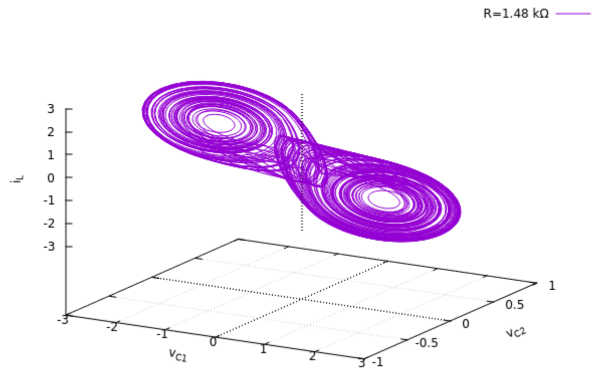
(e) $R = 1.6k\Omega$



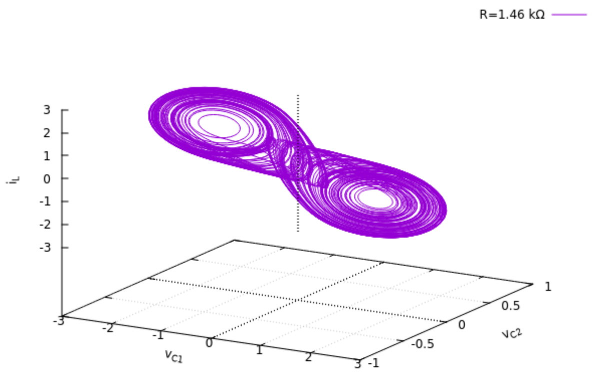
(f) $R = 1.55k\Omega$



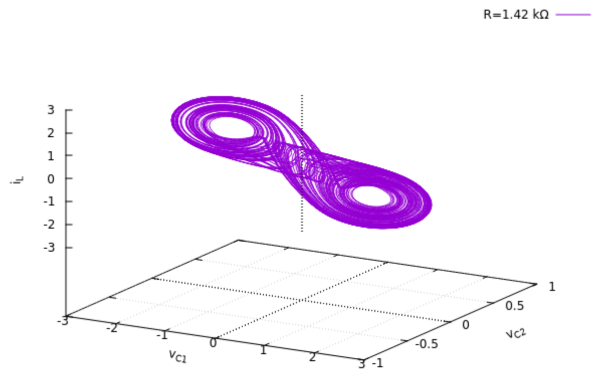
(g) $R = 1.5k\Omega$



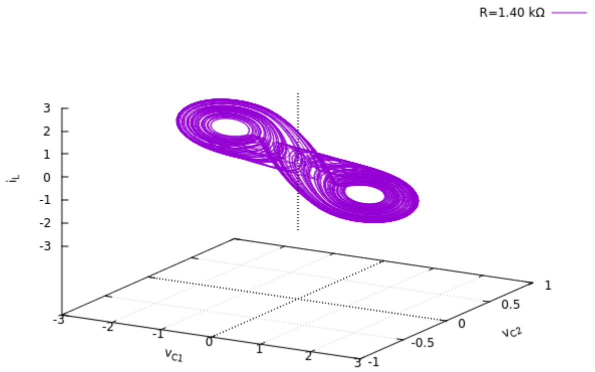
(h) $R = 1.48k\Omega$



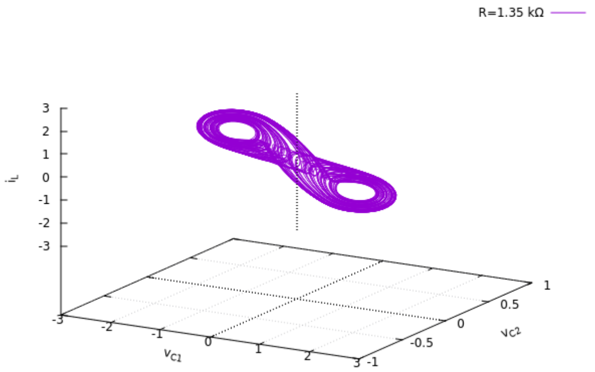
(i) $R = 1.46k\Omega$



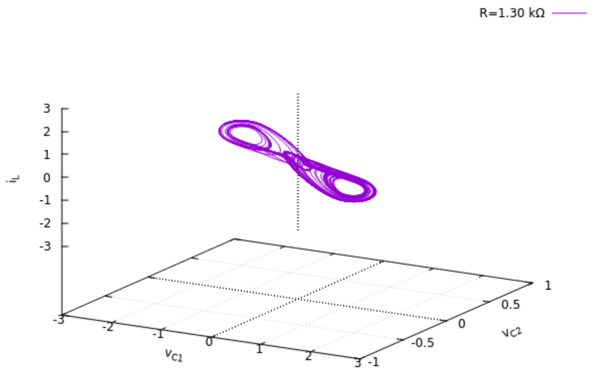
(j) $R = 1.42k\Omega$



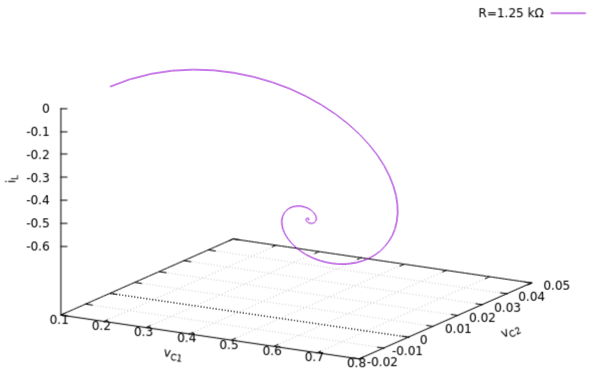
(k) $R = 1.4k\Omega$



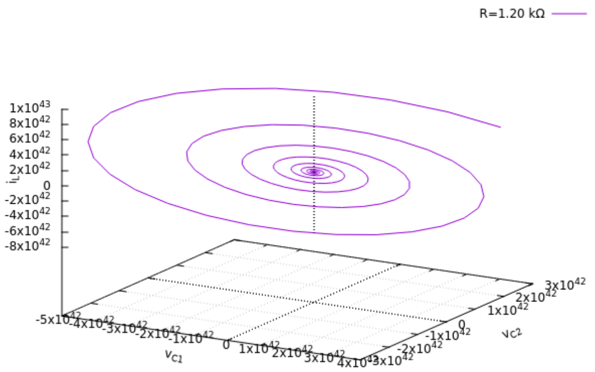
(l) $R = 1.35k\Omega$



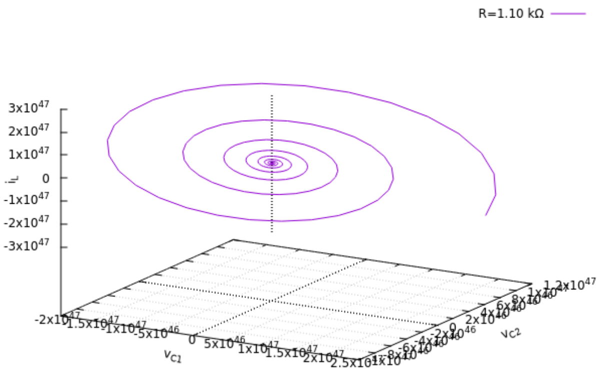
(m) $R = 1.3k\Omega$



(n) $R = 1.25k\Omega$



(o) $R = 1.2k\Omega$



(p) $R = 1.1k\Omega$

We observe that $R = 2.0k\Omega$ shows DC Equilibrium. As resistance is reduced, the characteristics looks like an unbounded spiral.

The Rossler Attractor forms around $R = 1.5k\Omega$ and the Double Scroll Attractor is first formed at $R = 1.48k\Omega$. As the resistance is further reduced, the size of the double scroll also decreases.

By $R = 1.25k\Omega$ the double scroll character is lost, and for lower resistances the graphs show a spiral with no upper bound. The values of v_{C_1}, v_{C_2} and i_L apparently take values in the ranges of 10^{43} and 10^{47}. This is not physically possible.

3.3 Problems with Simulation

The current simulation of the Chua Circuit is not physically viable, as is evident from the graphs obtained for various values of resistance. It predicts that except for a small window of resistance values (from $R = 1.5k\Omega$ to $R = 1.3k\Omega$) the values of voltages and current is not bounded.

The problems arises because we have failed to take into account the fact that all physical resistors are eventually passive, i.e., for large enough values of voltages applied across its terminals, the power consumed by the resistor becomes positive.
In our current equation (4) and graph of g(v), we see that the condition of eventual passivisity has not been taken into account. The power consumed by the resistor is negative for all values of voltages.

IV. Simulation to Practical Implementation

4.1 Negative Resistance

To implement a Chua Circuit practically we first need to construct a negative resistance which follows the characteristics of $g(v)$ graph. One way to construct a negative resistance is to connect three positive linear resistors to a voltage controlled voltage source (VCVS).

4.1.1 Voltage Controlled Voltage Source

A VCVS is defined to be an ideal circuit element with two input and two output terminals such that no current flows between the input terminals and voltage across output terminals is dependent on the volatge across input terminals.

$$v_o = f(v_d)$$

$f(.)$ could have any functional dependence, but the simplest non-trivial relation occurs when it is linear.

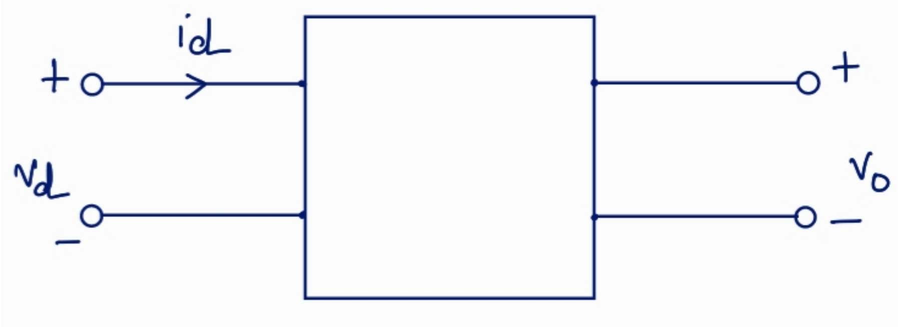


Fig 6. Voltage Controlled Voltage Source

4.1.2 Negative Resistance using ideal VCVS

The circuit diagram to build a negative resistance using an ideal VCVS and three linear resistors is given as follows :

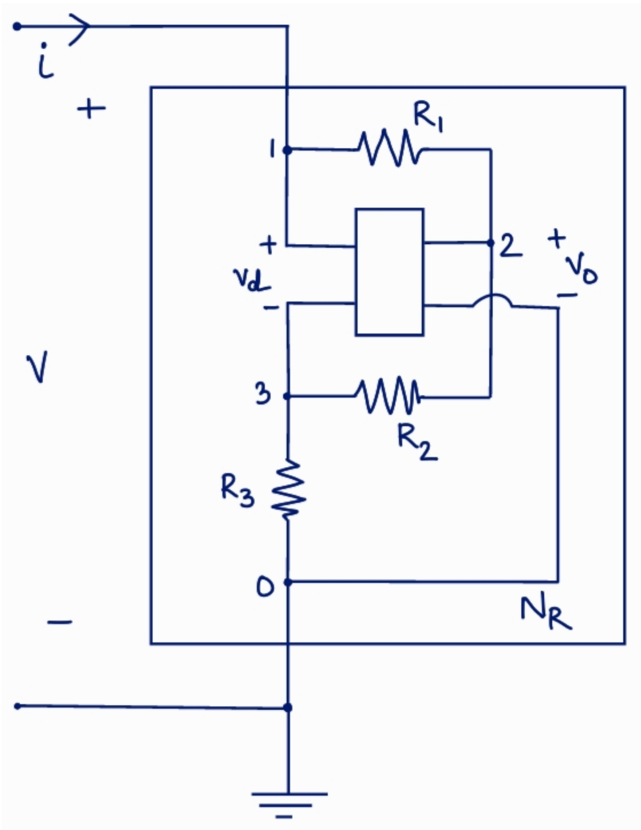


Fig 7. Negative Resistance using VCVS

Let us assume the VCVS follows a linear relationship :

$$v_o = Av_d \tag{8}$$

A is some proportionality constant.

Now, Kirchoff's Current Law (KCL) says that *for any node in an electrical circuit, the sum of the currents entering the node is equal to the sum of the currents leaving the node.*

Applying KCl on node (1) (in Fig 7):

$$i = i_1 \quad ; \quad i_1 = \frac{v - v_o}{R_1} \tag{9}$$

We can write this because no current goes inside VCVS.

Now, Kirchoff's Voltage Law (KVL) says that *if we move around a closed loop in a fixed direction then the sum of all the potential differences around the loop is zero.*

Applying KVL along loop 1-2-3-0 (in Fig 7):

$$v = v_d + i_3 R_3 \tag{10}$$

$$i_3 = \frac{v_o}{R_2 + R_3} \tag{11}$$

Using (11) in (10) :

$$v = v_d + \frac{v R_3 v_d}{R_2 + R_3} = \frac{R_2 + (1 + A)R_3}{R_2 + R_3} v_d$$

Now, using (8) in the above equation :

$$\therefore v = \left\{ \frac{R_2 + (1 + A)R_3}{A(R_2 + R_3)} \right\} v_o \tag{12}$$

We can now calculate the current i using (9) :

$$i = \frac{v}{R_1} - \frac{v_o}{R_1}$$

Using equation of v and v_o (12) :

$$\implies i = \frac{v}{R_1} - \frac{1}{R_1} \left\{ \frac{A(R_2 + R_3)}{R_2 + (1 + A)R_3} v \right\}$$

$$\implies i = \frac{vR_2 + vR_3 + vAR_3 - AR_2 v - AR_3 v}{R_1 \left[R_2 + (1 + A)R_3 \right]}$$

So we finally obtain :

$$\therefore i = \left\{ \frac{R_2(1 - A) + R_3}{R_1 \left[R_2 + (1 + A)R_3 \right]} \right\} \tag{13}$$

Now if we take A to be very large, $A \gg 1$, greater than R_1, R_2 and R_3, we can write :

$$\implies i \approx \frac{-R_2A + R_3}{R_1(R_2 + R_3A)}v$$

Also, as $R_2A \gg R_3$ and $R_1R_3A \gg R_1R_2$:

$$\implies i \approx -\frac{R_2A}{R_1R_3A}v \approx -\frac{R_2}{R_1R_3}v$$

We can now set $R_1 = R_2$, and we obtain :

$$i = -\frac{1}{R_3}v \tag{14}$$

Thus it now appears that the segment N_R (in Fig 7) now has negative resistance $-R_3$.

4.1.3 Op-Amps as VCVS

An opamp is the practical or real-life approximation of a VCVS. The voltage applied across inverting and non-inverting terminals produces voltage at output terminal, if we take the reference terminal to be ground.

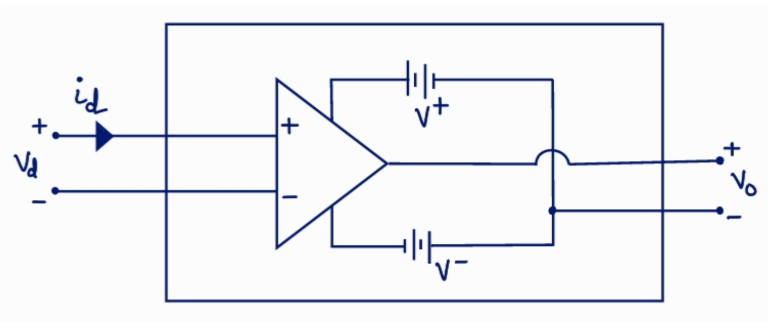


Fig 8. Op-amp as VCVS

Ideal opamps there is no current entering the circuit, i.e., $i_d = 0$ and the loop gain is infinite. But typically, most opamps produce output voltages 100,000 times larger than the potential difference between input terminals.

The output of a opamp becomes constant at some values of $v_d = \pm E_{sat}$.

$$\begin{aligned} \text{For } v_d &\geq \frac{E_{sat}}{A} + v_{OS} && : \text{positive saturation region} \\ \text{For } v_d &\leq -\frac{E_{sat}}{A} + v_{OS} && : \text{negative saturation region} \\ \text{For } -\frac{E_{sat}}{A} + v_{OS} < v_d &< \frac{E_{sat}}{A} + v_{OS} && : \text{linear region} \end{aligned}$$

v_{OS} is the offset voltage.

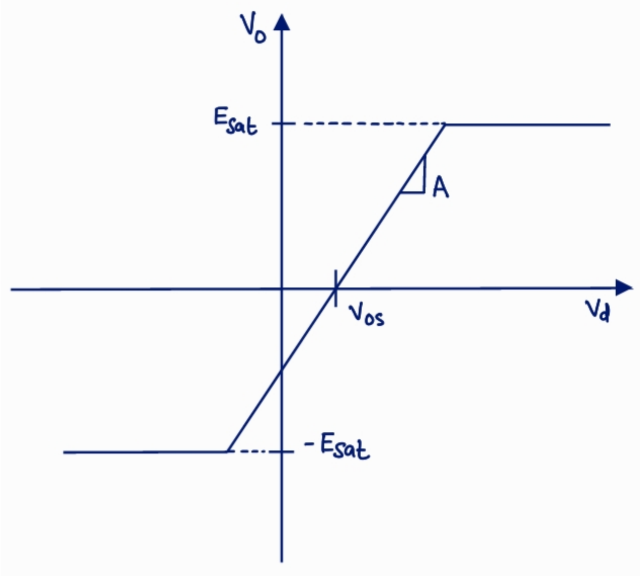


Fig 9. Output Characteristics of an Op-Amp

From here on, we will assume :

$$v_{OS} = 0 \quad ; \quad i_d = 0 \quad ; \quad v_o = f(v_d)$$

4.1.4 Negative Resistance using Op-Amps

We can build a negative resistance using the analysis we did previously, while using Op-Amp as VCVS.

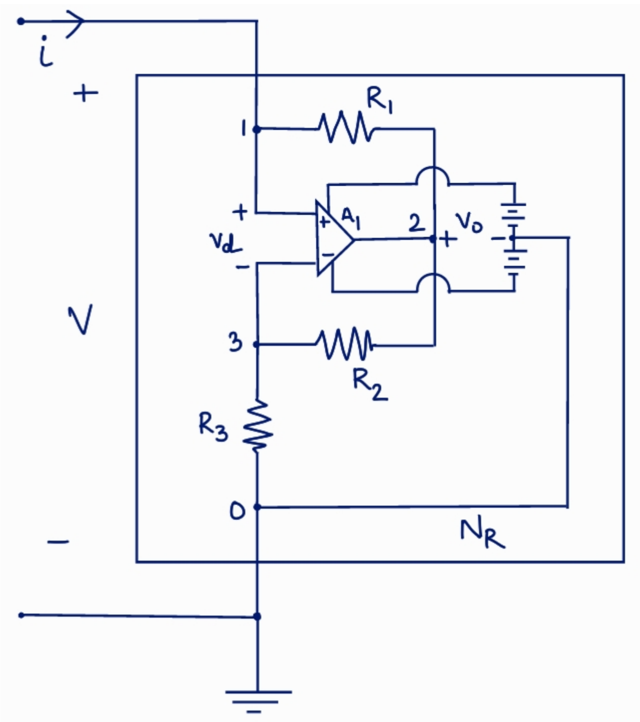


Fig 10. Negative Resistance using Op-Amps

From previous circuit analysis we have :

$$i = \frac{v - v_o}{R_1} \quad ; \quad v = v_d + \frac{R_3}{R_2 + R_3} v_o \quad ; \quad v_o = A v_d \tag{15}$$

As now we have an op-amp, there are three distinct regions depending on the voltage behavior :

$$\begin{aligned} \text{Negative Saturation :} \quad & v_o = -E_{sat} \quad ; \quad v_d \leq -\frac{E_{sat}}{A} \\ \text{Linear Region :} \quad & v_o = A v_d \quad ; \quad -\frac{E_{sat}}{A} < v_d < \frac{E_{sat}}{A} \\ \text{Positive Saturation :} \quad & v_0 = E_{sat} \quad ; \quad v_d \geq \frac{E_{sat}}{A} \end{aligned}$$

Positive Saturation

In the positive saturation region the output voltage is fixed at E_{sat}, even if the input changes.

$$v_o = E_{sat} \quad ; \quad v_d \geq \frac{E_{sat}}{A} \tag{16}$$

From (15) and (16), we have equation of current as :

$$i = \frac{v}{R_1} - \frac{E_{sat}}{R_1} \tag{17}$$

We can write the equation for voltage as :

$$\begin{aligned} & v = v_d + \frac{R_3}{R_2 + R_3} v_o \\ \Longrightarrow \; & v \geq \frac{E_{sat}}{A} + \frac{R_3}{R_2 + R_3} E_{sat} \\ \Longrightarrow \; & v \geq E_{sat} \left\{ \frac{R_2 + R_3 + A R_3}{A(R_2 + R_3)} \right\} \end{aligned}$$

So we finally obtain :

$$v \geq \left\{ \frac{R_2 + (1 + A) R_3}{A(R_2 + R_3)} \right\} E_{sat} \tag{18}$$

So the minimum value of valid v for positive saturation, or the positive breakpoint is given as :

$$B_P^+ = \frac{R_2 + (1 + A) R_3}{A(R_2 + R_3)} E_{sat} \tag{19}$$

Now for $A \longrightarrow \infty$, we have $(1 + A) R_3 + R_2 \longrightarrow A R_3$. So we obtain the breakpoint as :

$$B_P^+ \simeq \frac{R_3}{R_2 + R_3} E_{sat} \tag{20}$$