P443/P444 - Open Lab Experiment

Nd-YAG Laser

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Contents

Page

1. Introduction **1**

2. Theory **1**
2.1 Active Medium 2
2.1.1 Absorption and Emission 3
2.1.2 Population Inversion 7
2.1.3 Laser Rate Equations and Level Systems 9
2.2 Nd-YAG Laser System 14
2.2.1 Energy Levels of Nd-YAG 14
2.2.2 Nomenclature of Energy Levels 15
2.2.3 Selection Rules and Metastable State 15
2.3 Optical Resonator 16
2.3.1 Types of Optical Resonators 16
2.3.2 Longitudinal Mode of a Resonator 17
2.4 Laser Diode Pump 18
2.4.1 Diode Laser 18

3. Experimentation **19**
3.1 Experimental Set-Up 19
3.2 Experiments 21
3.2.1 Absorption Spectrum 21
3.2.2 $^4F_{3/2}$ Lifetime Measurement 22
3.3 Q-Switching 24
3.3.1 Methods of Q-Switching 24
3.3.2 Pockels Effect and Pockels Cell 24
3.3.3 Passive Q-Switching 27

4. Future Prospects **27**

Abstract

NdYAG crystal (Nd^{3+} ions doped in $Y_3Al_5O_2$) is one of the most widely solid-state lasers. In this experiment, we first discuss the theory of lasers has been along with laser rate equations and laser level systems. We then proceed to study the NdYAG laser system and its energy levels. After a discussion of the optical resonator and diode laser for the pump source (fundamental components of a laser system), we studied experimental setup and its components. We then discuss the experiment to determine the dependence of the wavelength of the laser diode on temperature and injection current via the measurements of absorption spectrum of NdYAG crystal and the experiment to determine the lifetime of the metastable state ($^4F_{3/2}$). We also discuss the theory of Q-switching and conclude with our future plans to practically implement the same.

I. Introduction

LASER stands for Light Amplification by Stimulated Emission of Radiation. It is a source of highly directional and intense light, that is monochromatic and coherent.
Lasers find a wide range of applications in the fields of research, medicine, communication, military etc.

In this open lab project, we intend to develop a Nd-YAG laser powered by a laser diode and use it implement Q-switching.

In this report we aim to discuss :

i.) the theory of laser action, Nd-YAG laser, cavity resonators and laser diodes

ii.) experimental setup and a discussion of the experiments that can be performed using the same

iii.) Q-switching and a proposal for implementation of passive Q-switching

II. Theory

A standard laser is primarily made up of three components :

i.) Pump

ii.) Active Medium

iii.) Cavity Resonator

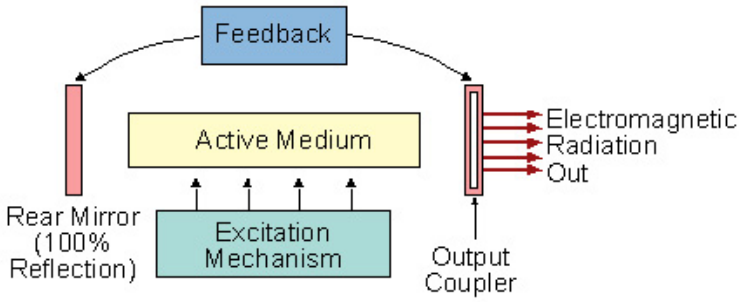

Fig 1. A standard Laser

Source: https://www.phys.ksu.edu/ksuper/

The pump provides suitable excitation to the active medium, which in turn provides amplification and the cavity resonators provide optical feedback.

The active medium is a collection of atoms which gets activated when the radiation from the pump interacts with it. Thus, the process of lasing is essentially the theory of interaction of radiation with matter.

2.1 Active Medium

The amplification in the active medium is generated in the active medium by the process of absorption, spontaneous emission and stimulated emission.
To talk about the processes in detail, we will first state the Planck's Law of Radiation and discuss the Boltzmann statistics for atomic population.

Planck's Law of Radiation : The spectral energy density is given as :

$$u(\nu)d\nu = \frac{8\pi h\nu^3 d\nu}{c^3}.\frac{1}{\exp\left(h\nu/kT\right)-1} \tag{1}$$

where,

$$\begin{aligned} \nu &\implies \text{frequency of radiation} \\ u(\nu) &\implies \text{total energy per unit volume between } \nu \text{ and } \nu + d\nu \\ c &\implies \text{speed of light} \\ h &\implies \text{Planck's constant} \\ k &\implies \text{Boltzmann's constant} \\ T &\implies \text{absolute temperature} \end{aligned}$$

Atomic population : It is the number of atoms per unit volume in a state or energy level. The distribution of atoms over energy levels is governed by the Boltzmann statistics.

In thermal equilibrium the occupation probability is given as :

$$P(E_i) = g_i \exp(-E_i/kT) \tag{2}$$

g_i is the degeneracy factor and is 1 for a non-degenerate system. For the simplicity of calculations we will assume our systems to be non-degenerate.

The average number of atoms in energy state E_i at absolute temperature T :

$$N(E_i) \propto \exp(-E_i/kT) \tag{3}$$

Thus, at constant temperature or thermal equilibrium, the population decreases exponentially with an increase in energy. So, the lower energy states have a greater probability of being occupied compared to higher energy states.

2.1.1 Absorption and Emission

Let us consider a two level system.

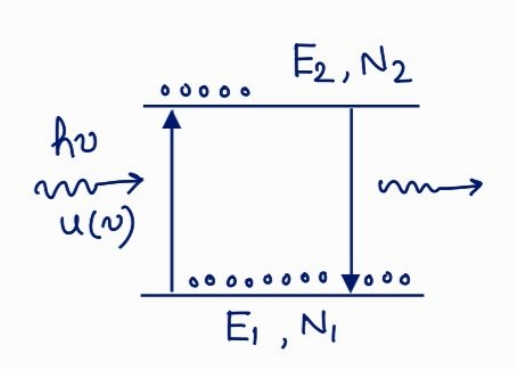

Fig 2. Two-level atomic system

Let N_1 be the number of atoms per unit volume in the ground state (E_1), N_2 be the number of atoms per unit volume in the excited state (E_2) and $u(\nu)$ is the spectral energy density.

Absorption

If a light of energy corresponding to the energy difference between the energy levels is incident on the system, there is a finite probability that atoms will transition from the ground state to the excited state.

The rate of absorption is proportional to the number of atoms present in the ground state and the spectral energy density.

$$\text{Rate of absorption} \propto N_1 u(\nu) = B_{12} N_1 u(\nu) \tag{4}$$

where, B_{12} is the proportionality constant.

Spontaneous Emission

Once in the excited state the atoms will tend to drop back down to the ground state. In this process, the energy difference will be emitted in the form of an electromagnetic wave of frequency :

$$\nu = \frac{E_2 - E_1}{h}$$

The process is known as spontaneous or radiative emission.
The rate of spontaneous emission is proportional to the number of atoms in the excited state.

$$\text{Rate of spontaneous emission} \propto N_2 = A_{21}N_2 \tag{5}$$

A_{21} and B_{12} are together called Einstein coefficients.

Stimulated Emission

If an atom is in the excited state, an incident photon, called a stimulating photon can stimulate the downward transition of such an atom. The photon emitted via such a process is of the same energy, polarization, phase and direction of propagation as of the stimulating photon. The process is called a stimulated emission. It provides coherent amplification of the incident photon.

The rate of stimulated emission is proportional to the amount of atoms in the excited state and the spectral energy density.

$$\text{Rate of stimulated emission} \propto N_2 u(\nu) = B_{21}N_2 u(\nu) \tag{6}$$

Calculating Einstein Coefficients

Considering the two-level system to be at equilibrium, the rate of upward transitions will be equal to the rate of downward transition.

$$\begin{aligned} A_{21}N_2 + B_{21}N_2 u(\nu) &= B_{12}N_1 u(\nu) \\ \implies A_{21}N_2 &= B_{12}N_1 u(\nu) - B_{21}N_2 u(\nu) \\ \therefore u(\nu) &= \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2} \end{aligned}$$

From (3) we can write :

$$\frac{N_1}{N_2} = \exp\left(\frac{E_2 - E_1}{kT}\right) = \exp\left(\frac{h\nu}{kT}\right)$$

Thus, we get the equation for spectral energy density for non-degenrate atomic system can be written as :

$$u(\nu) = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} \exp\left(\frac{h\nu}{kT}\right) - 1}$$

Comparing the above expression with the expression we have for spectral energy density from Planck's Law (1), we can get a relation for the Einstein coefficients as follows :

$$\boxed{\begin{aligned} \frac{A_{21}}{B_{21}} &= \frac{8\pi h\nu^3}{c^3} \\ B_{12} &= B_{21} \end{aligned}} \tag{7}$$

$$B_{12} = B_{21} \tag{8}$$

where,

$$\begin{aligned} A_{21} &\implies \text{rate constant for stimulated emission} \\ B_{12} &\implies \text{rate constant for (stimulated) absorption} \\ B_{21} &\implies \text{rate constant for stimulated emission} \end{aligned}$$

From (8) we see that the rate of absorption and stimulated emission is the same. From (7) we can also say that for frequency ν the two level system behaves as a black body.

Spontaneous Emission Lifetime

We can obtain an expression for the lifetime of spontaneous emission from the equation for rate of spontaneous emission (5) :

$$\begin{aligned} \frac{dN_2}{dt} = -A_{21}N_2 \quad &\implies \int \frac{dN_2}{N_2} = -\int A_{21}dt \\ \implies \ln N_2 &= -A_{21}t + c \\ \implies N_2 &= N_2(0)\exp\left(-A_{21}t\right) \end{aligned}$$

At $t = t_{sp} = 1/A_{21}$, called the spontaneous emission lifetime, the atomic population in the excited state reduces by value e.

$$\therefore N_2(t) = N_2(0)\exp\left(-\frac{t}{t_{sp}}\right) ; \qquad A_{21} = \frac{1}{t_{sp}} \tag{9}$$

So, for a large spontaneous emission lifetime, we have slower rate of the process.

Emission and Absorption Spectrum

When atoms interact with radiation, they do not just interact with a single frequency but over a range of frequencies around any transition frequency.

The probability that an atom interacts with radiation of frequencies between ν and $\nu + d\nu$ is

given by $g(\nu)d\nu$, which is called the atomic lineshape function. It gives us information about the strength of interactions at some particular frequencies.

Since atoms will interact with radiation of some frequency over the entire frequency, the lineshape function is normalized.

$$\int_0^\infty g(\nu)d\nu = 1 \tag{10}$$

In presence of the lineshape function, the number of atoms interacting with some particular frequency is given as $N_2 g(\nu)d\nu$.
So, we can write :

$$\text{No. of atoms interacting with all frequencies} = \int_0^\infty N_2 g(\nu)d\nu = N_2 \tag{11}$$

Rate of Emission and Absorption

Using the lineshape function we can now accurately write the rates of emission and absorption.

$$\text{Rate of spontaneous emission} = A_{21}N_2 g(\nu)d\nu \tag{12}$$

The rate of stimulated emission per unit volume (Γ_{21}) can be written as :

$$d\Gamma_{21} = B_{21}u(\nu)N_2 g(\nu)d\nu \tag{13}$$

Rate of (stimulated) absorption between per unit volume (Γ_{12}) can be written as :

$$d\Gamma_{12} = B_{12}u(\nu)N_1 g(\nu)d\nu \tag{14}$$

The rate equations are controlled by the width of $u(\nu)$, the spectral energy density and $g(\nu)$, the lineshape function, as we can take B_{12}, B_{21}, N_1 and N_2 to be constants.
We can have two distinct cases :

Case 1 : When $u(\nu)$ is the broadband radiation, i.e., it is much broader than $g(\nu)$, we can write :

$$\Gamma_{12} = B_{12}N_1 \int_0^\infty u(\nu)g(\nu)d\nu$$
$$\implies \Gamma_{12} = B_{12}N_1 u(\nu_0) \int_0^\infty g(\nu)d\nu = B_{12}N_1 u(\nu_0)$$

We perform the integration assuming $u(\nu_0)$ to be constant as $g(\nu)$ is non-zero in a very small region compared to $u(\nu)$. Using the value of B_{12} from (7), (8), and the value of A_{21} from (9), we get the final equation for Γ_{12} as :

$$\Gamma_{12} = \frac{c^3}{8\pi h\nu^3}\frac{1}{t_{sp}}u(\nu_0)N_1 \tag{15}$$

Case 2 : When $g(\nu)$ is much more spread out than $u(\nu)$, we can write as :

$$\Gamma_{12} = B_{12}N_1 \int_0^\infty u(\nu)g(\nu)d\nu$$
$$\Gamma_{12} = B_{12}N_1 g(\nu_0) \int_0^\infty u(\nu)d\nu = B_{12}N_1 g(\nu_0)U$$

where, U is the total energy per unit volume.
We have integrated keeping $g(\nu)$ constant as $u(\nu)$ is non-zero over a very small range. For stimulated emissions, we will have the same equations, except with N_2 instead of N_1.
We can finally write :

$$\boxed{\Gamma_{21} = W_{21}N_2\,; \quad \Gamma_{12} = W_{12}N_1} \tag{16}$$
$$\boxed{W_{12} = \frac{(c/n)^3}{8\pi h\nu^3 t_{sp}} g(\nu_0)U} \tag{17}$$

where,

$W \implies$ stimulated transition rate per atom
$U_\nu \implies$ total energy associated with the radiation field

2.1.2 Population Inversion

We have seen before than atoms naturally tend to occupy the lower energy states compared to the higher energy states. The phenomenon of population inversion occurs when this population of states is flipped, i.e., the higher energy states have a greater population than the lower energy state.

Population inversion is an essential condition for achieving amplification by stimulated emission. It is essential for the functioning of any standard laser.

Let us consider a thin slice of laser medium with width dz and cross-sectional area S.

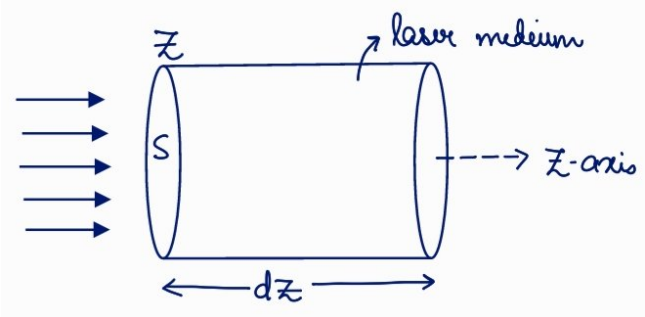

Fig 3. Thin slice of laser medium

Volume of the slice is given as Sdz. Now we can write :

$$\begin{aligned}
&\text{No. of stimulated emission per unit time in volume } Sdz \ = \Gamma_{21} Sdz \\
&\text{No. of stimulated absorptions per unit time in volume } Sdz \ = \Gamma_{12} Sdz \\
\therefore &\text{Energy generated } = \Gamma_{21} Sdz(h\nu) \\
\therefore &\text{Energy absorbed } = \Gamma_{12} Sdz(h\nu) \\
\therefore &\text{Net amount of energy generated in the volume } = (\Gamma_{21} - \Gamma_{12})\, Sdz(h\nu) \qquad (18)
\end{aligned}$$

If we consider the intensity of the light entering and leaving the medium to be I_ν, we can write :

$$\begin{aligned}
&\text{Energy entering the volume per unit time } = I_\nu(z) S \\
&\text{Energy leaving the volume per unit time } = I_\nu(z + dz) S \\
\therefore &\text{Net energy leaving the volume per unit time } = [I_\nu(z + dz) - I_\nu(z)]\, S = \frac{\partial I}{\partial z} Sdz \qquad (19)
\end{aligned}$$

We also know that energy and intensity can be related as :

$$I_\nu = vU = (c/n)U \qquad (20)$$

where,

$$\begin{aligned}
v &= \text{velocity of light in the laser medium} \\
c &= \text{speed of light in vacuum} \\
n &= \text{refractive index of laser medium} \\
U &= \text{total energy per unit volume}
\end{aligned}$$

Using (18), (19) and (20) :

$$\begin{aligned}
\frac{\partial I}{\partial z} &= \frac{(c/n)^3}{8\pi h\nu^3 t_{sp}} g(\nu)\, (N_2 - N_1)\, U(h\nu) \\
\implies \frac{\partial I}{\partial z} &= \frac{(c/n)^2}{8\pi h\nu^3 t_{sp}} g(\nu)\, (N_2 - N_1)\, I_\nu(h\nu)
\end{aligned}$$

Since intensity only depends on z, we can write :

$$\begin{aligned}
\frac{dI_z}{dz} &= \gamma(\nu) I_\nu(z) \\
\therefore I_\nu(z) &= I_\nu(0) \exp[\gamma(\nu) z] \qquad (21)
\end{aligned}$$

$\gamma(\nu)$ contains all parts of the equation which is independent of intensity.

$$\boxed{\gamma(\nu) = \frac{(c/n)^2}{8\pi h\nu^3 t_{sp}} g(\nu)\,(N_2 - N_1)\,(h\nu)} \tag{22}$$

For amplification, output intensity must be greater than the input intensity. If we consider the laser to be located between z and $z = z + L$, we can write for amplification :

$$\begin{aligned} I_\nu^{(out)} &> I_\nu^{(in)} \\ \implies I_\nu(z+L) &> I_\nu(z) \\ \implies I_\nu(0)\exp\left[\gamma(\nu)\,(z+L)\right] &> I_\nu(0)\exp\left[\gamma(\nu)z\right] \\ \implies \exp\left[\gamma(\nu)L\right] &> 1 \end{aligned}$$

So, from output intensity can be greater than incident intensity only if $\gamma(\nu) > 0$.
Now, $\gamma(\nu) > 0$ only if $(N_2 - N_1) > 0$ as all other quantities are positive.
Thus, $N_2 - N_1 > 0$ is a necessary condition for positive gain.

So, population inversion is the necessary condition for amplification by stimulated emission.

2.1.3 Laser Rate Equations and Level Systems

Laser rate equations are the equations that describe the rate of change of atomic population of energy levels in the presence of a pump.

2-Level System

Let us consider a 2-level atomic system in the presence of a pump.

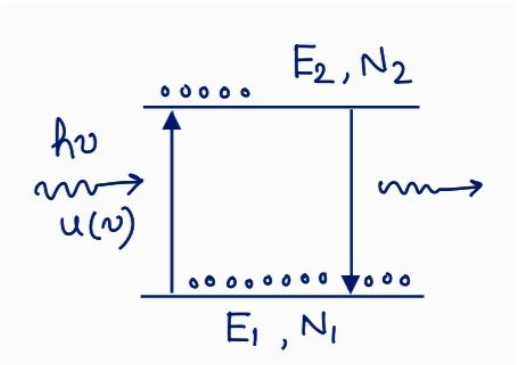

Fig 4. 2-Level Atomic System

The rate equations can be written as :

$$\frac{dN_2}{dt} = W_p N_1 - W_p N_2 - T_{21} N_1 \tag{23}$$

$$\frac{dN_1}{dt} = -W_p N_1 + W_p N_2 - T_{21} N_2 = -\frac{dN_2}{dt} \tag{24}$$

where,

$$W_p = W_{12} = W_{21} \implies \text{rate of stimulated transition per atom in the presence of pump}$$
$$T_{21} \implies \text{rate of spontaneous emission per atom}$$

At steady state, we have :

$$\frac{dN_1}{dt} = 0 = \frac{dN_2}{dt}$$

Solving the steady state equation we get :

$$\boxed{\frac{N_2}{N_1} = \frac{W_p}{W_p + T_{12}}} \tag{25}$$

Now as W_p and T_{21} are positive quantities, it means $W_p < W_p + T_{12}$ at all times.
So, $N_2 < N_1$ at all times. Population inversion is not possible in a 2-level system in steady state for any pumping rate.

3-Level System

Let us consider a 3-level atomic system in presence of a pump.

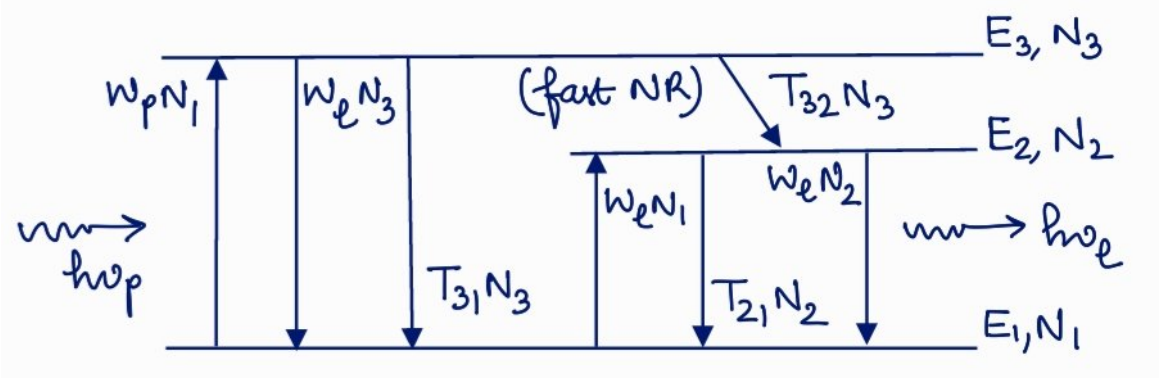

Fig 5. 3-Level Atomic System

Ground state and two excited states take part in this process. We can have population inversion between E_2 and E_1 if we pump to E_3 fast enough and the atoms drop down to E_2 faster than they drop to E_1 from E_2.
We can write the rate equations as :

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - T_{32}N_3 - T_{31}N_3 \tag{26}$$
$$\frac{dN_2}{dt} = W_l(N_1 - N_2) + T_{32}N_3 - T_{21}N_2 \tag{27}$$
$$\frac{dN_1}{dt} = W_p(N_3 - N_1) + W_l(N_2 - N_1) + T_{21}N_2 + T_{31}N_3 \tag{28}$$

where,

$W_p \implies$ rate of stimulated transitions per atom between E_1 and E_3 in the presence of pump

$W_l \implies$ rate of stimulated transitions per atom between E_1 and E_2 in the presence of pump

$T_{31} \implies$ rate of spontaneous emission per atom from E_3 to E_1

$T_{32} \implies$ rate of spontaneous emission per atom from E_3 to E_2

$T_{21} \implies$ rate of spontaneous emission per atom from E_2 to E_1

For population inversion, we need $T_{32} >> T_{31}$, i.e., atoms need to fall faster to level-2 than back to level-1 from level-3.
Using this assumption, we can solve the steady state equations :

$$\frac{dN_3}{dt} = \frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$$

From N_3 equation we have :

$$\frac{N_3}{N_1} = \frac{W_p}{W_p + T_{31} + T_{32}} \tag{29}$$

Similarly from N_2 equation, we have :

$$\frac{N_2}{N_1} = \frac{W_l}{W_l + T_{21}} + \frac{W_p T_{32}}{(W_l + T_{21})(W_p + T_{32} + T_{31})} \tag{30}$$

As we know, $N = N_1 + N_2 + N_3$, we can write :

$$N_1 = \frac{N}{\left(1 + \frac{N_2}{N_1} + \frac{N_3}{N_1}\right)} \tag{31}$$

The difference in the number of atoms between E_2 and E_1 can be written as :

$$\Delta N = N_2 - N_1 = N_1 \left(\frac{N_2}{N_1} - 1\right) \tag{32}$$

Using (29), (30), (31) and the assumption that $T_{32} >> T_{31}$:

$$\boxed{\frac{\Delta N}{N} = \frac{W_p (T_{32} - T_{21}) - T_{32} T_{21}}{W_l (3W_p + 2T_{32}) + T_{32} (W_p + T_{21}) + 2W_p T_{21}}} \tag{33}$$

Now to achieve population inversion :

$$W_p (T_{32} - T_{21}) > T_{32} T_{21} \quad \implies \quad T_{32} > T_{21}$$

So, for population inversion to take place, T_{21} should be as small as possible, i.e., lifetime of state E_2 must be as large as possible.
Assuming this condition holds, we can ignore T_{21} when it gets added to T_{32}, we get the

expression for ΔN as :

$$\frac{\Delta N}{N} = \frac{(W_p - T_{21})\, T_{32}}{W_l\,(3W_p + 2T_{32}) + T_{32}\,(W_p + T_{21}) + 2W_p T_{21}} \tag{34}$$

Now, for population inversion we get the condition :

$$N_2 > N_1 \quad \Longrightarrow \quad W_p > T_{21}$$

So, pumping rate per atom should be greater than T_{21} for population inversion to be achieved. Thus, there is a threshold pumping rate ($W_p = T_{21}$) that needs to be crossed.
While it is possible to achieve population inversion in a 3-level system, it requires a lot of pumping power.

4-Level System

Let us consider a 3-level atomic system.

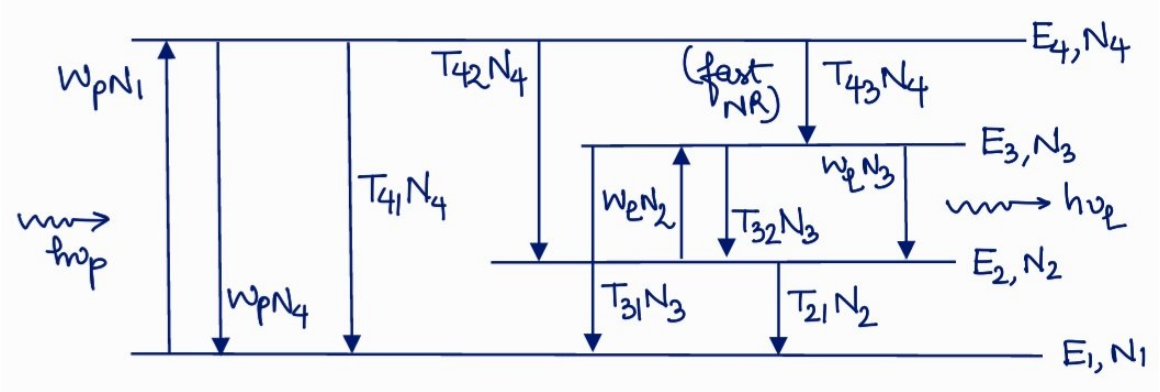

Fig 6. 4-Level Atomic System

We can achieve population inversion in this system between E_3 and E_2 if E_3 becomes the metastable state. The advantage of this is that we no longer have to pump a great amount of atoms to the excited state in an effort to reduce the number of atoms in the ground state to achieve population inversion. E_2 state ideally has no atoms in it, as it is not the ground state. Thus, achieving population inversion in a 4-level system is easier.

Assuming we would assume population inversion between E_3 and E_2, the rate of atoms falling to E_3 from E_4 must be greater than the rate of atoms falling to E_2 or E_1 (for both stimulated and spontaneous emission). Also, the atoms must stabilize at E_3, so atoms must fall to the ground state E_1 faster from E_2 than from E_3.
Summarizing these essential conditions we can write :

$$T_{43} >> T_{42}\,, T_{41}\,; \quad T_{21} >> T_{31} \tag{35}$$

To make calculations simpler, we will use the assumptions in (35) and ignore the transitions

T_{42}, T_{41} and T_{31}.
The rate equations can be written as :

$$\frac{dN_4}{dt} = W_p\left(N_1 - N_4\right) - T_{43}N_4 \tag{36}$$
$$\frac{dN_3}{dt} = W_l\left(N_2 - N_3\right) + T_{43}N_4 - T_{32}N_3 \tag{37}$$
$$\frac{dN_2}{dt} = W_l\left(N_2 - N_3\right) + T_{32}N_3 - T_{21}N_2 \tag{38}$$
$$\frac{dN_1}{dt} = W_p\left(N_4 - N_1\right) + T_{21}N_2 \tag{39}$$

Given N is the total number of atoms in the system, we can also write :

$$N = N_1 + N_2 + N_3 + N_4 = N_1\left(1 + \frac{N_2}{N_1} + \frac{N_3}{N_1} + \frac{N_4}{N_1}\right) \tag{40}$$

where,

$W_p \implies$ rate of stimulated transitions per atom between E_1 and E_4 in the presence of pump
$W_l \implies$ rate of stimulated transitions per atom between E_2 and E_3 in the presence of pump
$T_{43} \implies$ rate of spontaneous emission per atom from E_4 to E_3
$T_{32} \implies$ rate of spontaneous emission per atom from E_3 to E_2
$T_{21} \implies$ rate of spontaneous emission per atom from E_2 to E_1

Solving the steady state equations :

$$\frac{dN_4}{dt} = \frac{dN_3}{dt} = \frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$$

From (36), we can write :

$$\frac{N_4}{N_1} = \frac{W_p}{W_p + T_{43}} \tag{41}$$

From (39) and using (41) we can write :

$$\frac{N_2}{N_1} = \frac{W_p T_{43}}{\left(W_p + T_{43}\right)T_{21}} \tag{42}$$

From (37) and using (41), (42), we can write :

$$\frac{N_3}{N_1} = \frac{W_p T_{43}}{W_p + T_{43}}\left[\frac{W_l + T_{21}}{T_{21}\left(T_{32} + W_l\right)}\right] \tag{43}$$

As the required population inversion here is $N_3 > N_2$, we can write from (42) and (43) :

$$\boxed{\frac{\Delta N}{N_1} = \frac{N_3 - N_2}{N_1} = \frac{W_p T_{43}}{W_p + T_{43}}\left[\frac{T_{21} - T_{32}}{T_{21}\left(T_{32} + W_l\right)}\right]} \tag{44}$$

So, population inversion occurs as long as we have $T_{21} > T_{32}$. So, the transition to ground state from E_2 must be faster than the transition of atoms from E_3 to E_2. It means the lifetime of state E_3 must be longer than E_2.
There is no inherent condition on W_p (pumping rate). As soon as we start pumping we can achieve population inversion and lasing action.
Thus, 4-Level atomic systems are best suited for constructing lasers.

2.2 Nd-YAG Laser System

Nd-YAG laser is a solid state laser. It is made by doping Neodymium ions (Nd^{3+}) within Yttrium Aluminium Garnet ($Y_3Al_5O_{12}$) as host material. It has a refractive index of $n = 1.82$ and produces a laser beam of $\lambda = 1064nm$. It is a 4-Level laser system.

2.2.1 Energy Levels of Nd-YAG

The energy level diagram of Nd^{3+} is given as :

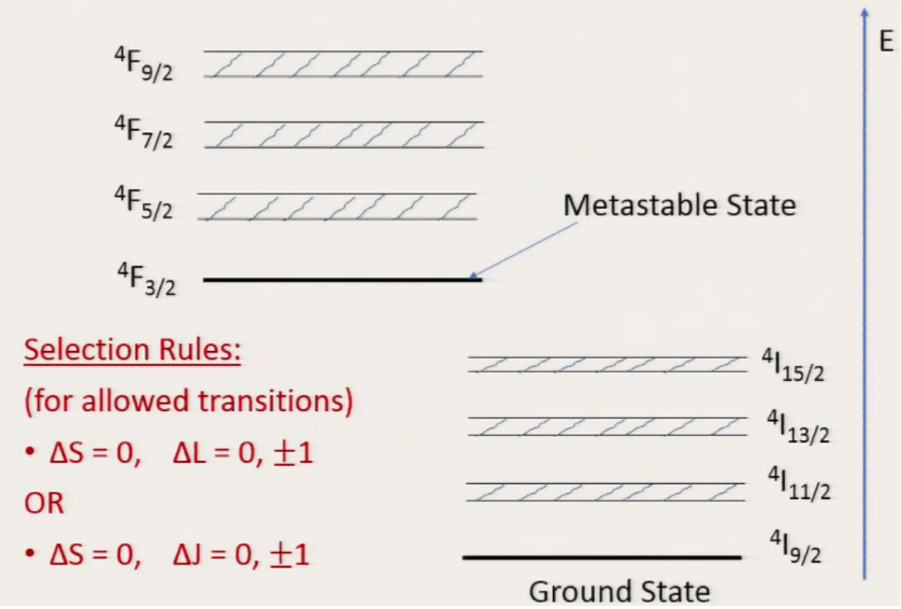

Fig 7. Energy levels of Nd-YAG Laser

Source : NPTEL Lectures on 'Introduction to LASERs' by Prof. Shenoy of IIT Delhi

The broader energy levels are the ones which have lesser lifetimes, as :

$$\Delta E = \Delta\nu h = \frac{1}{2\pi\tau}h \tag{45}$$

where, τ is the lifetime of the level. If lifetime is large, $\Delta\nu$ is small and the spread ΔE of the level is small, and vice-versa. The broader states are the unstable levels.

$^4F_{3/2}$ is the metastable state with a lifetime of about $100\mu s$.

2.2.2 Nomenclature of Energy Levels

Neodymium (Nd) is a rare earth element with atomic number 60. There are 57 electrons in Nd^{3+}.
The electronic configuration is given as :

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6 4f^3$$

The total spin of the ion is due to the $4f^3$, and is given as $S = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$.

The states are labelled as : ${}^{2S+1}L_J$, where S is the total spin, L is the orbital angular momentum and J is the total angular momentum. $2S + 1$ is also called the multiplicity.

For $4f^3$, $S = \frac{3}{2}$ and $2S + 1 = 4$. So, there are four levels of f with different angular momentum.
J can take all integer values from $|L - S|$ to $|L + S|$. For level F of atomic/molecular systems (analogous to f-orbital of electron systems), the orbital angular momentum L is 3.

So, for F : $L = 3$ and $S = \frac{3}{2}$:

$$J = 3 - \frac{3}{2}, 3 - \frac{1}{2}, 3 + \frac{1}{2}, 3 + \frac{3}{2} = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$$

Repeating the same calculations for level I (analogous to i-orbital with angular momentum 6), we can write :

$$\therefore \text{Levels for } F : {}^4F_{3/2}, {}^4F_{5/2}, {}^4F_{7/2}, {}^4F_{9/2}$$
$$\therefore \text{Levels for } I : {}^4I_{9/2}, {}^4I_{11/2}, {}^4I_{13/2}, {}^4I_{15/2}$$

2.2.3 Selection Rules and Metastable State

For the atomic system here, the selection rules for allowed transitions is given as :

$$\Delta S = 0, \Delta L = 0, \pm 1 \quad \text{OR} \quad \Delta S = 0, \Delta J = 0, \pm 1 \tag{46}$$

Transitions which do not satisfy the selection rules are called forbidden transitions. Metastable states are the ones which do not satisfy the selection rules for downward transition to lower energy levels. So, the atoms tend to accumulate there, increasing the lifetime of the state.

For the Nd-YAG system, the ground state is labelled as ${}^4I_{9/2}$. The levels ${}^4I_{11/2}$, ${}^4I_{13/2}$ and

$^4I_{15/2}$ form the second excited level or level-2. Level $^4F_{3/2}$ is the metastable state, labelled as level-3. And, the levels $^4F_{5/2}$, $^4F_{7/2}$, $^4F_{9/2}$ form the topmost level, called level-4.

In this system as per the selection rules for allowed transitions is given in (46) the transitions between the top three ^4F levels is allowed, but downward transitions from $4^F_{3/2}$ to the I levels is forbidden. Thus, $^4F_{3/2}$ becomes a metastable state.

2.3 Optical Resonator

An optical resonator or cavity resonator is an arrangement of mirrors that produces standing waves in the laser cavity. The cavity resonator is an integral part of the laser system.

We require an optical resonator in our apparatus even though the pump laser and active medium can generate lasing, because a single transit of the pump may not be enough to excite the atoms and obtain sufficient lasing.
In most solid-state laser, it is necessary to further enhance the gain by multiple passes of the radiation through the laser medium. Thus, an optical resonator is required to confine the radiation in the cavity.

2.3.1 Types of Optical Resonators

We have various kinds of optical resonators.

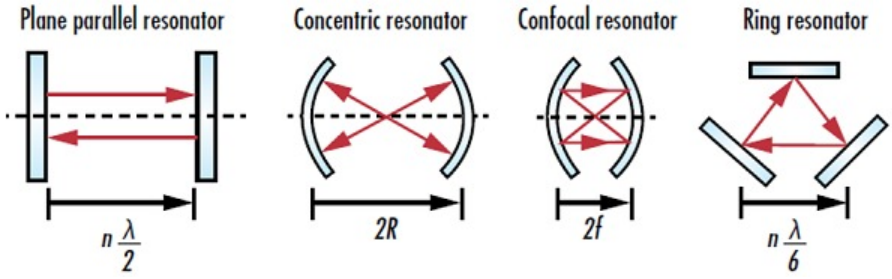

Fig 8. Different Types of Optical Resonators

Source : https://www.edmundoptics.com/knowledge-center/application-notes/lasers/laser-resonator-modes/

Plane-Parallel Resonator : It consists of two plane mirrors placed parallel to each other. The modes of this resonator can be taken to standing waves formed due to superposition of two EM waves travelling in opposite directions.

Concentric Resonator : It is also called spherical resonators and consists of two spherical mirrors with same radius R and separated by distance $L = 2R$ so that the centre of curvature of both mirrors lie on the same point. The modes for this resonator can be approximated to

be a superposition of two spherical waves travelling in opposite directions which originate at the centre.

Confocal Resonator : It consists of two spherical mirrors of the same radius of curvature and separated by distance $L = 2f$ such that foci of the two mirrors lie on the same point.

Ring Resonator : It is a resonator structure where the path of the optical rays is arranged in the form of a ring. The resonance mode is obtained by enforcing the condition that the total phase shift along the ring path is a multiple of 2π.

2.3.2 Longitudinal Mode of a Resonator

Let us consider a plane-parallel resonator.

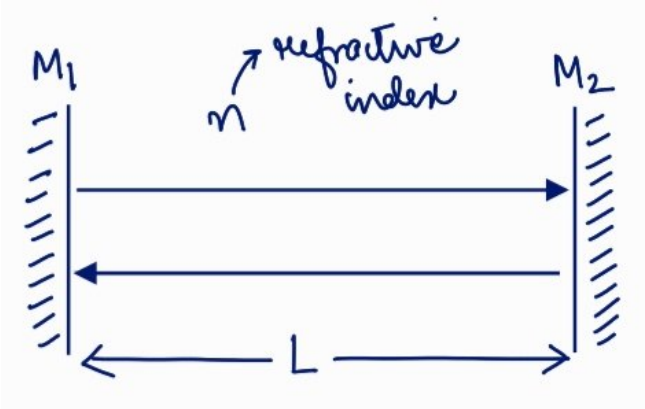

Fig 9. Plane-Parallel Resonator

Let us consider plane waves travelling between the mirrors. Longitudinal modes are standing waves along the optical axis of the setup.

The condition for resonance is that the round trip accumulated phase of the EM wave must be an integral multiple of 2π.

$$\boxed{\nu_q = q\frac{c}{2nL} \quad q = 1, 2, 3, \cdots} \tag{47}$$

where,

$\nu_q \implies$ resonance frequencies which form standing waves

$q \implies$ an integer

$c \implies$ speed of light in vacuum

$n \implies$ refractive index of the cavity

$L \implies$ length of the cavity

The separation between two resonant frequencies is given as :

$$\boxed{\delta_q = \nu_{q+1} - \nu_q = \frac{c}{2nL}} \tag{48}$$

It is called the free spectral range.

These are called the longitudinal modes and q represents the number of half-wavelengths of the mode inside the length of the cavity resonator.

2.4 Laser Diode Pump

A pump source is an integral part of any lasing system. It is required to excite the atoms of the active medium for lasing action to occur.
For our experimental set-up, we use a diode laser to pump our Nd-YAG crystal. This method is called laser pumping.

2.4.1 Diode Laser

Diode laser is a semi-conductor based laser in which a diode is directly pumped by electrical current to create lasing action at the diode junction. In our setup we use a *double heterostructure* diode laser.

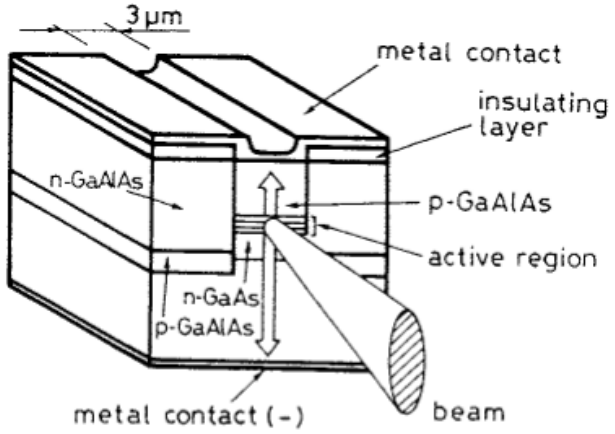

Fig 10. Single stripe based semiconductor depicting the double heterostructure

Source : Svelto, O. and Hanna, D.C. (1998). *Principles of lasers*,volume 4. Springer

Double Heterostructure

A heterostructure is a junction which consists of two dissimilar semiconducting materials with different energy band gaps. Usually, a heterojunction made up of gallium arsenide ($GaAs$) and aluminium gallium arsenide ($Al_xGa_{1-x}As$) is used. $GaAs$ is sandwiched between two layers of $Al_xGa_{1-x}As$, and hence it is called as double heterostructure.

The bandgap of $Al_xGa_{1-x}As$ is more than the band gap of $GaAs$, which creates a potential barrier in the material which can help the accumulation of charge carriers and creating a population inversion.

The active region of such a laser is where the electrons and holes recombine to release EM radiation, and it is confined to be a very thin middle layer ($GaAs$). The edges of the heterostructure is cleaved to ensure good reflectivity, which in turn makes it act like a cavity resonator. Thus we obtain a good level of amplification.

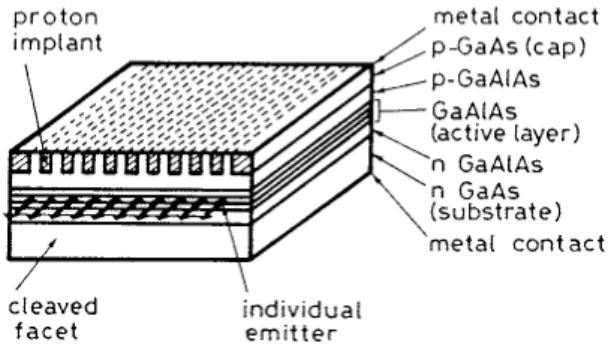

Fig 11. Schematic for the structure of double heterostructure lasers

Source : Svelto, O. and Hanna, D.C. (1998). *Principles of lasers*,volume 4. Springer

III. Experimentation

3.1 Experimental Set-Up

The experimental setup we use is shown below. It consists of multiple modules which play key roles in overall functioning of the experiment.

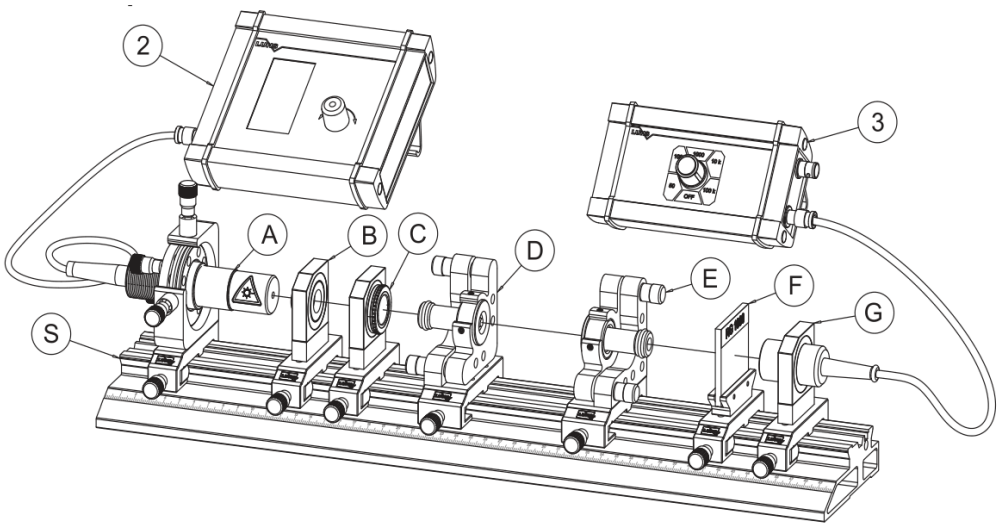

Fig 12. Schematic of the Experimental Set-up for NdYAG Laser

Module A : Diode Laser - 808nm 500mW

It consists of a laser light source along with a temperature controlled laser diode. The maximum output is 500mW at temperature 20^oC. The temperature of the laser diode can be changed between 15^oC to 40^oC using a Peltier's element. The diode head is connected to the ED-0020 controller to control both the injection current and temperature.

Module B : Collimator

It is an optical component which ensures that the incident light beams becomes parallel to one another after passing through it. It consists of a three lens system, with a short focal length of 8mm and a large aperture to collimate divergent beams.

Module C : Focussing Unit

It consists of lens of focal length 60mm and is required to focus the collimated beam onto the NdYAG rod (Module D).

Module D : Laser Mirror Adjustment Holder with NdYAG rod

Module D and E together form the optical resonator of the NdYAG rod. The NdYAG rod is 5mm in length and 3 mm in diameter. On one end of the rod, a highly reflective coating for 1064nm is applied to form the left end of the resonator mirror. The other end is coated with a high quality antireflection layer for 1064nm to ensure minimum internal losses. The back end is coated with a highly reflective layer for 532nm wavelength (green light) to redirect it to the resonator output.

Module E : Laser Mirror Adjustment Holder

It consists of the second resonant mirror which together with Module D forms the resonator cavity for the NdYAG laser. This mirror is marked with "SHG100".

Module F : Filter Plate Holder

There are two kinds of filters provided along with the holder :

i.) RG1000 colour filter to suppress the pump light of 808nm

ii.) BG39 filter to only allow light of wavelength 532nm to pass

Module G : SiPIN Photodiode

It consists of an SiPIN photodiode with an attached cable and a detector connected to the signal conditioner box.

3.2 Experiments

3.2.1 Absorption Spectrum

In this experiment we aim to measure the dependence of diode laser wavelength to its temperature and injection current. We can extract the absorption spectrum of the NdYAG crystal to to determine the dependence.
The NdYAG crystal shows four peaks in its absorption spectrum, namely at wavelengths 804.4nm, 808.4nm, 812.9nm and 817.3nm.

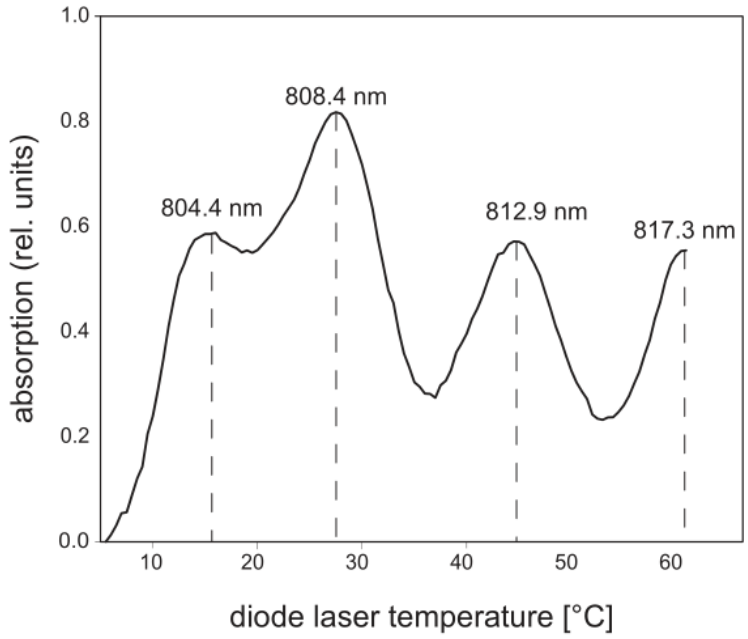

Fig 13. Absorption Spectrum as function of diode temperature

In this setup, the NdYAG laser is arranged such that the laser light from the pump hits it centrally. The photodetector is placed behind the NdYAG laser at an appropriate distance so that it does not get completely saturated.

The laser is turned on and the residual light is observed on the converter screen, after it passes through the NdYAG crystal. The diode temperature is then varied while injection current is kept constant and a change in the intensity of the residual light is observed due to a change in the wavelength of the diode laser.
The experimental plot obatined by our seniors is as follows :

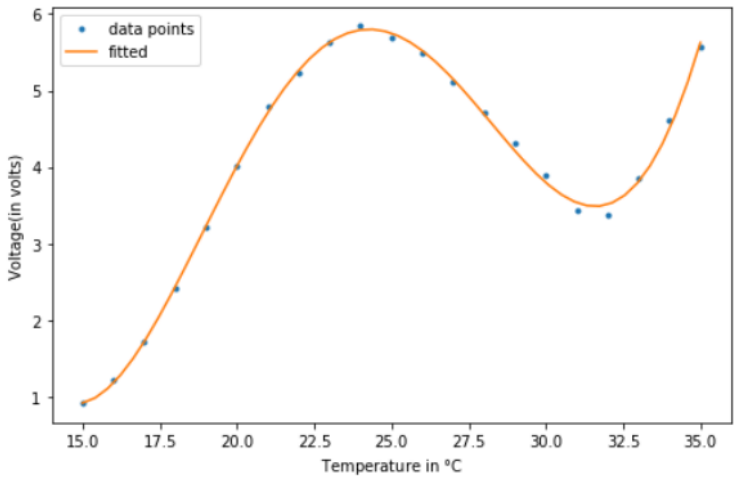

Fig 14. Measured absorption spectrum at injection current of 600mA

The absorption spectrum for variation of injection current has also been obtained by our seniors as follows :

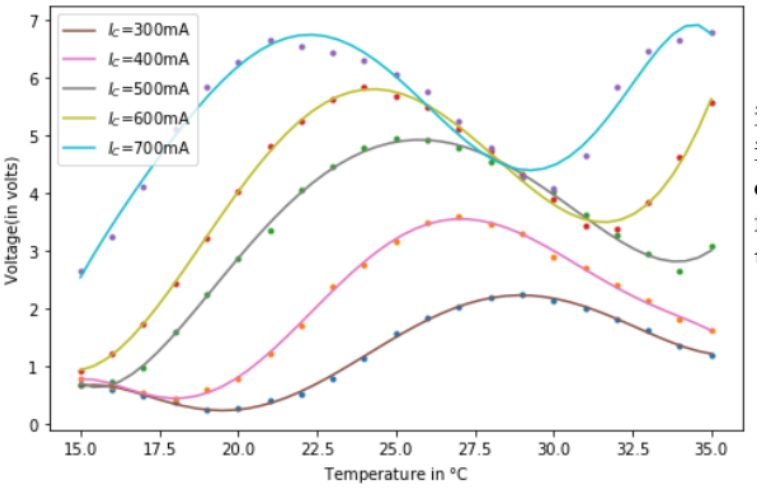

Fig 15. Measured absorption spectrum at various injection current values

3.2.2 $^4F_{3/2}$ Lifetime Measurement

In this experiment we aim to measure the lifetime of the metastable state of the NdYAG crystal, i.e., the $^4F_{3/2}$ state.

The lifetime of a state if defined to be the amount of time taken by the intensity of the transient light to reduce by a factor of $1/e$ of the original intensity.
As this is the metastable state of the system, it has a lifetime longer than the other optical transitions and from literature, we expect this value to be around $250\mu s$.

To measure the lifetime, the NdYAG is periodically pumped with short pulses and the variation of spontaneous emission is seen on the oscilloscope as a function of time. The pump

is pulsed with a certain frequency because if the laser is pumped continuously then the atoms will excited to the higher energy state faster than rate of spontaneous emission. This will mask the downward transition and hamper the measurement of the lifetime of the energy level.

The expected lifetime of $250\mu s$ is long enough to be directly observed on a simple oscilloscope.

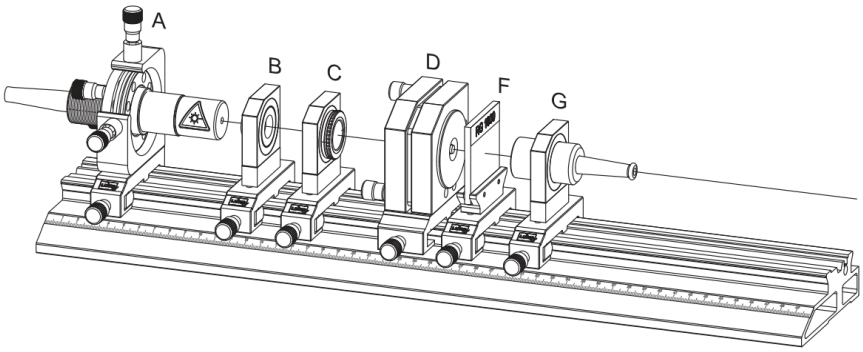

Fig 16. Experimental Set-up for ${}^4F_{3/2}$ Lifetime Measurement

The laser is turned on and off periodically with the required frequency using the knob on the front panel. The focussing unit focusses the pump light on the NdYAG crystal. Placing the RG1000 filter behind the NdYAG crystal suppresses the unabsorbed pump radiation. The resultant fluorescent light reaches the photodiode via the filter and the output of the photodetector amplifier along with the diode laser's injection current output signal is fed to the two-channel oscilloscope.
The fluorescent light is observed even after the pump is turned off. The time taken by the intensity of this fluorescent light to fall to $1/e$ of its original value gives us the required lifetime. The following image corresponds to an experimental measurement of the same.

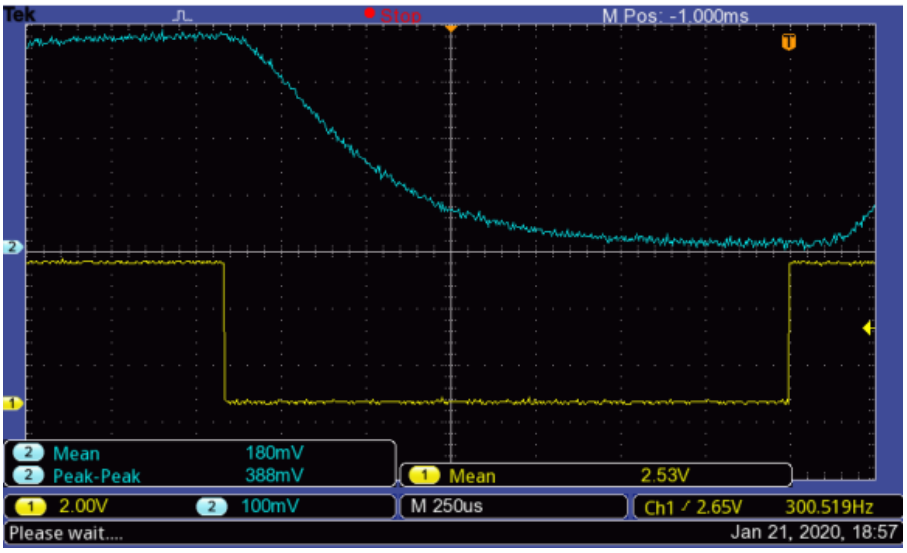

Fig 17. Measurement of lifetime using oscilloscope

Source : H. Chaudhury report

3.3 Q-Switching

Let us assume that we can introduce a shutter in our laser cavity. When the shutter is closed, the laser action is prevented via blocking resonance motion of the EM wave inside the cavity and population inversion can then reach a value a threshold value.
Now when the shutter is opened suddenly opened, a short and intense pulse of light is released due to a rapid gain that exceeds losses within the cavity. The energy stored is released all at once.

Since the process involves switching between low and high values of the quality factor or Q-factor of the cavity, it is called as Q-switching. It allows for the formation of light pulses with duration comparable to photon decay time (in nanoseconds) and high peak power (in megavolts).

3.3.1 Methods of Q-Switching

Q-Switching can be achieved by several methods. The most commonly used for the same are :

i.) Electro-optical shutters

ii.) Rotating Prisms

iii.) Acousto-optical Switches

iv.) Saturable Absorbers

All such methods of implementing Q-switching can be categorized into two main groups : active and passive Q-switches.
For active Q-switching one is required to apply an external active operation to the device to generate Q-switching whereas for passive Q-switching, the switching operation is performed automatically by via the optical nonlinearity of the element used.

3.3.2 Pockels Effect and Pockels Cell

The devices used for implementing Q-switching involve the use of a cell which utilises Pockels effect.
Pockels effect describes the phenomenon of change in the birefringence of a non-linear crystal when an external electric field is applied to it. It is also called a linear electro-optic effect.
If we have an anisotropic material, the relation between field vectors D and E are given as :

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \epsilon_0 \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{49}$$

where,

$$D_i \implies \text{components of displacement current}$$
$$\epsilon_0 \implies \text{permittivity of free space}$$
$$\epsilon_{ij} \implies \text{components of permittivity tensor } \epsilon$$
$$E_i \implies \text{components of externally applied electric field}$$

So from (49) we can say that a for nonlinear crystal with second-order susceptibility $\chi^{(2)}$, Pockels effect is linear in external electric field.
Thus we can conclude that the change in the birefringence of the material is directly proportional to the first power of applied electric field.

A Pockels cell consists of a crystal which has the capability to exhibit Pockels effect. Non-linear crystals such as KDP ($K_2H_2PO_4$) or lithium niobate ($LiNbO_4$) are usually chosen for the visible to near infrared frequency range. In the given frequency range, an applied DC voltage induces a change in the crystal's refractive index along the respective ordinary and extraordinary axes.

Consider the setup as shown below :

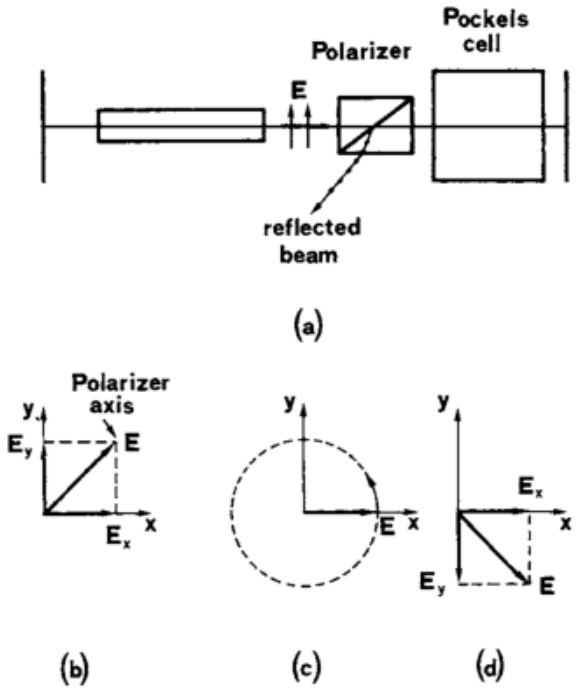

Fig 18. Pockels Cell Setup

Source : Svelto, O. and Hanna, D.C. (1998). *Principles of lasers*,volume 4. Springer

In this setup the Pockels cell is oriented such that the x and y axes of the induced birefringence due to the applied electric field are lying on a plane orthogonal to the optic axis

of the crystal. The optic axis of the crystal is the direction along which the incident light does not suffer any birefringence.
The polarizer makes a 45^o angle to the axes of birefringence.

We will now consider a laser beam propagating from the active medium towards the polarizer-Pockels-cell setup and make the assumption that the polarization of the beam is parallel to the polarizer axis.
Thus the beam will get transmitted completely through the polarizer and will then be incident onto the Pockels cell.
The electric field of the beam will make an angle of 45^o to the birefringence axes of the Pockels cell. So it can be resolved into components E_x and E_y such that their respective oscillations are in phase.

Once they travel through the Pockels cell, the components will experience differing phase shifts which will give rise to a difference in phase :

$$\Delta\phi = k\Delta n L' \tag{50}$$

where,

$$k = \frac{2\pi}{\lambda} \implies \text{wave vector}$$
$$\Delta n = |n_x - n_y| \implies \text{value of induced birefringence}$$
$$L' \implies \text{length of the crystal}$$

The DC voltage applied to the Pockels cell is chosen such it introduces a phase difference given as : $\Delta\phi = \pi/2$. Thus, the two E-field components will differ by a phase of $\pi/2$. Physically, it means that the light wave will become circularly polarized.

The wave then gets reflected off the highly reflective mirror and passes through the Pockels cell once more. The components then acquire additional phase difference of $\pi/2$.
Thus, the total phase difference between E_x and E_y now becomes π.
So, the overall field $\vec{E_i}$ is again linearly polarized but now the polarization axis makes an angle of 90^o with the original polarization axis of the wave.

Thus, the beam gets reflected out of the cavity instead of being transmitted through the transmitter. This corresponds to the Q-factor of the cavity being low. If we remove the DC voltage, the Q-switch is then opened. The induced birefringence in the crystal disappears and the incident light is transmitted through it without any change in polarization.

The technique involving the use of Pockels cells relied on the external intervention to turn the switch on or off. This method is called as active Q-switching.

3.3.3 Passive Q-Switching

In this section we will briefly discuss the method of passive Q-switching which can be implemented in our laboratory setup. Here, the Q-switching part is implemented directly via physical processes involved during the experiment and does not require any external interventions.

In this setup a saturable absorber is used as the Q-switch. It is a material whose transmission increases after the intensity of the incident light exceeds a threshold value. An ion-doped crystal like CrYAG is a suitable candidate for such a material and is used in our laboratory setup.

Initially the loss of the absorber material is very high. It is low enough to permit some lasing action once the active medium achieves a threshold population and some resultant amount of energy. As the power of the laser increases, the absorber material gets saturated, and the loss in the resonator cavity rapidly reduces. This in turn helps the power to increase at a faster rate. In an ideal situation, the absorber material reaches a state of low loss and efficient extraction of the stored energy via a laser pulse is possible.

After a pulse is released, the absorber goes back to its original high loss state. Thus, there is a delay before the energy in the active medium is fully restored.

We can control this repetition of pulse indirectly by varying the power of pump source, or the amount of absorber material in the cavity.

IV. Future Prospects

We were not able to perform the experiment physically and had to refer to our seniors work of previous years. Now that the current situation has improved, we primarily intend to implement the passive Q-switching experimentally using CrYAG crystal in our laboratory. We also wish to investigate the TEM modes and non-linear phenomena such as second harmonic generation.