Name Ashna Khemani Section # 101

Forces and Motion:
Oscillation of a Mass-Spring System

Introduction to Mechanics Lab
University of Pennsylvania

Write out your answers for each question below. You may talk with other students about this assignment, use a calculator and other tools, ask the teaching team questions, and consult outside sources; however, what you write down must be your own work. On computational problems, please show all steps and box your answer.

The post-lab is due the Saturday a week after the in person lab session. Please do not attach this post-lab to your lab, as these two assignments are graded separately. Late submissions will be penalized by 10 points for each day that transpires between the deadline and your submission. This assignment is worth a total of 50 points.

1. A spring with a diameter of 1 centimeter has a total length (end to end) of 13.6 centimeters when pulled with 3.5 newtons. Explain why you cannot determine the spring's stiffness from this information. *(3 points)*

You can't find k [N/m] if you don't know the force [N]

2. In a second test, the same spring has a total length of 18.2 centimeters when pulled with 9.2 newtons. What is its approximate stiffness in N/m? Show your work and box your answer. You do not need to calculate an uncertainty for this estimate. *(3 points)*

$x_2 = 0.182\,m$ $F_2 = 9.2\,N$ $k = ?$

$x_1 = 0.136\,m$ $F_1 = 3.5\,N$

$F = kx$

$$k = \frac{\Delta F}{\Delta x} = \frac{9.2 - 3.5}{0.182 - 0.136} = 123.9 \approx \boxed{124\,N/m}$$

3. What is this spring's rest length? *(3 points)*

$F = k \Delta x$ $k = 123.9 \frac{N}{m}$ $x = 0.182m$ $F = 9.2N$

$F = k(x - x_0)$ $9.2 = 123.9(0.182 - x_0)$

$x_0 = 0.182 - \frac{9.2}{123.9} = 0.107m \approx \boxed{0.11m}$

4. What length would you expect this spring to have (in total) if you hung a 1.5 kilogram mass on it and allowed it to come to equilibrium? *(3 points)*

$m = 1.5kg$ $g = 9.81 m/s^2$ $k = 123.9$ $x_0 = 0.11m$ $x = ?$

$F = k(x - x_0) \rightarrow x = \frac{F}{k} + x_0 = \frac{mg}{k} + x_0 = 0.227 \approx \boxed{0.23m}$

5. For the same spring, what behavior would you expect to see if you pulled down on it with a force of 500 newtons? Explain. *(3 points)*

You'd probably break the spring b/c 500N is a lot.

$F = k\Delta x$ $F = 500N$ $k = 123.9 \ N/m$ $\Delta x = ?$

$\Delta x = \frac{F}{k} = 4.04m$

This is a lot to stretch.

6. In lab you applied Newton's Second Law to an oscillating mass-spring system. You found that the resulting motion is sinusoidal with a natural angular frequency ω_n given by the equation below. Show that the units on the two sides of this equation are equivalent when stiffness is expressed in newtons per meter and mass is in kilograms. *(3 points)*

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\sqrt{\frac{\frac{kg \cdot \cancel{m}}{s^2 \cdot \cancel{m}}}{\cancel{kg}}} = \sqrt{\frac{1}{s^2}} = \frac{1}{s} = Hz$$

Hz is the unit for angular frequency ω

7. At what frequency would you expect the 1.5 kilogram mass to oscillate when hung on the spring discussed in questions 1 through 5? Express your answer in hertz. Show your work and box your answer. *(3 points)*

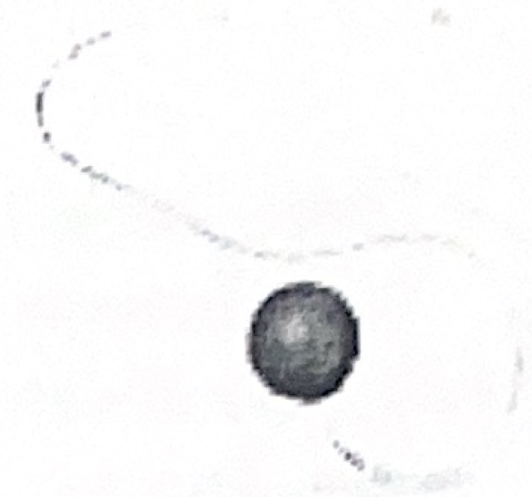
$m = 1.5\ kg \quad k = 123.9\ N/m \quad \omega = ?$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{123.9}{1.5}} = 9.088 \approx \boxed{9.1\ Hz}$$

8. If you start a mass-spring system bouncing, it will oscillate for a long time, but it will eventually stop. This behavior is not predicted from the analysis we completed in lab. Explain why real mass-spring systems eventually stop oscillating. Be as specific as you can be. *(3 points)*

There would be some friction between the spring and the support it's suspended from. Some of the energy will be lost to that friction, thereby dampening and eventually stopping the oscillations.

9. Imagine you are an engineer at a toy design company. You are designing a paddle ball game like the one shown at right. You want to give this toy a natural frequency of 2 hertz so that it is easy and fun to play with. You are planning to use balls that have a mass of 45 grams. What should the stiffness of the cord be? Be careful with units, show your work, and box your answer. *(3 points)*



$\omega = 2\,Hz \quad m = 0.045\,kg \quad k = ?$

$\omega = \sqrt{\frac{k}{m}} \rightarrow k = \omega^2 m = \boxed{0.18\ N/m}$

10. Your manager at the toy design company has found a supplier that will sell you a certain type of stretchy cord very cheaply. For a piece the length you want in the paddle ball toy, its stiffness will be 200 N/m. What ball mass would you need to give the toy a natural frequency of 2 Hz with this cord? Would this be a good toy? Explain. *(3 points)*

$k = 200\ N/m \quad \omega = 2\,Hz \quad m = ?$

$\omega = \sqrt{\frac{k}{m}} \rightarrow m = \frac{k}{\omega^2} = \boxed{50\,kg}$

$\omega^2 = \frac{k}{m}$

$\omega^2 m = k$

$m = \frac{k}{\omega^2}$

This is an awful toy. It's a little heavy to carry.

For the next set of questions, imagine that you are a mechanical engineer at a company that makes small bathroom appliances. Interplanetary space travel is becoming commonplace for both business and pleasure, and these space travelers need to measure their mass every day so they can know how their body is responding to the changing levels of gravity. Loss of bone density is a particular health problem associated with space travel; if it is detected, it can be re-mediated with additional weight-bearing exercise, but there are not yet any suitable scales avail-able for this application on the commercial market.

You have been given the task of designing a mass measurement device that will work in space and on all planets and moons in our solar system, without requiring reconfiguration or re-calibration. Note that the acceleration due to gravity varies from 0 m/s2 to approximately 23 m/s2 across this range of locations. Objects are weightless in space, but they are never mass-less.

11. Use the Internet to research how a typical analog bathroom scale works. These are bathroom scales with a rotating needle (pictured below). Focus on the mechanism that transfers the weight of the human into a needle angle. Write a quick summary of your findings here, and name your primary source. (4 points)

Weight of person stretches a spring, which moves a rack (attached to the other end with another spring) that then turns a pinion connected to the needle/dial on the display.

Source: Jared Owen on YouTube: "How does a Mechanical Scale work? (Spring Scale)"

12. Would an analog bathroom scale designed for use on Earth work correctly in space and on other planets? Explain why or why not. (3 points)

No. It depends on the weight force, which depends on acceleration due to gravity, which can vary widely in our scenario.

13. Building on what you learned about mass-spring systems in this lab, propose a design for an interplanetary mass measurement device that does not require reconfiguration or re-calibration when moved between planets or into space. Explain how it would work. (5 points)

$$ma = -kx$$

unknown

Find from $x = \frac{1}{2}at^2$ $\Rightarrow a = \frac{2x}{t^2}$

Given Given

(t is measured)

Make person sit in a chair attached to a spring w/ a known stiffness k, and push themselves against a stable support for a set distance x. Time the process so you can derive acceleration. Using this information we can find m.

14. What questions, comments, and suggestions do you have about this lab and its associated assignments? Your responses will help us understand what you enjoyed and how we could improve the lab for the future. Your answers to this question will not affect your grade. (0 points)