Assignment - 1

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Problem 1 →

let A : "Dog is brown."

let B : "Dog is fluffy."

let C : "Dog is good."

let D : "Dog loves treats."

let E : "Dog is white."

1 → A dog that is brown or fluffy is a good dog.

Logic statement : $(A \lor B) \land C$

2 → If a dog loves treats then they are a good dog.

Logic statement : $D \to C$

3 → Fluffy white Dogs don't like treats.

Logic statement : $(B \land E) \land \neg D$

4 → Every dog is fluffy or loves treats or is a good dog.

Logic statement : $(B \vee C \vee D)$

5 → A dog which does not like treats and is not fluffy is a bad dog.

Logic statement : $(\neg D \wedge \neg B) \longrightarrow \neg C$

① $(A \lor B) \land C$

A	B	C	$(A \lor B)$	$(A \lor B) \land C$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

② $D \rightarrow C$

D	C	$D \rightarrow C$
T	T	T
T	F	F
F	T	T
F	F	T

③ $(B \land E) \land \neg D$

B	D	E	$(B \land E)$	$(B \land E) \land \neg D$
T	T	T	T	F
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

④ $B \vee C \vee D$

B	C	D	$B \vee C \vee D$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

⑤ $(\neg D \wedge \neg B) \rightarrow \neg C$

B	C	D	$(\neg D \wedge \neg B)$	$\neg C$	$(\neg D \wedge \neg B) \rightarrow \neg C$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	F	F	T
F	T	F	T	F	F
F	F	T	F	T	T
F	F	F	T	T	T

A	B	C	D	E	$(A \vee B) \wedge C$	$D \rightarrow C$	$(B \wedge E) \wedge \neg D$	$B \vee C \vee D$	$(\neg D \wedge \neg B) \rightarrow \neg C$
T/F	T	T	F	T	T	T	T	T	T

$$((A \vee B) \wedge C) \wedge (D \rightarrow C) \wedge ((B \wedge E) \wedge \neg D) \wedge (B \vee C \vee D) \wedge ((\neg D \wedge \neg B) \rightarrow \neg C) = T$$

This set of propositions is consistent because a truth assignment exists for each propositional variable $(A, B, C, D, E) = (T, T, T, F, T)$ or (F, T, T, F, T), such that the set of propositions is consistent.

A	B	C	D	E	(A∨B)∧C	D→C	(B∧E)∧¬D	B∨C∨D	(¬D∧¬B)→¬C	Con
T	T	T	T	T	T	T	F	T	~~F~~ T	F
T	T	T	T	F	T	T	F	T	~~F~~ T	F
T	T	T	F	T	T	T	T	T	~~F~~ T	T
T	T	T	F	F	T	T	F	T	~~F~~ T	T
T	T	F	T	T	F	F	F	T	~~F~~ T	F
T	T	F	T	F	F	F	F	T	~~F~~ T	F
T	T	F	F	T	F	T	T	T	~~F~~ T	F
T	T	F	F	F	F	T	F	T	~~F~~ T	F
T	F	T	T	T	T	T	F	T	~~F~~ T	F
T	F	T	T	F	T	T	F	T	~~F~~ T	F
T	F	T	F	T	T	T	F	T	F	F
T	F	T	F	F	T	T	F	T	F	F
T	F	F	T	T	F	F	F	T	T	F
T	F	F	T	F	F	F	F	T	T	F
T	F	F	F	T	F	T	F	F	T	F
T	F	F	F	F	F	T	F	F	T	F
F	T	T	T	T	T	T	F	T	~~F~~ T	F
F	T	T	T	F	T	T	F	T	~~F~~ T	F
F	T	T	F	T	T	T	T	T	~~F~~ T	T
F	T	T	F	F	T	T	F	T	~~F~~ T	F
F	T	F	T	T	F	F	F	T	~~F~~ T	F
F	T	F	T	F	F	F	F	T	~~F~~ T	F
F	T	F	F	T	F	T	T	T	~~F~~ T	F
F	T	F	F	F	F	T	F	T	~~F~~ T	F
F	F	T	T	T	F	T	F	T	T	F
F	F	T	T	F	F	T	F	T	T	F
F	F	T	F	T	F	T	F	T	F	F
F	F	T	F	F	F	T	F	T	F	F
F	F	F	T	T	F	F	F	T	T	F
F	F	F	T	F	F	F	F	T	T	F
F	F	F	F	T	F	T	F	F	T	F
F	F	F	F	F	F	T	F	F	T	F

Question 2 →

Let A : "A mushroom is poisnous."
Let B : "A mushroom doesn't have gills."
Let C : "A mushroom is red in colour."
Let D : "A mushroom has pores."
Let E : "A mushroom is found growing out of a living tree."
Let F : "A mushroom is bioluminescent."

Rules for poisonous mushroom :-

1. $A \rightarrow B$
2. $A \rightarrow C$
3. $\neg A \rightarrow D$
4. $\neg A \rightarrow E$
5. $F \rightarrow A$

Charactertics of the three mushrooms transated into propositional logic :

Mushroom A : $D \& E \& F \& \neg C \& C$
Mushroom B : $D \& E \& \neg F$
Mushroom C : $\neg E \& C \& B \& \neg D$

Mushroom A is Poisonous since emits a faint glowing light which makes it bioluminescent. As per the given rules, if a mushroom is bioluminescent, then it is poisonous. i.e., $F \rightarrow A$. Also, the uncertainity about the color of the mushroom suggests that there is a chance that it is a dark shade of red, which as per the rules ($A \rightarrow C$, converse being $C \rightarrow A$) further suggests that Mushroom A is poisonous. It is not safe to eat.

Mushroom B is Not Poisonous since it was gathered from a living tree, has pores, is dark brown in color, and is not bioluminescent, ~~implie~~ ~~suggests that it is not poisonous~~. Rules that are given ($\neg A \rightarrow D$, converse: $D \rightarrow \neg A$; $\neg A \rightarrow E$, converse : $E \rightarrow \neg A$; $F \rightarrow A$, converse $A \rightarrow F$) further help to determine the characteristics of this non-poisonous mushroom. It is safe to eat.

Mushroom C is poisonous since it was gathered from a fallen tree, is red coloured, and neither has pores or gills. Converse of rules ($B \rightarrow A$, $C \rightarrow A$, $\neg D \rightarrow A$, $\neg E \rightarrow A$) further help to establish that it is not safe to eat.

Question 3 →

1) ① $n \le n^2$

This statement is true ; Let $F(n) = n \le n^2$

Proof : By direct proof.

Predicate Logic Statement : $\forall n \, (n < n^2)$ or $\forall n F(n)$

proof → Let the domain of n be all integers. Since n is an integer, and n^2 is $n \times n$ which is n times n. The square of any integer is always greater than or equal to the given integer. Therefore, n^2 is always greater than or equal to n, i.e., $n^2 \ge n$. Thus, the statement $n \le n^2$ is true for all or any integers.

A few examples include:

Let $n = -3$, $n^2 = (-3)^2 = 9$;

$-3 < 9$

Let $n = 4$, $n^2 = (4)^2 = 16$;

$4 < 16$.

~~Hence, for any integer n, $n \le n^2$~~

Let $n = 1$, $n^2 = (1)^2 = 1$;

$1 = 1$

Hence, for any integer n, $n \le n^2$ is true by direct proof.

Furthermore, $(n^2 - n) \ge 0$ or $n(n-1) \ge 0$

Case 1 : if $n > 0$,

(i) $n - 1 \ge 0$ (ii) As n are only integers

The product of equation (i) and (ii) will be 0 or greater than 0.

(Multiplying two positives gives a positive)

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Case 2 : if $n \le 0$

(i) $n-1 < 0$ (ii) As n are only integers the product of equations (i) and (ii) will 0 or greater than 0.

(multiplying two negatives gives negative)

From Case 1 and 2, we can clearly say that for every value of $n \in$ integer,

$n(n-1) \geq 0 \Rightarrow (n^2 - n) \geq 0 \Rightarrow n^2 \geq n$.

2 Four Integers : a, b, c, d

Predicate Logic Stament :

$$\forall a,b,c,d\,(((a+b)\%2=1) \iff ((c+d)\%2=1))$$

Let the domain be a set of Integers : Z

Proof : proof by ~~Contrapositive~~ Cases

~~if (c+d) is even if and only if (a+b) is even~~.

Case 1 : a, b are odd and c, d are even.

Here, (a+b) is even and (c+d) is even

For example, Let $a = \cancel{5K}\ 2K+1$ and $b = \cancel{7K}\ 2L+1$

$(a+b) = (\cancel{5K+7K}\ 2K+1+2L+1) = \cancel{12K}\ 2(L+K)+2\ \cancel{= 2(6K)}$

$\Rightarrow (a+b)$ is even.

Let $c = 2K$ and $d = 2L$

$(c+d) = 2K+2L = 2(K+L)$

$\Rightarrow (c+d)$ is ~~odd~~ even.

Case 2 : a, b are even and c, d are odd

Here, (a+b) is even and (c+d) is even.

For example, Let $a = 2m$ and $b = 2n$

$(a+b) = 2m + 2n = 2(m+n)$

Let $c = 2m+1$ and $d = 2n+1$

$(c+d) = 2m+1 + 2n+1 = 2(m+n)+2$

$\Rightarrow (a+b)$ is even and $(c+d)$ is even.

Case 3 : a, c are even and b, d are odd

Here, (a+b) is odd and (c+d) is odd

Since the addition of even and odd is odd and because there are two even and two odd integers off the four integers.

For example, Let $a = 2m$ and $b = 2n+1$

$a+b = 2m + 2n+1 = 2(m+n)+1$

$\Rightarrow (a+b)$ is odd.

Let $c = 2n$ and $d = 2m+1$

$(c+d) = 2n+2m+1 = 2(m+n)+1$

$\Rightarrow (c+d)$ is odd.

Hence, $(a+b)$ is odd when $(c+d)$ is odd

Alternatively, ~~any~~ two variables like even b, d could be taken as even and a, c as odd to prove the above shown proof.

~~Therefore, by proof by contrapositive, (a+b) is odd if and only if (c+d) is odd~~. This statement is true.

(3) Let the first integer be a
Let the second integer be b.
Let the domain be a set of Integers : $\mathbb{Z}^+$
predicate logic statement : ~~$\forall a \in \mathbb{Z}^+, \exists b \in \mathbb{Z}^+,$~~

$$\forall a \, \exists b \, (a^2 \leq b < a^3)$$

~~Proof : by proof by Cases~~ This statement is False
Counterexample : Let a be 1 ~~and b be 1~~
$\Rightarrow 1 \leq b < 1$

Here, b might greater than or equal

to $a^2=1$, but it cannot be less than $a^3=1$. When $b=1$, the predicate logic statement $1 \leq 1 \nless 1$ is false. Similarily, for $b=2$, $1 \leq 2 \nless 1$ is false, and so on.

Therefore, it is not the case that for any positive integer, there exists a second positive integer which is greater than or equal to the square of first integer, but smaller than the cube of first integer as shown in the above example where $a=1$ and $b=1, 2, 3 \ldots\ldots$ The statement is hence false.