COMPSCI 2209B

Assignment 1

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<u>Exercise 1 :-</u>

1. If a request occurs, then either it will eventually be acknowledged, or a requesting process won't ever be able to make progress.

Let p = a request occurs.

Let q = request will eventually be acknowledged

Let r = requesting process will make progress.

Propositional formula : if $\boxed{p}$, then $\boxed{q}$ or $\boxed{\text{not } r}$

ϕ : $p \to (q \vee \neg r)$

2. Cancer will not be cured unless its cause is determined and a new drug for cancer is found.

Let p = Cancer will be cured

Let q = Cause of Cancer is determined.

Let r = A new drug for Cancer is found.

Propositional formula :

: $\cancel{(q \wedge r) \to p}$

: $\cancel{(\neg p) \to (\neg q \wedge \neg r)}$

if $\boxed{\text{not } q}$ or $\boxed{\text{not } r}$, then $\boxed{\text{not } p}$: $\neg(q \wedge r) \to \neg p$

3. if interest rates go up, share prices go down.

Let p = Interest rates go up.

Let q = share prices go down.

Propositional formula : if $\boxed{p}$, then $\boxed{q}$.

: $p \to q$

4. if Smith has installed Central heating, then he has sold his car, or he has not paid his mortgage.

Let p : Smith installed Central heating

Let q : Smith sold his Car.

Let r : Smith has paid his mortgage.

Propositional formula : if $\boxed{p}$, then $\boxed{q}$ or $\boxed{\text{not } r}$

: $p \to (q \vee \neg r)$

5. Today, it will rain or shine, but not both.

Let p = Today, it will rain.

Let q = Today, it will Shine.

Propositional formula : $\boxed{p}$ or $\boxed{q}$ and $\boxed{\text{not } p}$ or $\boxed{\text{not } q}$

: $(p \vee q) \wedge (\neg p \vee \neg q)$

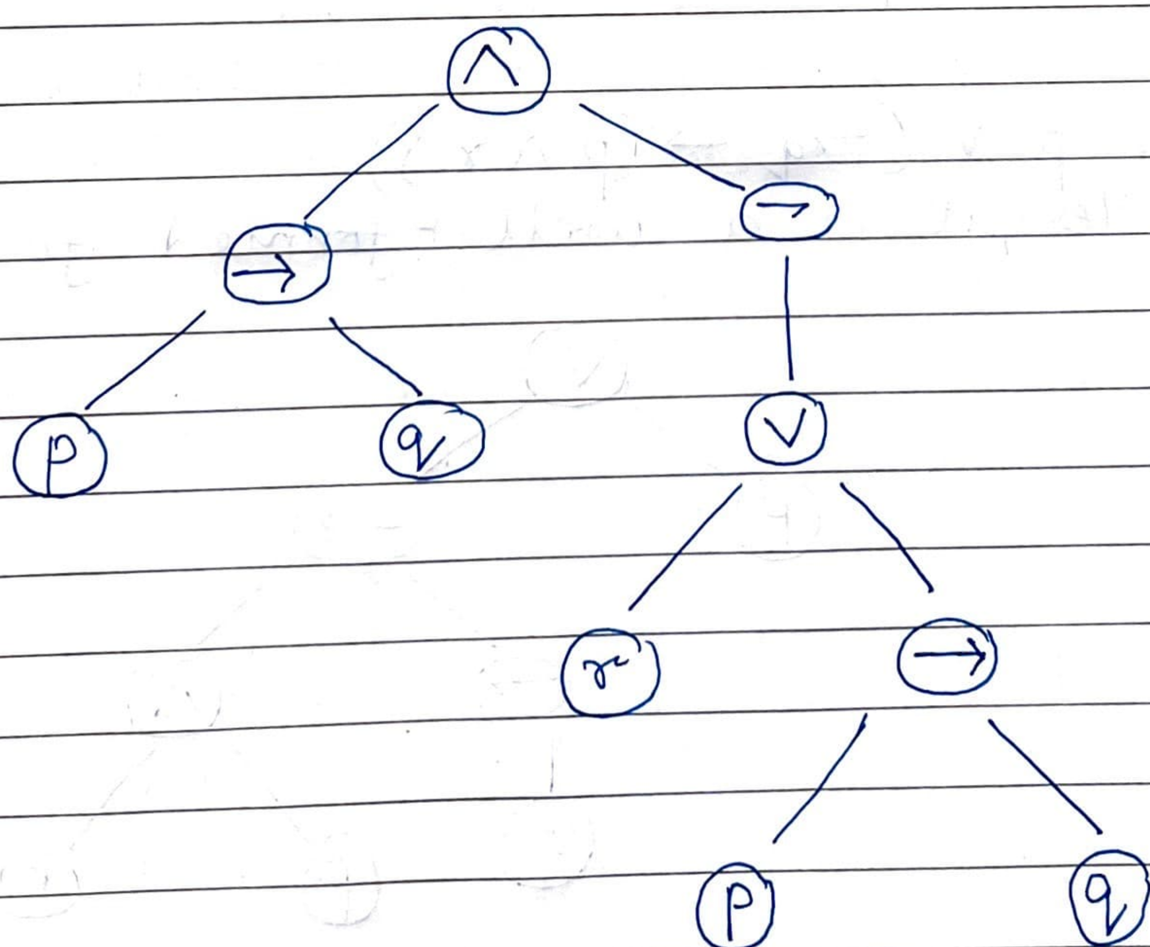
Exercise 2 :-

1 $\neg p \wedge \rightarrow r$

No, not a well-formed formula (WFF) since the '$\wedge$' conjunctive is used between two propositions. However, here it is used between a proposition 'p' and a connective '$\rightarrow$'. Hence, not a WFF.

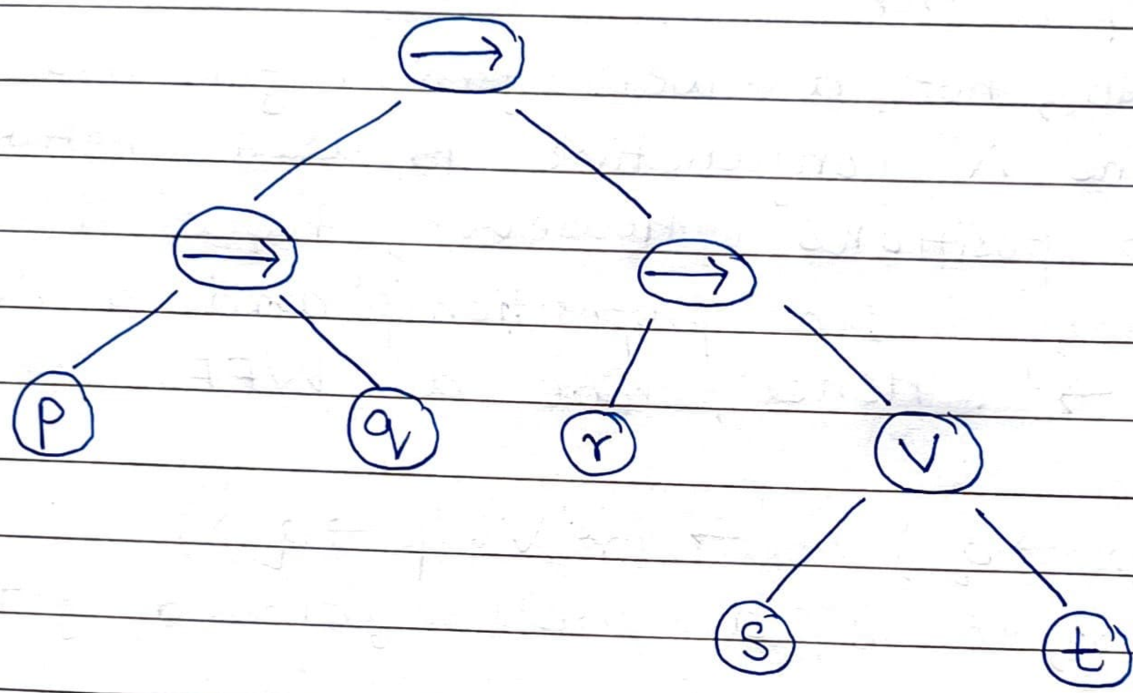
2. $(p \rightarrow q) \wedge \neg (r \vee (p \rightarrow q))$

Yes, it is a well-formed formula (WFF)



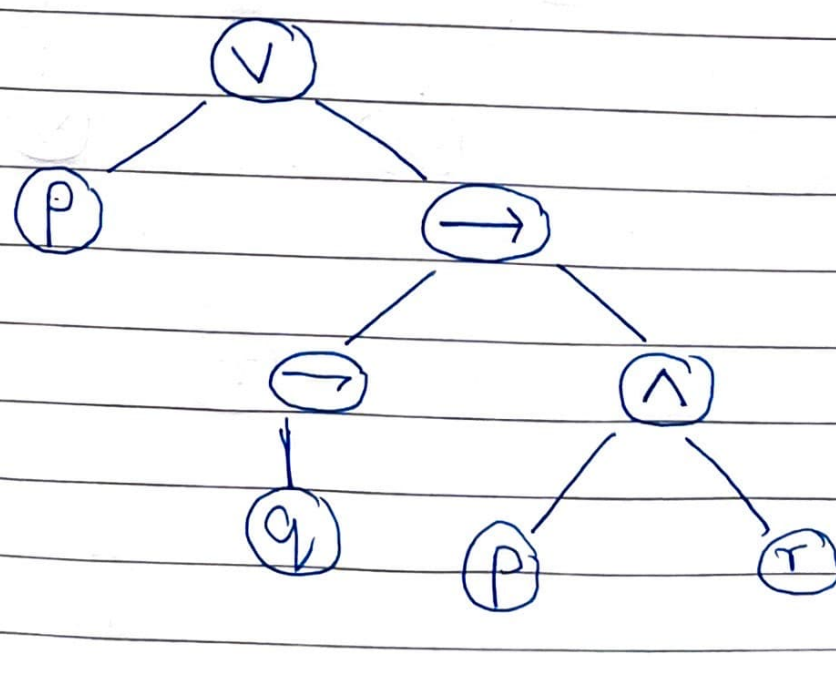
3. $((p \to q) \to (r \to (s \lor t)))$

Yes, it is a well-formed formula (WFF).



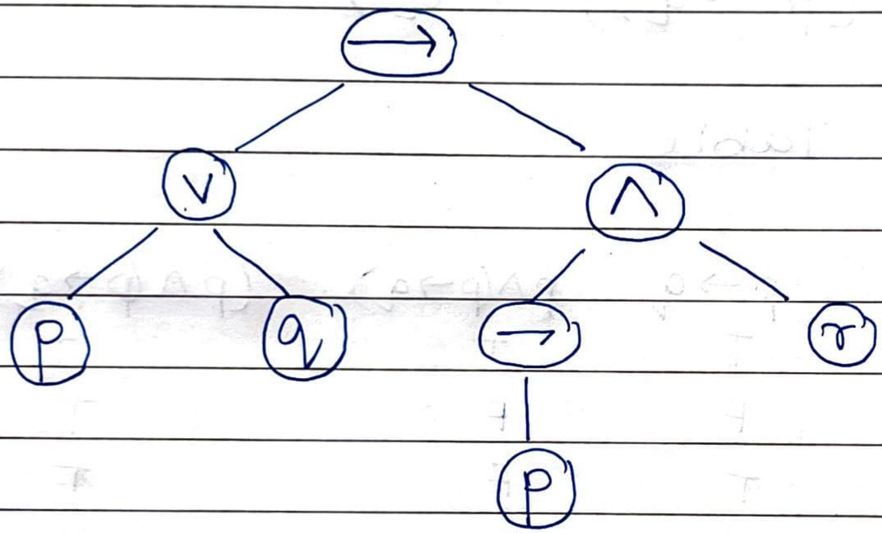
4. $p \lor (\neg q \to (p \land r))$

Yes, it is a well-formed formula.



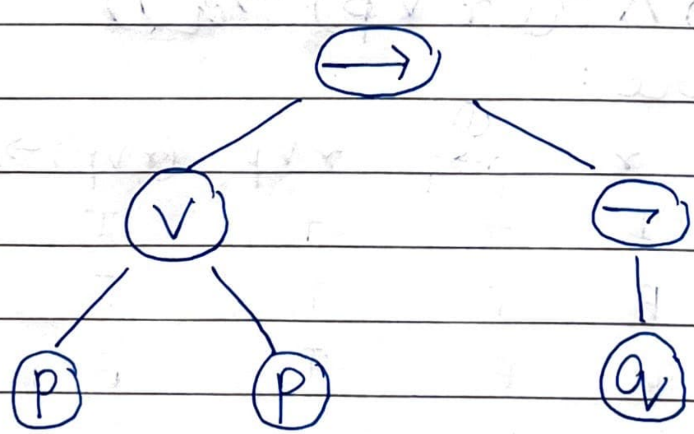
5. $(p \vee q) \rightarrow (\neg p \wedge r)$

Yes, it is a well-formed formula. (WFF)



6. $((p \vee p) \rightarrow \neg q)$

Yes, it is a well-formed formula (WFF)



Exercise 3 :-

1. $((p \wedge (p \to q)) \to q)$

Truth Table.

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since, $((p \wedge (p \to q)) \to q)$ hold all 'T' true value, hence it is a TAUTOLOGY.

2. $((p \to q) \wedge \neg((r \vee p) \to q))$

Truth Table :

p	q	r	① $p \to q$	$r \vee p$	$(r \vee p) \to q$	② $\neg((r \vee p) \to q)$	① $\wedge$ ②
T	T	T	T	T	T	F	F
T	T	F	T	T	T	F	F
T	F	T	F	T	F	T	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	F	F
F	T	F	T	F	T	F	F
F	F	T	T	T	F	T	T
F	F	F	T	F	T	F	F

$((p \to q) \wedge \neg((r \vee p) \to q))$ is SATISFIABLE since it is true for $p = F$, $q = F$ and $r = T$.

3. $(((p \to q) \land \neg q) \to p)$

Truth Table:

p	q	$p \to q$	$\neg q$	$(p \to q) \land \neg q$	$((p \to q) \land \neg q) \to p$
T	T	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

$(((p \to q) \land \neg q) \to p)$ is SATISFIABLE since it holds True for $(p = T, q = T, r = T)$ and $(p = T, q = F, r = F)$ and $(p = F, q = T, r = T)$.

Exercise 4 :-

ϕ : WFF formula

The definition of count(ϕ, pr) using induction is :-

Base Cases :

① if $\phi = pr$, then count (ϕ, pr) = 1.
Therefore, if the formula ϕ is exactly equal to the proposition 'pr', then count = 1

② if $\phi \neq pr$, then count (ϕ, pr) = 0.
Therefore, if the formula ϕ is not equal to the proposition 'pr', then count = 0.

Inductive steps :

if ϕ is a compound formula, such as $\neg\psi$, $\psi_1 \wedge \psi_2$, $\psi_1 \vee \psi_2$, ~~[illegible]~~ & $\psi_1 \rightarrow \psi_2$, then:
count(ϕ, pr) = count (ψ, pr) + count(ψ_1, pr) + count(ψ_2, pr)
where ψ, ψ_1, ψ_2 are sub-formules of ϕ, and so we have to calculate the count of 'pr' in each sub-formula and add them.

count ($\neg\psi$, pr) = count(ψ, pr) ; count($\psi_1 \vee \psi_2$, pr) = count(ψ_1, pr) + count(ψ_2, pr) ; count ($\psi_1 \wedge \psi_2$, pr) = count(ψ_1, pr) + count(ψ_2, pr) ; count ($\psi_1 \rightarrow \psi_2$, pr) = count($\psi_1$, pr) + count($\psi_2$, pr).

$\Rightarrow$ By the principal of Mathimatical Induction, count (ϕ, pr) is well defined for any WFF ϕ and propositional symbol 'pr'.

Exercise 5 :-

1. $(((p \vee q) \wedge (r \vee p)) \wedge (\neg q \vee \neg r \vee p)) \equiv p$

p	q	r	$p \vee q$	$r \vee p$	$(p \vee q) \wedge (r \vee p)$ ①	$\neg q \vee \neg r \vee p$ ②	① $\wedge$ ②
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	F
F	F	T	F	T	F	T	F
F	F	F	F	F	F	T	F

Since $p \equiv (((p \vee q) \wedge (r \vee p)) \wedge (\neg q \vee \neg r \vee p))$, therefore, they are logically equivalent as they hold the same truth value in every possible world.

2. $((\neg q \vee \neg r \vee p) \wedge (p \vee q) \wedge (r \vee p)) \equiv (p \vee q)$

p	q	r	$(\neg q \vee \neg r \vee p)$	$(p \vee q)$	$(r \vee p)$	$(\neg q \vee \neg r \vee p) \wedge (p \vee q) \wedge (r \vee p)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	F
F	F	T	T	F	T	F
F	F	F	T	F	F	F

Since, $(p \vee q) \not\equiv ((p \vee \neg q \vee \neg r) \wedge (p \vee q) \wedge (r \vee p))$
is not logically equivalent,
Let $\phi = ((\neg q \vee \neg r \vee p) \wedge (p \vee q) \wedge (r \vee p))$
Let $\psi = (p \vee q)$

we know that, ϕ logically entails ψ $(\phi \models \psi)$
if and only if every evaluation that
satisfies ϕ also satisfy ψ.

$((\neg q \vee \neg r \vee p) \wedge (p \vee q) \wedge (r \vee p))$		$(p \vee q)$
T	$\longleftrightarrow$	T
T	$\longleftrightarrow$	T
T	$\longleftrightarrow$	T
T	$\longleftrightarrow$	T
F	× $\longleftarrow$	T
F	× $\longleftarrow$	T
F		F
F		F

Thus, $\phi \models \psi$ but $\psi \not\models \phi$.

Exercise 6 :-

$((\neg p \to \neg q) \vee ((r \to \neg q) \wedge s))$

① Eliminating implication using $(a \to b) = (\neg a \vee b)$

$= ((\neg(\neg p) \vee q) \vee ((\neg r \vee \neg q) \wedge s)$

Using Double Negation law, $\neg(\neg a) = a$

$= ((p \vee q) \vee ((\neg r \vee \neg q) \wedge s))$

~~Using De Morgan's law,~~

$\cancel{= ((p \vee q) \vee (\neg(\neg r \vee \neg q) \vee \neg s)}$

Using Distributivity Law, $(p \vee (q \wedge r)) \equiv (p \vee q) \wedge (p \vee r)$

$= ((p \vee q) \vee ((\neg r \wedge s) \vee (\neg q \wedge s)))$

Using Associativity law, $((p \vee q) \vee r) = (p \vee (q \vee r))$

$= (\neg r \wedge s) \vee (p \vee q) \vee (\neg q \wedge s)$

$= (\neg r \wedge s) \vee (p \vee q \vee \neg q \wedge s)$

$= (\neg r \wedge s) \vee (p \vee (q \vee \neg q \wedge s))$

$= (\neg r \wedge s) \vee (p \vee q \vee s)$

$= p \vee q \vee (s \vee s \wedge \neg r)$

Using Absorption law, $(p \vee (q \wedge p)) \equiv p$

$= p \vee q \vee s.$

The final formula : $p \vee q \vee s$.

Note : The equivalence could be checked by a truth Table which proves that these are logically equivalent.

(2) $\neg\phi \equiv (\neg\phi \vee \bot) \equiv (\phi \to \bot)$

Using Implicate rule : $(p \to q) = (\neg p \vee q)$

$\equiv \neg(\neg p \to q) \to ((r \to \neg q) \wedge s))$

Using the given formula,

$\equiv (\neg q \to p) \to ((r \to \neg q) \wedge s)$

$\equiv ((q \to \bot) \to p) \to ((r \to (q \to \bot)) \wedge s)$

Using $\neg(\neg a \vee \neg b) = a \wedge b$.

$\equiv ((q \to \bot) \to p) \to \neg(\neg(r \to (q \to \bot)) \vee \neg s)$

$\equiv ((q \to \bot) \to p) \to \neg((r \to (q \to \bot)) \to \neg s)$

$\equiv ((q \to \bot) \to p) \to \neg(r \to (q \to \bot)) \to (s \to \bot))$

$\equiv ((q \to \bot) \to p) \to (((r \to (q \to \bot)) \to (s \to \bot))) \to \bot$

The final formula with only $\to, \bot$

$\equiv ((q \to \bot) \to p) \to (((r \to (q \to \bot)) \to (s \to \bot)) \to \bot$

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Exercise 7 :-

To prove: $(p \leftrightarrow q) \equiv (\neg p \leftrightarrow \neg q)$

① Truth Table:

p	q	$p \leftrightarrow q$	$\neg p \leftrightarrow \neg q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

Since $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ in all values of truth Table, they are logically equivalent

(2) Substitution:

LHS:- $(p \leftrightarrow q)$

Using double implication formula: $(p \rightarrow q) \wedge (q \rightarrow p)$

Using implication formula: $(\neg p \vee q) \wedge (\neg q \vee p)$

Using commutative law: $(\neg q \vee p) \wedge (\neg p \vee q)$

Using Commutative law: $(p \vee \neg q) \wedge (q \vee \neg p)$

Using Double negation law: $(\neg(\neg p) \vee \neg q) \wedge (\neg(\neg q) \vee \neg p)$

Using implication law: $(\neg p \rightarrow \neg q) \wedge (\neg q \rightarrow \neg p)$

Using double Implication formula: $(\neg p \leftrightarrow \neg q)$

Therefore, $(p \leftrightarrow q) \equiv (\neg p \leftrightarrow \neg q)$

Exercise 8 :-

if $(\phi \rightarrow \psi)$ is a contradiction, then it is false for all truth values of ϕ and ψ. For this, ϕ is a tautology, meaning having all true possible values and ψ is a contradiction, meaning having all false possible values. This is necessary because only $T \rightarrow F$ gives a F and it is given that the true Table for $\phi \rightarrow \psi$ is a contradiction (holds only F), rest any combination (like $T \rightarrow T$, $F \rightarrow F$, $F \rightarrow T$) gives a T value.

Truth Table :-

ϕ	ψ	$\phi \to \psi$
T	F	F
T	F	F
T	F	F
T	F	F

Contradiction / unsatisfiable

Tautology

Contradiction.

Exercise 9 :-

p	q	r	ϕ_1	$(\neg p \wedge r)$	$\phi_2 = ((\neg p \wedge r) \vee \neg q)$	$\phi_1 \to \phi_2$	$\phi_2 \to \phi_1$
T	T	T	F	F	F	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	T	T	F
T	F	F	F	F	T	T	F
F	T	T	T	T	T	T	T
F	T	F	F	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	F	F	T	T	F

$\phi_1 \not\equiv \phi_2$ since the truth table of the two do not hold all the same values

$\phi_1 \nvDash \phi_2$ since every evaluation that satisfies ϕ_1 does not satisfy ϕ_2. Therefore, $\phi_1 \to \phi_2$ would not give a tautology.

$\phi_2 \nvDash \phi_1$, since every evaluation that satisfies ϕ_2 does not satisfy ϕ_1. Therefore, $\phi_2 \to \phi_1$ would not give a tautology.

Therefore, none of the given three conditions hold, i.e., $\phi_1 \not\equiv \phi_2$, $\phi_1 \nvDash \phi_2$, $\phi_2 \nvDash \phi_1$