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Batch - 3CS-6
Subject - Predictive analytics using statistics (UCS654)

1) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-u)^2}{2\sigma^2}}$

Here, $x_1, x_2, x_3, \ldots x_n$ sample of size 'n'

$L(x_1, x_2, \ldots x_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$

$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_1-u)^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-u)^2}{2\sigma^2}} \right) \ldots$

Taking ln on both sides

$\ln(L) = \frac{-n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left| \frac{(x_i-u)^2}{2\sigma^2} \right|$ — ①

$\frac{\partial \ln(L)}{\partial u} = 0 + \sum_{i=1}^{n} - \left(\frac{2(x_i-u)^2}{2\sigma^2} \right) = 0$

$\sum_{i=1}^{n} (x_i - u) = 0$

$n\bar{x} - nu = 0$

$\bar{x} = u$

$Q_1 = \bar{x}$ is therefore sample mean

$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} - \frac{(x_i-u)^2}{2\sigma^2} = 0$

$$n = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2} = 0$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

(2.) Binomial Distribution $\; {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

$$\log L = \sum_{i=1}^{n} \left(\log({}^{n}C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

$$\log L = \sum_{i=1}^{n} \log({}^{n}C_{x_i}) + \log \theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n - x_i)$$

(i) $\dfrac{d \log(L)}{d\theta} = 0$

$$\frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n - x_i) \geq 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i \geq 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta}$$

$$\theta = \frac{\sum x_i}{n^2}$$