Foundation for your future

But its not always intuitive

But Its not Always Intuitive

A patient has a positive Zika test.

What is the probability they have zika?

- *0.8% of people have zika*
- *Test has 90% positive rate for people with zika*
- *Test has 7% positive rate for people without zika*

The right answer is 9%

Probability = Important + Needs Study

Delayed gratification

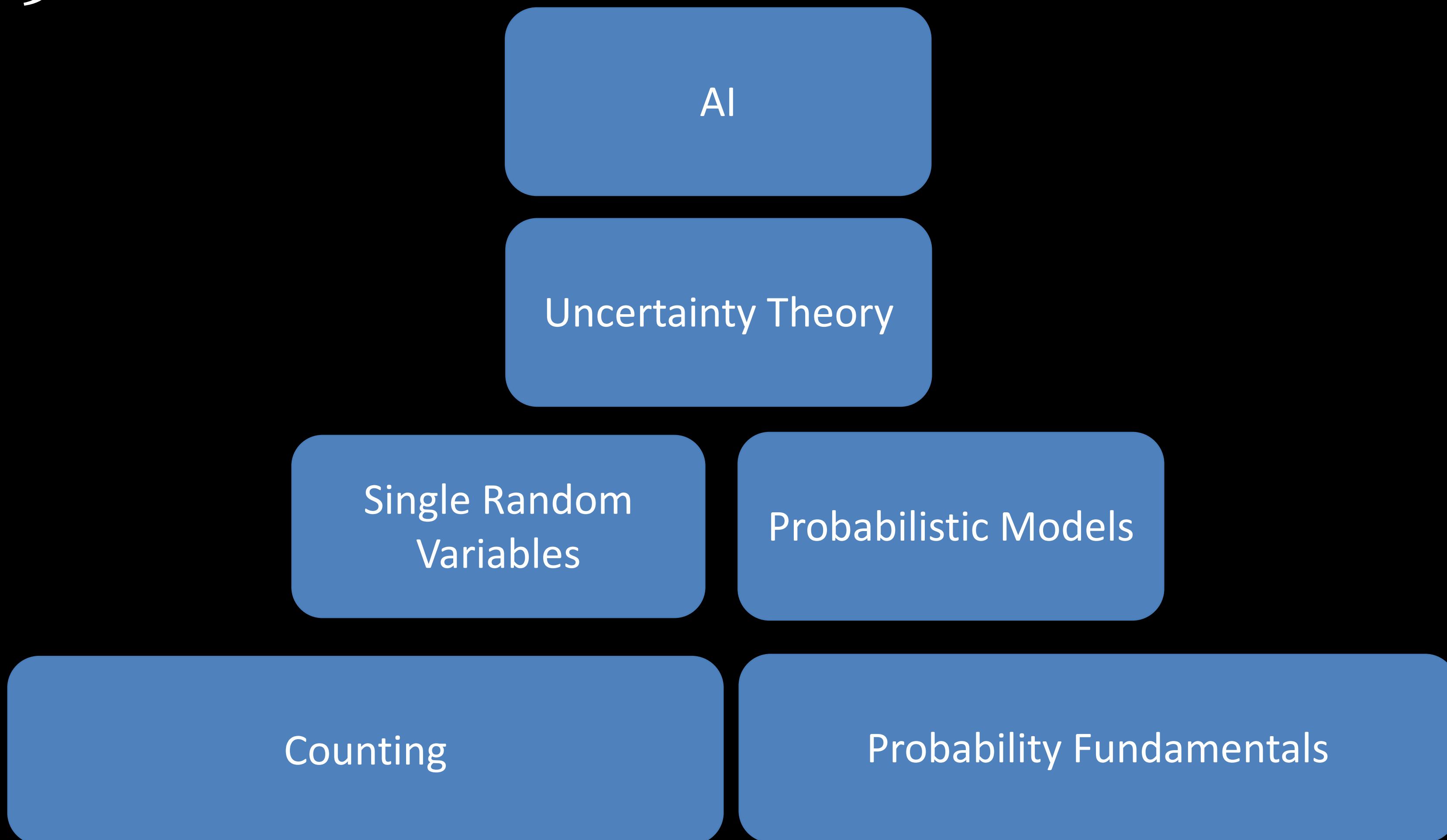
CS109 View of Probability

Teach you how to write programs
that most people are not able to write.

CS109 View of Probability

Teach you the theory you need to do the math that most people are not able to do.

CS109


AI
Uncertainty Theory
Single Random Variables
Probabilistic Models
Counting
Probability Fundamentals

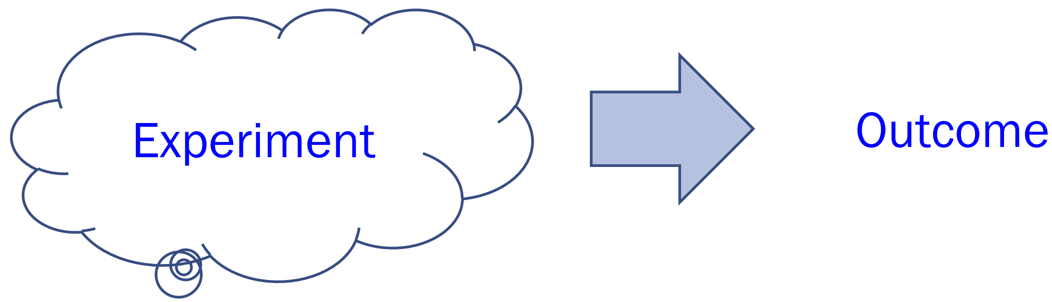
Lets dive in…

2 min pedagogic pause.

Counting I

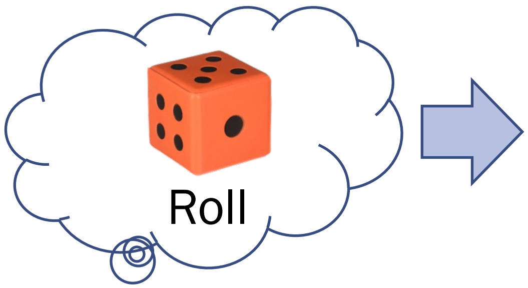
What is Counting?

An experiment in probability:



Counting: How many possible outcomes satisfy some event?

What is Counting?

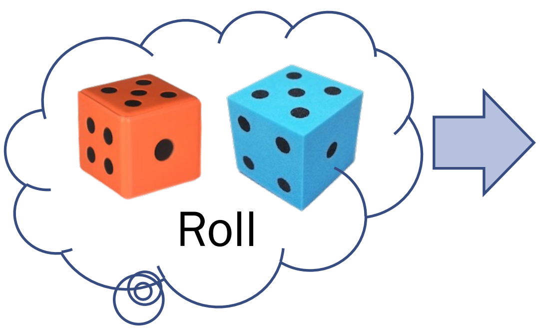


6

{1, 2, 3,
4, 5, 6}

3

{2, 4, 6}



36

{(1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two steps, where

The first step's outcomes are from Set A, where $|A| = m$,
and the second step's outcomes are from Set B, where $|B| = n$,

and |B| is unaffected by outcome of first step.

Then the number of outcomes of the experiment is

$$|A||B| = mn.$$

Two-step experiment

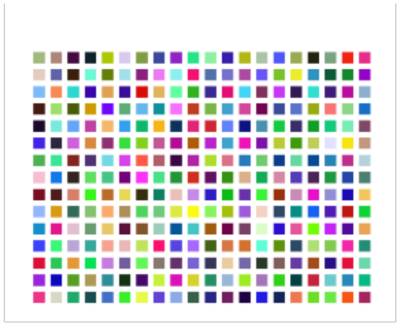
$\rightarrow A \rightarrow B$

[On board]

How Many Unique Images?

Each pixel can be one of 17 million distinct colors

(a) 12 million pixels



(b) 300 pixels

(c) 12 pixels

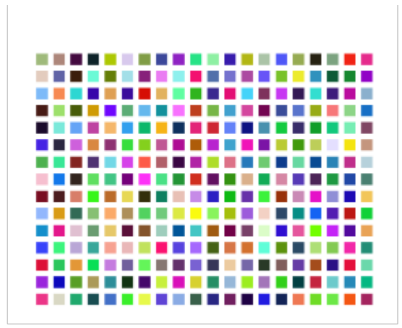
$$(17 \text{ million})^n$$

How Many Unique Images?

Each pixel can be one of 17 million distinct colors

(a) 12 million pixels

$$\approx 10^{86696638}$$



(b) 300 pixels

$$\approx 10^{2167}$$

(c) 12 pixels

$$\approx 10^{86}$$

$(17 \text{ million})^{n}$

Sum Rule of Counting

If the outcome of an experiment can be either from

Set A, where $|A| = m$,

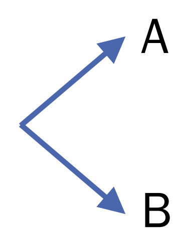
or Set B, where $|B| = n$,

where $A \cap B = \emptyset$,

Then the number of outcomes of the experiment is

$|A| + |B| = m + n$.

One experiment



How many toys?

Question: All of Freya's toys are either Balls **OR** Plush Animals. She has 2 Balls and 3 Plush Animals. How many toys does she have?

Answer: 20 + 10


OFFICIAL
3
SIZE

How Many Bit Stings?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Answer

2^4 start with 01	2^4 end with 10
010000	000010
010001	000110
010010	001010
010011	001110
010100	010010
010101	010110
010110	011010
010111	011110
011000	100010
011001	100110
011010	101010
011011	101110
011100	110010
011101	110110
011110	111010
011111	111110
Set A	Set B

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010100	**010010**
010101	**010110**
010110	**011010**
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Answer

$$N = |A| + |B| - |A \text{ and } B|$$
$$= 16 + 16 - 4$$
$$= 28$$

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010001	000110
010010	001010
010011	001110
010100	**010010**
010101	**010110**
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011100	110010
011101	110110
011110	111010
011111	111110
Set A	Set B

Or Rule of Counting (aka Inclusion/ Exclusion)

If the outcome of an experiment can be either from

Set A, where $|A| = m$,

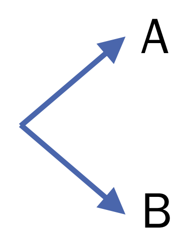
<u>or</u> Set B, where $|B| = n$,

where $A \cap B$ $\ may\ not\ be\ empty$,

Then the number of outcomes of the experiment is

N = $|A| + |B| \ - |A \cap B|$.

One experiment



Core Counting

Counting with steps

Definition: Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of m outcomes and the second part can result in one of n outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is $m \cdot n$.

Counting with "or"

Definition: Inclusion Exclusion Counting

If the outcome of an experiment can either be drawn from set A or set B, and sets A and B may potentially overlap (i.e., it is not the case that A and B are mutually exclusive), then the number of outcomes of the experiment is $|A \text{ or } B| = |A| + |B| - |A \text{ and } B|$.

Challenge Problem

1. Strings
   - How many *different* orderings of letters are possible for the string BOBA?

BOBA, ABOB, OBBA…

🤔

Incredible time and school at which to study probability! Exciting.