PROJECT 2: BUILD A GAME PLAYING AGENT

GENERAL FORM OF HEURISTIC FUNCTIONS

The heuristic functions used in this project all take the form of a weighted linear combination of features extracted from a game board. This is shown in the formula below where $H(g)$ denotes a heuristic function for the game board g, and the weights w_i determine how to combine the game board features f_i.

$$H(g) = \sum_{i=1}^{L} w_i * f_i(g)$$

Note that the number and complexity of the game board features f_i should be limited because a heuristic function that takes too long to evaluate will limit the depth reached by our chosen adversarial search method.

BASELINE AGENT PERFORMANCE

The mean Win Ratio of the “ID Agent” for seven tournaments is 66 +/- 2.9%. Consequently, a heuristic function that allows an agent to achieve a Win Ratio greater than 69% will be considered an improvement over the baseline.

HEURISTIC 1: WEIGHTED MOVE DIFFERENCE (WMD)

The first heuristic function I evaluated is like the *improved_score* heuristic with the exception that the weights w_1 and w_2 are applied to the number of moves available to the agent and the opponent respectively. I will refer to this heuristic as the Weighted Move Difference (WMD) heuristic, and its equation is below.

$$H_{WMD}(g) = w_1 * f_1(g) + w_2 * f_2(g)$$

$$H_{WMD}(g) = w_1 * NumMyMoves + w_2 NumOpponentMoves$$

To determine the weights w_1 and w_2, I evaluated the win ratio of an agent that uses the H_{WMD} heuristic for w_1 and w_2 ranging from 1 to 3 and -1 to -3 respectively for $|w_1| \neq |w_2|$. The results in the table below show that for the selected range of weights the choice $w_1 = 2$ and $w_2 = -1$ yield the highest win ratio of 71.43%.

w_1	w_2	Win Ratio (Percent)
1	-2	68.57
1	-3	67.86
2	**-1**	**71.43**
2	-3	67.86
3	-1	67.14
3	-2	68.57

HEURISTIC 2: CORNERING (C) + WEIGHTED MOVE DIFFERENCE (WMD)

The second heuristic function combines the Weighted Move Difference heuristic described above ($H_{WMD}(g)$) with a game board feature I will refer to as *Cornering*. The Cornering (C) feature measures how much closer an opponent is to a corner of the game board relative to the agent. The idea being that game states where the agent has pushed the opponent to a corner of the board are more likely to result in the agent winning.

To compute the Cornering (C) feature, we subtract the smallest distance between the opponent's current position (P_{opp}) and the four corners of the game board (c_k for $k = 1 - 4$) from the smallest distance between the agent's current position (P_{agent}) and the four corners of the game board. When this difference is positive, it implies that the opponent is closer to a game board corner than the agent. Expressed mathematically we have

$$C = \min_{c_k} Distance(P_{agent}, c_k) - \min_{c_k} Distance(P_{opp}, c_k)$$

Now we can write our second heuristic as a combination of the WMD heuristic and the Cornering feature as shown in the equation below. ***Note that the heuristic $H_{WMD}(g)$ in this case will use the optimal choice of weights $w_1 = 2$ and $w_2 = -1$ discovered in the previous section.***

$$H_{WMD+C}(g) = w_3 H_{WMD}(g) + w_4 C$$

To determine the weights w_3 and w_4, I evaluated the win ratio of an agent that uses the H_{WMD+C} heuristic for w_3 and w_4 both ranging from 1 to 3 for $|w_3| \neq |w_4|$. The results in the table below show that for the selected range of weights the choice $w_3 = 3$ and $w_4 = 2$ yield the highest win ratio of 75%.

w_3	w_4	Win Ratio (Percent)
1	2	73.57
1	3	68.57
2	1	72.86
2	3	66.43
3	1	60.71
3	**2**	**75**

HEURISTIC 3: SPACE ACCESS (AS) + CORNERING (C) + WEIGHTED MOVE DIFFERENCE (WMD)

The third heuristic function combines the Weighted Move Difference + Cornering heuristic described above ($H_{WMD+C}(g)$) with a game board feature I will refer to as *Space Access* (AS). The Space Access feature measures the difference between the average distance of the opponent to blank space and the average distance of the agent to blank space. The motivation being that if this difference is positive then the agent is closer to blank space and thus is less likely to get cornered and lose the game.

To compute the Space Access (AS) feature, we subtract the average distance between the agent's current position (P_{agent}) and the game board's blank spaces from the average distance between the opponent's current position

(P_{opp}) and the game board's blank spaces. If we let $b_k$for $k = 1 - N$ denote the N blank spaces on a given game board then we can express the Space Access feature as follows.

$$AS = \frac{1}{N}\left(\sum_{k=1}^{N} Distance\left(P_{opp}, b_k\right) - \sum_{k=1}^{N} Distance\left(P_{agent}, b_k\right)\right)$$

Now we can write our third heuristic as a combination of the WMD+C heuristic and the Space Access feature as shown in the equation below. ***Note that the heuristic $H_{WMD+C}(g)$ in this case will use the optimal choice of weights $w_1 = 2$, $w_2 =$ -1, $w_3 = 3$, $w_4 = 2$ discovered in the previous section.***

$$H_{WMD+C+AS}(g) = w_5 H_{WMD+C}(g) + w_6 AS$$

To determine the weights $w_5$and w_6, I evaluated the win ratio of an agent that uses the $H_{WMD+C+AS}$ heuristic for $w_5$and $w_6$both ranging from 1 to 3 for $|w_3| \neq |w_4|$. The results in the table below show that for the selected range of weights the choice $w_5 = 3$ and $w_6 = 1$ yield the highest win ratio (75%).

w_5	w_6	Win Ratio (Percent)
1	2	72.86
1	3	70.71
2	1	69.29
2	3	71.43
3	**1**	**75**
3	2	72.86

HEURISITC 4: OPPONENT PURSUIT (OP) + SPACE ACCESS (AS) + CORNERING (C) + WEIGHTED MOVE DIFFERENCE (WMD)

The fourth heuristic function combines the Weighted Move Difference + Cornering + Space Access heuristic described above ($H_{WMD+C+AS}(g)$) with a game board feature I will refer to as *Opponent Pursuit* (OP). The Opponent Pursuit feature measures the distance between the agent and the opponent. The idea being that if the agent pursues the opponent and minimizes the distance between them, the agent might pin the opponent and win the game. With the opponent and agent positions ($P_{opp}$and P_{agent}), the Opponent Pursuit feature is $OP = Dist(P_{opp} - P_{agent})$.

Now we can write our fourth heuristic as a combination of the WMD+C+AS heuristic and the Opponent Pursuit feature as shown in the equation below. ***Note that the heuristic $H_{WMD+C+AS}(g)$ in this case will use the optimal choice of weights $w_1 = 2$, $w_2 =$ -1, $w_3 = 3$, $w_4 = 2$, $w_5 = 3$, $w_6 = 1$ discovered in the previous section.***

$$H_{WMD+C+AS+OP}(g) = w_7 H_{WMD+C+AS}(g) + w_8 OP$$

Once again, to determine the weights $w_7$and w_8, I evaluated the win ratio of an agent that uses the $H_{WMD+C+AS+OP}$ heuristic for $w_7$and $w_8$both ranging from 1 to 3 for $|w_7| \neq |w_8|$. The results in the table below show that for the selected range of weights the choice $w_6 = 3$ and $w_7 = 2$ yield the highest win ratio (74.29%).

w_7	w_8	Win Ratio (Percent)
1	2	74.29
1	3	68.57
2	1	70.71
2	3	67.14
3	1	71.43
3	**2**	**74.29**

WHICH DO WE CHOOSE?

We should choose the simplest heuristic that maximizes the Win Ratio. Recall that given a fixed amount of time, a cheap heuristic allows our agent to search deeper down the game tree. ***The maximum Win Ratio recorded during my simulations was 75% and the simplest scoring function that produced that success rate was H_{WMD+C} with weights $w_1 = 2$, $w_2 = -1$, $w_3 = 3$, $w_4 = 2$.*** Since this win ratio is significantly greater that 69% (the upper limit of the ID agent Win Ratio), we consider H_{WMD+C} as an advantageous heuristic that will be implemented in the *custom_score* method.