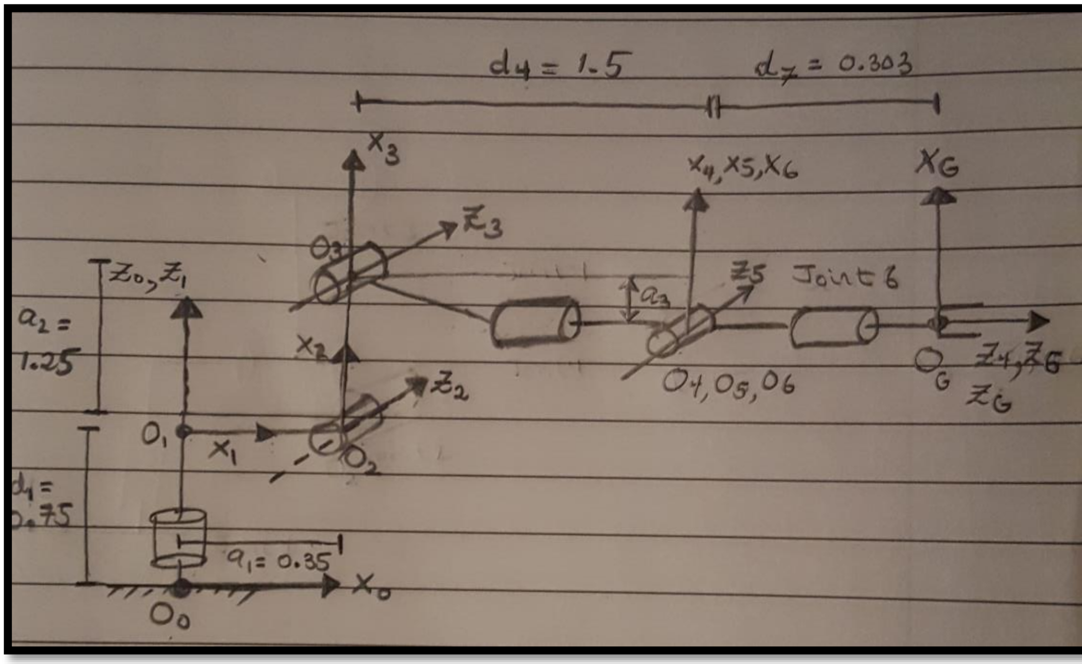
PICK AND PLACE PROJECT

The image below is a sketch of the Kuka KR210 arm. The sketch highlights the 6 revolute joints of the robot as well as the coordinates frames used to define the modified DH parameters. The origins $O_0, O_1, O_2, \dots, O_G$ correspond to the origins of the base frame and the 6 revolute joints; O_G corresponds the origin of the gripper's coordinate frame. Associated with each origin are the axes $X_0, X_1, X_2, \dots, X_G$ and $Z_0, Z_1, Z_2, \dots, Z_G$ which are necessary for defining the DH parameters $\alpha_{i-1}, a_{i-1}$and d_i, θ_i respectively.



The robot's modified DH parameter table is illustrated below. To fill out this table one must consult the robot's URDF file since in many instances the a_{i-1} and d_i parameters are sums of individual values in the URDF file. For example, d_1 is the sum of the z-axis displacements from the base to Joint 1 and from Joint 1 to Joint 2. Since all the robot's joints are revolute the parameters θ_i are the variable joint rotation angles.

Link (i)	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0.75	θ_1
2	$-\pi/2$	0.35	0	$\theta_2 - \pi/2$
3	0	1.25	0	θ_3
4	$-\pi/2$	-0.054	1.5	θ_4
5	$\pi/2$	0	0	θ_5
6	$-\pi/2$	0	0	θ_6
G	0	0	0.303	0

TRANSFORMATION MATRICES

The following matrices represent the transformation matrices linking one joint to the next. These were derived using the robot's modified DH parameter table (previous section) and the general expression for deriving the transformation matrix ${}^{i-1}_{i}T$ given the parameters $\alpha_{i-1}, a_{i-1}, d_i, \theta_i$. This expression is

$$ {}^{i-1}_{i}T = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & a_{i-1} \\ \sin(\theta_i)\cos(\alpha_{i-1}) & \cos(\theta_i)\cos(\alpha_{i-1}) & -\sin(\alpha_{i-1}) & -\sin(\alpha_{i-1})d_i \\ \sin(\theta_i)\sin(\alpha_{i-1}) & \cos(\theta_i)\sin(\alpha_{i-1}) & \cos(\alpha_{i-1}) & -\cos(\alpha_{i-1})d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

The method *get_tranform_matirx* in the script *IK_server.py* is responsible for calculating the individual transform matrices. The final transformation (T_{corr}) corresponds to the rotation required to align the gripper coordinate frame with that defined in the URDF file.

$$ {}^{0}_{1}T = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^{1}_{2}T = \begin{pmatrix} \sin(\theta_2) & cos(\theta_2) & 0 & 0.35 \\ 0 & 0 & 1 & 0 \\ \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}_{3}T = \begin{pmatrix} \cos(\theta_3) & -sin(\theta_3) & 0 & 1.25 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^{3}_{4}T = \begin{pmatrix} \cos(\theta_4) & -sin(\theta_4) & 0 & -0.054 \\ 0 & 0 & 1 & 1.50 \\ -\sin(\theta_4) & -\cos(\theta_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{4}_{5}T = \begin{pmatrix} \cos(\theta_5) & -sin(\theta_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^{5}_{6}T = \begin{pmatrix} \cos(\theta_6) & -sin(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_6) & -\cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{6}_{G}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.303 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T_{corr} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

The total transformation matrix from the base to gripper expressed in the gripper's world position (x, y, z coordinates) and pose (roll, pitch, yaw angles) is shown below

$$ \begin{pmatrix} \cos(pitch)\cos(yaw) & \sin(pitch)\sin(roll)\cos(yaw) - \sin(yaw)\cos(roll) & \sin(pitch)\cos(roll)\cos(yaw) + \sin(roll)\sin(yaw) & p_x \\ \sin(\mathrm{yaw})\cos(\mathrm{pitch}) & \sin(\mathrm{pitch})\sin(\mathrm{roll})\sin(\mathrm{yaw}) + \cos(\mathrm{roll})\cos(\mathrm{yaw}) & \sin(pitch)\sin(yaw)\cos(roll) - sin(roll)\cos(yaw) & p_y \\ -\sin(\mathrm{pitch}) & \sin(\mathrm{roll})\cos(\mathrm{ptich}) & \cos(pitch)\cos(roll) & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

INVERSE KINEMATICS

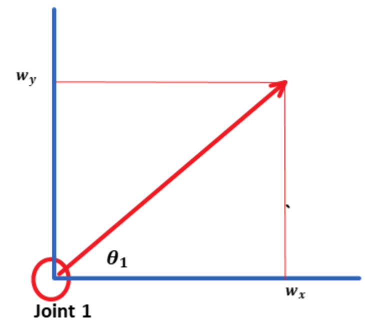
Inverse kinematics involves determining the six joint angles $(\theta_1, \theta_2, \cdots, \theta_6)$ given the gripper's world position (x, y, z coordinates) and pose (roll, pitch, yaw angles). Since our robot has a spherical wrist with joint axes that intersect at a single point, we can independently solve for the wrist center position (determined by $\theta_1, \theta_2, \theta_3$) and pose (determined by $\theta_4, \theta_5, \theta_6$).

Determining Wrist-Center Coordinates

The wrist-center (WC) is taken to be at Joint 5. Based on the robot's geometry, we know the gripper's coordinate frame is translated 0.303 meters along the x-axis relative to joint 5. Given the grippers position (p_x, p_y, p_z) and pose $({}^0_6R)$ in the world frame, we can determine the wrist center's world-coordinates (w_x, w_y, w_z) using the equation below.

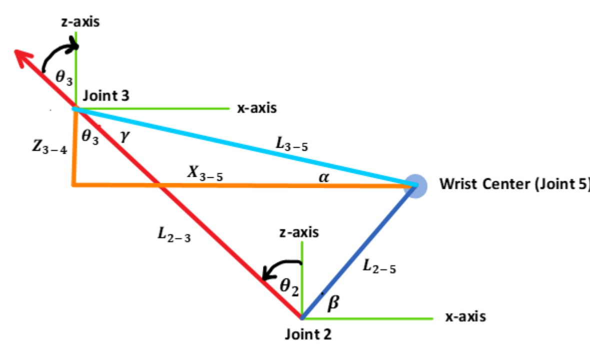
$$\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - 0.303 \cdot {}^0_6R \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Determining $(\boldsymbol{\theta_1, \theta_2, \theta_3})$



To determine the angle of Joint 1 (θ_1), we take advantage of the fact that a top-down view of the robot's arm reduces the problem to geometry shown in the figure to the right. This allows to easily solve for the angle θ_1 using the following equation

$$\theta_1 = atan2(w_y, w_x)$$



To determine the angles θ_2 and θ_3, we use the geometry illustrated in figure to the right. Here we are looking at the coordinate frames of Joints 2 and 3 in the Z-X plane as well as the wrist-center (Joint 5).

To compute θ_3 we make use of the cosine law for the triangle formed by the sides L_{2-3} (distance from Joint 2 to 3), L_{2-5} (distance Joint 2 to 5), L_{3-5} (distance form Joint 3 to 5). Writing the cosine law for the angle γ we get:

$$\gamma = \ cos^{-1}(\frac{L_{2-3}^2 + L_{3-5}^2 - L_{2-5}^2}{2L_{2-3}L_{3-5}})$$

Now we use the triangle formed by the sides Z_{3-4} (displacement form Joint 3 to 4 in Z-axis of Joint 3), X_{3-5} (displacement from Joint 3 to 5 in X-axis of Joint 3), and L_{3-5} (distance form Joint 3 to 5). In this triangle, the following relations hold:

$$\alpha = \tan^{-1}\left(\frac{Z_{3-4}}{X_{3-5}}\right)$$

$$\theta_3 = 90 - \gamma - \alpha$$

To compute θ_2 we make use of the cosine law for the triangle formed by the sides L_{2-3} (distance from Joint 2 to 3), L_{2-5} (distance Joint 2 to 5), L_{3-5} (distance form Joint 3 to 5). Writing the cosine law for the angle subsuming θ_2 we get:

$$\theta_2 + (90 - \beta) = \ cos^{-1}(\frac{L_{2-3}^2 + L_{2-5}^2 - L_{3-5}^2}{2L_{2-3}L_{2-5}})$$

The quantities L_{2-3} (distance from Joint 2 to 3), Z_{3-4} (displacement form Joint 3 to 4 in Z-axis of Joint 3), and X_{3-5} (displacement from Joint 3 to 5 in X-axis of Joint 3) can be directly inferred form the robot's URDF file. On the other hand, the link lengths L_{2-5} (distance from Joint 2 to 5), L_{3-5} (distance form Joint 3 to 5) and the angle β must be computed. To facilitate these computations, determine the wrist-center coordinates in the Joint 2 coordinate frame as follows:

$$\begin{bmatrix} {}^2_x w \\ {}^2_y w \\ {}^2_z w \end{bmatrix} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} - \begin{bmatrix} X_{1-2}\cos(\theta_1) \\ X_{1-2}\sin(\theta_1) \\ Z_{0-2} \end{bmatrix}$$

Now use the wrist-center coordinates in the Joint 2 coordinate frame to compute the remaining quantities:

$$L_{3-5} = \sqrt{Z_{3-4}^2 + X_{3-5}^2}$$

$$L_{2-5} = \sqrt{{}^2_x w^2 + {}^2_y w^2 + {}^2_z w^2}$$

$$\beta = tan^{-1}\left(\frac{{}^2_z w}{\sqrt{{}^2_x w^2 + {}^2_y w^2}}\right)$$

<u>Determining ($\theta_4, \theta_5, \theta_6$)</u>

To determine the angles $\theta_4, \theta_5, \theta_6$ we equate the analytical expression and numerical value of the rotation matrix ${}^3_6R$ and then solve for the desired angles. The analytical form of this rotation matrix is derived by multiplying the homogenous transforms linking the coordinate frames of joints 3 through 6 and then extracting the rotational component. This leads to the following analytical expression:

$${}^3_6R = \begin{pmatrix} -\sin(\theta_5)\cos(\theta_4) & \sin(\theta_4)\cos(\theta_6) + \sin(\theta_6)\cos(\theta_4)\cos(\theta_5) & -\sin(\theta_4)\sin(\theta_6) + \cos(\theta_4)\cos(\theta_5)\cos(\theta_6) \\ \cos(\theta_5) & \sin(\theta_5)\sin(\theta_6) & \sin(\theta_5)\cos(\theta_6) \\ \sin(\theta_4)\sin(\theta_5) & -\sin(\theta_4)\sin(\theta_6)\cos(\theta_5) + \cos(\theta_4)\cos(\theta_6) & -\sin(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_6)\cos(\theta_4) \end{pmatrix}$$

The numerical value for the rotation matrix ${}^3_6R$ can be obtained using the desired gripper pose (${}^0_6R$) and the rotation matrix linking the coordinate frames of the base and joint 3 (${}^0_3R$). Specifically, this accomplished using the following equation

$${}^3_6R = {}^0_3R^T\,{}^0_6R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Now the angles $\theta_4, \theta_5, \theta_6$ can be determined as follows

$$\theta_4 = tan^{-1}\left(\frac{r_{31}}{-r_{11}}\right)$$

$$\theta_5 = tan^{-1}\left(\frac{\sqrt{r_{31}^2 + r_{11}^2}}{r_{21}}\right)$$

$$\theta_6 = tan^{-1}\left(\frac{r_{22}}{r_{23}}\right)$$

The value of ${}^0_6R$ is determined by compounding elementary rotation matrices about the z-axis, y-axis, and x-axis given the desired gripper yaw, pitch, and roll angles as follows:

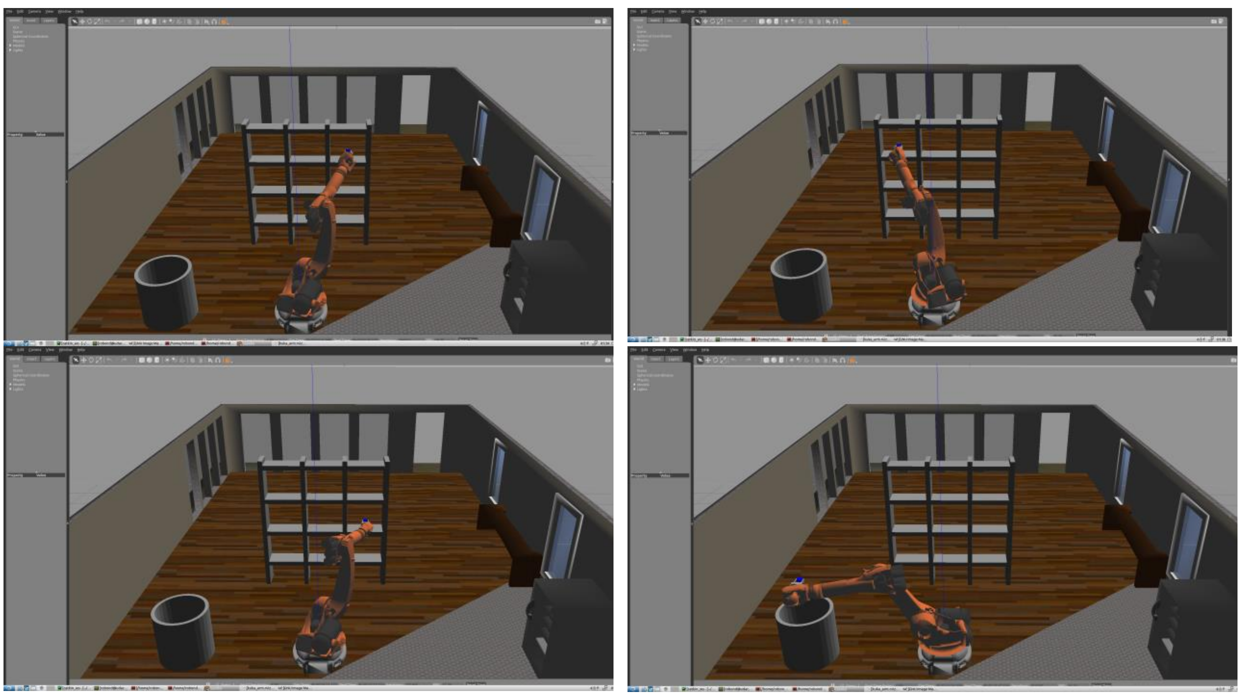
$$ {}^0_6R = R_z(yaw)R_y(pitch)R_x(roll) $$

The value of ${}^0_3R$ is determined by evaluating the analytical expression for this matrix using the values of $\theta_1, \theta_2, \theta_3$ that were determined in the previous section. The analytical is expression for ${}^0_3R$ is derived by multiplying the homogenous transforms linking the coordinate frames of the base through joint 3 and then extracting the rotational component. This leads to the following analytical expression:

$$ {}^0_3R = \begin{pmatrix} \sin(\theta_2+\theta_3)\cos(\theta_1) & \cos(\theta_1)\cos(\theta_2+\theta_3) & -\sin(\theta_1) \\ \sin(\theta_2+\theta_3)\sin(\theta_1) & \sin(\theta_1)\cos(\theta_2+\theta_3) & \cos(\theta_1) \\ \cos(\theta_2+\theta_3) & -\sin(\theta_2+\theta_3) & 0 \end{pmatrix} $$

RESULTS

The arm can successfully reach and grab items from the shelf as well as drop those items in the bin. This is illustrated in the screen shots below.



On occasion, the arm will fail to grasp an item from the shelf even though the gripper will have assumed the correct position and orientation. Lastly, the robot arm will often perform a needless 360-degree rotation of join 4 as it heads towards a desired position. This can be explained by the fact that the code is not checking for the equivalence of the two solutions θ and $\theta + 2\pi$.