Intro Geometry *2025*

UNSW Competitive Programming and Mathematics Society

Questions

1. Convince yourself that all of the theorems are true in all different cases we didn't cover (or alternatively prove that they are true).
2. Let ABC be a triangle where D is the midpoint of segment BC. Show that $\angle BAC = 90°$ if and only if $AD = \frac{1}{2}BC$.
3. (Radical Axis) Let two circles, ω_1 and ω_2, intersects at A and B. Prove that AB is perpendicular to the line connecting the centre of the two circle.
4. (Miquel's Theorem): Let ABC be a triangle with points D, E, and F on sides BC, AC, and AB respectively. Show that the circles (AEF), (BDF), and (CDE) all intersect at a point.
5. Let ω_a be a circle tangent to the line L at the point A, and ω_b be a second circle tangent to the same line L at the point B. Furthermore, C_a and C_b lie on the same side of L and are tangent to each other. They touch on the point P. Show that $\angle BPA = 90°$.
6. Let Γ_1 and Γ_2 be two circles that intersect at points X and Y. A, B lie on Γ_1 and P, Q lie on Γ_2 such that AP and BQ intersect at X. Show that triangles YAB and YPQ are similar.
7. (Simson's Line): Let A, B, C, D be points on a circle. Let the feet of the perpendiculars from D to BC, AC, AB be X, Y, Z respectively. Prove that X, Y, Z are colinear.
8. Let ABC be a triangle with incentre I and A-excentre I_A. Prove that the points I, I_A, B, C lie on a circle with centre on (ABC).
9. (Miquel point): Let $ABCD$ be a convex quadrilateral, with $AB \cap CD = E$, $AD \cap BC = F$. Prove that:
   (a) The 4 circles (BCE), (ADE), (ABF), (CDF) all intersect at a point
   (b) If $ABCD$ is cyclic, then the intersection point lies on line EF.
10. (INAMO 2023) Let ABC be an acute triangle where BC is its longest side. Points D, E respectively lie on AC, AB such that $BA = BD$ and $CA = CE$. The point A' is the reflection of A against line BC. Prove that the circumcircles of ABC and $A'DE$ have the same radius.
11. (Anti-Steiner) Let triangle ABC have an orthocentre H, and ℓ be a line passing through H. Denote ℓ_A, ℓ_B, ℓ_C be the reflections of ℓ over BC, AC, and AB respectively. Prove that the 3 lines ℓ_A, ℓ_B, ℓ_C concur on (ABC).