

GATE Question Paper 2018, CS Question Number 49

Q.49 Consider the minterm list form of a Boolean function F given below.

$$F(P, Q, R, S) = \sum m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$$

Here, m denotes a minterm and d denotes a don't care term. The number of essential prime implicants of the function F is _____.

ANSWER

1. Create the 4 variable K-map:

- We have a function $F(P, Q, R, S)$ with 4 variables, so we'll need a 4-variable K-map.

RS	00	01	11	10
PQ 00	m_0	m_1	m_3	m_2
PQ 01	m_4	m_5	m_7	m_6
PQ 11	m_{12}	m_{13}	m_{15}	m_{14}
PQ 10	m_8	m_9	m_{11}	m_{10}

2. Fill in the minterms (m) and don't cares (d):

- Minterms (m): 0, 2, 5, 7, 9, 11
- Don't cares (d): 3, 8, 10, 12, 14

RS	00	01	11	10
PQ 00	1	0	1	1
PQ 01	0	1	1	0
PQ 11	d	0	0	d
PQ 10	d	1	1	d

Table 1: K-map for (p, q, r, s)

	$00(r' \cdot s')$	$01(r' \cdot s)$	$11(r \cdot s)$	$10(r \cdot s')$
$00(p' \cdot q')$	1		X	1
$01(p' \cdot q)$		1	1	
$11(p \cdot q)$	X			X
$10(p \cdot q')$	X	1	1	X

3. Identify the prime implicants:

- **Prime implicant:** A group of 1's (or 1's and don't cares) that cannot be further combined into a larger group.
- Let's look for the largest possible groups:
- **Group 1:** The 1's in cells 8, 9, 10 and 11 can be combined: PQ'
- **Group 2:** The 1's in cells 0, 2, 8 and 10 can be combined: PS'
- **Group 3:** The 1's in cells 2, 3, 10 and 11 can be combined: RQ'

4. Identify the essential prime implicants:

- **Essential prime implicant:** A prime implicant that covers at least one minterm that is not covered by any other prime implicant.
- Let's check each minterm:
- **Minterm 0:** Covered only by $P'Q'S'$ (Essential)
- **Minterm 2:** Covered only by $P'Q'S'$ (Essential)
- **Minterm 9:** Covered only by $PQ'S$ (Essential)
- **Therefore, all for all three implicants are essential.**
- **Answer:** The number of essential prime implicants of the function F is 3.