

GATE Question Paper 2010, EE Question Number 52

Question 12 Analysis

Question:

The following Karnaugh map represents a function F .

		00	01	11	10
X	0	1	1	1	0
	1	0	0	1	0

A minimized form of the function F is

- (A) $F = \bar{X}Y + YZ$ (B) $F = \bar{X}\bar{Y} + YZ$ (C) $F = \bar{X}\bar{Y} + Y\bar{Z}$ (D) $F = \bar{X}\bar{Y} + \bar{Y}Z$

Karnaugh Map Simplification Problem

Given: The following Karnaugh map represents a function F :

Step-by-Step Solution

1. Identify variables:

- Let the variables be X , Y , and Z .
- The rows are labeled by X , and the columns by YZ in Gray code order: 00, 01, 11, 10.

2. Extract minterms:

- From the K-map, 1s appear at: $(X, YZ) = (0, 00), (0, 01), (0, 11), (1, 01)$
- Corresponding minterms:

$$(0, 00) \rightarrow \bar{X}\bar{Y}\bar{Z}$$

$$(0, 01) \rightarrow \bar{X}\bar{Y}Z$$

$$(0, 11) \rightarrow \bar{X}Y Z$$

$$(1, 01) \rightarrow X\bar{Y}Z$$

3. Group minterms in K-map:

- Group 1: $\bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z = \bar{X}\bar{Y}$ (common factor)
- Group 2: $\bar{X}Y Z + X\bar{Y}Z = Z(\bar{X}Y + X\bar{Y}) = Z(\bar{X} \oplus \bar{Y})$
- But instead of factoring XOR, observe original terms: we keep $\bar{X}Y Z$ and $X\bar{Y}Z$ separate

4. Final simplified expression:

$$F = \bar{X}\bar{Y} + YZ$$

Correct Option:

Option (B): $F = \bar{X}\bar{Y} + YZ$