

MODULE 1**BASIC CONCEPTS****NETWORK:**

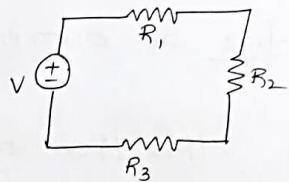
Interconnection of electrical components is called a network.

**CIRCUIT:**

An electrical ckt. is an interconnection of electrical elements linked together in a closed path so that an electric current may continuously flow.

(or)

An electric circuit is a pipeline that facilitates the transfer of charge from one point to another.



- * **NOTE:-** All circuits are networks but all networks are not circuit.

ELECTRIC CURRENT:

charge is the quantity of electricity responsible for electric phenomena.
Charge is denoted by 'q' & is -1.602×10^{-19} coulombs.
 6.24×10^{18} electrons.
 $\therefore -1$ coulomb is charge on

The net displacement of charge carriers through the cross-sectional area

of a conductor, such as copper wire is called electric current & is denoted by 'I' (or) ' i '

(or)
"Current is the rate of flow of charge."

i.e.
$$I = \frac{Q}{t}$$

If the charge flowing in the conductor is varying with time, then

$$\dot{i} = \frac{di}{dt}$$

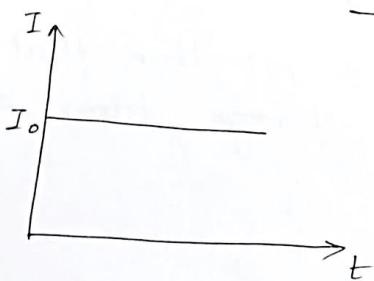


fig1: dc current

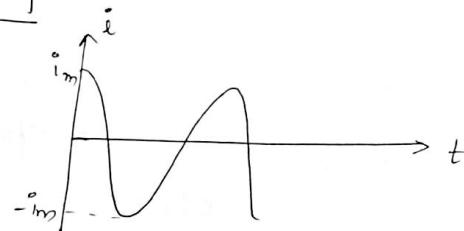


fig2: ac current

When the magnitude & direction of current flowing in an element does not change with respect to time is called direct current [fig. 1]

If the current in an element has a continuously varying magnitude with time & changes in its direction of flow is called alternating current [fig. 2]

VOLTAGE:

The voltage or potential energy difference between two points in an electric circuit is the amount of energy to move a unit charge.

$$V = \frac{W}{Q} = \frac{\text{Work (or) energy}}{\text{charge}}$$

POWER:

Power is the rate of doing work.

$$P = \frac{W}{t}$$

$$\begin{aligned} \text{DC: } P &= VI \\ \text{AC: } P &= VI \cos\phi \\ \cos\phi &\rightarrow \text{power factor} \end{aligned}$$

$$(or) P = \frac{W}{t} = \frac{W}{Q} \cdot \frac{Q}{t} = V \cdot I$$

If the sign of power is positive, power is being absorbed by the element; if the sign is negative, power is being supplied by the element.

CIRCUIT ELEMENTS:

Any individual electrical component like resistor, inductor, capacitor, voltage source, current source with terminals is an electric circuit elements.

BRANCH:

A branch is a single ckt. element (or) ckt. elements connected in series. [AB, BC, CD, AD, CF, EF, DE]

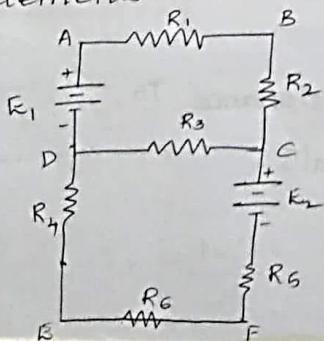


fig 3: An electrical ckt.

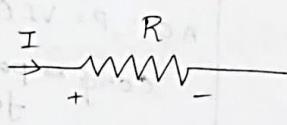
• Node :-

A node is a junction of two (or) more ckt. elements (or) branches [A, B, C, D, E, F]

• Junction point :-

A point where three or more branches meet is called junction point [C & D]

• Resistor :-



→ Resistor opposes the flow of current through it.

→ It is denoted by R & measured in terms of "ohms" (Ω)

→ Resistance of a given material is given by.

$$R = \frac{\rho l}{a}$$

Where,

ρ → resistivity in ohm meter

l → length in metre

a → cross-sectional area in square metre

R → Resistance in Ω

→ Relation b/w. voltage, current & resistance is given by ohm's law.

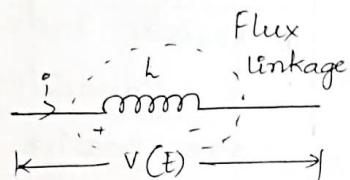
i.e. $V = IR$
or $R = \frac{V}{I}$

→ Power absorbed by a resistance is,

$$P = VI = \frac{V^2}{R} = I^2 R$$

Watts

• Inductance :



→ Energy is stored in the form of electromagnetic field, it is denoted by 'L' & the unit is Henry. Voltage across inductor is

given by,

$$V = L \frac{di}{dt}$$

and,

$$i = \frac{1}{L} \int_{-\infty}^t v dt$$

∴ Power is, $P = vi = L \frac{di}{dt} \cdot i$

→ If inductor has 'N' turns & flux ' ϕ ' is produced which is produced by current $i(t)$, then

$$V(t) = N \frac{d\phi}{dt}$$

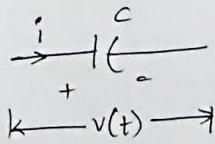
∴ Total flux linkage is proportional to current through the coil,

$$N\phi = Li$$

$$L = \frac{N\phi}{i}$$

And energy stored is, $W = \frac{1}{2} L i^2(t)$

• Capacitance : → Energy is stored in the form of electrostatic field.



→ It is denoted by 'C' & the unit is farad. (F)

∴ $q = C \frac{dv(t)}{dt}$ & $C = \frac{q}{V}$ (or) $V(t) = \frac{1}{C} \int_{-\infty}^t i dt$

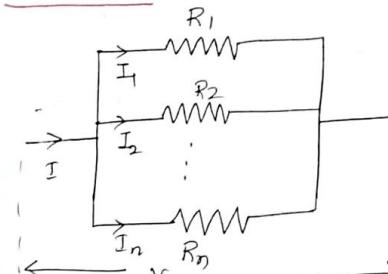
* Series & parallel combinations

Circuit elements & formulae

Parallel

- Current gets divided
- voltage remains same across all the element

Resistor:-



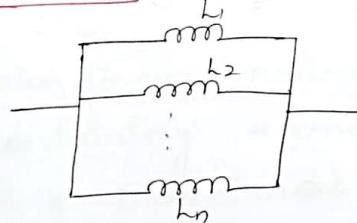
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

$$I = I_1 + I_2 + \dots + I_n$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \dots + \frac{V}{R_n}$$

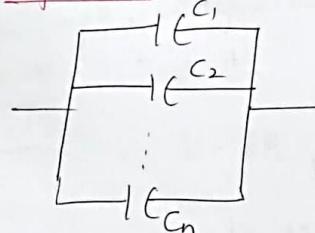
$$I = V \left[\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right]$$

Inductor :-



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$$

Capacitor :-

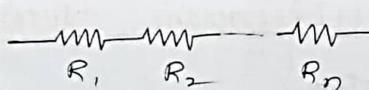


$$C_{eq} = C_1 + C_2 + \dots + C_n$$

Series

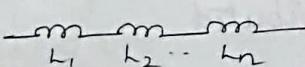
- voltage gets divided
- current remains same.

Resistor:-



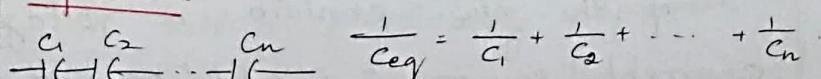
$$R_{eq} = R_1 + R_2 + \dots + R_n$$

Inductor:-



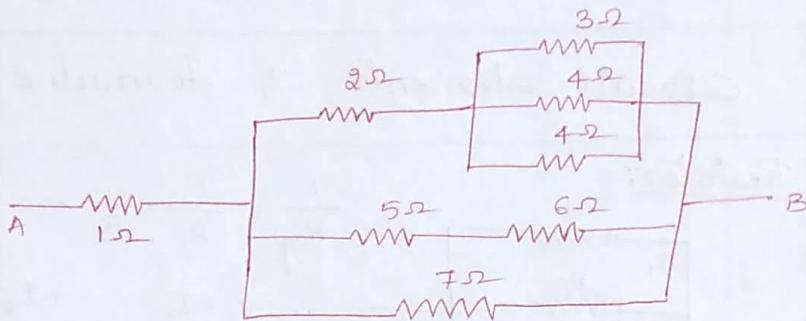
$$L_{eq} = L_1 + L_2 + \dots + L_n$$

Capacitor:-



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

1. Find the equivalent resistance b/w A' & B'.



→ Solⁿ: $R_{eq} = R_1 + R_2 = 5 + 6 = 11 \Omega$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{4} + \frac{1}{4} = \frac{10}{12}$$

$$\therefore R_{eq}'' = 1.2 \Omega$$

$$R_{eq}''' = 2 + 1.2 = 3.2 \Omega$$

$$\frac{1}{R_{eq}'''} = \frac{1}{3.2} + \frac{1}{11} + \frac{1}{7}$$

$$R_{eq}''' = 1.8304 \Omega$$

$$\therefore R_{AB} = 1 + 1.8304$$

$$\boxed{R_{AB} = 2.8304 \Omega}$$

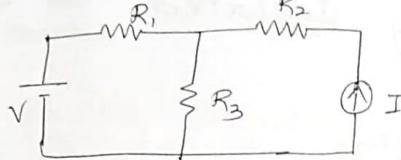
* CLASSIFICATION OF ELECTRICAL NETWORK :-

1. Active & Passive
2. Linear & Non-linear
3. Unilateral & Bilateral
4. Lumped & distributed

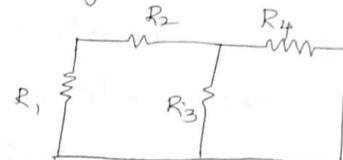
Active & Passive nws :-

If the nws contain a voltage source or current source in addition to passive element

is called active n/w [fig(a)]



fig(a)



fig(b).

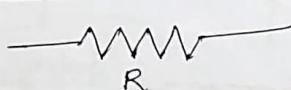
A n/w. containing only passive elements is called passive n/w. [fig(b)].

* NOTE:-

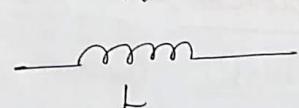
Voltage sources & current sources are "active sources".

2. Linear & Non-linear network :-

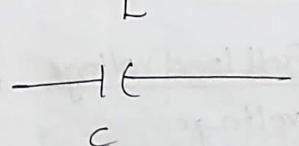
A ckt containing elements like resistance, capacitance & inductance whose parameters are always constant irrespective of the change in time, voltage, temperature is known as linear network.



$$V = iR ; i = \frac{V}{R}$$



$$V = L \frac{di}{dt} ; i = \frac{1}{L} \int v dt$$



$$V = \frac{1}{C} \int i dt ; i = C \frac{dv}{dt}$$

Nonlinear n/w. is a ckt. whose parameters change their values with change in time, temperature, voltage etc.,

3. Lumped & distributed n/w. :-

A n/w. in which physically separate R, L (or) C can be represented is called Lumped n/w.

In this n/w R, L & G cannot be electrical separated & individually isolated as a separate ckt. element.
 Eg: A transmission line.

4. Unilateral ckt. & bilateral ckt:-
 In a unilateral ckt, the relation b/w voltage & current changes with the change in direction of current.

Eg: Diode

In a bilateral ckt., the relation b/w voltage & current remains the same for either directions of current flow.

Eg: R, L & G are bilateral ckt.

- NOTE:
 A n/w containing R, L & G is called a Linear bilateral n/w.

* Regulation & loading of sources:-

Regulation is given by,

$$\% \text{ Regulation} = \frac{\text{No load voltage} - \text{Full load voltage}}{\text{Full load voltage}} \times 100$$

$$\boxed{\% \text{ Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100}$$

- If the source is loaded in such a way that the load voltage falls below specified full load value & the regulation is higher

than that specified for the source, then the source is said to be 'loaded'

* ENERGY SOURCES:-

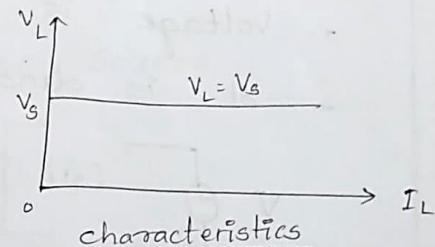
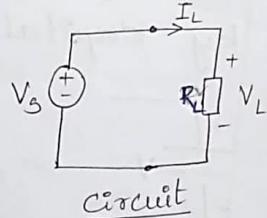
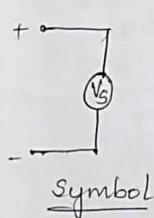
The two basic energy sources are:

- Voltage Source
- Current source

Independent sources:-

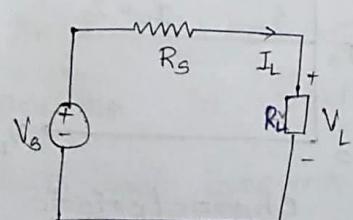
i) Voltage source:-

- A V.S. can be ideal (or) practical.
- Ideal voltage source is defined as the energy source which gives constant voltage 'V_L' independent of terminal current 'I_L'.
- The symbol of ideal V.S. is shown below.

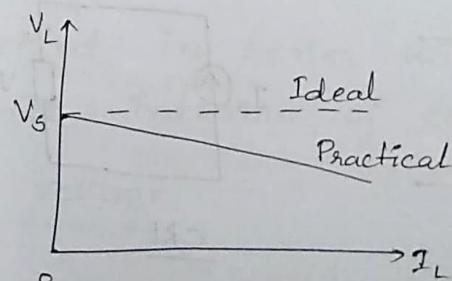


i) Ideal Voltage source.

- All practical voltage sources will have internal resistance called source resistance.
- Due to the presence of source resistance, 'V_L' depends on 'I_L' as shown below,



$R_s \rightarrow$ internal resistance.



$$\therefore \text{From fig, } I_L = \frac{V_s}{R_s + R_L}$$

$$V_L = V_s - I_L R_s.$$

\therefore For ideal v.s., $R_s = 0$, $\therefore [V_L = V_s]$

→ V.S can be classified as,

- Time invariant sources.

- Time variant sources.

- Time variant source [AC Source] :-

- * Voltage varies w.r.t. time.

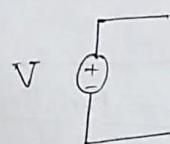
- * It is denoted by small letters



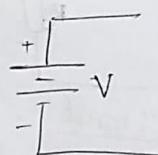
- Time invariant source [DC source] :-

- * Voltage is not varying w.r.t. time.

- * It is denoted by capital letters.



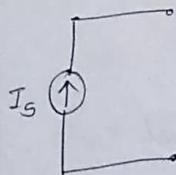
(or)



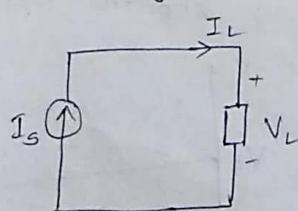
ii) Current source :-

→ A current source can be ideal/practical

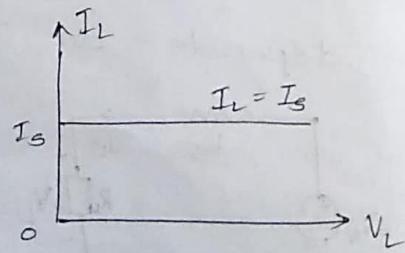
→ In an ideal c.s. ' I_L ' remains constant independent of ' V_L ' & it is as shown below,



Symbol

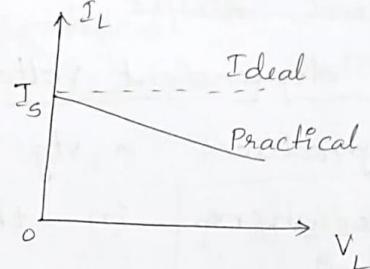
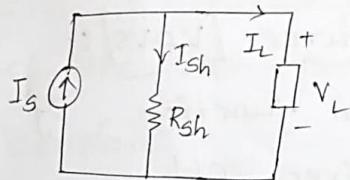


ckt.



characteristic

- In a practical C.S. an internal resistance is present known as source resistance.
- Due to the presence of source resistance, I_L depends on V_L .



- C.S. can be classified as,

- Time variant C.S.
- Time invariant C.S.

<u>Time invariant C.S.</u>	<u>Time variant C.S.</u>
* Current does not vary w.r.t time	* Current varies w.r.t time
* DC Source	* AC Source
*	*
* Capital letters	* Small letters.

* NOTE :-

- Any element connected across [like] voltage source is redundant in terms of voltage but not in terms of current.
- Any element connected in series with current source is redundant in terms of current but not in terms of voltage.

Dependent source :-

The value of source that depends on voltage / current in the ckt is called dependent source.

i) Voltage dependent voltage source [VDVS] :-

It produces a vtg. as a function of vtg. elsewhere in the given ckt.

ii) current dependent current source [CDCS] :-

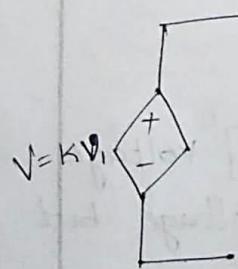
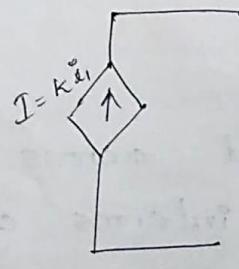
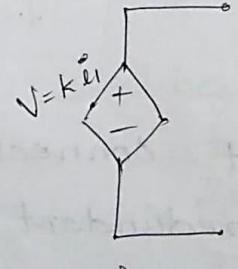
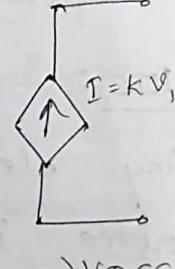
It produces a current as a function of current elsewhere in the given ckt.

iii) current dependent voltage source [CDVS] :-

It produces a voltage as a function of current elsewhere in the given ckt.

iv) Voltage dependent current source [VDCS] :-

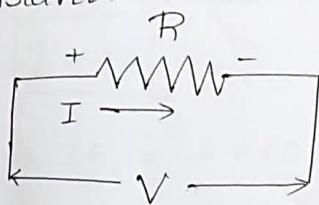
It produces a current as a function of voltage elsewhere in the given ckt.

i) VDVSii) CDCSiii) CDVSiv) VDCS

Dependent sources are also called "controlled sources".

* Ohm's Law :- [Georg Simon Ohm]

Ohm's law is stated as, "the current flowing through the electric ckt. is directly proportional to the potential difference across the ckt. & inversely proportional to the resistance of the ckt, provided the temperature remains constant."



i.e.

$$I \propto \frac{V}{R}$$

ohm's law states that the voltage across conducting material is

$$I = \frac{V}{R}$$

$$V = IR$$

$$R = \frac{V}{I}$$

$$V \propto I ; V = IR$$

The limitations of ohm's law are,

- It is not applicable to the non linear devices such as diodes, zener diodes etc.,
- It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is,

$$V = k I^m$$

Where,

$k, m \rightarrow$ constant

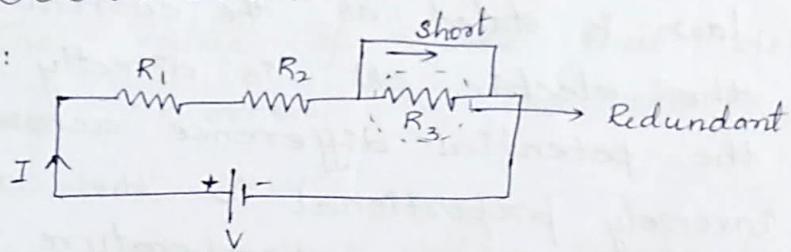
* NOTE:-

- The v.tg. & resistance across short ckt. is "zero" i.e. $V_{sc} = R_{sc} I_{sc} = 0, I_{sc} = 0$,
- The resistance of open ckt. is infinity & current is zero.

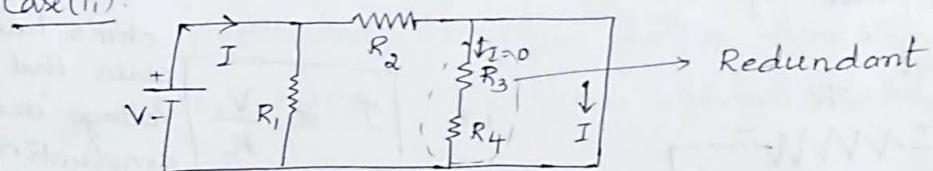
$$\text{i.e. } I_{oc} = \frac{V}{R_{oc}} = \frac{V}{\infty} = 0,$$

* Redundant branches :-

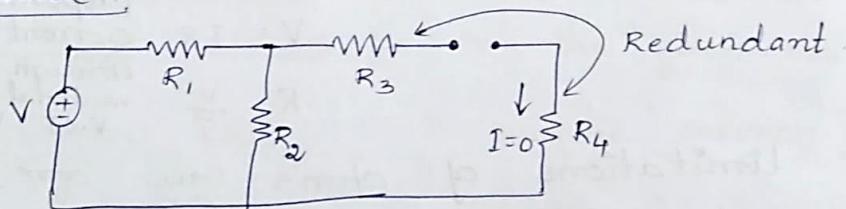
Case(i):



Case(ii):

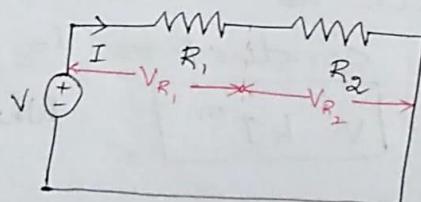


Case(iii):



* VOLTAGE DIVISION :-

Consider the ckt,



Apply KVL

$$V = IR_1 + IR_2$$

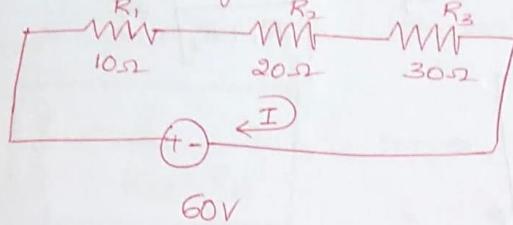
$$V = I(R_1 + R_2)$$

$$I = \frac{V}{R_1 + R_2}$$

$$\therefore V_{R_1} = IR_1 = \frac{VR_1}{R_1 + R_2}$$

$$V_{R_2} = IR_2 = \frac{VR_2}{R_1 + R_2}$$

Find the v.tg. across R_1 , R_2 & R_3 .



→ Soln:-

$$I = \frac{V}{R_1 + R_2 + R_3} = \frac{60}{10 + 20 + 30} = \frac{60}{60}$$

$$\boxed{I = 1A}$$

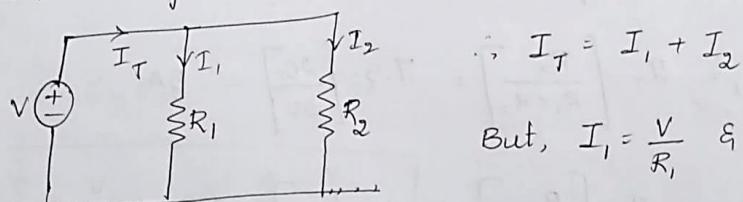
$$V_{R_1} = IR_1 = 1 \times 10 = 10V$$

$$V_{R_2} = IR_2 = 1 \times 20 = 20V$$

$$V_{R_3} = IR_3 = 1 \times 30 = 30V$$

* CURRENT DIVISION :-

Consider a parallel ckt.



$$\therefore I_T = I_1 + I_2$$

$$\text{But, } I_1 = \frac{V}{R_1} \text{ & } I_2 = \frac{V}{R_2}$$

i.e.

$$V = I_1 R_1 = I_2 R_2$$

$$I_1 = I_2 \left(\frac{R_2}{R_1} \right)$$

Subs. ' I_1 ' in I_T

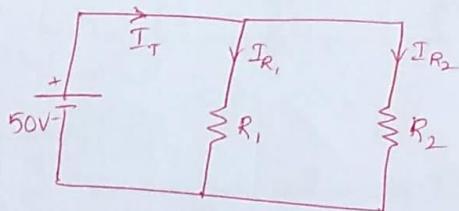
$$\therefore I_T = \left(\frac{R_2}{R_1} \right) I_2 + I_2 = I_2 \left[\frac{R_2}{R_1} + 1 \right] = I_2 \left[\frac{R_2 + R_1}{R_1} \right]$$

$$\therefore \boxed{I_2 = \frac{R_1}{R_1 + R_2} I_T}$$

$$\text{Now, } I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1 + R_2} \right] I_T$$

$$\boxed{I_1 = \frac{R_2 I_T}{R_1 + R_2}}$$

Find the magnitude of total current I_R & I_{R_2}
if $R_1 = 10\Omega$, $R_2 = 20\Omega$ & $V = 50V$



→ Soln:- $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 20}{30} = 6.67\Omega$

$$I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = 7.5A$$

$$\text{Now, } I_1 = I_T \left[\frac{R_2}{R_1 + R_2} \right] = 7.5 \cdot \left[\frac{20}{30} \right] = 5A$$

$$I_2 = I_T \left[\frac{R_1}{R_1 + R_2} \right] = 7.5 \left[\frac{10}{30} \right] = 2.5A$$

* SOURCE TRANSFORMATION :-

Consider voltage source in fig(a). having internal resistance R_{se} , connected to load resistance R_L .

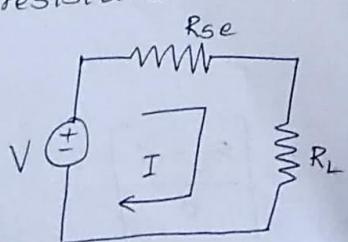


fig.(a)

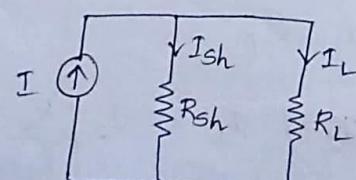


fig.(b)

The voltage source is replaced by equivalent current source as shown in fig.(b)

$$\boxed{I = \frac{V}{R_{se} + R_L}} \rightarrow (1)$$

Load current I_L is given by,

$$\boxed{I_L = I \cdot \frac{R_{sh}}{R_{sh} + R_L}} \rightarrow (2)$$

Now equating (1) & (2)

$$I = I_L$$

$$\frac{V}{R_{se} + R_L} = \frac{I \times R_{sh}}{R_{sh} + R_L}$$

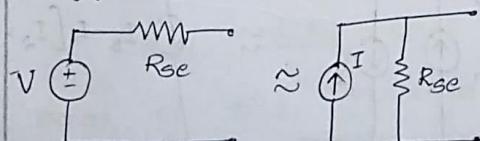
\therefore (Equating denominator) ; $R_{se} + R_L = R_{sh} + R_L$

$$\boxed{R_{se} = R_{sh}}$$

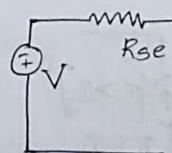
$$V = I \times R_{sh} = I \times R_{se}$$

$$\boxed{I = \frac{V}{R_{sh}} \text{ (or)} \quad I = \frac{V}{R_{se}}} \quad (3)$$

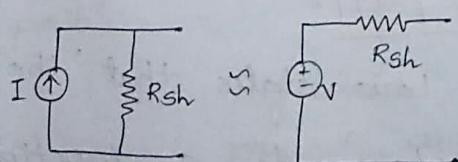
* Different transformed sources are:



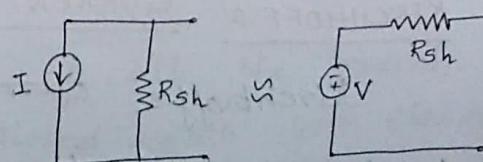
$$(a) \quad I = \frac{V}{R_{se}}$$



$$(b) \quad I = \frac{V}{R_{se}}$$

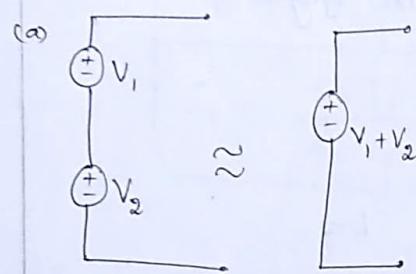


$$(c) \quad V = I \times R_{sh}$$

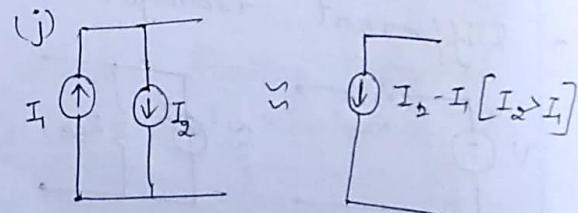
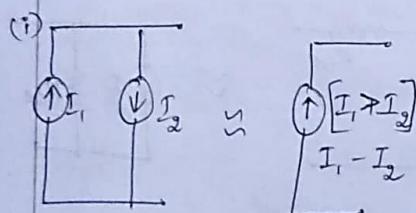
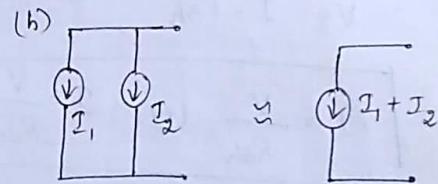
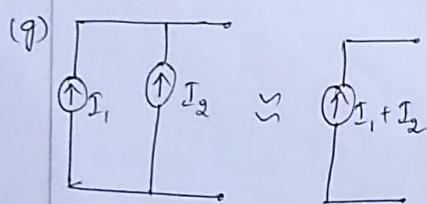
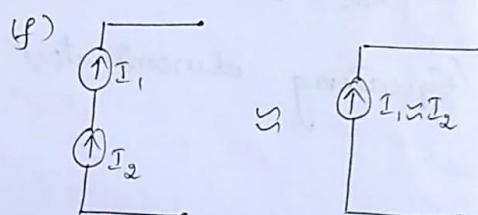
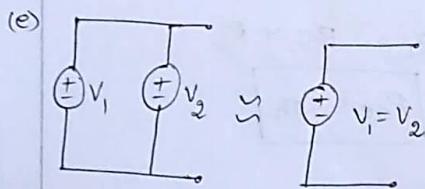
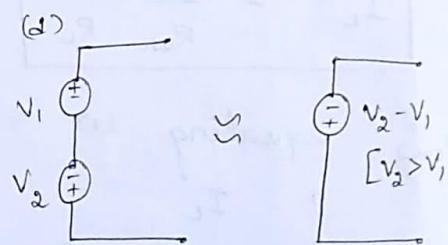
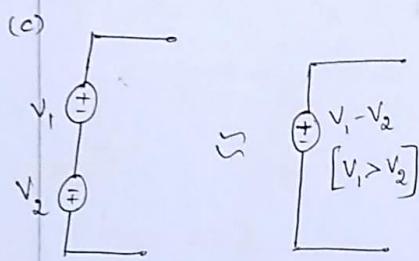
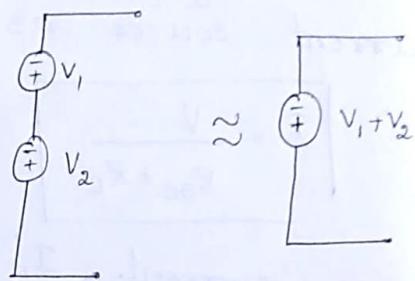


$$(d) \quad V = I \times R_{sh}$$

• Combinations :-



(b)



* KIRCHHOFF'S CURRENT LAW [KCL] :-

Kirchhoff's current law states that, the total current flowing towards a junction

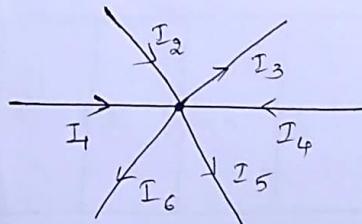
point is equal to the total current flowing away from that junction point."

(OR)

The algebraic sum of all the current meeting at a junction point is always zero.

$$\text{i.e. } \boxed{\sum I \text{ at junction point} = 0}$$

Consider a junction point shown in fig.(a)



$$\left. \begin{aligned} I_1 + I_2 - I_3 + I_4 - I_5 - I_6 &= 0 \\ I_1 + I_2 + I_4 &= I_3 + I_5 + I_6 \end{aligned} \right\}$$

* NOTE :-

Current flowing towards a junction point is assumed to be positive while current flowing away from a junction point is assumed to be negative.

* KIRCHHOFF'S VOLTAGE LAW [KVL] :-

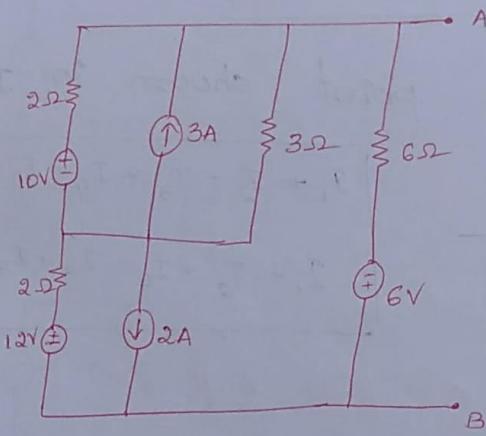
KVL states that, "In any n/w., the algebraic sum of the voltage drop across the ckt. elements of any closed path (or) loop is equal to the algebraic sum of the e.m.f's in the path".

(OR)

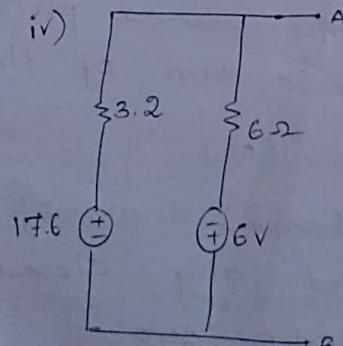
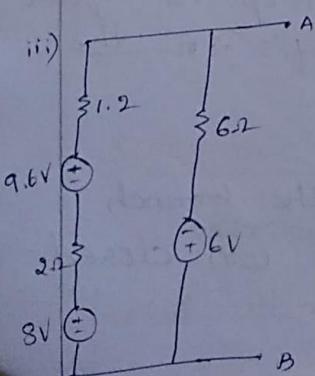
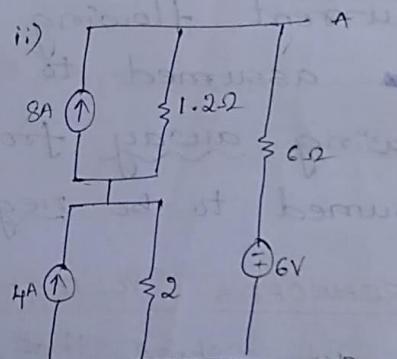
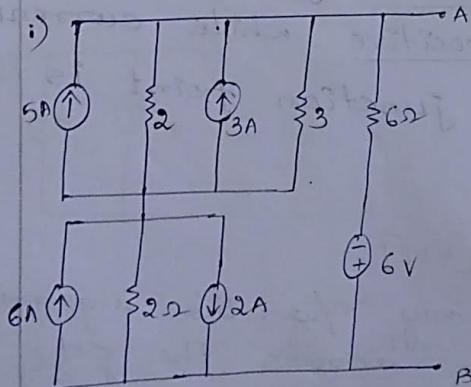
The algebraic sum of all the branch voltages around any closed path (or) closed loop is always zero.

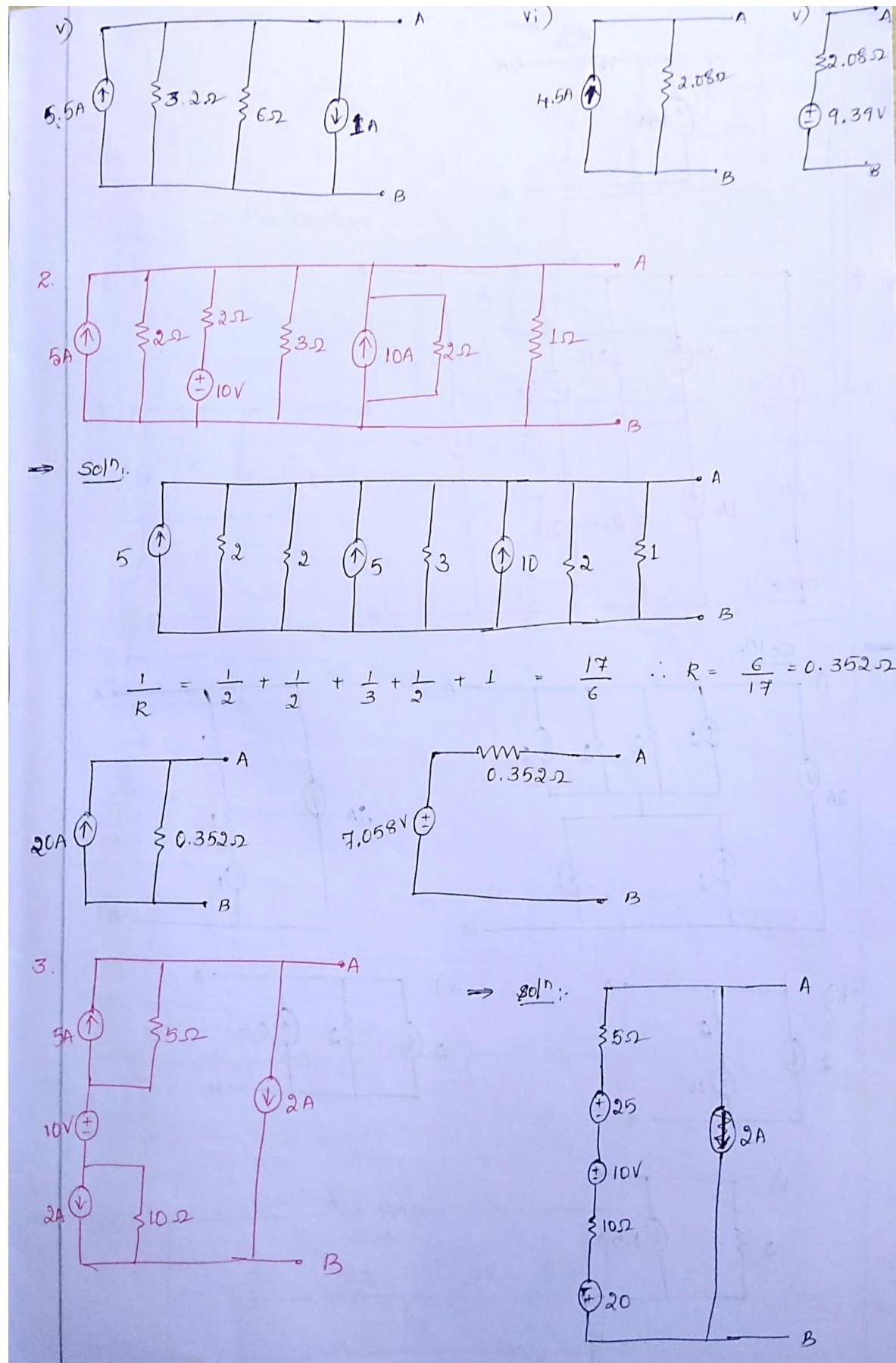
Sum of all the potential rises must be equal to the sum of all the potential drop while tracing any closed path of the ckt. The total change in potential along a closed path is always zero.

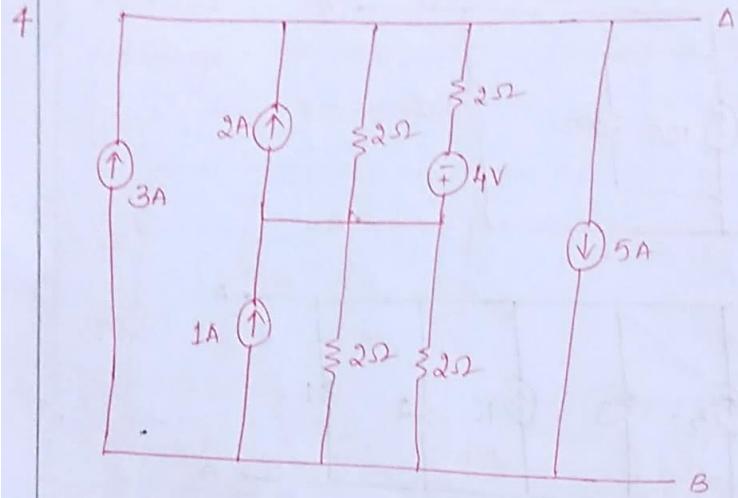
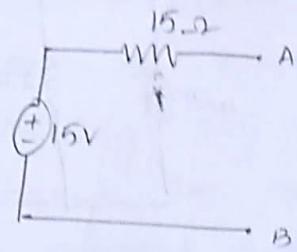
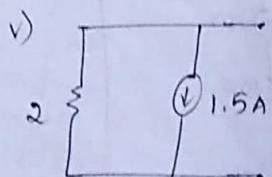
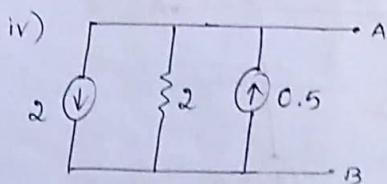
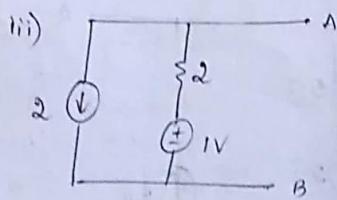
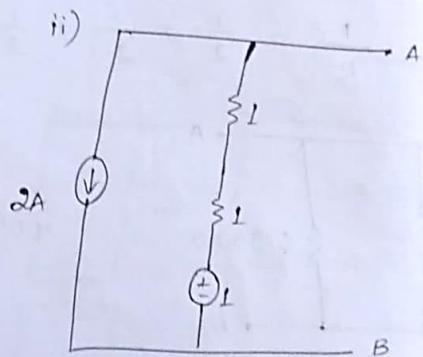
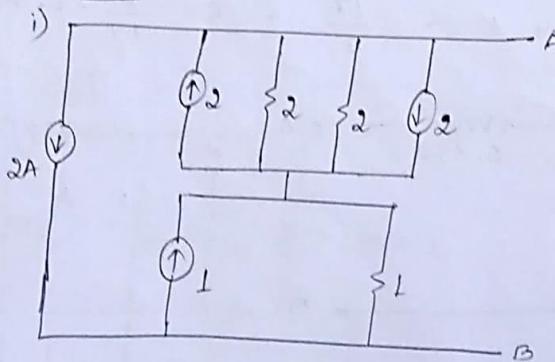
- 1 Reduce the n/w. given below into a single voltage source in series with the resistance b/w the terminals A & B.

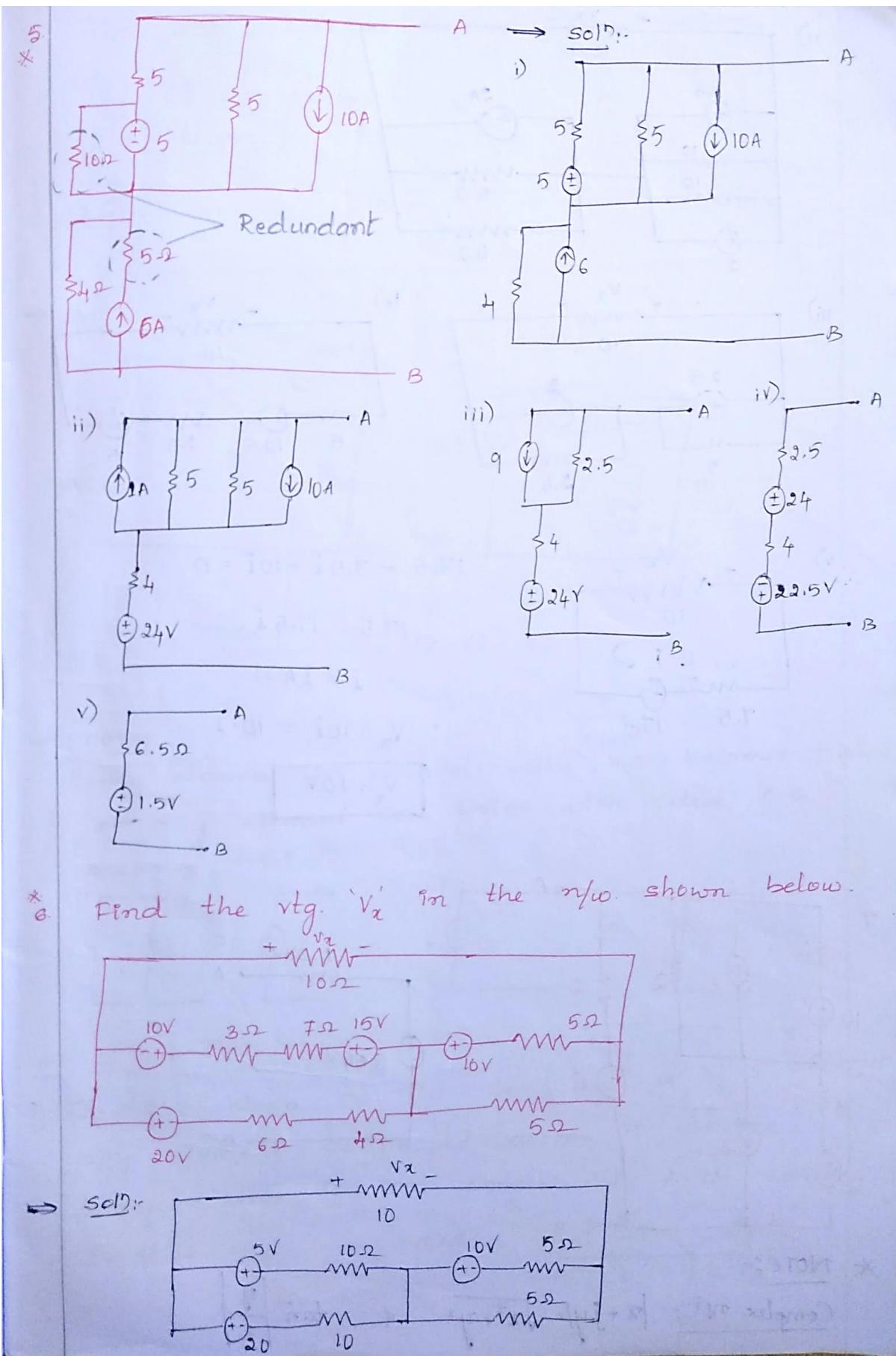


Sol^D:

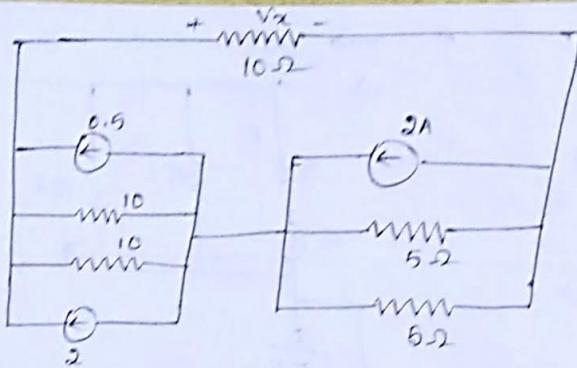




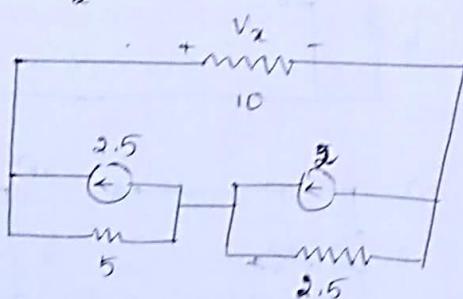
SOLN:-



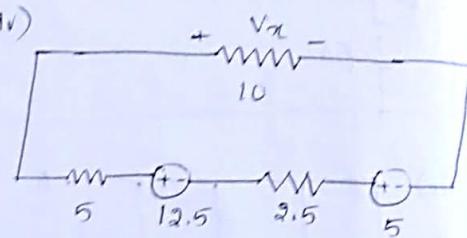
iii)



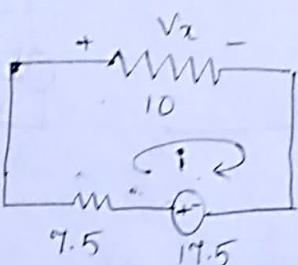
iv)



v)



v)



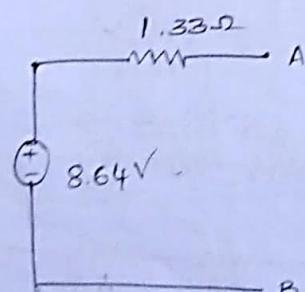
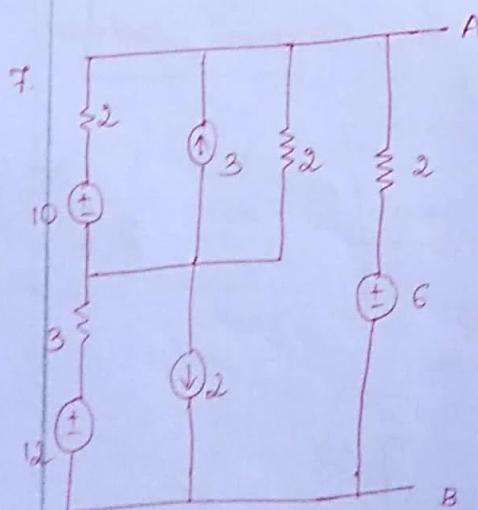
$$17.5 - 7.5i - 10i = 0$$

$$17.5 = 17.5i$$

$$\therefore i = 1A$$

$$\therefore V_x = 10i = 10 \times 1$$

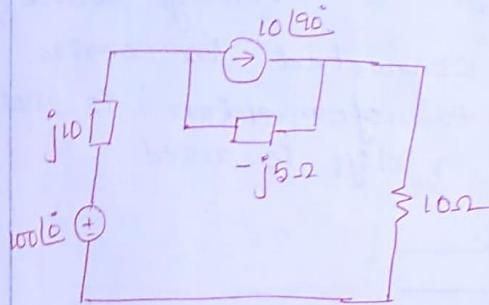
$$\boxed{V_x = 10V}$$



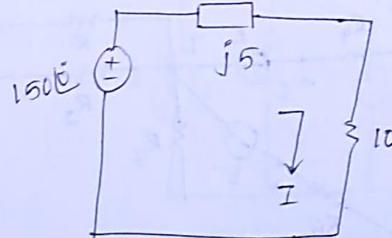
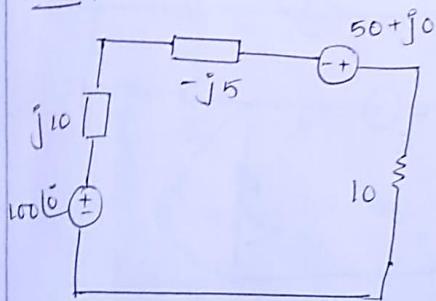
* NOTE :-

$$\text{Complex no. : } |x+iy| = \sqrt{x^2+y^2} \quad \& \quad \tan^{-1} \left(\frac{y}{x} \right)$$

8. Find the current through 10Ω resistor



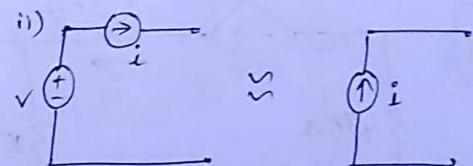
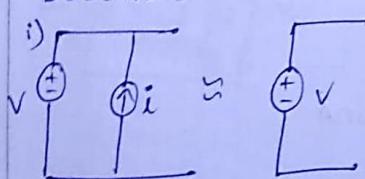
→ Soln.



$$\therefore I = \frac{150[0^\circ]}{10 + j5} = (12 - 6j) \text{ A}$$

* NOTE:

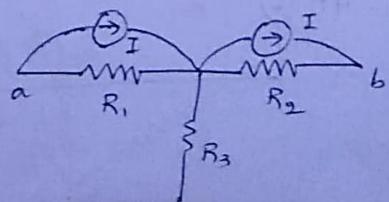
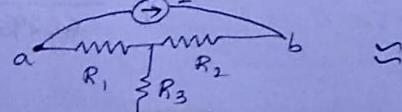
Any element in parallel with ^{ideal}V.S. becomes trivial
if any element in series with ideal C.S. becomes trivial.



* SOURCE SHIFTING:-

I-shift: When a n/w. contains a C.S. without any element connected across it, source transformation is not possible. In such cases transformation can be used.

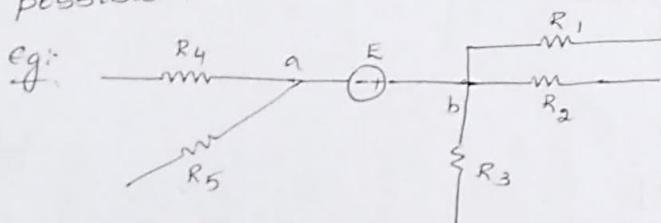
I-shift can be used.



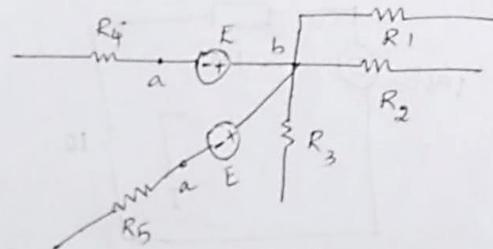
V-shift :-

When a netw. contain a voltage source without any element connected in series with it, then source transformation is not possible. In such cases v-shift is used.

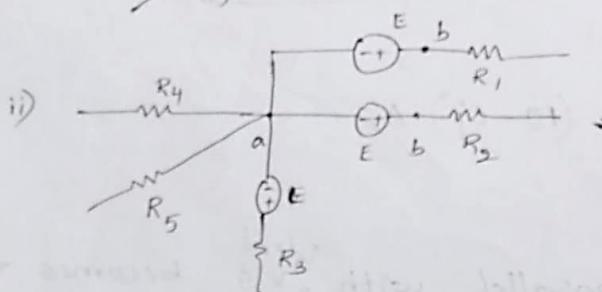
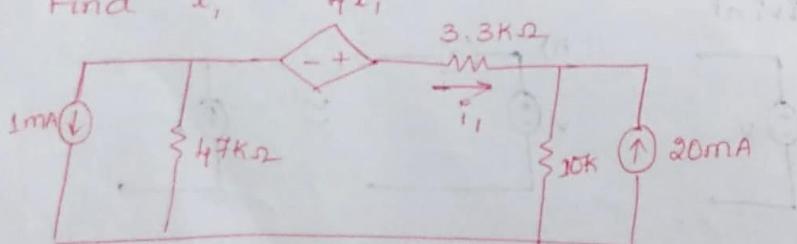
Eg:-



i)

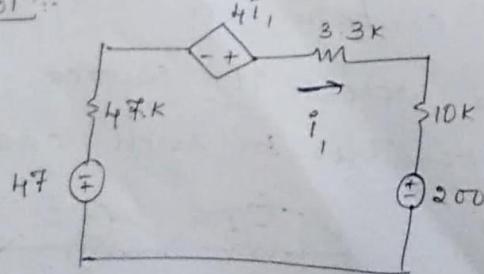


ii)

9. Find i_1 , $4i_1$,

→

Soln:-

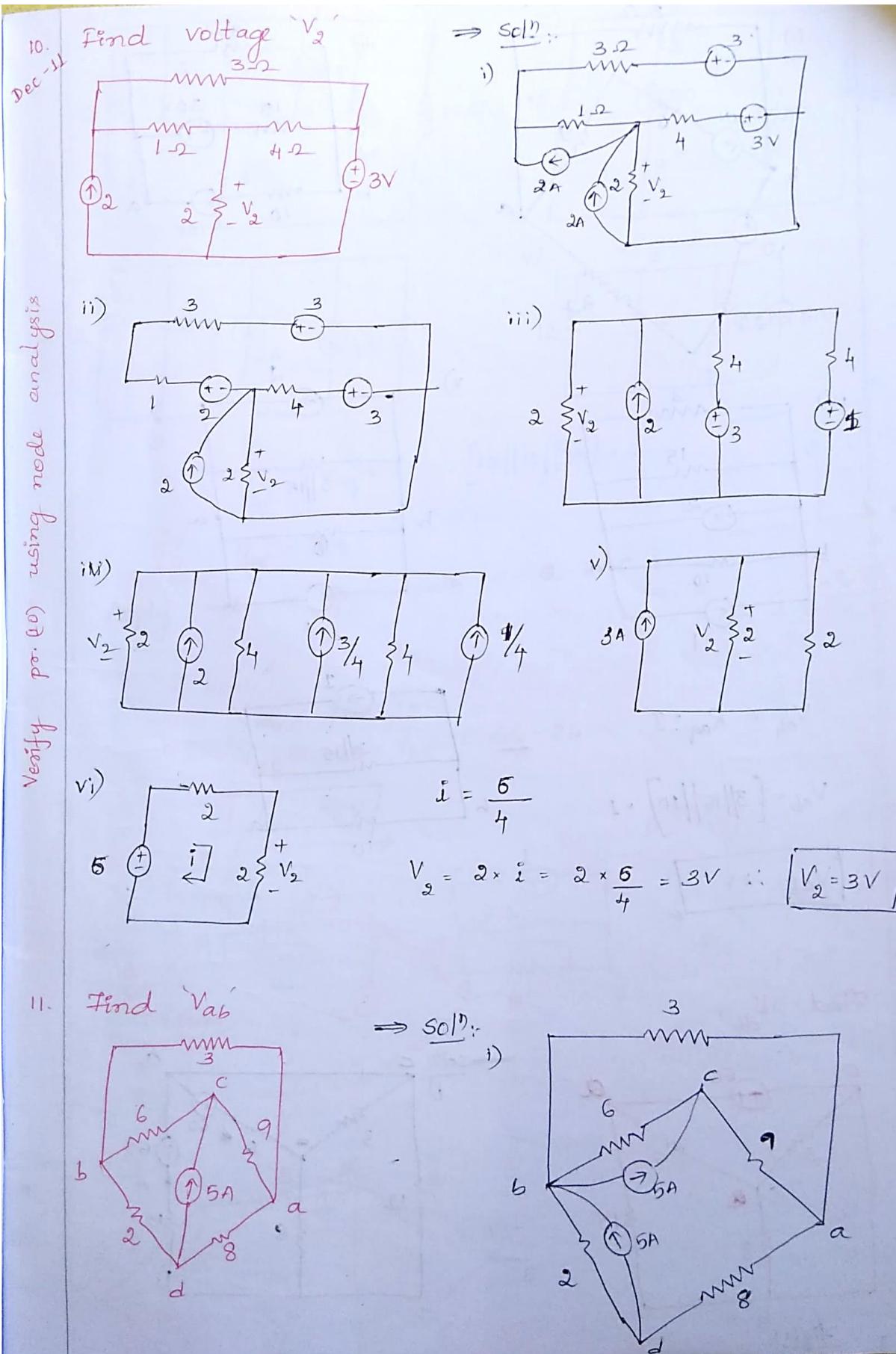


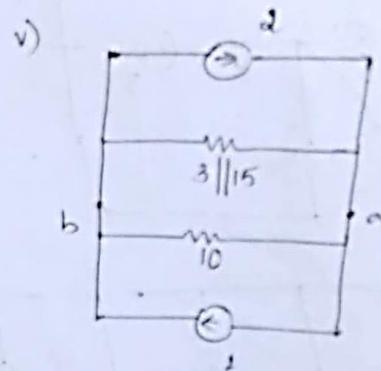
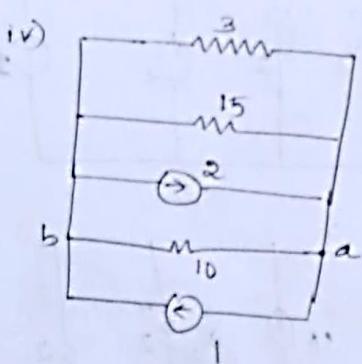
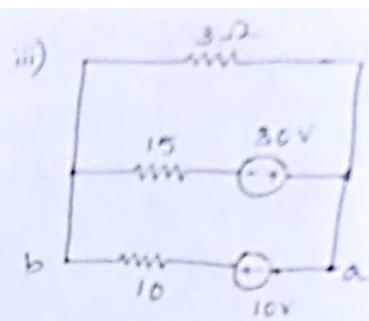
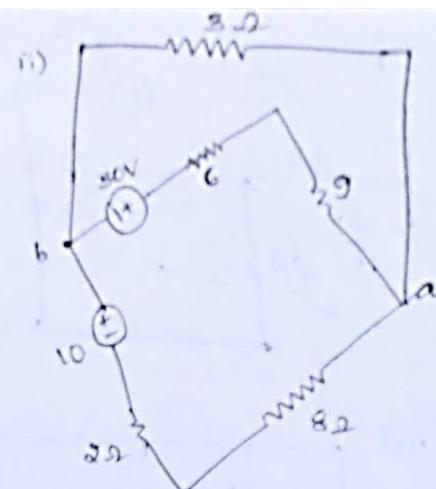
$$-47 - 47 \times 10^3 i_1 + 4i_1 - 3.3 \times 10^3 i_1$$

$$-10 \times 10^3 i_1 - 200 = 0$$

$$-247 - 60296 i_1 = 0$$

$$i_1 = \frac{-247}{60296} = -4.09 \text{ mA}$$

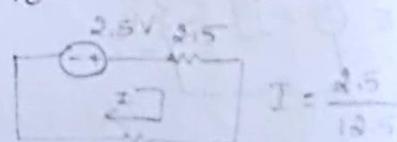
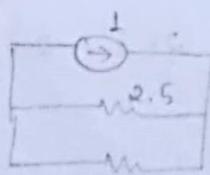
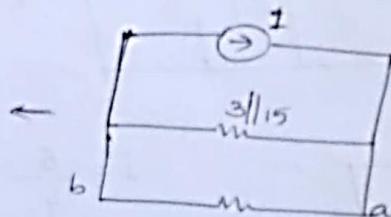




$$V_{ab} = R_{eq} \cdot I$$

$$V_{ab} = [3 \parallel 15 \parallel 10] \cdot I$$

$$\boxed{V_{ab} = 2V}$$

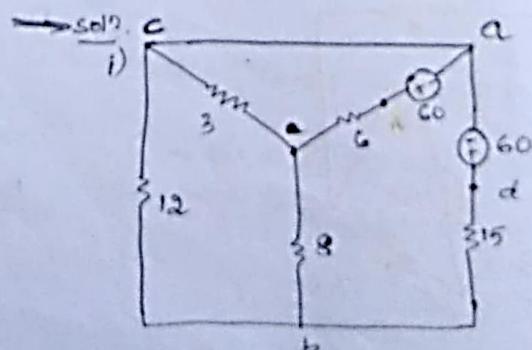
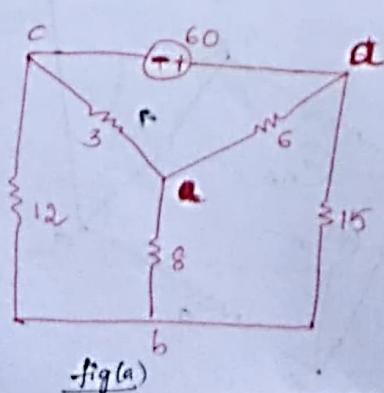


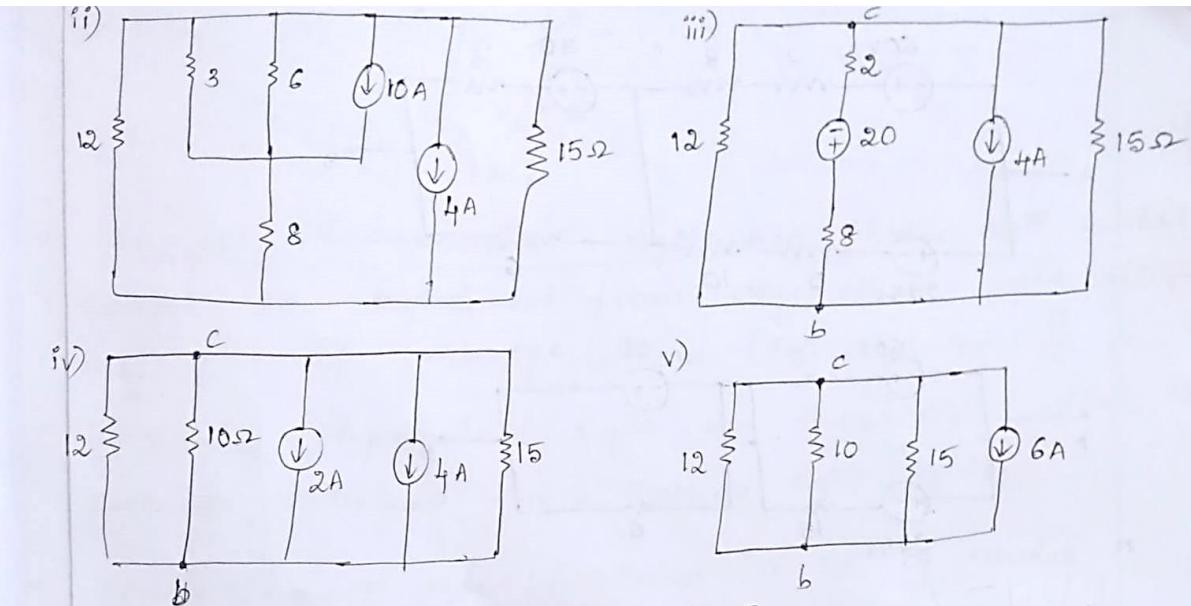
$$I = \frac{2.5}{12.5}$$

$$I_A = 0.2$$

$$V_{AB} = 10 \cdot 0.2$$

#12

Find V_{db} 



$$\therefore V_{bc} = R_{eq} \times I = [12 \parallel 10 \parallel 15] \times 6$$

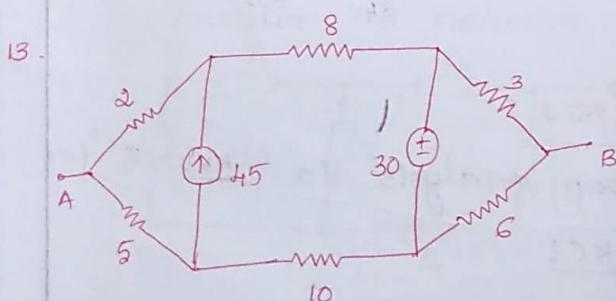
$$V_{bc} = 24V$$

Apply KVL to fig(a) path c-a-b-c

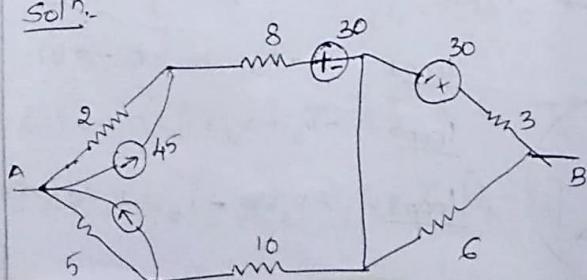
$$V_{ab} = V_{ac} - V_{bc}$$

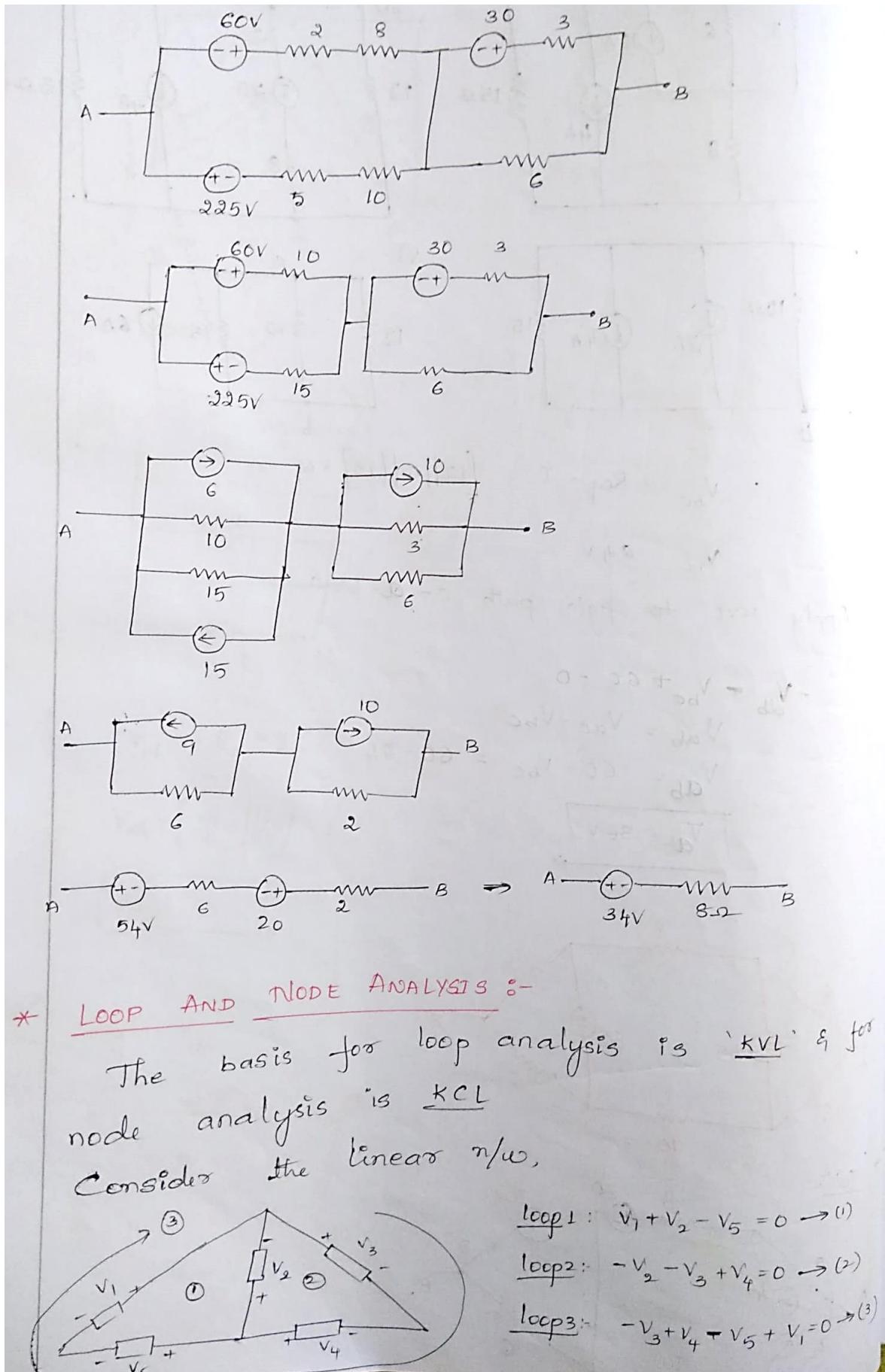
$$V_{ab} = 60 - V_{bc} = 60 - 24$$

$$\boxed{V_{ab} = 36V}$$



→ Sol'n.





$$(1) + (2), V_1 + V_2 - V_5 - V_2 - V_3 + V_4 = 0$$

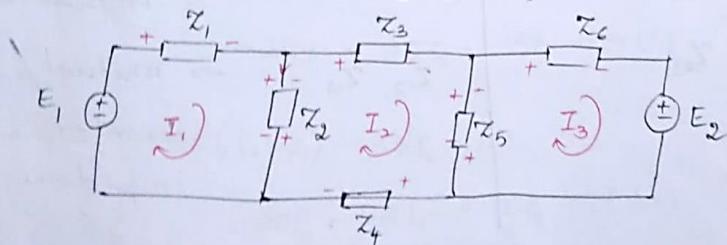
$$V_1 + V_4 - V_5 - V_3 = 0.$$

$$\therefore (1) + (2) \Rightarrow (3)$$

- * Linearly independent eqns are those eqns which cannot be obtained from any other eqns. after any form of manipulations [Eqn.(1) & Eqn.(2)]
- * Linearly dependent eqns are those eqns. which can be obtained from other eqns. [Eqn.(3)]
- * Assuming a n/w. contains 'n' no of nodes & 'b' branches, \therefore minimum no of eqns. necessary to perform nodal analysis is "n-1"
- * In node analysis one node is considered as reference / ground / datum node & the remaining nodes are called independent/principle nodes.
- * Minimum no. of equations necessary to perform loop analysis is "b-(n-1)"

Loop analysis

Consider a network,



$$\text{Loop 1: } E_1 - Z_1 I_1 - Z_2 [I_1 - I_2] = 0$$

$$(Z_1 + Z_2) I_1 - Z_2 I_2 = E_1 \rightarrow 0$$

$$\text{Loop 2: } -Z_2 [I_2 - I_1] - Z_3 I_2 - Z_5 [I_2 - I_3] - Z_4 I_2 = 0$$

$$-Z_2 I_1 + (Z_3 + Z_4 + Z_5 + Z_2) I_2 - Z_5 I_3 = 0 \quad \rightarrow (2)$$

loop 3: $-Z_6 I_3 - E_2 - Z_5 (I_3 - I_2) = 0$

$$-Z_5 I_2 + (Z_5 + Z_6) I_3 = -E_2 \quad \rightarrow (3)$$

In matrix form,

$$\begin{bmatrix} (Z_1 + Z_2) & -Z_2 & 0 \\ -Z_2 & (Z_2 + Z_3 + Z_4 + Z_5) & -Z_5 \\ 0 & -Z_5 & (Z_5 + Z_6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \\ -E_2 \end{bmatrix}$$

$$[Z] [I] = [E]$$

where, $[Z]$ = Impedance matrix

$[I]$ = Current matrix

$[E]$ = Column matrix containing v.s.

The general form of $[Z]$ is,

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \dots \\ Z_{21} & Z_{22} & Z_{23} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \begin{array}{l} Z_{11}, Z_{22}, \dots \rightarrow \text{self impedance.} \\ Z_{12}, Z_{13}, \dots \rightarrow \text{mutual / transfer impedance.} \end{array}$$

* Node analysis:-

Assuming a n/w contain 3 independent source & a datum node, the 3 nodal eqns can be written in general form as,

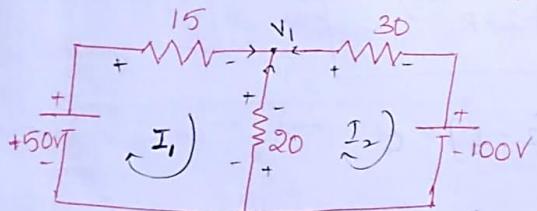
$$\begin{bmatrix} y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \\ y_{21}v_1 + y_{22}v_2 + y_{23}v_3 \\ y_{31}v_1 + y_{32}v_2 + y_{33}v_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Where,
[y] = admittance matrix

$$\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$[y] [v] = [I]$$

Q4 Find I_1 & I_2 by using KVL & KCL



→ Soln:-

KVL :-

$$\text{Loop 1: } 50 - 15I_1 - 20(I_1 - I_2) = 0$$

$$50 - 35I_1 + 20I_2 = 0$$

$$-35I_1 + 20I_2 = -50 \rightarrow (1)$$

$$\text{Loop 2: } -20(I_2 - I_1) - 30I_2 - 100 = 0$$

$$20I_1 - 50I_2 = 100 \rightarrow (2)$$

$$\therefore \boxed{I_1 = 0.37A, I_2 = -1.85A}$$

$$\text{KCL :- } \frac{50 - V_1}{15} + \frac{0 - V_1}{20} + \frac{100 - V_1}{30} = 0$$

$$200 - 4V_1 - 3V_1 + 200 - 2V_1 = 0$$

$$400 - 9V_1 = 0 \\ \boxed{V_1 = 44.4V}$$

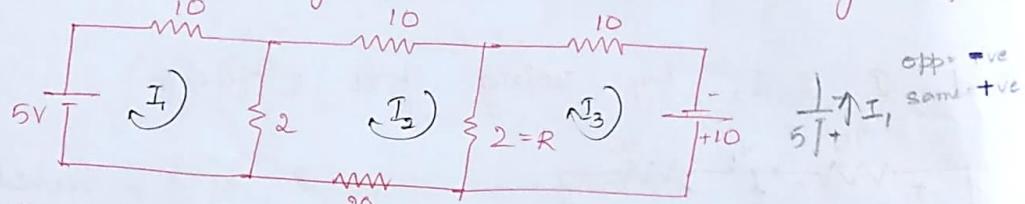
$$\therefore I_1 = \frac{50 - V_1}{15} = 0.37A_1$$

$$I_1 - I_2 = \frac{0 - V_1}{20}$$

$$0.37 - I_2 = \frac{-44.4}{20}$$

$$\therefore I_2 = -1.85A_2$$

15. Find the voltage across $R=2\Omega$ using loop analysis.



Sol: Loop 1 :- $5 - 10I_1 - 2(I_1 - I_2) = 0$

$$-12I_1 + 2I_2 = -5 \rightarrow (1) \quad \text{x by } -1$$

Loop 2 :- $-2(I_2 - I_1) - 10I_2 - 2(I_2 - I_3) - 20I_2 = 0$

$$2I_1 - 34I_2 + 2I_3 = 0 \rightarrow (2) \quad \text{x by } -1$$

Loop 3 :- $-2(I_3 - I_2) - 10I_3 + 10 = 0$

$$2I_2 - 12I_3 = -10 \rightarrow (3) \quad \text{x by } -1$$

$$\begin{bmatrix} 12 & -2 & 0 \\ -2 & 34 & -2 \\ 0 & -2 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix}$$

$$V_{(R=2\Omega)} = 2(I_2 - I_3) = 2(I_3 - I_2)$$

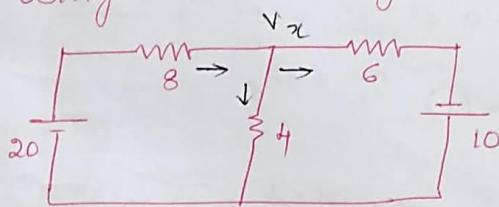
$$\Delta = \begin{vmatrix} 12 & -2 & 0 \\ -2 & 34 & -2 \\ 0 & -2 & 12 \end{vmatrix} = 12(34 \times 12 - 4) + 2[-24] + 0 = 4800$$

$$I_2 = \frac{\begin{vmatrix} 12 & 5 & 0 \\ -2 & 0 & -2 \\ 0 & 10 & 12 \end{vmatrix}}{\Delta} = \frac{12[20] - 5[-24]}{4800} = 0.075$$

$$I_3 = \frac{\begin{vmatrix} 12 & -2 & 5 \\ -2 & 34 & 0 \\ 0 & -2 & 10 \end{vmatrix}}{\Delta} = \frac{12[340] + 2[-20] + 5[4]}{4800} = 0.845$$

$$\therefore V_{(R=2\Omega)} = 2[0.845 - 0.075] = 1.54V$$

16. Using node analysis find all branch currents.



$$\Rightarrow \text{Soln: } \frac{20 - V_x}{8} = \frac{V_x - 0}{4} + \frac{V_x - (-10)}{6}$$

$$\frac{20 - V_x}{8} = \frac{V_x}{4} + \frac{V_x + 10}{6}$$

$$\therefore I_{8\Omega} = \frac{20 - V_x}{8} = 2.3077A$$

$$\frac{20 - V_x}{8} = \frac{6V_x + 4V_x + 40}{24}$$

$$I_{4\Omega} = \frac{V_x}{4} = 0.3845A$$

$$3[20 - V_x] = 10V_x + 40$$

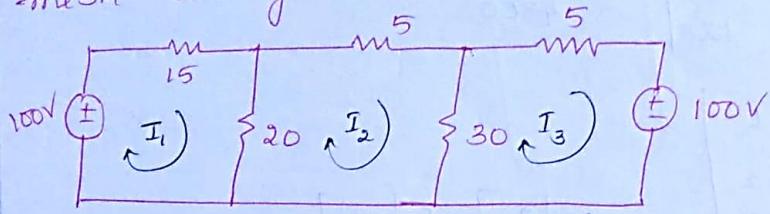
$$I_{6\Omega} = \frac{V_x + 10}{6} = 1.9231A$$

$$60 - 3V_x = 10V_x + 40$$

$$20 = 13V_x$$

$$V_x = \frac{20}{13} = 1.5384$$

17. Find current through 30Ω resistor using mesh analysis.



\Rightarrow Solⁿ:

$$\text{Loop 1: } 100 - 15I_1 - 20(I_1 - I_2) = 0$$

$$-35I_1 + 20I_2 = -100 \rightarrow (1)$$

$$\text{Loop 2: } -20(I_2 - I_1) - 5I_2 - 30(I_2 - I_3) = 0$$

$$20I_1 - 55I_2 + 30I_3 = 0 \rightarrow (2)$$

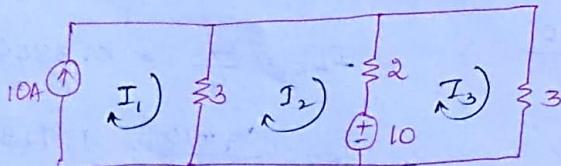
$$\text{Loop 3: } -30(I_3 - I_2) - 5I_3 - 100 = 0$$

$$30I_2 - 35I_3 = 100 \rightarrow (3) \quad I_1 = 1.9A$$

$$\therefore I_2 = -1.6A; I_3 = -4.228A$$

$$\therefore I_{30\Omega} = I_2 \cup I_3 = \pm 2.628A$$

18. Find mesh currents.



\Rightarrow Solⁿ: Loop 1: $I_1 = 10A$

$$\text{Loop 2: } -3(I_2 - I_1) - 2(I_2 - I_3) - 10 = 0$$

$$-5I_2 + 2I_3 = -20 \rightarrow (1)$$

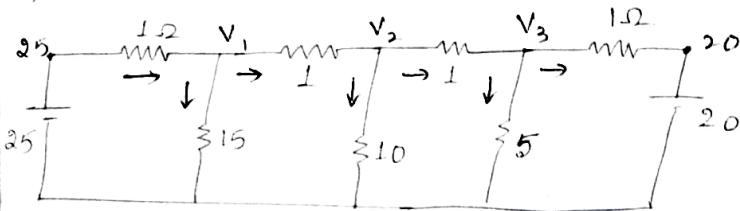
$$\text{Loop 3: } 10 - 2(I_3 - I_2) - 3I_3 = 0$$

Method 2:
 $\Delta = 21,875$
 $\Delta_2 = -35000$
 $\Delta_3 = -92500$

$$2I_2 + 5I_3 - 10 \rightarrow (1)$$

$$\left[\begin{array}{l} I_1 = 10A \\ I_2 = 5.9142A \\ I_3 = 4.2857A \end{array} \right]$$

19. find node vgs



→ soln: Node(1): $\frac{25 - V_1}{1} = \frac{V_1}{15} + \frac{V_1 - V_3}{1}$

$$31V_1 - 15V_2 = 375 \rightarrow (1)$$

Node(2):

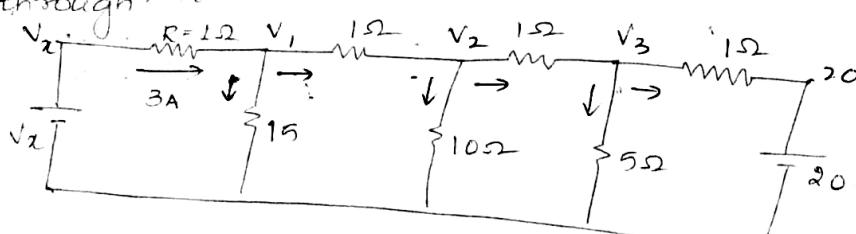
$$\frac{V_1 - V_2}{1} = \frac{V_2}{10} + \frac{V_2 - V_3}{1} ; 10V_1 - 21V_2 + 10V_3 = 0 \rightarrow (2)$$

Node(3):-

$$\frac{V_2 - V_3}{1} = \frac{V_3}{5} + \frac{V_3 - 20}{1} \Rightarrow 5V_2 - 11V_3 = -100 \rightarrow (3)$$

$$\left[\begin{array}{l} V_1 = 20.92V \\ V_2 = 18.24V \\ V_3 = 17.38V \end{array} \right]$$

20. Find the value of voltage V_x such that current through $R = 1\Omega$ is 3A



→ soln: Node(1): $\frac{V_x - V_1}{3} = \frac{V_1 - 0}{15} + \frac{V_1 - V_2}{1}$

$$3 = \frac{V_1 + 15V_1 - 15V_2}{15}$$

$$16V_1 - 15V_2 = 45 \rightarrow (1)$$

Node 2:-

$$\frac{V_1 - V_2}{1} = \frac{V_2}{10} + \frac{V_2 - V_3}{1}$$

$$-10V_1 + 21V_2 - 10V_3 = 0 \rightarrow (2)$$

Node 3:-

$$\frac{V_2 - V_3}{1} = \frac{V_3}{5} + \frac{V_3 - 20}{1}$$

$$-5V_2 + 11V_3 = 100 \rightarrow (3)$$

$$V_1 = 18.57V$$

$$V_2 = 16.81V$$

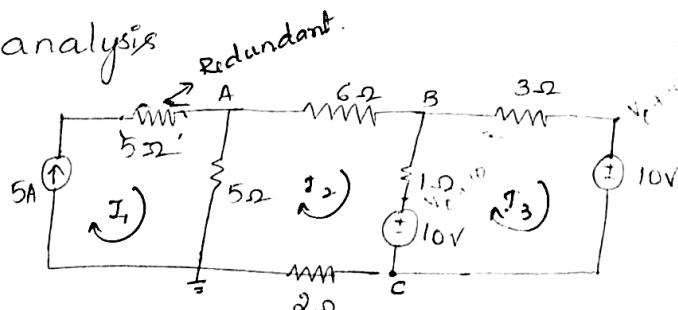
$$V_3 = 16.73V$$

$$\therefore \frac{V_x - V_1}{1} = 3$$

$$\therefore V_x = 3 + V_1 = 3 + 18.57$$

$$\boxed{V_x = 21.57V}$$

21. Determine voltages at 'A' & 'B' using loop & node analysis



loop analysis:-

$$I_1 = 5A$$

loop 2:- $-5[I_2 - I_1] - 6I_2 - 1[I_2 - I_3] - 10 - 2I_2 = 0$

$$-5I_1 + 14I_2 - I_3 = -10 \rightarrow (1)$$

For both loop & node analysis
gnd. should be
the common
ground.

$$\text{Loop 3: } -3I_3 - 10 + 10 - 1 [I_1 - I_2] = 0$$

$$-I_2 + 4I_3 = 0 \rightarrow (2)$$

$$\therefore I_1 = 5A; I_2 = 1.09A; I_3 = 0.27A$$

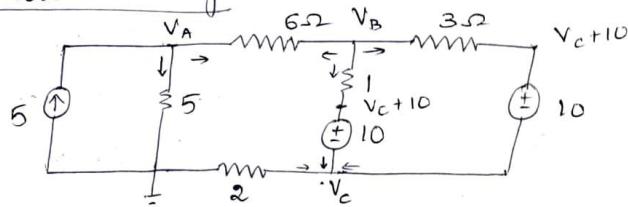
$$\therefore V_A = 5 [I_1 - I_2] = 5 [5 - 1.09] = 19.55V$$

$$V_B = -5 [I_2 - I_1] = 6I_2 = -5 [5 + 1.09] = 6 \times 1.09 = 2 [I_1 - I_2] - 6$$

$$V_B = 13.01V$$

$$V_C = 2I_2 = 2 \times 1.09 = 2.18V$$

Node analysis :-



$$\text{Node A: } 5 = \frac{V_A}{5} + \frac{V_A - V_B}{6}$$

$$11V_A - 5V_B = 150 \rightarrow (1)$$

$$\text{Node B: } \frac{V_B - V_A}{6} + \frac{V_B - (V_C + 10)}{1} + \frac{V_B - (V_C + 10)}{3} = 0$$

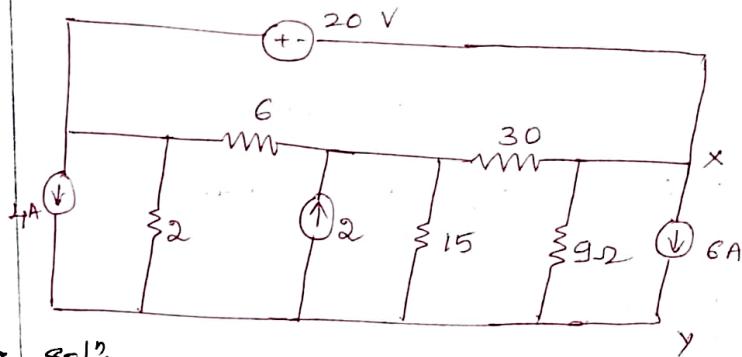
$$-V_A + 9V_B - 8V_C = 80 \rightarrow (2)$$

$$\text{Node C: } \frac{0 - V_C}{2} + \frac{V_B - (V_C + 10)}{1} + \frac{V_B - (V_C + 10)}{3} = 0$$

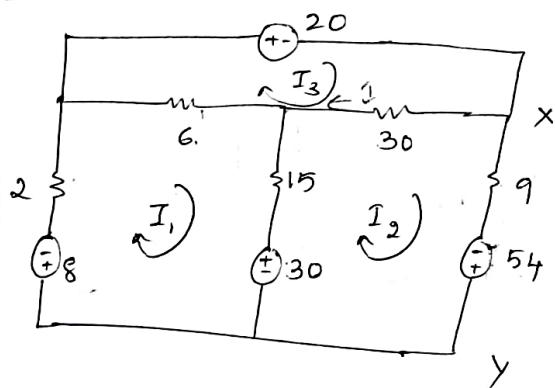
$$-8V_B + 11V_C = -80 \rightarrow (3)$$

$$\therefore \begin{cases} V_A = 19.54V \\ V_B = 13V \\ V_C = 2.18V \end{cases}$$

22. Find the v_{tg}. & power developed across OA current source in the n/w shown below. Use Source transformation & mesh analysis.



→ Sol:



$v_{xy} \rightarrow 'x'$ is at higher potential than 'y'.
∴ We shld. go from 'y' to 'x'

$$v_{xy} = -54 + 9I_2$$

loop1:

$$-8 - 2I_1 - 6[I_1 - I_3] - 15[I_1 - I_2] - 30 = 0$$

$$23I_1 - 15I_2 - 6I_3 = -38 \rightarrow (1)$$

loop2:

$$30 - 15[I_2 - I_1] - 30[I_2 - I_3] - 9I_2 + 54 = 0$$

$$-15I_1 + 54I_2 - 30I_3 = 84 \rightarrow (2)$$

loop3:

$$-6[I_3 - I_1] - 20 - 30[I_3 - I_2] = 0$$

$$-6I_1 - 30I_2 + 36I_3 = -20 \rightarrow (3)$$

$$\therefore I_1 = 0.62 \text{ A}$$

$$I_2 = 0.75 \text{ A}$$

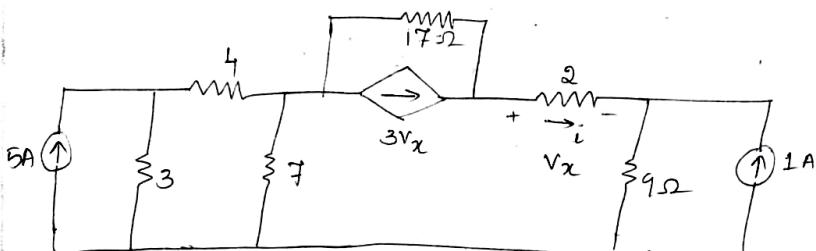
$$I_3 = 1.84 \text{ A}$$

$$\therefore v_{xy} = -54 + 9 \times 2.75$$

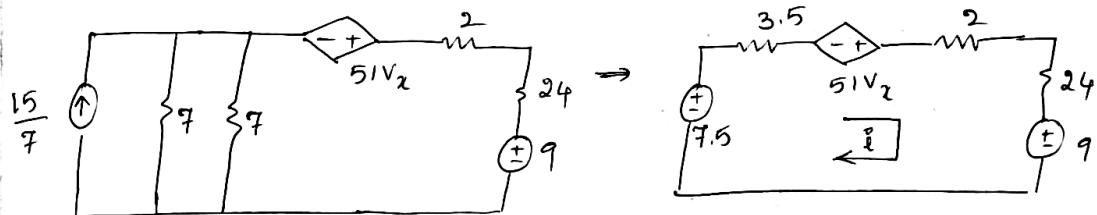
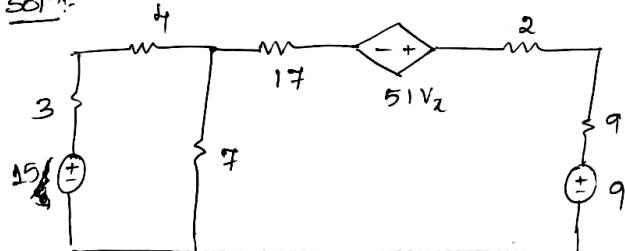
$$\boxed{v_{xy} = -29.25 \text{ V}}$$

$$P = VI = -29.25 \times 6 = -175.5 \text{ W}$$

23 Dec calculate the current through 2Ω resistor S.T.



→ Sol'n:-



$$i = \frac{7.5 + 51V_x - 9}{3.5 + 2 + 24}$$

$$(or) i = \frac{7.5 + 51V_x - V_x - 9}{3.5 + 24}$$

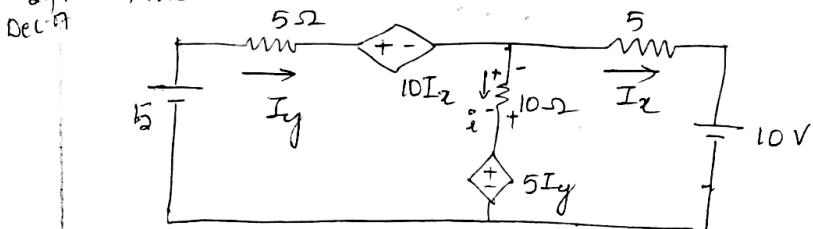
$$31.5i = 51V_x - 15$$

$$\text{But } V_x = 2i$$

$$31.5i = 51(2i) - 15 ; i = 0.02127 \text{ A}$$

$$\boxed{i = 21.27 \text{ mA}}$$

24 Dec find current through 10Ω.



$$\Rightarrow \text{Soln: } i = I_y - I_x$$

$$\text{Loop 1: } 5 - 5I_y - 10I_x - 10[I_y - I_x] - 5I_y = 0$$

$$5 - 10I_y - 10I_x - 10I_y + 10I_x = 0$$

$$5 - 20I_y = 0$$

$$I_y = \frac{5}{20}$$

$$I_y = 0.25A$$

Loop 2:

$$5I_y - 10[I_x - I_y] - 5I_x - 10 = 0$$

$$15I_y - 15I_x - 10 = 0$$

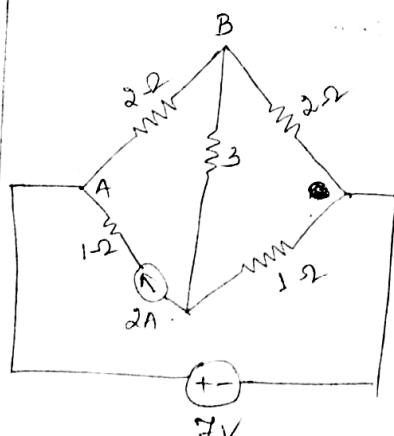
$$15(0.25) - 15I_x - 10 = 0$$

$$I_x = -0.4166A$$

$$\therefore i = I_y - I_x = 0.25 - (-0.4166)$$

$$\boxed{i = 0.666A}$$

*5. Find current in branch AB'



3. Super mesh analysis.

When a n/w. contain a c.s. without any element

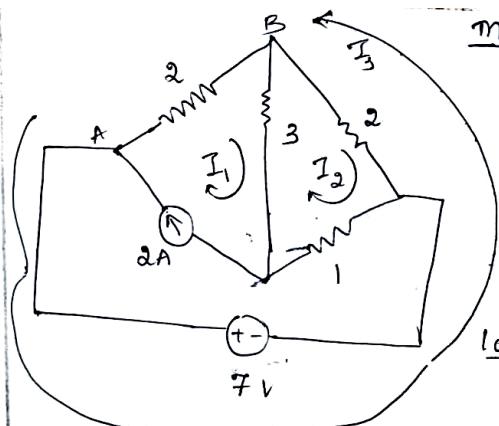
connected across it, kvl

cannot be applied directly

Remedies:-

1. Choose loop currents such that only one current flows through c.s.

2. By assuming arbitrary v.s. across c.s.

Method 1:

$$\text{loop 1: } I_1 = 2A$$

$$\text{loop 2: } -3[I_2 - I_1] - 2[I_2 - I_3] = 0$$

$$-I_2 = 0$$

$$6I_2 - 2I_3 = 6 \rightarrow (1)$$

$$6I_2 + 2I_3 = 6$$

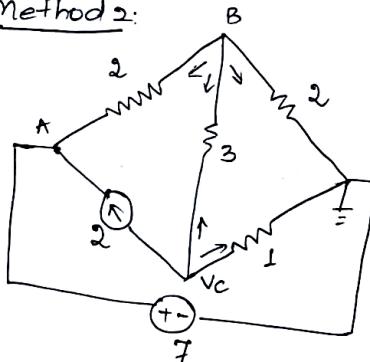
$$-7 - 2[I_3 - I_2] - 2[I_3 - I_1] = 0$$

$$-2I_2 + 4I_3 = -3 \rightarrow (2)$$

$$\therefore I_2 = 0.9A, I_3 = 0.3A$$

$$2I_2 + 4I_3 = 3$$

$$I_{AB} = I_1 + I_3 = 2 + 0.3 = 2.3A,$$

Method 2:

$$\text{Node A: } V_A = 7V$$

$$\text{Node B: } \frac{V_B - V_A}{2} + \frac{V_B - V_C}{2} + \frac{V_B - V_C}{3} = 0$$

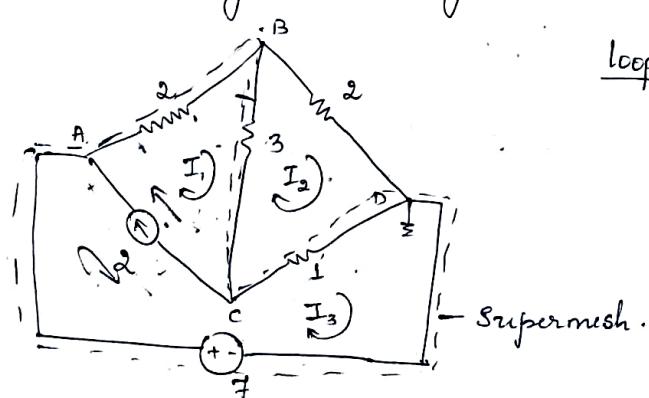
$$8V_B - 2V_C = 21 \rightarrow (1)$$

$$\text{Node C: } 2 + \frac{V_C - V_B}{3} + \frac{V_C}{1} = 0$$

$$-V_B + 4V_C = -6 \rightarrow (2)$$

$$\therefore V_B = 2.4V; V_C = -0.9V$$

$$\therefore i_{AB} = \frac{V_A - V_B}{2} = \frac{7 - 2.4}{2} = 2.3A,$$

Method 3:- By assuming arbitrary v.s.

$$\text{loop 2: } -I_2 - 1(I_2 - I_3) - 3(I_2 - I_1) = 0$$

$$-3I_1 + 6I_2 - I_3 = 0 \rightarrow (1)$$

$$I_1 - I_3 = 2 \rightarrow (2)$$

Let an arbitrary v_{tg} across 2A be V_{AC}

$$\text{loop}_1: -2I_1 - 3(I_1 - I_2) + V_{AC} = 0 \rightarrow (3)$$

$$\text{loop}_3: -1(I_3 - I_2) + 7 - V_{AC} = 0 \rightarrow (4)$$

$$(3) + (4)$$

$$-2I_1 - 3I_1 + 3I_2 + V_{AC} - I_3 + I_2 + 7 - V_{AC} = 0$$

$$5I_1 - 4I_2 + I_3 = 7 \rightarrow (5)$$

Solving (1), (2) & (5)

$$I_1 = 2.3A$$

$$I_2 = 1.2A$$

$$I_3 = 0.3A$$

Method 4: SUPER MESH

Supermesh is superposition of two or more meshes to avoid c.s.

∴ Mesh 1: 2-2, 3-2, 2A

Mesh 3: 2A, 1-2, 7V

Super mesh contains all the elements in mesh (1) & mesh (2) except '2A' c.s.

∴ Supermesh eqn is,

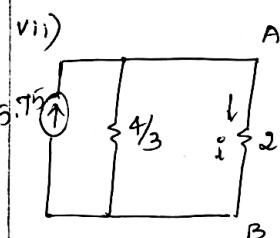
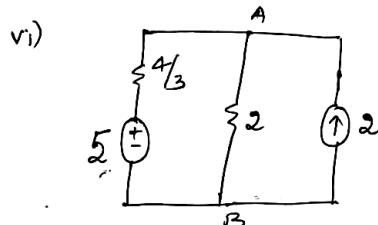
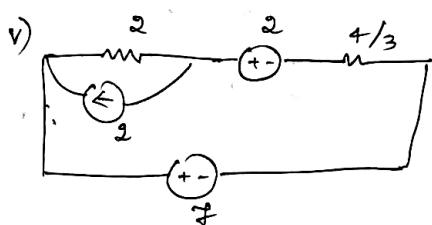
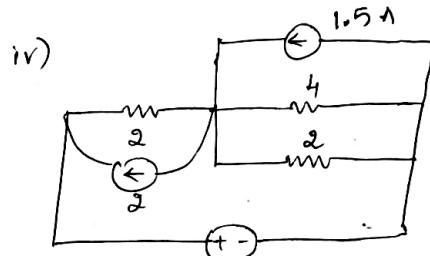
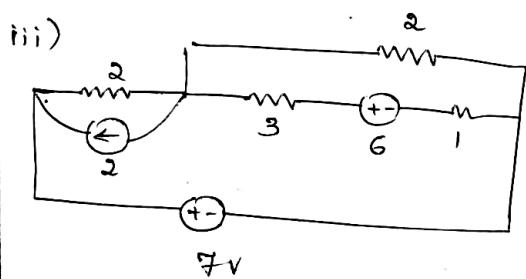
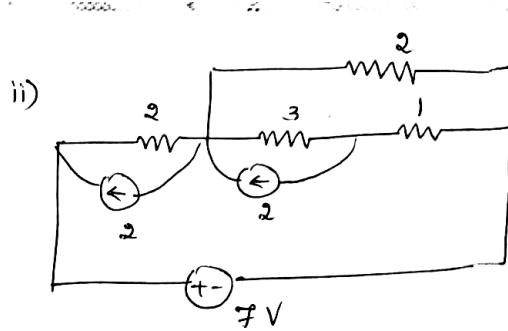
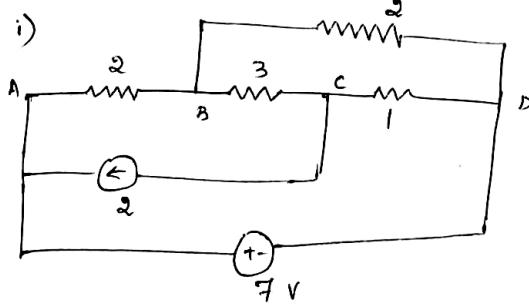
$$-2I_1 - 3(I_1 - I_2) - 1(I_3 - I_2) + 7 = 0$$

$$5I_1 - 4I_2 + I_3 = 7 \rightarrow (3)$$

$$\text{loop}_2: -3I_1 + 6I_2 - I_3 = 0 \rightarrow (1)$$

$$\therefore I_1 - I_3 = 2 \rightarrow (2)$$

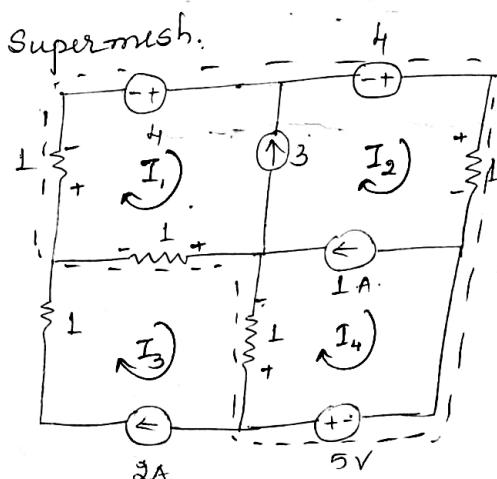
$$\therefore I_1 = 2.3A, I_2 = 1.2A, I_3 = 0.3A$$

Method 5 : Source Shift

$$\therefore i = I_{AB} = \frac{4/3 \times 5.75}{4/3 + 2}$$

$$i = 2.3A$$

26.



⇒ Soln:

$$I_3 = 2A$$

$$I_2 - I_1 = 3 \rightarrow (1)$$

$$I_2 - I_4 = 1 \rightarrow (2)$$

$$\text{mesh 1: } 1\Omega, 1\Omega, 4V, 3A$$

$$\text{mesh 2: } 4V, 1\Omega, 1A, 3A$$

$$\text{mesh 4: } 1A, 5V, 1\Omega$$

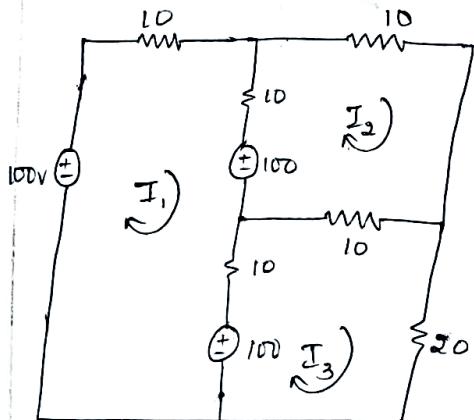
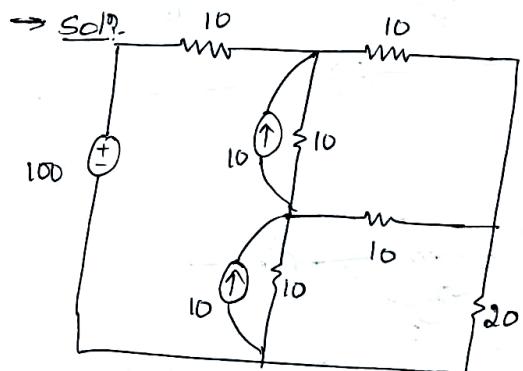
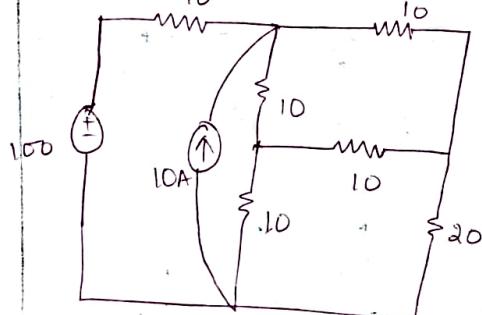
Supermesh eqn. is,

$$-I_1 + 4 + I_4 - I_2 + 5 - [I_4 - I_3] - J[I_1 - I_3] = 0$$

$$\therefore 2I_1 + I_2 + I_4 = 17 \rightarrow (3)$$

$$\boxed{I_1 = 3A; I_2 = 6A; I_3 = 2A; I_4 = 5A}$$

27. Find current through 20Ω



Loop 1:-

$$100 - 10I_1 - 10[I_1 - I_2] - 100 - 10[I_1 - I_3] - 100 = 0$$

$$30I_1 - 10I_2 - 10I_3 = -100 \rightarrow (1)$$

Loop 2:-

$$100 - 10[I_2 - I_1] - 10I_2 - 10[I_2 - I_3] = 0$$

$$-10I_1 + 30I_2 - 10I_3 = 100 \rightarrow (2)$$

Loop 3:-

$$100 - 10[I_3 - I_1] - 10[I_3 - I_2] - 20I_3 = 0$$

$$-10I_1 - 10I_2 + 40I_3 = 100 \rightarrow (3)$$

$$I_1 = -0.833$$

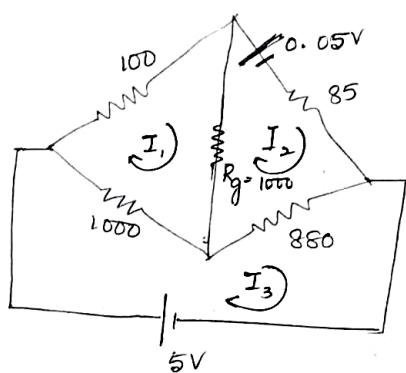
$$I_2 = 4.16$$

$$I_3 = 3.33A$$

$$\boxed{\frac{V}{20\Omega} = 20I_3 = 66.6V}$$

$$\therefore I_{20\Omega} = I_3 = 3.33A$$

28. Determine the current through galvanometer of the bridge n/w, assume galvanometer resistance $R_g = 1000\Omega$, use loop analysis.



\rightarrow Soln:

loop 1:

$$2100I_1 - 1000I_2 - 1000I_3 = 0 \rightarrow (1)$$

loop 2:

$$-1000I_1 + 1965I_2 - 880I_3 = -0.05 \rightarrow (2)$$

loop 3:

$$-1000I_1 - 880I_2 + 1880I_3 = +5 \rightarrow (3)$$

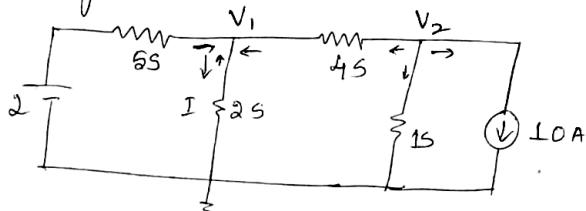
$$\therefore I_1 = 0.0267518\text{A}$$

$$I_2 = 0.026762505\text{A}$$

$$I_3 = 0.029416422\text{A}$$

$$\therefore I_g = I_1 - I_2 = -10.635\text{mA}.$$

29. Find the current in conductance of '2S' in n/w using node analysis.



$$\text{Conductance, } G = \frac{1}{R} \text{ Siemens/mho}$$

\rightarrow Soln:

$$i = \frac{V_1 - V_2}{R} = (V_1 - V_2)G$$

$$\text{Node 1: } (2 - V_1)5 + (0 - V_1)2 + (V_2 - V_1)4 = 0.$$

$$11V_1 - 4V_2 = 10 \rightarrow (1)$$

$$\text{Node 2: } (V_2 - V_1)4 + (V_2)1 + 10 = 0$$

$$\therefore I_{2S} = 2V_1 = 2 \times 0.2564$$

$$-4V_1 + 5V_2 = -10 \rightarrow (2)$$

$$\therefore V_1 = 0.2564\text{V}; V_2 = -1.79\text{V}.$$

$$\boxed{I_{2S} = 0.5128\text{A}}$$

* Supernode analysis :-

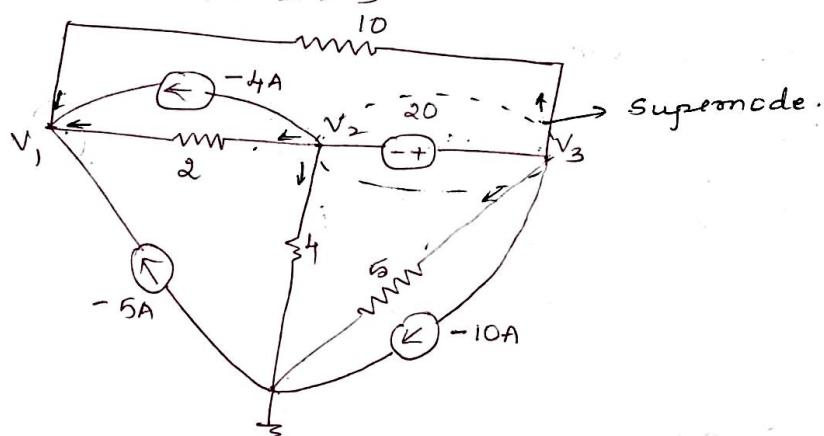
When a n/w. contains a v.s. without an element connected in series with it, kcl cannot be applied directly.

Remedies :-

- * By assuming an arbitrary current through v.s.
- * Super node analysis

A super node analysis is superposition of two / more nodes to avoid v.s.

30. Find v_1, v_2, v_3



⇒ Soln:-

$$\text{Node 1: } -4 - 5 + \frac{v_2 - v_1}{2} + \frac{v_3 - v_1}{10} = 0$$

$$-0.6v_1 + 0.5v_2 + 0.1v_3 = 9 \rightarrow (1)$$

$$v_3 - v_2 = 20 \rightarrow (2)$$

$$\text{Node 2: } 2v_1, 4v_2, 2v_2, -4A$$

$$\text{Node 3: } 10v_1, 20V, 5v_2, -10A$$

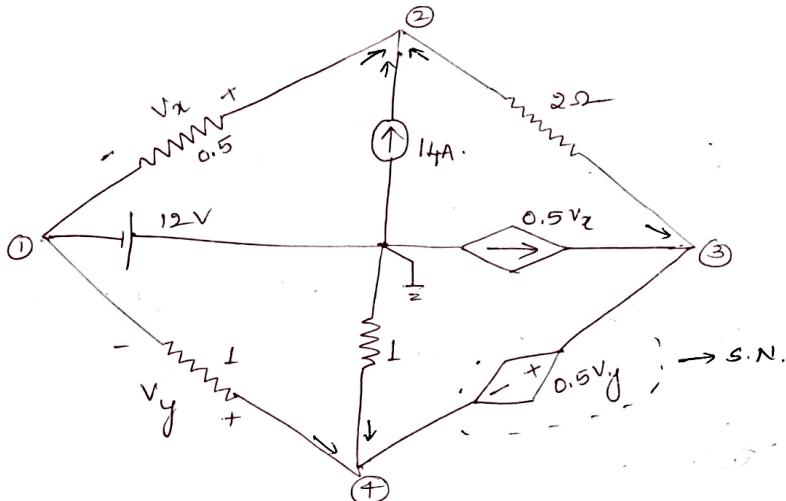
∴ Supernode eqn. is,

$$-4 + \frac{v_2 - v_1}{2} + \frac{v_2}{4} + \frac{v_3}{5} - 10 + \frac{v_3 - v_1}{10} = 0$$

$$-0.6V_1 + 0.95V_2 + 0.3V_3 = 14 \rightarrow (3)$$

$$\therefore \boxed{V_1 = -9.44V; V_2 = 2.22V; V_3 = 22.22V}$$

31

 $\Rightarrow \underline{\text{Sol'n.}}$

$$\underline{\text{Node 1}}: V_1 = -12V$$

$$\underline{\text{Node 2}}: 14 + \frac{V_1 - V_2}{0.5} + \frac{V_3 - V_2}{2} = 0$$

$$-2.5V_2 + 0.5V_3 = 10 \rightarrow (1)$$

$$V_3 - V_4 = 0.5V_y$$

$$\text{But } V_y = V_4 - V_1 = V_4 + 12$$

$$\therefore V_3 - V_4 = 0.5 [V_4 + 12]$$

$$V_3 - 1.5V_4 = 6 \rightarrow (2)$$

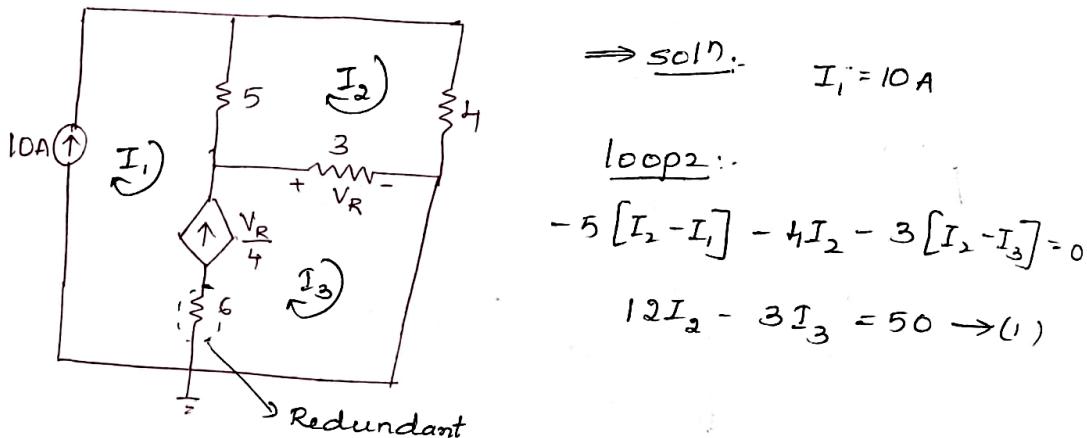
Supernode eqn is,

$$\frac{V_2 - V_3}{2} + 0.5V_x + \frac{0 - V_4}{1} + \frac{V_1 - V_4}{1} = 0. \quad ; \quad \begin{aligned} \text{put } V_x &= V_2 - V_1 \\ V_1 &= -12V \end{aligned}$$

$$V_2 - 0.5V_3 - 2V_4 = 6 \rightarrow (3)$$

$$\boxed{\begin{aligned}V_1 &= -12 \text{ V} \\V_2 &= -4.24 \text{ V} \\V_3 &= -1.224 \text{ V} \\V_4 &= -4.8 \text{ V}\end{aligned}}$$

32. Find I_1 , I_2 & I_3



loop 3:-

$$I_3 - I_1 = \frac{V_R}{4}$$

$$\text{But } V_R = 3(I_3 - I_2)$$

$$\therefore 4(I_3 - I_1) = 3(I_3 - I_2)$$

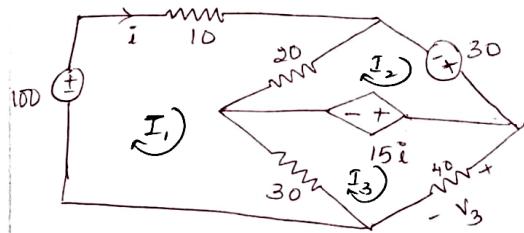
$$4I_3 - 3I_3 - 4I_1 + 3I_2 = 0$$

$$-4I_1 + 3I_2 + I_3 = 0$$

$$3I_2 + I_3 = 40 \rightarrow (2)$$

$$\therefore \boxed{\begin{aligned}I_1 &= 10 \text{ A} \\I_2 &= 8.095 \text{ A} \\I_3 &= 15.714 \text{ A}\end{aligned}}$$

33. Use mesh analysis to find V_3



→ Soln:-

$$\text{Loop 1: } 100 - 10I_1 - 20[I_1 - I_2] - 30[I_1 - I_3] = 0$$

$$60I_1 - 20I_2 - 30I_3 = 100 \rightarrow (1)$$

Loop 2:-

$$-20[I_2 - I_1] + 30 - 15I_2 = 0$$

$$\text{But } i = I_1$$

$$-20I_2 + 20I_1 + 30 - 15I_2 = 0$$

$$5I_1 - 20I_2 = -30$$

$$-5I_1 + 20I_2 = 30 \rightarrow (2)$$

Loop 3:-

$$-30[3 - I_1] - 40I_3 + 15I_1 = 0$$

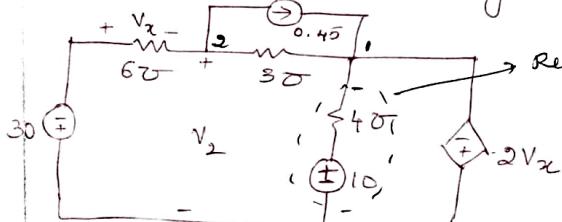
$$45I_1 - 70I_3 = 0 \rightarrow (3)$$

$$\therefore I_1 = 3.64 \text{ A}, I_2 = 2.41 \text{ A}, I_3 = 2.34 \text{ V}$$

$$\therefore V_3 = 40I_3 = 40 \times 2.34$$

$$\boxed{V_3 = 93.6 \text{ V}}$$

34. Find V_2 & V_x using node analysis.



redundant 4Ω is in series with $10V$ is redundant in terms of vtg. \therefore this element can be neglected.

$$\text{Node 2: } 0.45 + (V_2 - V_1)3 + (V_2 + 30)6 = 0.$$

$$0.45 + 3V_2 - 3V_1 + 6V_2 + 180 = 0$$

$$-3V_1 + 9V_2 = -180.45$$

$$-3(-2V_2) + 9V_2 = -180.45$$

$$6V_2 + 9V_2 = -180.45 \rightarrow (1)$$

$$\text{But } V_1 = -2V_2$$

$$\therefore V_2 = -30 - V_2$$

\therefore put in (1)

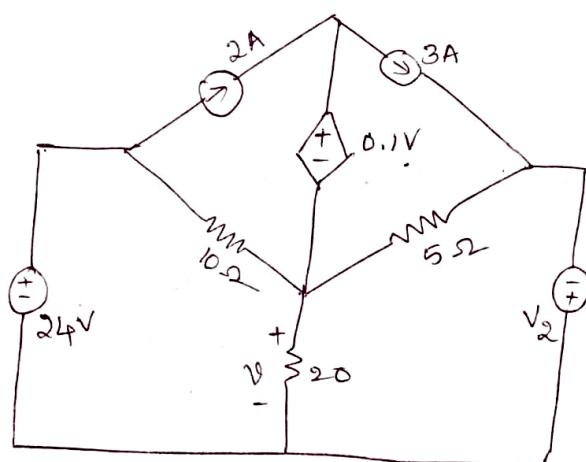
$$6[-30 - V_2] + 9V_2 = -180.45$$

$$\therefore \boxed{V_2 = -0.15V}$$

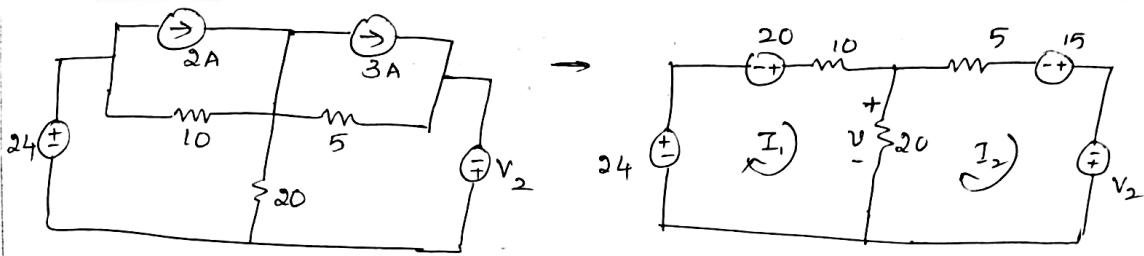
$$\therefore V_x = -30 - V_2 = -30 + 0.15$$

$$\boxed{V_x = -29.85V}$$

35. Use mesh analysis to find what value of V_2 in the n/w. causes $v=0$, where 'v' is voltage across 20Ω .



Since it is necessary to find V_2 to make $v=0$, dependent v.s. $0.1V$ becomes '0' [sc]. The two e.s. can be converted into v.s.



$$V = 0 = 20 [I_1 - I_2]$$

$$I_1 - I_2 = 0 ; I_1 = I_2$$

Loop 1 :- $24 + 20 - 10I_1 - 20 [I_1 - I_2] = 0$

$$44 = 10I_1$$

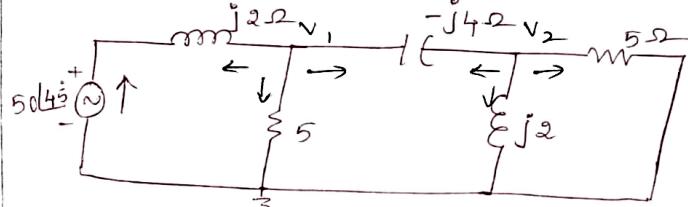
$$I_1 = 4.4 A = I_2$$

Loop 2 :- $-5I_2 + 15 + V_2 - 20 = 0$

$$-5(4.4) + 15 + V_2 = 0$$

$$\therefore \boxed{V_2 = 7V}$$

36. Find current through 5Ω using node analysis



Soln:-

Node 1 :- $\frac{V_1 - 50 \angle 45}{j2} + \frac{V_1}{5} + \frac{V_1 - V_2}{-j4} = 0$

$$(0.2 - j0.25)V_1 - j0.25V_2 = 25 \angle -45^\circ \rightarrow (1)$$

$$\begin{cases} j = \sqrt{-1} ; j^2 = -1 \\ jL = 1 \angle 90^\circ \\ -jL = 1 \angle -90^\circ \\ \frac{1}{j} = -j ; \frac{1}{-j} = j \end{cases}$$

$$\text{Node 2: } \frac{V_2 - V_1}{-j4} + \frac{V_2}{j2} + \frac{V_2}{5} = 0$$

$$-j0.25V_1 + (0.2 - j0.25V_2) = 0$$

$$\begin{bmatrix} (0.2 - j0.25) & -j0.25 \\ -j0.25 & (0.2 - j0.25) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 25[-45] \\ 0 \end{bmatrix}$$

$$I_{52} = \frac{V_1}{5}$$

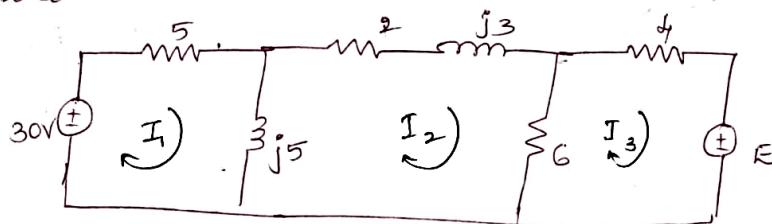
$$V_1 = \frac{\begin{vmatrix} 25[-45] & -j0.25 \\ 0 & (0.2 - j0.25) \end{vmatrix}}{\Delta}$$

$$V_1 = \frac{(25[-45])(0.2 - j0.25)}{(0.2 - j0.25)^2 + (0.25)^2} = 74.31[-28.1]$$

$$= (65.5 - j35.03) V$$

$$\therefore I_{52} = (13.01 - 7j) A$$

* 37 Use mesh analysis to determine current in $(2+j3)\omega$ if $E = 20V$. Calculate E to make the current in $(2+j3)\omega = 0$



\Rightarrow Soln: i) $E = 20V$

$$\begin{bmatrix} (5+j5) & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -20 \end{bmatrix}$$

$$I_2 = \frac{(5+j5)(-120) - 30(-j50)}{(5+j5)[(80+j80)-30] + j5(-j50)} = \frac{-600+j900}{70+620j} = \frac{-600+j900}{70+620j},$$

$$I_2 = 1.733 \angle 40.13^\circ A = 1.32 + j1.127$$

i) $I_2 = 0$

$$\begin{bmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -E_2 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 5+j5 & 30 & 0 \\ -j5 & 0 & -6 \\ 0 & -E_2 & 10 \end{vmatrix}}{\Delta} = 0.$$

$$(5+j5)(-6E) - 30[-10(5j)] = 0$$

$$E = 35.35 \angle 45^\circ V$$

38. The vtg of a node of a n/w is given by

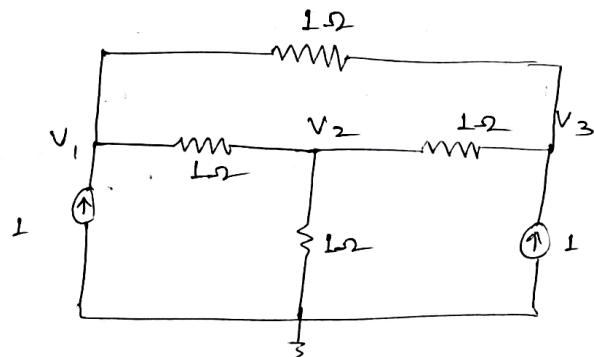
$$V_2 = \frac{\begin{vmatrix} 2 & 1 & -1 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{vmatrix}}$$

Draw the electrical n/w.

Hint:
If vtg is
given:- Node
analysis

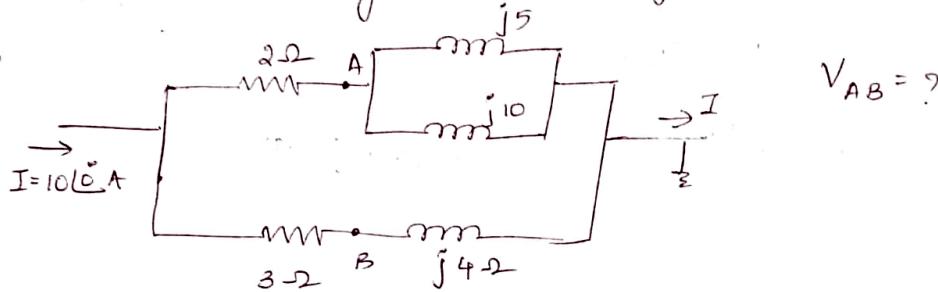
→ Soln: $[Y][V] = [I]$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$



39. Find V_{AB} using node analysis.

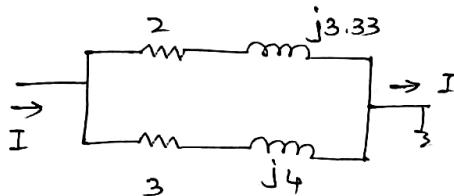
May
June-10



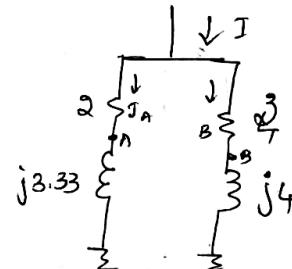
$$V_{AB} = ?$$

→ Soln:- $V_{AB} = V_A - V_B$

$$j5 \parallel j10 = \frac{j5 \cdot j10}{j15} = \frac{-50}{j15} = j3.33\angle -2.57^\circ$$



(or)

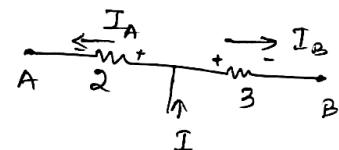


$$I_A = \frac{(3 + 4j) \cdot 10\angle 0^\circ}{5 + 7.33j}$$

$$I_A = 5.635 \angle -2.57^\circ = (5.62 - 0.25j) A$$

$$I_B = \frac{(2 + j3.33) \cdot 10\angle 0^\circ}{5 + 7.33j}$$

$$I_B = 4.377 \angle 3.309^\circ A$$

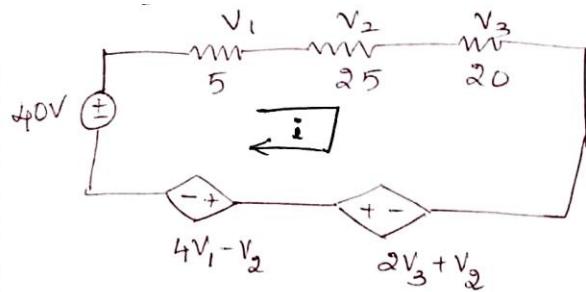


$$V_{AB} = 3I_B - 2I_A$$

$$\hookrightarrow = 2.2562 \angle 34.264^\circ V$$

$$\hookrightarrow = 1.8647 + j1.2703 V$$

40 Find power across $5\Omega, 20\Omega, 25\Omega$ using KVL



$$40 - 5i - 25i - 20i + 2V_3 + V_2 - 4V_1 + V_2 = 0$$

$$40 - 50i + 2V_3 + 2V_2 - 4V_1 = 0$$

But $V_1 = 5i$, $V_2 = 25i$, $V_3 = 20i$

$$40 - 50i + 2(5i) + 2(25i) - 4(20i) = 0$$

$$\therefore i = \frac{-40}{26} = -2A$$

$$P_{5,2} = V_1 i = 5i \times i = 20W$$

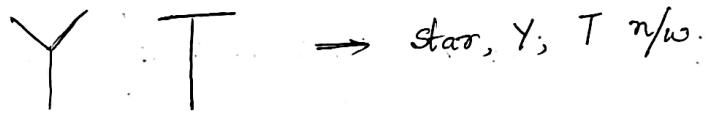
$$P_{25,2} = V_2 i = 25i \times i = 100W$$

$$P_{20,2} = V_3 i = 20i \times i = 80W$$

$$P_{(2V_3+V_2)} = (2V_3 + V_2) i = [40i + 25i] i = 220W.$$

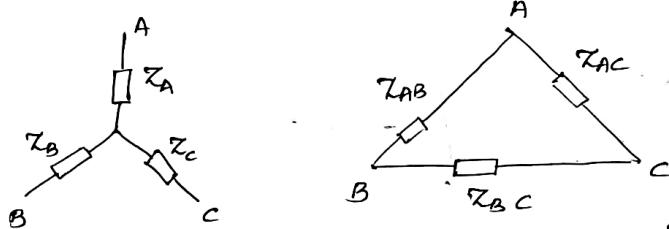
$$P_{(4V_1-V_2)} = (4V_1 - V_2) i = -20W.$$

* STAR - DELTA TRANSFORMATION :-



When a n/w. contains large no. of loops (or) nodes, application of KVL & KCL becomes complicated. In such cases it is possible to simplify the n/w. by using star- Δ transform.

Consider the star & delta n/w. given below.



For two n/w's to be equivalent, they should have same i/p, o/p & transfer impedances. The impedance b/w 'A' & 'B' are:

$$Z_A + Z_B = Z_{AB} \parallel (Z_{BC} + Z_{AC})$$

$$\hookrightarrow = \frac{Z_{AB} (Z_{BC} + Z_{AC})}{Z_{AB} + Z_{BC} + Z_{AC}} = \frac{Z_{AB} Z_{BC} + Z_{AB} Z_{AC}}{\sum Z_{AB}} \rightarrow (1)$$

Impedance b/w 'B' & 'C',

$$Z_B + Z_C = Z_{BC} \parallel (Z_{AC} + Z_{AB}) = \frac{Z_{BC} (Z_{AC} + Z_{AB})}{\sum Z_{AB}}$$

$$\hookrightarrow = \frac{Z_{AC} Z_{BC} + Z_{AB} Z_{BC}}{\sum Z_{AB}} \rightarrow (2)$$

$$Z_C + Z_A = Z_{AC} \parallel (Z_{BC} + Z_{AB}) = \frac{Z_{AC} Z_{BC} + Z_{AC} Z_{AB}}{\sum Z_{AB}} \rightarrow (3)$$

(i) Delta - star conversion :-

(1) - (2)

$$Z_A + Z_B - Z_C = \frac{Z_{AB}Z_{AC} + Z_{AB}\cancel{Z_{BC}} - Z_{AB}\cancel{Z_{AC}} - Z_{BC}Z_{AC}}{\sum Z_{AB}}$$

$$Z_A - Z_C = \frac{Z_{AB}Z_{AC} - Z_{BC}Z_{AC}}{\sum Z_{AB}} \rightarrow (4)$$

(3) + (4)

$$Z_A + Z_C + Z_A - Z_C = \frac{Z_{AC}Z_{AB} + Z_{AC}\cancel{Z_{BC}} + Z_{AB}Z_{AC} - Z_{BC}\cancel{Z_{AC}}}{\sum Z_{AB}}$$

$$2Z_A = \frac{2Z_{AB}Z_{AC}}{\sum Z_{AB}}$$

$$\therefore \boxed{Z_A = \frac{Z_{AB}Z_{AC}}{\sum Z_{AB}}} \rightarrow (5)$$

$$Z_B = \frac{Z_{AB}Z_{BC}}{\sum Z_{AB}} \rightarrow (6)$$

$$Z_C = \frac{Z_{AC}Z_{BC}}{\sum Z_{AB}} \rightarrow (7)$$

Eqns. (5), (6) & (7) are used for Δ -Y conversionii) Star - delta conversion :-

(5) x (6)

$$Z_A Z_B = \frac{Z_{AB}^2 Z_{BC} Z_{AC}}{\sum^2 Z_{AB}} \rightarrow (8)$$

(6) x (7)

$$Z_B Z_C = \frac{Z_{BC}^2 Z_{AB} Z_{AC}}{\sum^2 Z_{AB}} \rightarrow (9)$$

(6) + (7)

$$Z_A \cdot Z_C = \frac{Z_{AB} Z_{BC} Z_{AC}^2}{\sum Z_{AB}} \rightarrow (10)$$

(8) + (9) + (10)

$$Z_A Z_B + Z_B Z_C + Z_A Z_C = \frac{Z_{AB} Z_{BC} Z_{AC} (Z_{AB} + Z_{BC} + Z_{AC})}{\sum Z_{AB}}$$

$$L = \frac{Z_{AB} Z_{BC} Z_{AC}}{\sum Z_{AB}}$$

Substitute eqn. (7)

$$Z_A Z_B + Z_B Z_C + Z_A Z_C = Z_{AB} Z_e$$

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_C}$$

$$\therefore Z_{AB} = Z_A + Z_B + \frac{Z_A Z_B}{Z_C} \rightarrow (11)$$

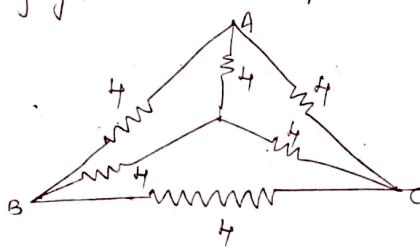
$$Z_{BC} = Z_B + Z_C + \frac{Z_B Z_C}{Z_A} \rightarrow (12)$$

$$Z_{AC} = Z_A + Z_C + \frac{Z_A Z_C}{Z_B} \rightarrow (13)$$

(11), (12) & (13) are used for γ to Δ

42. Six equal resistors of 4Ω are connected as shown in fig. Find equivalent Δ b/w any two nodes.

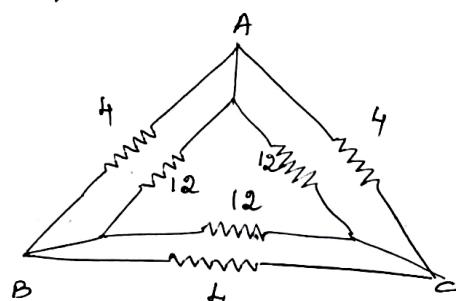
* NOTE:
Start simplification from innermost part of the n/w.

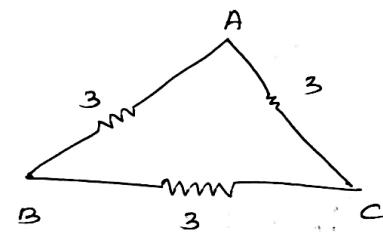


\Rightarrow Soln: $\gamma - \Delta$.

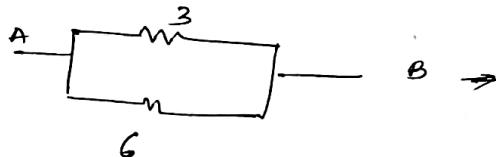
$$R_{AB} = R_{BC} = R_{AC} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$\begin{aligned} &= 4 + 4 + \frac{16}{4} \\ &\Rightarrow = 12 \Omega \end{aligned}$$

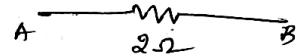




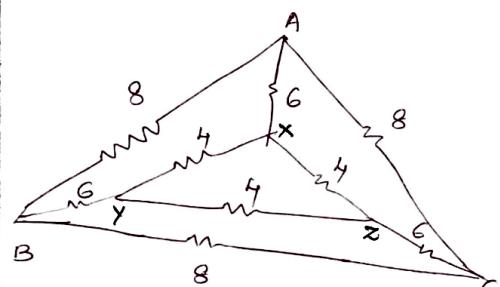
$$12 \parallel 4 = \frac{12 \times 4}{16} = 3 \Omega$$



$$R_{AB} = 3 \parallel 6 = \frac{3 \times 6}{9} = 2 \Omega$$



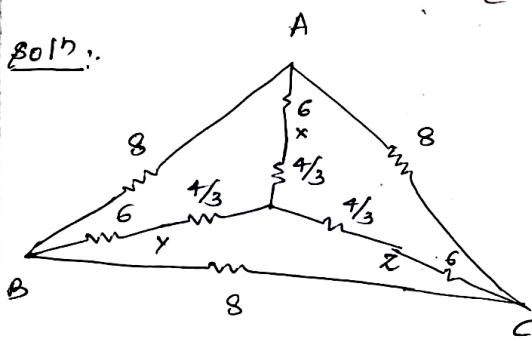
43.

A to Y :-

$$R_x = R_y = R_z$$

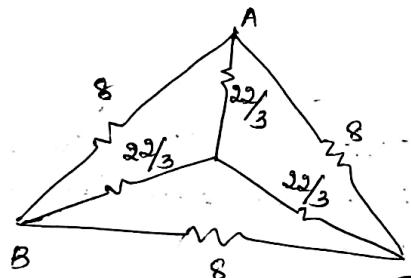
$$\therefore R_x = \frac{R_{xy} R_{xz}}{R_{xy} + R_{xz}}$$

→



$$L = \frac{4 \times 4}{12} = \frac{4}{3}$$

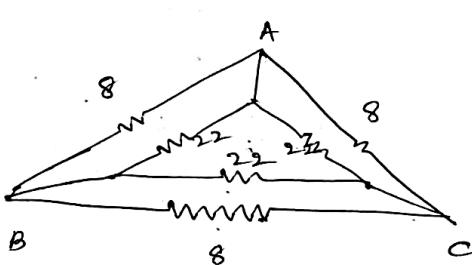
$$\frac{4}{3} + 6 = \frac{22}{3}$$

Y to A :-

$$R_{AB} = R_{BC} = R_{CA}$$

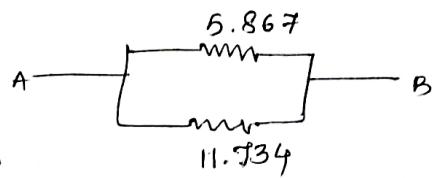
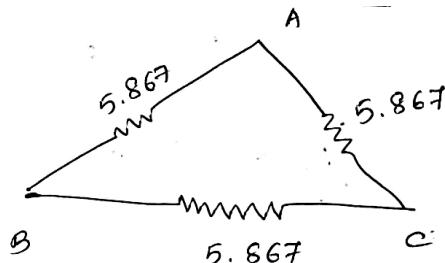
$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$= \frac{22}{3} + \frac{22}{3} + \frac{(22/3)^2}{22/3}$$



$$R_{AB} = 22 \Omega$$

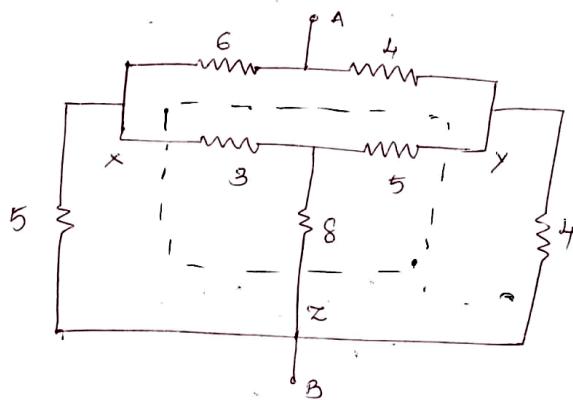
$$8 \parallel 22 = \frac{8 \times 22}{30} = 5.867 \Omega$$



$$R_{eq} = R_{AB} = 5.867 \parallel 11.734$$

$$\boxed{R_{AB} = 3.9113 \Omega}$$

*44 Determine \triangle b/w A & B



$$R_{xy} = R_x + R_y + \frac{R_x R_y}{R_z}$$

$$\therefore = 3 + 5 + \frac{15}{8} = 9.875$$

$$R_{yz} = R_y + R_z + \frac{R_y R_z}{R_x}$$

$$\therefore = 5 + 8 + \frac{40}{3}$$

$$\therefore = 26.33 \Omega$$

$$R_{xz} = R_x + R_z + \frac{R_x R_z}{R_y}$$

$$= 3 + 8 + \frac{24}{5}$$

$$\therefore = 15.8 \Omega$$

$$5 \parallel 15.8 = 3.79 \Omega$$

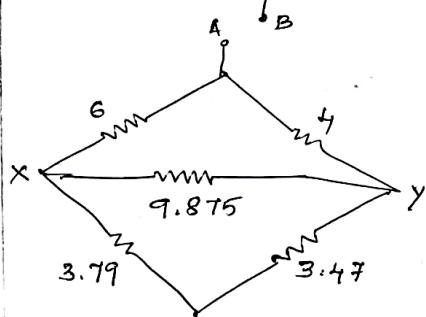
$$4 \parallel 26.33 = 3.472 \Omega$$

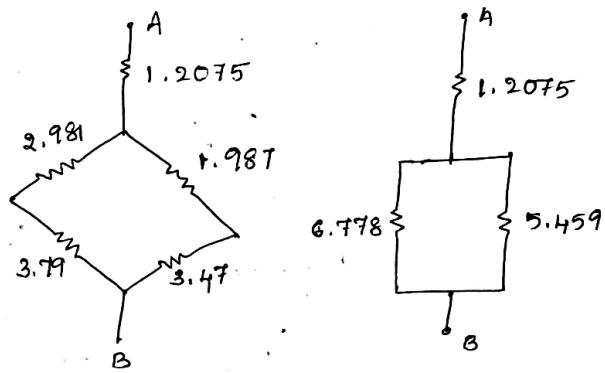
Δ to \triangle

$$R_A = \frac{R_{xy} R_{xz}}{R_{xy} + R_{xz}} = \frac{6 \cdot 4}{19.875} = 1.2075 \Omega$$

$$R_X = \frac{R_{xy} \cdot R_{xz}}{R_{xy} + R_{xz}} = \frac{9.875 \cdot 6}{19.875} = 2.981 \Omega$$

$$R_Y = \frac{R_{xy} \cdot R_{xz}}{R_{xy} + R_{xz}} = \frac{9.875 \cdot 4}{19.875} = 1.981 \Omega$$

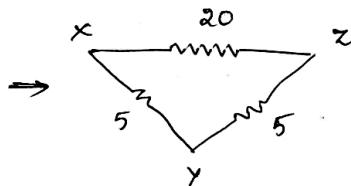
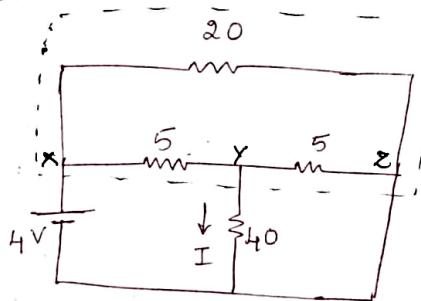




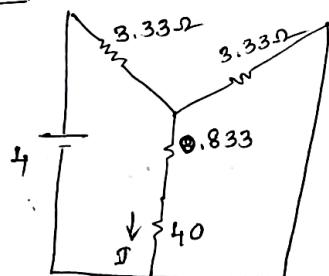
$$R_{AB} = 1.2075 + \left[6.778 // 5.459 \right]$$

$$\boxed{R_{AB} = 4.231\Omega}$$

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calculate current in 40Ω resistor.

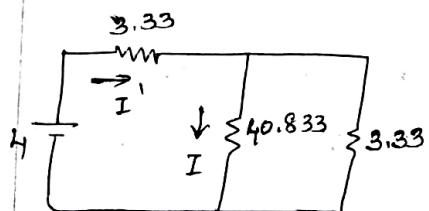
→ soln:-



$$R_x = \frac{R_{xy} R_{xz}}{R_{xy} + R_{xz}} = \frac{5 \cdot 20}{30} = 3.33\Omega$$

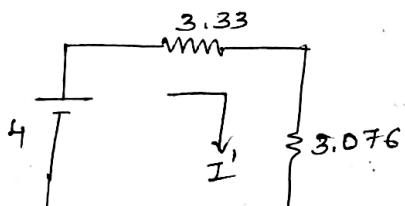
$$R_y = \frac{R_{xy} R_{yz}}{R_{xy} + R_{yz}} = \frac{20 \cdot 5}{30} = 0.833\Omega$$

$$R_z = \frac{R_{xz} R_{yz}}{R_{xz} + R_{yz}} = \frac{20 \cdot 5}{30} = 3.33\Omega$$



$$I' = \frac{3.33 \times I}{40.833 + 3.33} = 0.0754 I'$$

$$40.833 // 3.33 = 3.076\Omega$$

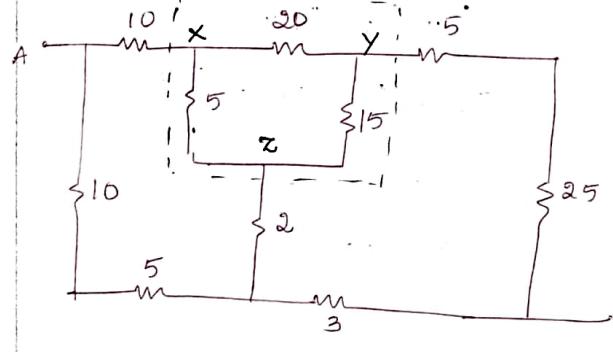


$$I' = \frac{4}{3.33 + 3.076} = 0.624A$$

$$\therefore I = 0.0754 \times 0.624$$

$$\boxed{I = 0.047A}$$

Q6. Find resistance b/w A & B

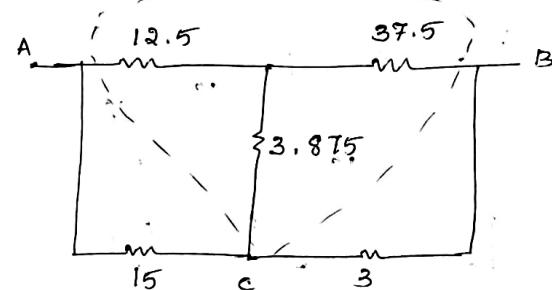
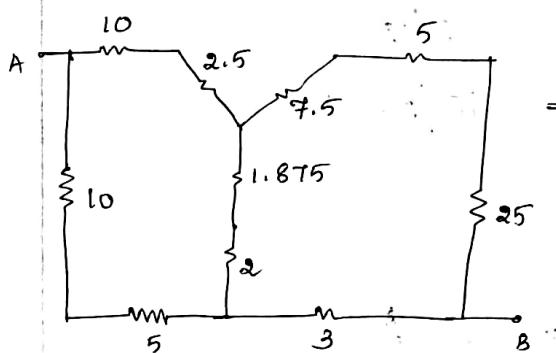


A to Y:

$$R_X = \frac{R_{XY} R_{XZ}}{\Sigma R_{XY}} = \frac{5 \cdot 20}{40} = 2.5$$

$$R_Y = \frac{R_{XY} R_{YZ}}{\Sigma R_{XY}} = \frac{20 \cdot 15}{40} = 7.5$$

$$R_Z = \frac{R_{YZ} R_{ZX}}{\Sigma R_{YZ}} = \frac{15 \cdot 5}{40} = 1.875$$



Y to Δ

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} = 12.5 + 37.5 + \frac{12.5 \cdot 37.5}{3.875}$$

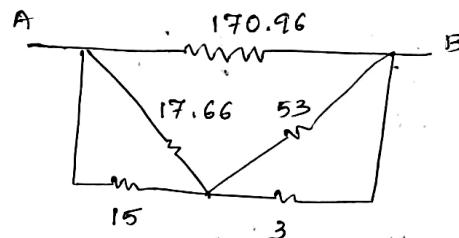
$$\therefore R_{AB} = 170.96 \Omega$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} = 37.5 + 3.875 + \frac{37.5 \cdot 3.875}{12.5}$$

$$\therefore R_{BC} = 53 \Omega$$

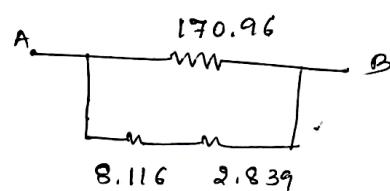
$$R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B} = 12.5 + 3.875 + \frac{12.5 \cdot 3.875}{37.5}$$

$$\therefore R_{AC} = 17.66 \Omega$$



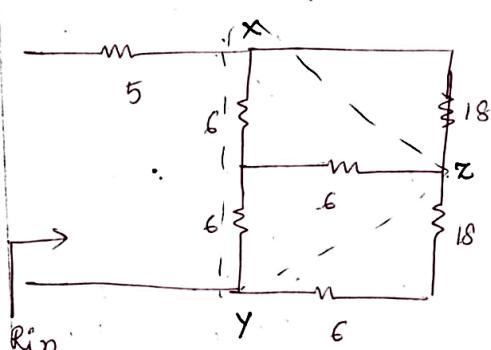
$$15 \parallel 17.66 = 8.116 \Omega$$

$$53 \parallel 3 = 2.839 \Omega$$



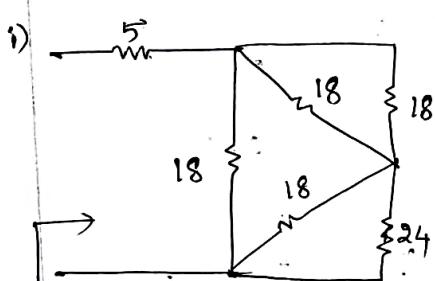
$$\therefore R_{AB} = (8.116 + 2.839) \parallel 170.96$$

$$\therefore R_{AB} = 10.29 \Omega$$

77. Find R_{in} 

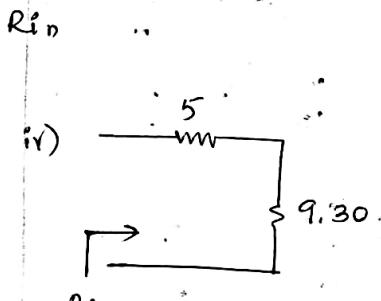
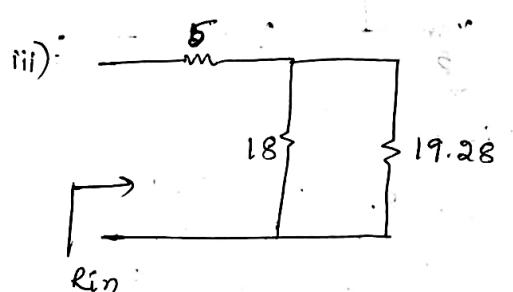
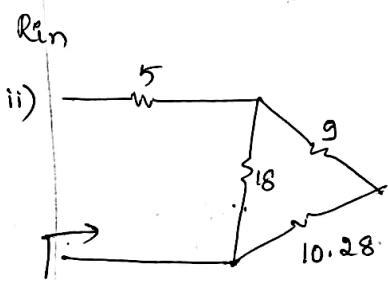
$$R_{xy} = R_{yz} = R_{zx}$$

$$\begin{aligned} R_{xy} &= R_x + R_y + \frac{R_x R_y}{R_z} \\ &= 6 + 6 + \frac{6 \times 6}{6} \\ \therefore &= 18 \end{aligned}$$



$$18 \parallel 18 = 9\text{-}\Omega$$

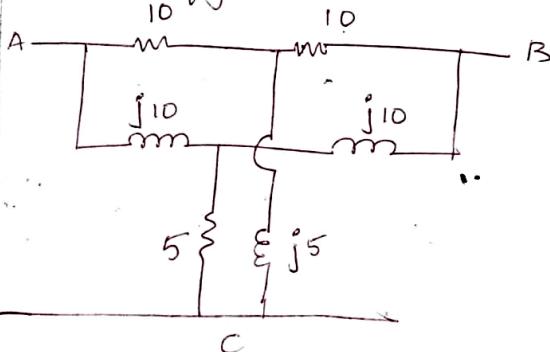
$$18 \parallel 24 = 10.28$$

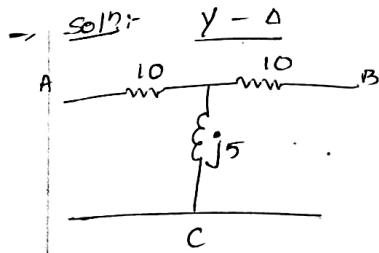


$$\therefore R_{in} = 14.30\text{-}\Omega$$

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Simplify the given twin-T n/w.





$$Z_{AB} = Z_A + Z_B + \frac{Z_A Z_B}{Z_\Delta}$$

$$= 10 + 10 + \frac{100}{j5}$$

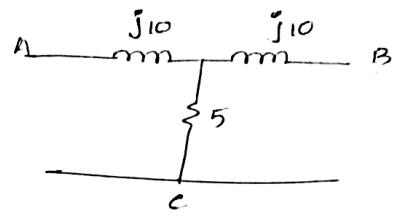
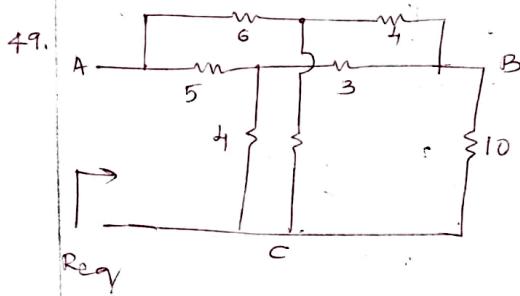
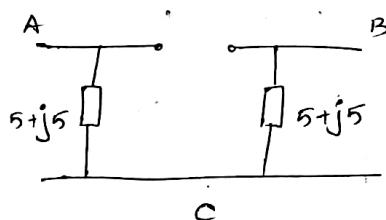
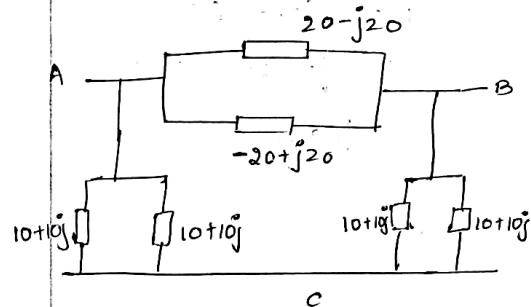
$$\hookrightarrow = 20 - j20$$

$$Z_{BC} = 10 + j5 + \frac{50j}{10}$$

$$\hookrightarrow = 10 + 10j$$

$$Z_{AC} = 10 + j5 + \frac{50j}{10}$$

$$\hookrightarrow = 10 + 10j$$



$$Z_{AB} = j10 + j10 + \frac{100j^2}{5}$$

$$\hookrightarrow = 20j - 20$$

$$Z_{BC} = j10 + 5 + \frac{50j}{10j}$$

$$\hookrightarrow = 10 + j10$$

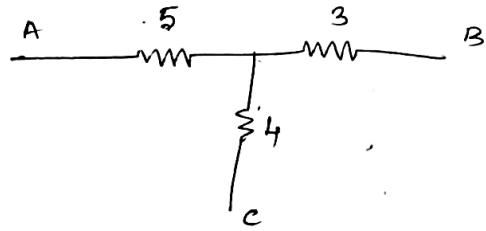
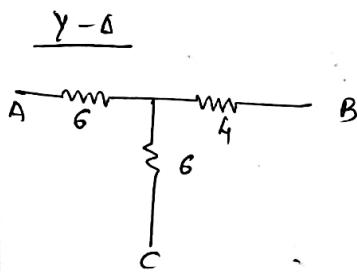
$$Z_{AC} = j10 + 5 + 5$$

$$\hookrightarrow = 10 + 10j$$

$$(20 - j20) // (-20 + 20j)$$

$$\frac{(20 - j20) \cdot (-20 + 20j)}{20 - j20 - 20 + j20} = \infty [0.C]$$

$$(10 + j10) // -(10 + j10) = (5 + j5) \Omega$$



$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_\text{e}}$$

$$= 6 + 4 + \frac{24}{6}$$

$$\hookrightarrow = 14\text{-}\Omega$$

$$R_{AB} = 5 + 3 + \frac{15}{4} = 11.75$$

$$R_{BC} = 4 + 6 + \frac{24}{6}$$

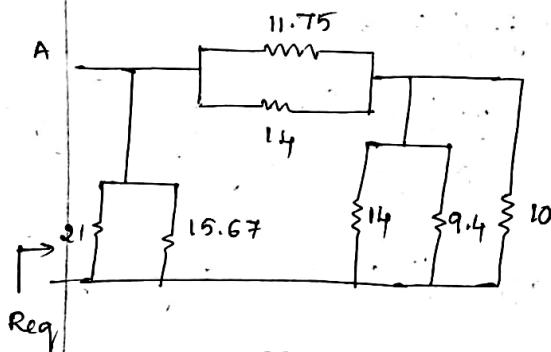
$$\hookrightarrow = 14\text{-}\Omega$$

$$R_{BC} = 3 + 4 + \frac{12}{5} = 9.4\text{-}\Omega$$

$$R_{AC} = 6 + 6 + \frac{36}{5}$$

$$\hookrightarrow = 21\text{-}\Omega$$

$$R_{CA} = 5 + 4 + \frac{20}{3} = 15.67\text{-}\Omega$$

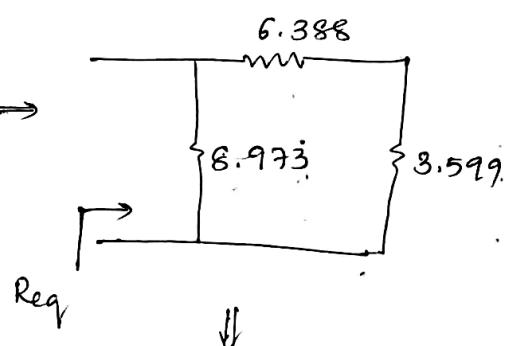
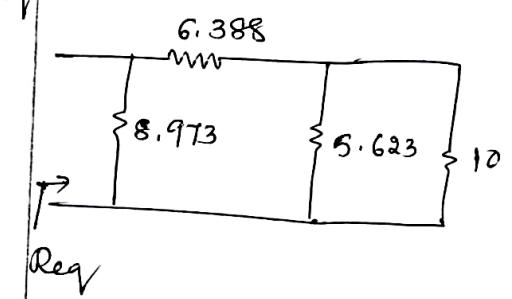


$$11.75 \parallel 14 = 6.388\text{-}\Omega$$

$$21 \parallel 15.67 = 8.973\text{-}\Omega$$

$$14 \parallel 9.4 = 5.623\text{-}\Omega$$

• 8.9



$$\therefore \text{Req} = \frac{89.6133}{18.96}$$

$$\boxed{\text{Req} = 4.72\text{-}\Omega}$$

