PORTFOLIO OPTIMISATION

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Title and Abstract:

The main aim of this project is to minimize the risk and maximize the returns of an investment in stocks and financial bonds.

In today's word financial crisis is considered as one of the major problems in human life. People wants to earn/save money with low risk but they don't have any idea where to invest. Among them 55% of the population in developed countries will invest their money in stocks to get high returns. Most of the people will invest money without any knowledge of stock market and other assets. People will lose their money because of Inappropriate allocation of resources and bad decisions. This project will give an idea about assets and proper allocation of money on good stocks. User can expect good returns with minimum risk and we are also providing quarterly based returns based on his/her risk level criteria. We will also show some graphs of financial assets and volatility v/s return charts so, user can easily understand his/her risk level and returns.

I. INTRODUCTION

The portfolio management is one of the major ways of minimizing risk and maximize returns. It motivates the customer/users to invest money on specific assets and customer can expect good returns. Portfolio is not a new topic in financial industry, people are trying to optimize stocks, bonds and financial assets. We can see lot of companies adopted optimization technique and they are providing their own solutions to customers. In this project we will provide some good assets to invest and also, we will suggest bonds for quarterly based returns.

Portfolio is essentially a basket of security the stocks of some financial instruments. An asset is an object which is regularly sold and bought in a financial market. If we have X_0 is our initial investment, So that we buy 'n' different stocks of different companies asset distribute X_0 to other company assets.

$$X_0w_1, X_0w_2, X_0w_3 \dots \dots \dots \dots \dots \; X_0w_n$$

$$\Rightarrow \sum_{i=1}^{n} w_i = 1$$

Portfolio also be viewed by collection of weights

$$\text{Portfolio} = (w_1, w_2, w_3, w_4 \dots \dots w_n)^T$$

The goal of the portfolio management is to provide a good optimal solution to customer to select his/her assets and make sure he will get good return with low risk. In other words, portfolio management is used to select, prioritize and control company programmes and assets. The goal of the implementation of portfolio is to maintain business in stable away and make sure we will get good returns.

To achieve good optimization, we are proposing Markowitz's model, this model will give us an idea of investment on selected assets and also minimizes the risk and increase the returns. Apart from Markowitz's model we will draw the efficient frontier using Monte Carlo model. Before optimization we are going to select some assets from the market, for selecting those assets we will Calculate ROCE, PER, EPS and liquidity ratio. Based on these key terms we will choose assets for optimization.

Once we are done with the optimization, we are going to represent our results in graphs and charts so customer can easily understand returns and risk of the choosing those assets. We will also represent our results in text format so user can read and understand. This models also shows the covariance of assets means relationship between selected assets and annual growth of each asset. We will show the sharpe ratio, minimum and maximum sharpe ratio, return vs Volatility charts, expected return, Expected volatility. This all results will give and idea about good investment returns.

Now a days in financial market we can see lot of optimization technique for stock assets, But our implementation is different compare to all other models, because we used Double stage evolutionary algorithm. The idea of double stage evolutionary algorithm is to two ways of filtering stocks and generate optimization report for selected stocks. In the first stage we will select assets based on key terms, we already mentioned those key terms (ROCE, PER, EPS, Liquidity ratio). In the second stage we will generate 10000 portfolios for these assets and we will choose maximum sharpe ratio from this portfolio. And we will apply Markowitz's model to minimize the risk and increase the return rates. We will also see a bit of Monte carlo model to generate minimum volatility rate and maximum return rate.

II. BACKGROUND ANALYSIS

Our optimization is related to several approaches in the assets and other investments, but we will mainly concentrate on stocks and bonds. Unifying and demystifying these seemingly disparate frameworks is one of our theoretical contributions. An important area of recent research is the use of shrinkage to estimate variance–covariance [10]. Random matrix theory is also one of the methods to improve variance-covariance model [3]. We found that these approaches are outperforms because optimization techniques use much bigger shrinkages for estimating volatility and returns.

Secondly, Black and Litterman (1992) they focus only on returns and they didn't even care about risk, including they assumed some parameters are constant (mean). People will find very difficult to apply and failed to understand where the expected results come from. Although our approach is completely different from these approaches, we show our mathematical functions and risk analysis is different. Indeed, our optimization function is simple to understand and user can interact with our application. Our expression, correlation matrices can help to address the expected returns and relationship among the stocks.

Thirdly, as a regression coefficient, the standard MVO measures how well a constant performs in regressing on realized returns on a constant. In Machine learning techniques we have lot of way to regularize the regressions. Some analysts proves that LASSO regression is significantly increases the performance. To generate most optimized portfolio, we must consider some points related to variance-covariance matrix, elastic net regression. We are not going deeply into regression part because we are doing minimization of risk by using Markowitz model.

Our optimization results and enhancement of Monte Carlo and Markowitz model will give us the better performance in real data. Risk-return relationship is also playing an important role to generate good optimization model. Some researchers tried to enhance Markowitz's model in trivial strategies but I found some bugs in those cases because they are not properly selecting dataset and they don't apply any methodology to select good assets. So at the end if we choose wrong stock or asset there is no point of Enhancing models.

There is one common question in mind, How we can calculate risk or volatility across different risky assets and how we can reduce the risk as well as increase the rate of return rate? After the introduction of Markowitz's theory researchers can able to find out the way to minimize the risk. These model mainly focuses on measuring a portfolio's on each assets and calculate rate of return rate and it gives an facility to minimize the risk in mathematical way, And Markowitz is succeeded to prove this will be the best model till date and he got Nobel prize for this achievement.

Moreover, a study of original work by Markowitz (1952) is an way of measuring a risk by variance. This is the one of the most convenient ways to justify normally scatter the financial returns. There is another researcher proposed his facts in 2011 (Danielson 2011) variance is a yield of inaccurate risk estimation and it turns into wrong estimation of portfolio. The reason beyond this research is deficiency of risk calculation in asset allocation is strongly diverted when we try to truly apply the optimization. And another main reason is Markowitz model doesn't well suited for bad assets and it gives flaw report if we apply this model to inconsistent stocks or assets.

In our case we no need to worry about choosing bad assets or insufficient risks because we are filtering assets before applying Markowitz or Monte Carlo model. Monte Carlo model is also an one of the best designed algorithm in the way of identifying low-risk portfolio. It uses some Average daily return, Periodic daily return and Next day's Price for calculation.

I got inspired by Markowitz idea of evaluation of risk and minimization of risk. We just need to utilizes those function and we need to apply or define our own way of calculation of rate of return then it performs much better than earlier models. The main thing we have to consider for Markowitz's model is asset selection. It plays a vital role in the optimization. we should not include more than 3 bad assets in the optimization algorithm. I tried with 10 – 15 stocks but its performance is going down as I started including bad assets into an optimization. So finally, I ended up to optimize these portfolios by using Six stocks. I am considering last 5 years of historical data of these stocks and I will apply minimization function.

There are number of portfolio optimization techniques are available in the market and they are following their own way of giving solution to customers. Our optimization technique is different from other portfolio's because we are using double stage genetic algorithm, we are show casing our stocks to customers and customers can propose their own stocks based on his analysis or he can go-ahead with our selected stocks. Basically, we are providing flexible way of choosing his risk level and he can know the complete details of our analysis.

For selecting good stocks from market, we are calculating Return of capital employed, price earning ratio, Earning per share and liquidity ratio. Based on these four key terms we will choose stocks to play around in optimization. Customer also propose his own stocks based on his analysis in the market but we prefer our stocks for good optimal solution. Once we are done with selection, we will start collecting data of stocks and we apply our optimization technique.

We are proposing Markowitz's model for optimization because this model is considered has one of the best models from 1952. It eliminates the number of estimation and it only suggests the good optimal results, it will reduce the risk of investment and increase the return rate of amount. We also have other methods but Markowitz's uses weights to reduce the risk. So, it is considered has one of the feasible optimization techniques till date. We will also see Monte carlo method of optimization but it will give us only minimum volatility and maximum returns in generated portfolio it doesn't have an advantage of minimizing risk.

Markowitz's model also called as mean-variance model because it considers volatility, liquidity and transaction cost-based theory. In mean-variance framework it analyses portfolio frontier characteristics with consideration of individual background risk of an assets. In this technique we are using historical data to optimize results. We are taking 5 years of historical data and based on this data we will generate feasible results to get high returns. We will also represent log return, sharpe ratio, expected return and expected volatility. These terms will give us better understanding of investment.

III. AIMS & OBJECTIVES

Basically, we will discuss portfolio methodologies that include different risk evaluation while maintaining a target portfolio return. By using this approach, the ability of handling risk is different.

The main aim of this thesis is to handling the key terms such as maximize the return and minimize the volatility. We are going to list down some official aims in this paper

- Use Double stage algorithm to find out best optimized portfolio's
- Enhance Markowitz's model in order to minimize the risk as well as increase the return rate.
- Make the best and more effective resource for optimization to get good results
- Make sure investors must be happy to work with us by giving optimized results.
- Processing of stocks and bonds and managing historical data by yahoo finance API and make best flawless model for optimization.
- Consider Monte Carlo Model to Grab the minimum risk rate weights and compare with Markowitz's model.
- Identifying desired stocks for portfolio analysis by considering some financial key terms.
- Generate Efficient frontier by using Markowitz model (After applying minimization function).

Objectives:

We have a lot of ways to implement Markowitz and Monte Carlo model. We can use Excel sheet data and other json files to get the historical data of stock. But I am using online APIs to download historical data. We can find lot of way to calculate expected return, risk and Sharpe ratio. I am using mathematical approaches to calculate these terms and I am using Expected log return to process the optimization models. We will use some basic user interaction forms to know the user risk level and his investment amount.

By keeping all the above things (aims and background research) we have adopting some Objective points to develop better portfolio optimization.

The basic plan is to develop good user interface for customer to choose and analyze his investment. So investor gain some basic knowledge about our models. And also, we are giving an option for customer to come up with his/her own stocks/assets. As per the risk level chooses by investor, we are dividing the money into two parts, One for bonds and another for stocks. Firstly, bonds will give quarterly returns to the customer. But stocks don't have an option for quarterly return but customer have a flexibility to allocate money into other stocks.

We will show some evaluation of bonds and how it will work and when customer can get his/her return in the bond Evaluation section. And also, we are providing investors to choose risk level. We are proposing double stage genetic algorithm for optimizing portfolio.

Double stage genetic algorithm works in two phases. In the first stage we are going to select stocks and bonds based on some financial key terms. Bonds are selected based on the highest credit and interest rate. We are considering only UK bonds for this evaluation. For stocks we are doing some research while selecting stocks. We are calculating some financial key terms based on last year return statement and balance sheet. By considering four financial key terms we are selecting stocks to enter optimization model.

After applying Double stage genetic algorithm, we are generating co-variance matrix and other graphs to analyze the asset performance. Co-variance matrix is one of the key terms for optimization because if two stocks are co-related each other then put complete model is going to be wrong. So, we must be careful while analyzing co-variance matrix.

We are generating multiple graphs to analyze Expected return, Expected volatility and Sharpe Ratio. And we are also generating performance of stocks based on yearly data. Generating a Risk vs Return chart is one of the ways to understand the basic analysis of portfolio. After calculating all these financial key terms, we will implement our models based on Monte Carlo and Markowitz's theory.

Monte Carlo model is well suited for generating portfolio weights for low-volatility assets. First, we will implement our model using Monte Carlo theory and we will grab low-volatility weights. Based on these weights we will get an amount to be invested on particular asset. But Monte Carlo model doesn't suit for high return assets.

Markowitz's theory is one of the most robust models for portfolio from 1952. By using Markowitz concept, we will grab high-return assets and we will also apply minimization function then will grab weights generated by minimization function. Finally, we will draw Efficient frontiers of portfolio based on Markowitz model.

We have some different ways to shows the results generated by Monte Carlo and Markowitz's model. We will create separate charts for both models and different portfolios. We are also providing Return vs Volatility chart understand the relationship between risk and returns, it will helps to understand the basic portfolio optimization. We can see this graphs and charts in the Result section.

IV. EXPERIMENTAL DESIGN AND METHODS

As I mentioned we are using two models for this portfolio optimization. Before applying this models, we need to follow some steps. We have to collect the data, remove unwanted data from the frame and generate some graphs whether this data is showing proper results or not. Now we will see the complete flow of the project from user input to model build. Please see the below steps for more details.

- ⇨ Getting Input from User
- ⇨ Bonds Calculation
- ⇨ Choose Stocks to Invest
- ⇨ HOW DID WE PREFER THESE 6 STOCKS FOR SELCTION?
    - Select assets based on ROCE, PER, EPS, Liquidity Ratio.
    - Get income statement and Annual report of stocks
- ⇨ Download the stock data from yahoo finance:
- ⇨ Plot Growth of selected stocks:

- ⇨ Correlation among stocks:
- ⇨ Calculate Required key terms to build a model
  - Expected return
  - Log return
  - Expected volatility
  - Sharpe ratio
- ⇨ Return vs Volatility chart
- ⇨ Monte Carlo Efficient frontier
- ⇨ Markowitz's frontiers

A.Getting Input from User

In the first step we are going to ask the user to enter his/her amount for investment.

Figure 1 : Customer window for amount

Once user decided how much amount he/she wants to invest, next we will show the risk levels to and let him to choose the risk levels.
We have 3 types of risk levels.

1. High risk (80% for stocks and 20% for bonds)
2. Risk (70% for stocks and 30% for bonds)
3. Low risk (60% for stocks and 40% for bonds)

In the below picture user selected his/her risk level 1.

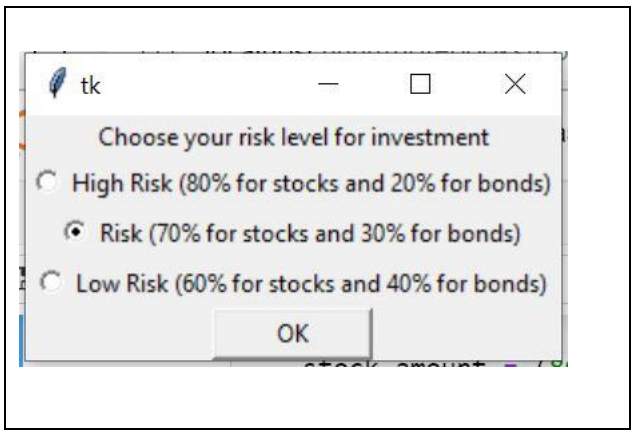


Figure 2: Customer window for Risk Selection

Once we got the risk level and his amount. We are going to split his amount into two parts, one is for stocks and another one is for bonds. Based on above inputs we will allocate 70% of money to stocks and 30% of money to bonds.
The below figure shows how much money is allocated to stock and bonds.

```
Amount You want to invest :5000
You Risk level is :2
<class 'int'> <class 'int'>
3500.0 Will goto Stocks
1500.0 Will goto Bonds
```

Figure 3: Allocation of funds (Bonds & Stocks)

B.Bonds Calculation

We have taken top 3 bonds in UK based on crediting rating and interest. We are going to invest money on these bonds. Below 3 are selected bonds and its interest rates:

1. Barclays Bank plc: coupon rate = 9% , Current value/Face value = 107.37
2. Co-operative Group Ltd : coupon rate = 11%, current value/Face value = 110.15
3. EnQuest plc : coupon rate = 9%, current value/ Face value = 95.00

The below figure shows the bond present value and number of bonds we are going to buy.

	Bonds	Interest Rates	Current Price	Number of Bonds
0	BARCLAYS	9	107.37	4
1	CO-OPERATIVE	11	110.15	4
2	ENQUEST	9	95.00	5

Figure 4: Number of bonds allocation

We are also showing how much amount is going to return in next 6 months.
Based on interest rate we calculate the return amount. As per the above figure user will get 190.5288 for every 6 months. This price may vary based on current price of the bonds but as per the understanding it won't vary in larger or smaller amounts.
We are also showing how much amount is going to return in next 6 months.

```
# Bond Price After 6 months
price_of_bonds = [ ((((float(bond_interest_rates[i])/100) * float(bonds_current_val[i])) * 0.5) ) * no_of_bonds[i]
                  for i in range(len(no_of_bonds))]

print(price_of_bonds)
print("Fixed amount will be return in next 6 months:" + str(sum(price_of_bonds)))
```

```
[57.9798, 72.699, 59.849999999999994]
Fixed amount will be return in next 6 months:190.5288
```

Figure 5 : Amount will be returned in next 6 months

C.Choose Stocks to Invest

We are allowing customer to choose stocks, as well as we also provide some default stocks (preferred stocks). Customer can choose anything that is comfortable to him. The below picture shows the stocks selection for his investment.

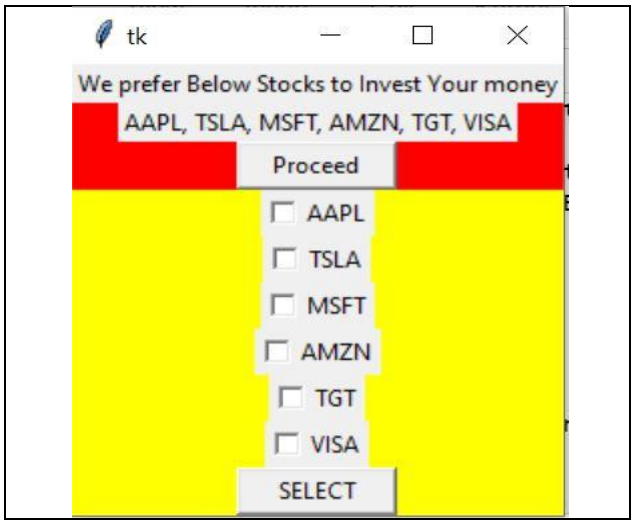


Figure 6: Stock selection window

From the above figure we are preferring 6 stocks to proceed or customer can choose any of the stocks as listed below and go through those stocks. In this example we will go with preferred stocks and will see the results.
The below picture shows the selected stocks from the customer:

```
Selected stock lists are:
['AAPL', 'TSLA', 'MSFT', 'AMZN', 'TGT', 'V']
```

Figure 7: Selected stocks from users.

D.How did we prefer These 6 stocks

According to market analysis and research data from last 3 years these 6 stocks growth rate is very good. In the below section we also shows the growth rate, annualized return and correlation of the stocks. Lets have a brief discussion about these stocks.

- Apple Inc (AAPL) is one of the best specializes in electronics, it provides software service.
- Tesla, Inc (TSLA) is an automotive and it is best in manufacturing electronic vechicles.
- Microsoft Corporation (MSFT) is more related to producing consumer software and personal computers
- Amazon.com (AMZN) its main focus on e-commerce, cloud commuting, AI
- Target Corporation (TGT) it is one of the largest retailers.
- Visa (V) it facilitates to electronic fund transfer.

D.01: Select assets based on ROCE, PER, EPS, Liquidity Ratio.

By using stockanalysis.com website to get income statement and balance sheet and using these statements I calculated ROCE, EPS etc.

D.02: Get Income statement & Annualized report

To get Income statement use below line

```
requests.get(f"https://stockanalysis.com/stocks/{stock}/financials/", headers=headers)
```

To get Balance sheet use below line

```
requests.get(f"https://stockanalysis.com/stocks/{stock}/financials/balance-sheet/", headers=headers)
```

The below table shows the calculation of ROCE, Price earning ratio, EPS and liquidity ratio

	ROCE	PRICE_EARNING_RAIO	EPS	Liquidity_RATIO
AAPL	172.688223	2580.321781	5.670	1.074553
GOOG	31.281022	2564.146513	113.880	2.928113
MSFT	49.240781	3315.293636	8.120	2.079994
AMZN	17.996311	5113.538530	3.298	1.135760
TSLA	21.607208	19113.213675	5.600	1.375285
TGT	69.743510	1516.802983	14.230	0.991999
UNH	33.403010	2734.066994	18.330	0.788816
V	42.044215	4005.034262	5.630	1.754050
DIS	3.393448	16147.705918	1.090	1.083020
KO	44.819340	2562.612010	2.260	1.130075

Figure 8: Asset selection Information

Based on the above values we are selecting AAPL, MSFT, AMZN, TSLA, V, TGT for portfolio optimization.

E.Download Stock data from Yahoo finance

We have taken selected stocks and download the last 6 years of data from yahoo finance.
{'AAPL', 'CISCO', 'MSFT', 'AMZN', 'TGT', 'VISA'}

We will combine all data into single data frame. We are using only Adj Close column from the stock data.

```
stocks.head()
```

Out[9]:

Date	AAPL	TSLA	MSFT	AMZN	TGT	V
2014-12-31	27.594999	44.481998	46.450001	15.5175	75.910004	65.550003
2015-01-02	27.332500	43.862000	46.759998	15.4260	75.330002	66.254997
2015-01-05	26.562500	42.018002	46.330002	15.1095	73.980003	64.792503
2015-01-06	26.565001	42.256001	45.650002	14.7645	73.970001	64.375000
2015-01-07	26.937500	42.189999	46.230000	14.9210	76.769997	65.237503

Figure 9: Head of selected stock Data

F.Plot growth of Selected Stocks

The below line/graph shows if we invest £100 on individual stock it will shows the growth over time period.

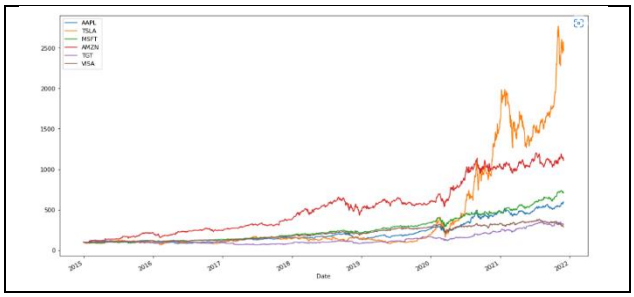


Figure 10: Growth of each stock from last 5 years

Annualized Return:

As we can see from the below chart TSLAs performs better than other stocks (yearly performance)

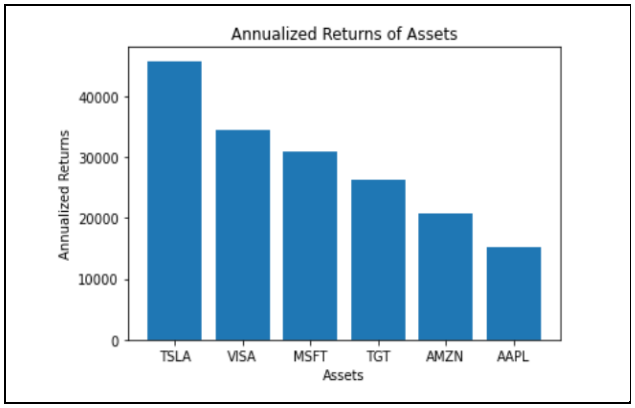


Figure 11: Annualized return of stocks

G.Correlation among stocks

We will see the correlation between stocks, by generating co-variance matrix.

The below plot shows the correlation of stocks. Here each stock correlated to each other and main thing we have to see, no stock correlation goes to 0.0 it shows the we have taken right stocks to optimization.

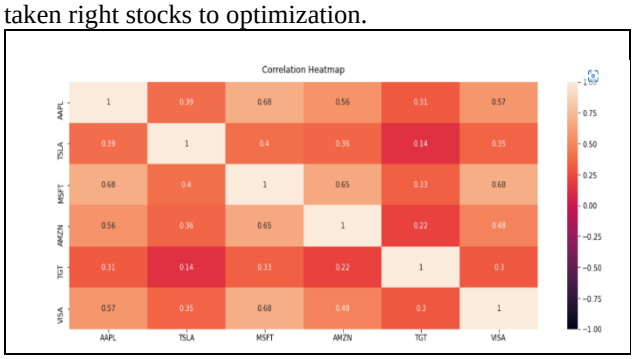


Figure 12: Correlation of stocks

H. Key Terms To Build a Model

H.1: Returns

It represents change in price of an asset.

⇨ Current price / previous price

$$r_t = \frac{p_t}{p_{t-1}}$$

Returns = stocks/stocks.shift(1)

H.2: Log Returns

Log returns measures the rate of exponential growth, instead of measuring the percentage of price change. Suppose if we have two stocks with high and low prices this log returns is useful to compute them.

$$logr_t = log\ \frac{p_t}{p_{t-1}}$$

⇨ Np.log(returns)

H.3: Expected Returns

The total amount of money they expect to gain or lose on a particular investment or portfolio.

$$R(w) = W^T \log(r)$$

H.4: Expected Volatility

It measures the risk of an asset.

$$\sigma(w) = \sqrt{W^T \sum W}$$

H.5: Sharpe Ratio

The Sharpe ratio adjusts a portfolio's past performance or expected future performance for the excess risk that was taken by the investor. A Sharpe ratio between 1 and 2 is considered good. A ratio between 2 and 3 is Very good, and any result higher than 3 is excellent.

$$SR(w) = \frac{W^T \log(r)}{\sqrt{W^T \sum W}}$$

Where:

R(w) = return of Portfolio

Rf = risk-free Param

$\sigma(w)$=standard deviation of portfolio or volatility

W = [w1, w2, w3, w4, w5, w6]

Sharpe ratio is propositional to Expected return / Expected volatility.

$$SR(w) \sim \frac{R(w) - R_f}{\sigma(w)}$$

The maximum sharpe ratio is:

print("Maximum Sharpe ratio:
{}".format(sharpe_arr.max()))

Maximum Sharpe ratio: 1.2983740113853646

I. Monte Carlo Efficient Frontieir

We have a way to evaluate how well our portfolio is allocated. It involves calculating expected returns, the expected volatility, and then from here each sharpe ratio to quantify how well our portfolio is allocated based on a risk perspective.

Better portfolio allocation: First, we could test a bunch of different random allocations and see which ones produces the highest sharpe ratio. Second, we could use mathematical optimization defined by constraints to arrive at the optimal allocation.

"Monte carlo simulations are used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of a random variable. It is a technique used to understand the impact of risk and uncertainty in prediction and forecasting models."

Steps To Implement Monte Carlo Simulation [7]:

Step 1: To forecast the price orientation of stocks we can use historical data of stocks and generate daily returns using logarithm

$$\text{PDR} = \ln\left(\frac{Daily\ price}{Yesterday's price}\right)$$

PDR Periodic daily return

Step 2: Now we will use Average, standard deviation, Value at risk functions to obtain the result of Drift

$$\text{Drift} = \text{ADR} - \left(\frac{Variance}{2}\right)$$

ADR Average Daily Return

Step 3: Generate random weights and add to drift to calculate or find out next day's price.

Weights = $\sigma * np.rand()$

Next day's price = Current_Price * $e^{(Drift+Weights)}$

Step 4: By using all these above terms generate random number of simulations, then find out minimum volatility ratio and display the results (weights of the minimum volatility ratio).

Pros and Cons of Monte Carlo Method:

It helps the customers or investors estimate the possibilities of gain or loss on an investment. It especially built for estimating a probable outcome. The difference is compared to other models from Monte Carlo model is it generates random numbers and take average but other models are starting with average.

Monte Carlo model is good to suggesting portfolio but it don't have an ability to minimize the risk. It also suggests for maximum sharpe ratio but Markowitz's model is more accurate compared to Monte Carlo model.

J. Markowitz Theory:

Markowitz theory can be built a portfolio to minimize risk and increase the amount of returns. In the same way investors can be increase the return by proper way of asset allocation of their investments.

To start with, in his theory of risk aversion, Markowitz assumed investors were risk-averse in their hearts. When there is less risk, they are more comfortable, whereas when there is more risk, they are nervous and anxious. Furthermore, it implies that losing money is preferable to finding or gaining it. Most people naturally prefer portfolios with a lower return possibility, even if they mean a lower return, when given the choice between a higher return possibility and greater risk.

The Markowitz theory was published in 1952, Markowitz stated portfolio can be optimize by using relationship between expected return and risk. The risk can be measured as the value of return. Markowitz also demonstrate the limit called "Efficient frontier". He suggested the process of choosing portfolio in two ways. First observe the stocks and experience with beliefs with available performance resources (Ex. Historical data, balance sheet, annual report etc). Second, Select future performance with good choice of portfolio weights.

We already know $X_{0i} = w_i X_0$

X_0 is the initial amount

w_i is the weights

Let R_i denote the total return on a net i.

$R_i\ X_{0i}$ = Money generated by trading in the asset

$R_i\ X_{0i} = R_i\ w_i\ X_i$

Total Money = $\sum_{i=1}^{n} R_i\ w_i X_0$

$R_i = \frac{\sum_{i=1}^{n} R_i w_i X_0}{X_0}$

$R_i = \sum_{i=1}^{n} R_i w_i$

Since $\sum_{i=1}^{n} w_i = 1$ we got relative return is r = $\sum_{i=1}^{n} w_i r_i$

Once we got the relative return and volatility (we will use same terms to calculate volatility from section Expected volatility) will grab Maximum sharpe ratio and will apply minimization function to that portfolio. As a result we will get Minimum volatility with maximum return portfolio.

Maximize sharpe ratio

We will use SciPy to minimize function is used to minimize the risk.

w_opt = minimize (negativeSR, w0, method='SLSQP', bounds=bounds, constraints=constraints)

NegativeSR = It will return negative sharpe ratio by weights.

Minimize $W^T \sum w$

Subject to $P^{-T} \geq r_{min}$

$w \geq 0$

Minimize -SR(w)

Subject to $0 \leq w_i \leq,\ i = 1, \ldots .6$

V. Results

A. Return vs Volatility Chart

We collected a total 10000 portfolios with random generated weights, now we will see in graphical representation of volatility and returns. Volatility in a portfolio is a risk instalment, it guides to a positive relationship between return and volatility. In other words, negative returns increase risk and it acts in the other direction. By simple terms we can say when the asset prices fall there will be an increase in volatility. The negative correlation ship is dissimilarity with positive correlation volatility and expected return is done by intertemporal capital asset model [6].

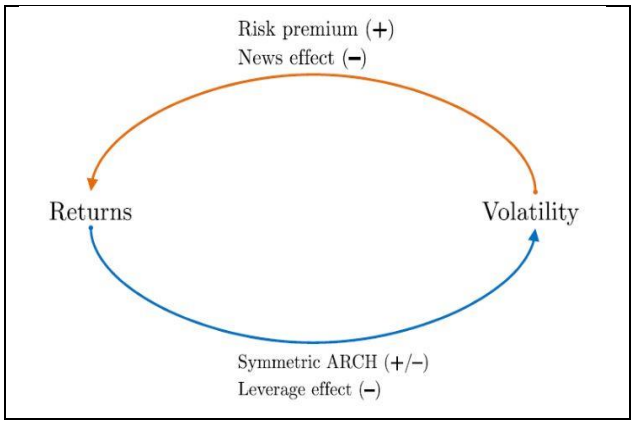


Figure 13: Interaction between Return and Volatility

To generate Return vs volatility chart in our stocks we will consider following points.

The below picture shows the Return vs Volatility Chart

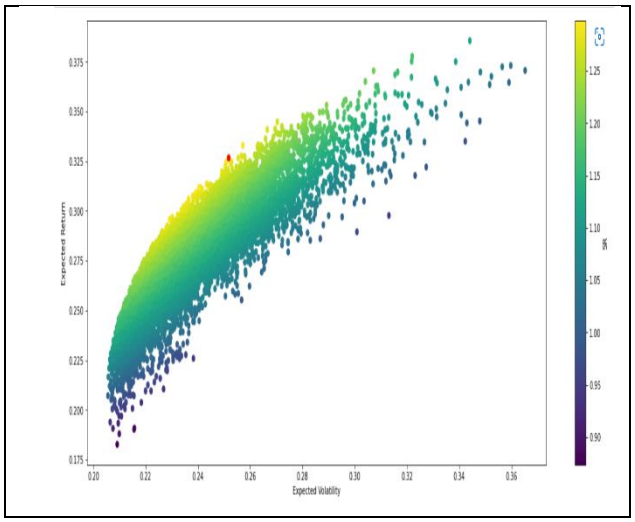


Figure 14: Return Vs Volatility chart

From the above figure we can identify the maximum sharpe ratio by red dot. We can also fetch weights of maximum sharpe ration and we can get an amount to be invested in each stocks.

B. Monte Carlo Simulation Results:

Once we are done with the calculation of Monte carlo model, we will generate 10000 random portfolios and will grab minimum volatility from generated portfolios. Now we will see the graph of generated portfolios and will point out the minimum volatility in graph.

According to the Modern Portfolio Theory, the Efficient Frontier is a set of optimal portfolios in the risk-return spectrum. This means that the portfolios on the frontier:

Offer the lowest level of risk for a given level of expected returns

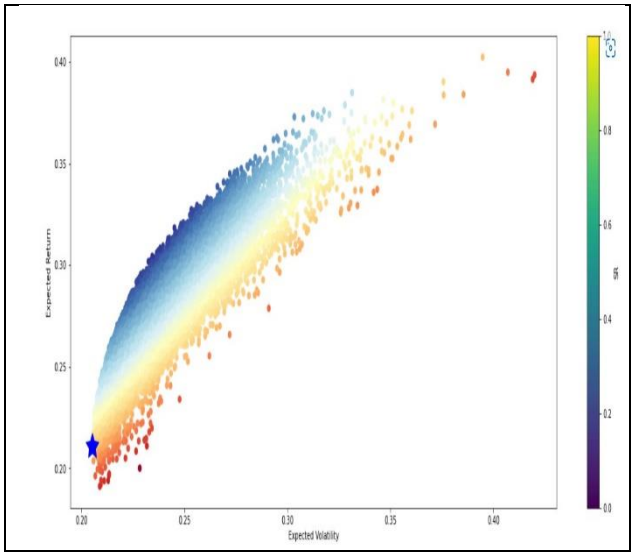


Figure 15: Efficient Frontier Using Monte Carlo Model

The blue dot shows the Minimum Volatility rate and its w eights are [0.10247633 0.00532469 0.02021858 0.178325 87 0.33890881 0.35474572].

The below figure shows the normalized amount of investments suggested by Monte Carlo simulation.

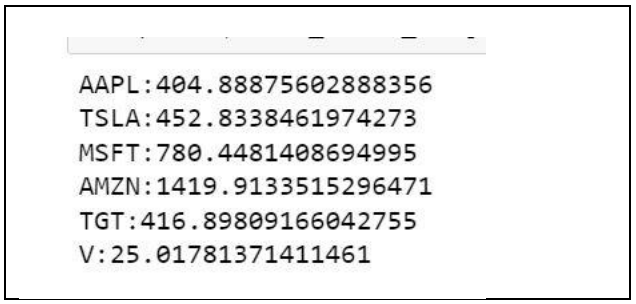
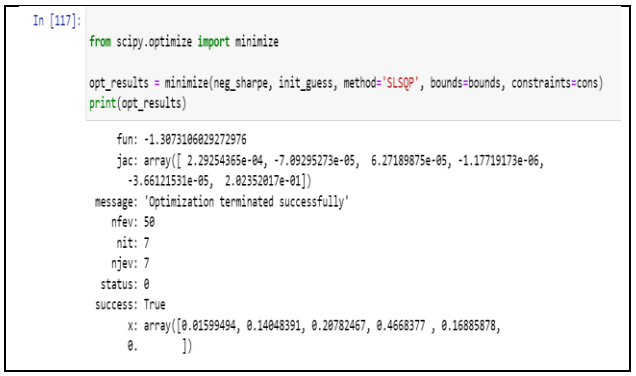


Figure 16: Amount suggested From Monte Carlo

Based on the Monte Carlo weights we calculated amount t o be invested in each stock.

C. Markowitz Simulation Results:

Once we applied optimization function or minimization function to randomly generated portfolio. By the end of the results, it will give us new weights with maximum sharpe ratio. Here we are using 'SLSQP' method (Sequential Least Squares Programming). We are also passing neg_sharpe(), This function uses random weights generated by Markowitz theory of analysis and it will consider covariance of each stock, mean of log return and finally it gives a weights with highest sharpe ratio. We can see these weights in below picture (opt_results.x). Once we got the weights, we will apply percentage function based on user amount and we will calculate amount per each stock then will display these results to users.



```
In [117]: from scipy.optimize import minimize

          opt_results = minimize(neg_sharpe, init_guess, method='SLSQP', bounds=bounds, constraints=cons)
          print(opt_results)

              fun: -1.3073106029272976
              jac: array([ 2.29254365e-04, -7.09295273e-05,  6.27189875e-05, -1.17719173e-06,
                -3.66121531e-05,  2.02352017e-01])
          message: 'Optimization terminated successfully'
             nfev: 50
              nit: 7
             njev: 7
           status: 0
          success: True
                x: array([0.01599494, 0.14048391, 0.20782467, 0.4668377 , 0.16885878,
                0.       ])
```

Figure 17: Minimization Function Using SLSQP

The above code snippet shows the minimization of risk or maximization of returns using Markowitz's frontier model (we used SciPy tool to solve this problem).The array 'x' suggests us to invest percentage of money should be invest on corresponding to stocks to get good return with minimum risks.

```
In [119]: get_ret_vol_sr(opt_results.x)
Out[119]: array([0.31887513, 0.24391688, 1.3073106 ])
```

Above array shows the return, volatility and sharpe ratio. As we can see sharpe ratio is maximized compared to earlier scenario.
Now we will apply percentile function to calculate the amount to be invest on each stock. The below picture shows the amount of individual stocks from £3500.

```
AAPL:55.99455417832166
TSLA:491.69193162793437
MSFT:727.382869813022
AMZN:1633.9275412327024
TGT:591.0031031480192
V:2.0415526907902048e-13
```

Figure 18: Amount suggested by Markowitz

Based on the Markowitz's theory we will draw efficient frontier for generated portfolio.

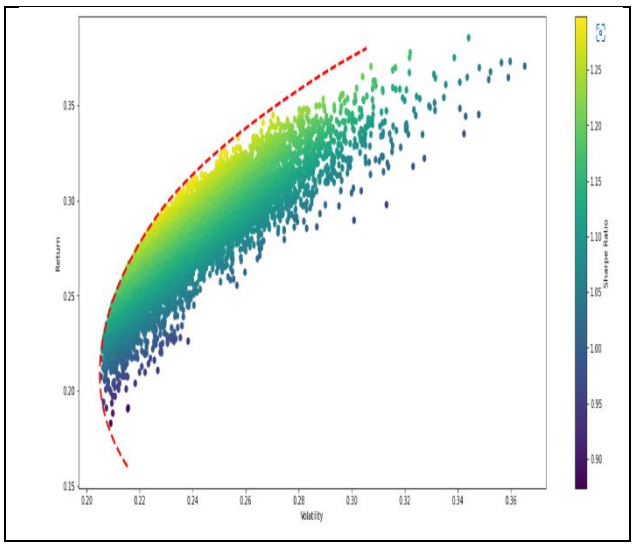


Figure 19: Markowitz's Efficient frontier after minimizing risk

The above figure shows the Efficient frontier from the Markowitz model with user selected stocks.
The red line represents the efficient frontier line, it tells use to invest anywhere on red passing dot. Each dot represents portfolio. Suppose user wants to invest particular point then we can grab those portfolio weights and we will apply percentile function to extract amount and we will display amounts for the user. Or user can go with the maximum sharpe ratio i.e preferred by Markowitz minimization function.

VI. Measuring Performance

Normally portfolio can be measured by Sharpe ratio, if the Sharpe is more than 1 then it is good portfolio, we can trust those weights. If SR is in between 2 and 3 then Very good, if it is more than 3 portfolio is excellent. We have two models lets grab the Sharpe ratio of Markowitz's and Monte Carlo model and let's decide which one is performing better.

First will see the Monte Carlo model Sharpe Ratio

```
print("Maximum Sharpe ratio of Monte Carlo Model: {}".format(sharpeRatio.max()))
#print("Its location of the array: {}".format(sharpeRatio.argmax()))
```

```
Maximum Sharpe ratio of Monte Carlo Model: 1.2958672669698
```

From the above statement we can see Monte Carlo model maximum sharpe ratio is 1.29. This states that our Monte Carlo model is good.

Now let's calculate Sharpe ratio for Markowitz's model, We are not extracting maximum Sharpe ratio from Markowitz's model. We are extracting Sharpe ratio of weights i.e suggested by after applying minimization function.

```
val = get_ret_vol_sr(opt_results.x)
print("Sharpe Ratio of Markowitz's after appying minimization functio:",val[2])
```

```
Sharpe Ratio of Markowitz's after appying minimization functio: 1.3073106021152863
```

We can see from the above statement our minimization function weights performs better than Monte Carlo model weights. So, we can easily trust our minimization function and we can invest money based on suggested weights.
Other Measures:

We are going to measure out portfolio by using exchange-traded fund (ETF). ETF is a kind of security it is used to track certain sector, index, assets and other cash flow events. We will download ETF terms from yahoo resource (we will use same start and end date from stocks data). After that we will merge etf returns and our portfolio returns into a single data frame then we will apply model regression.

$$retf=\alpha\hat{}+\beta\hat{}rm+et$$
$$et\sim N(0,\sigma\hat{}2e)$$

```
linear_reg_measure = merge_etf_total_return.apply(lambda x: stats.linregress(merge_etf_total_return['SPY'], x),
                                                  result_type='expand').rename(index={0: 'slope', 1:
                                                                                      'intercept', 2: 'rvalue', 3:
                                                                                      'p-value', 4:'stderr'})
linear_reg_measure
```

	ARKK	GLD	QQQ	SPY	XLE	XLF	Portfolio
slope	1.231037e+00	-0.003644	1.089853	1.0	1.247107e+00	1.158529	1.043211e+00
intercept	4.632606e-04	0.000257	0.000243	0.0	-5.965615e-04	-0.000130	6.581234e-04
rvalue	7.039061e-01	-0.004612	0.920698	1.0	7.060668e-01	0.664858	7.563169e-01
p-value	9.748438e-261	0.847519	0.000000	0.0	6.242764e-263	0.000000	7.805050e-323
stderr	2.978113e-02	0.018848	0.011079	0.0	2.999374e-02	0.016015	2.163877e-02

For calculating model regression we are using stats.lingress. As we can see from above picture the slope of SPY is 1.0 and intercept is 0.0, these values are proving good portfolio weights. And we see standard error is 0.0 means our model is accurate but rvalue is 1.0 we are expecting less than 0. By consideration of slope intercept and stderr our model performs well in the real time scenarios [3].

These standard errors are not residual now we will calculate residual error by using Information ratio.

$$IR = \frac{\propto_p}{\sigma_{e_p}}$$

```
linear_reg_measure['ARKK'].intercept / standardized_error
```

```
0.033501713744304204
```

```
# This calculates the predicted value for each observed value
final_observation = merge_etf_total_return['Portfolio']
final_prediction = linear_reg_measure['Portfolio'].slope * merge_et

# This prints the residual for each pair of observations
final_error = final_observation - final_prediction
final_std_error = np.std(final_error)

linear_reg_measure['Portfolio'].intercept / final_std_error
```

```
0.06549960204173248
```

As we can see from above statements our Portfolio is good compare to ARK etf. This states that our portfolio is performing well.

VII. Future work

One basic idea for future plan is to introduce a greater number of stocks to the list of optimizations. As of now I am using only 6 stocks but I am planning to include up to 15-20 stocks for optimization. And also planning to optimize bonds, in this paper we are evaluating bonds and showing the results based on interest rate and credit score but in future we need to consider all rules for bonds optimization then we can also earn more fixed returns from bonds. Another plan is to introduce more assets for optimization. Currently we are only optimizing stocks and bonds but some customers don't have faith in these assets so we are trying to give alternate option for this problem. Other assets like cash flow, land acquisition, mutual funds, bank deposits. Land acquisition means predicting land price after 1 or 2 years.

Another point is I am planning to upload this complete application into some website or blog. So, some users can use this application and use for their portfolio analysis.

VIII. Conclusion

In Summary, Double stage genetic algorithms play an important role in this theory. This algorithm makes investors to filtering their assets in two stages. First stage is selecting based on financial key terms and Second stage is applying Markowitz & Monte Carlo model for selected assets. Our Mathematical terms, financial indicators are standardized by the financial derivatives and results are well defined. In our thesis investors have a flexibility to choose his/her assets and risk level. Apart from that we are also suggested stocks for optimization, if user don't have any knowledge on the stocks they can go with our preferred stocks. We are also providing bonds for fixed returns, investor have an idea about these bonds and they can expect returns in every quarter. Our portfolio optimization performs better compared to other available portfolio management because online available optimization techniques are not calculating mathematical key terms and we are also using models to optimize the portfolio. Minimization function is one of the highlighted point in our optimization, It performs risk reduction and maximizes the return rate of portfolio. But we can't apply minimization function for single asset. Co-relation of two asset also play an important role in this theory, suppose if two assets are strongly co-related to each other then we have to remove either one of the asset from optimization otherwise we will ended up with bad allocation. One of the major points in our theory is investors can choose asset for optimization, this makes out technique of selecting asset is unique because no other techniques have a flexibility for customers to choose their assets. By Considering all these points our portfolio optimization is one of the best methods earn money. Some people still don't have faith to invest money in stocks, I will strongly suggest them to allocate some small funds based on our optimization suggestion.

Monte Carlo model is one of the good ways to optimize the assets because it considers wide range of weights and helps us to suggest uncertainty in stocks. This model is very flexible and it allows us to examines the risk assumptions under all params and it give wide range of possible outcomes. Monte Carlo model also good in corporate finance and management related to personal finance. But from the studies of Monte Carlo model, it is well suited for safe optimization suggestion means if customer don't want to take any risk, he can go with Monte Carlo model. As the same way it is safe compared to all other models and investors shouldn't expect more returns because it is suggested by considering all risk matters.

Markowitz model is one of the best designed models for portfolio, it provides lot of functionalities related to financial risk, asset and gives frontiers of risk vs returns. One thing from Markowitz's theory, he expands to suggest us the theory of portfolio could be used for further studies and analysis. I found Markowitz model is very interesting because he never suggests any particular rules from paradise but still this model plays an important role in financial areas. We can see lot of researchers developed their own portfolio based on Markowitz model. I am also got inspired by Markowitz model and other researchers and I came up with my own portfolio optimization.

IX. Declaration

Originality: Having read and understood Exeter's plagiarism policy, I hereby agree to comply with it, unless otherwise noted by referencing, all work in this assignment is mine.

Ethical Concerns: My thesis is not going to raise any ethical issues. I am not using any particular person and personal data of any website. All data are accessible by public.

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