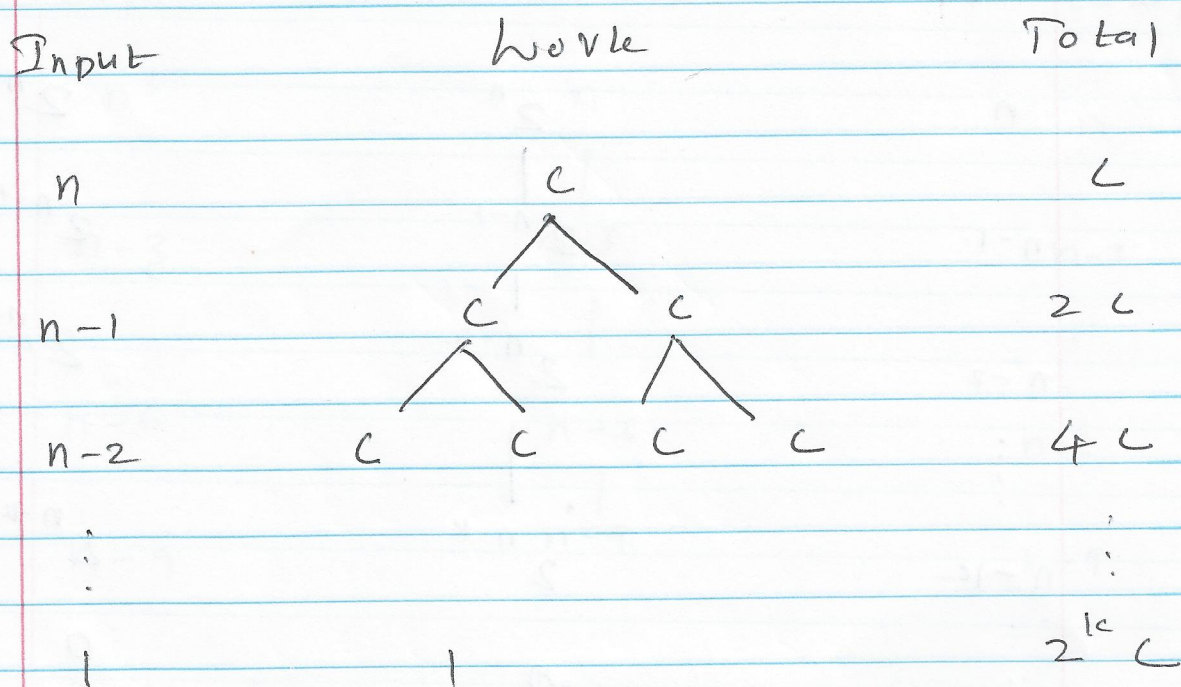


a.)  $T(n) = C + 2T(n-1), T(1) = C$

Recursion Tree Method:



Since  $T(1) = C$ , the recursion ends at  $k = n-1$   
Hence

$$T(n) = C + 2C + 4C + \dots + C \cdot 2^{(n-1)}$$

$$= C(1 + 2 + 4 + \dots + 2^{(n-1)})$$

$$= C(2^{n-1+1} - 1)$$

$$= C(2^n - 1) \quad \text{Via Summation of Series}$$

$$T(n) = O(2^n)$$



6.)  $T(n) = T(n-3) + n, T(0) = 0$  ( $n$  multiple of 3)

Recursion tree method

Input	Work	Total
$n$	$n$	$n$
$n-3$	$n-3$	$n-3$
$n-6$	$n-6$	$n-6$
$n-9$	$n-9$	$n-9$
$\vdots$	$\vdots$	$\vdots$
$n-k$	$n-k$	$n-k$
$\vdots$	$\vdots$	$\vdots$
$0$	$0$	$0$

$$T(n) = n + n-3 + n-6 + \dots + n-k \text{ to } 0$$

Since  $T(0) = 0$ , Consider  $n=k$ ,  $k=n$  & divisible of 3

$$T(n) = n + n-3 + n-6 + \dots + n-n+3 + n-n$$

$$= n + n-3 + n-6 + \dots + 3 + 0$$

$$= \frac{n}{2} (0 + (n-1)(3)) \text{ By Summation of A.P}$$

$$= \frac{3}{2} (n^2 - n)$$

$$T(n) = O(n^2)$$



$$(.) \quad T(n) = 2^n + T(n-1), \quad T(1) = 0$$

Recursion Tree method:

Input	Work	Total
n	$2^n$	$2^n$
n-1	$2^{n-1}$	$2^{n-1}$
n-2	$2^{n-2}$	$2^{n-2}$
$\vdots$	$\vdots$	$\vdots$
n-k	$2^{n-k}$	$2^{n-k}$
$\vdots$	$\vdots$	$\vdots$
1	0	0

$$T(n) = 2^n + 2^{n-1} + 2^{n-2} + \dots + 2^{n-k} + 0$$

$$= 2^n \left( 1 + \frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^k} \right)$$

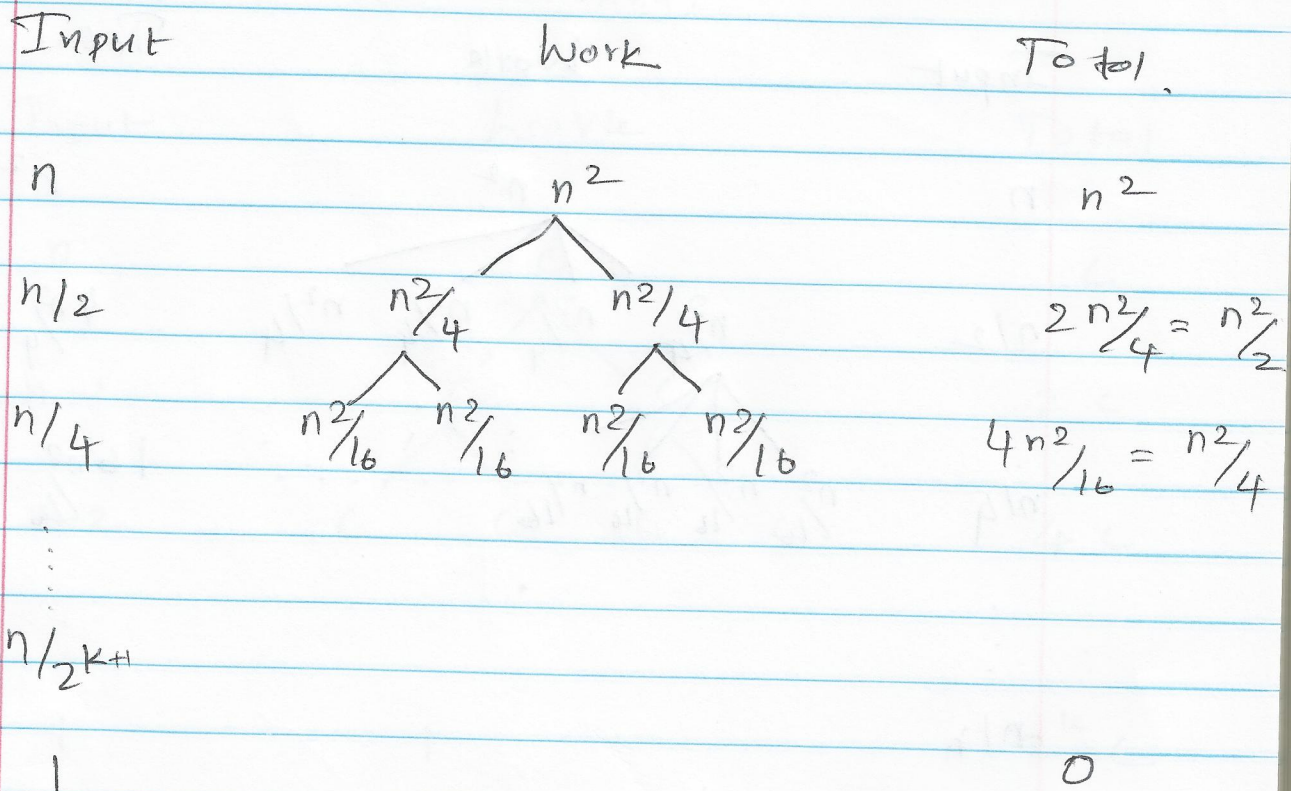
= Cannot be  $> 2$

$$T(n) = O(2^n)$$



d.)  $T(n) = n^2 + 2T(n/2)$ ,  $T(1) = 0$

Recursion Tree Method:



Here  $n = 2^{k+1}$   $k = \log n$

$$T(n) = n^2 + n^2/2 + n^2/4 + \dots + n^2/2^k$$

$$= n^2 \left( 1 + 1/2 + 1/4 + \dots + 1/2^k \right)$$

cannot be more than 2

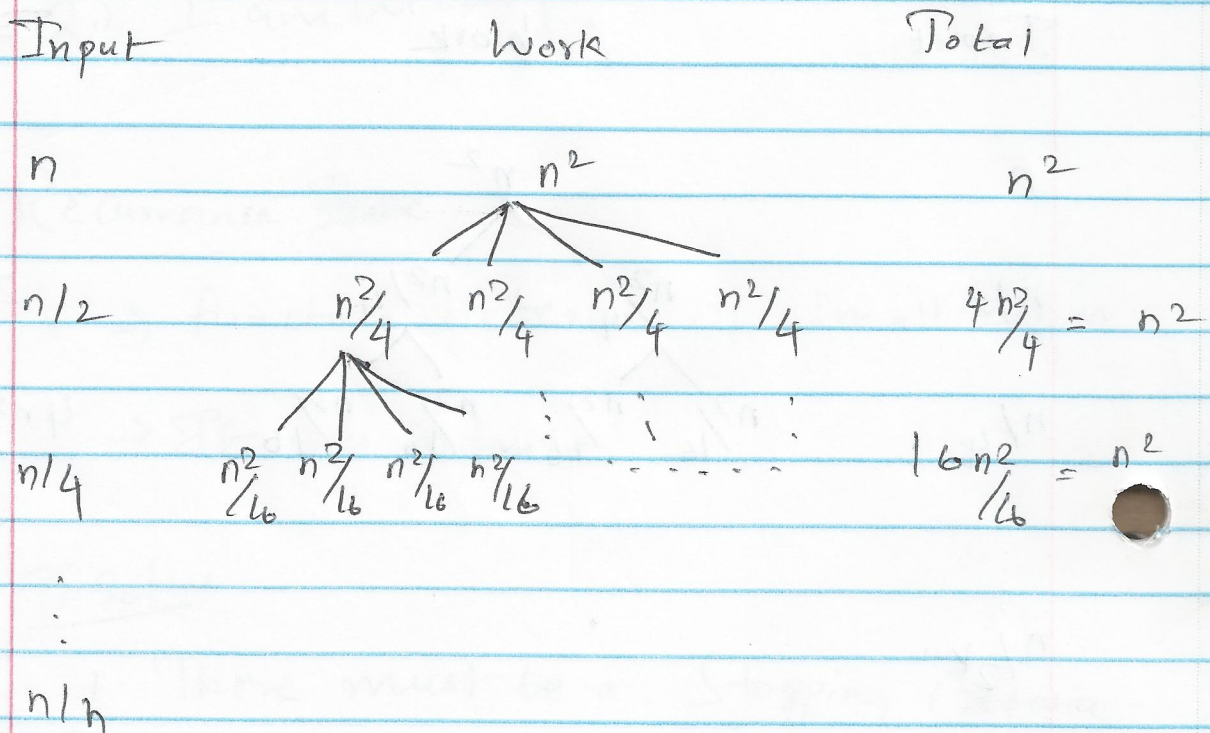
$$T(n) = n^2(2)$$

$$T(n) = O(n^2) \text{ where } C=2$$



e-)  $T(n) = n^2 + 4T(n/2)$

Recursion Tree method:



Here  $n = 2^{k+1}$  Since ~~n=0~~, When  $n=0$ , it must be  $2^0$

$$\begin{aligned}
 T(n) &= n^2 + n^2 + n^2 + \dots + n^2_{2^{k+1}} \\
 &= n^2 (\log n \text{ times})
 \end{aligned}$$

Hence  $T(n) = \Theta(n^2 \log n)$