9.) $T(n) = c + 2T(n-1), \quad T(1) = c$

Recursion Tree Method:

Input	Work	Total
n	c	c
$n-1$	$c \quad c$	$2c$
$n-2$	$c \quad c \quad c \quad c$	$4c$
$\vdots$		$\vdots$
1	1	$2^k c$

Since $T(1)=c$, the recursion ends at $k=n-1$

Hence

$$T(n) = c + 2c + 4c + \dots + C_2(n-1)$$

$$= c(1+2+4+\dots 2^{(n-1)})$$

$$= c(2^{n-1+1} - 1)$$

$$= c(2^n - 1)$$ Via Summation of Series

$T(n) = O(2^n)$

6.) $T(n) = T(n-3) + n$, $T(0) = 0$ (n multiple of 3)

Recursion tree method

Input	Work	Total
n	n	n
$n-3$	$n-3$	$n-3$
$n-6$	$n-6$	$n-6$
$n-9$	$n-9$	$n-9$
$\vdots$	$\vdots$	$\vdots$
$n-k$	$n-k$	$n-k$
$\vdots$		
0	0	0

$T(n) = n + n-3 + n-6 + \cdots n-k + 0$

Since $T(0) = 0$, Consider $n = k$, $k = n$ is divisible of 3

$T(n) = n + n-3 + n-6 + \cdots + n-n+3 + n-n$

$= n + n-3 + n-6 + \cdots 3 + 0$

$= \frac{n}{2}(0 + (n-1)(3))$ By Summation of A.P

$= 3/2(n^2 - n)$

$T(n) = O(n^2)$

(c) $T(n) = 2^n + T(n-1)$, $T(1) = 0$

Recursion Tree method:

Input	Work	Total
n	2^n	2^n
$n-1$	2^{n-1}	2^{n-1}
$n-2$	2^{n-2}	2^{n-2}
$\vdots$		
$n-k$	2^{n-k}	2^{n-k}
$\vdots$		
1	0	0

$$T(n) = 2^n + 2^{n-1} + 2^{n-2} + \dots + 2^{n-k} + 0$$

$$= 2^n \underbrace{\left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \dots \frac{1}{2^k}\right)}_{\text{Cannot be } > 2}$$

$=$

$$T(n) = O(2^n)$$

d.) $T(n) = n^2 + 2T(n/2)$, $T(1) = 0$

Recursion Tree Method:

Input	Work	Total
n	n^2	n^2
$n/2$	$n^2/4 \quad n^2/4$	$2n^2/4 = n^2/2$
$n/4$	$n^2/16 \quad n^2/16 \quad n^2/16 \quad n^2/16$	$4n^2/16 = n^2/4$
$\vdots$		
$n/2^{k+1}$		
1		0

Here $n = 2^{k+1}$ $k = \log n$

$$T(n) = n^2 + n^2/2 + n^2/4 + \dots + n^2/2^k$$

$$= n^2 \underbrace{(1 + 1/2 + 1/4 + \dots 1/2^k)}_{\text{cannot be more than } 2}$$

$$T(n) = n^2(2)$$

$$Tn = O(n^2) \quad \text{where } c = 2$$

e-) $T(n) = n^2 + 4T(n/2)$

Recursion Tree method:

Input	Work	Total
n	n^2	n^2
$n/2$	$n^2/4 \quad n^2/4 \quad n^2/4 \quad n^2/4$	$4n^2/4 = n^2$
$n/4$	$n^2/16 \quad n^2/16 \quad n^2/16 \quad n^2/16 \quad \ldots\ldots$	$16n^2/16 = n^2$
$\vdots$		
n/n		

Here $n = 2^{k+1}$ Since , when $n=0$, it must be 2^1

$$T(n) = n^2 + n^2 + n^2 + \ldots + n^2 2^{k+1}$$

$$= n^2 (\log n \text{ times})$$

Hence $T(n) = \Theta(n^2 \log n)$