NCERT QUESTIONS WITH SOLUTIONS

EXERCISE : 10.1

1. How many tangents can a circle have?

Sol. There can be infinitely many tangents to a circle.

2. Fill in the blanks :

(i) A tangent to a circle intersects it in ..... point (s).

(ii) A line intersecting a circle in two points is called a.......

(iii) A circle can have .... parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called........

Sol. (i) One (ii) Secant

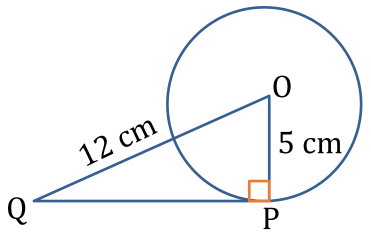
(iii) Two (iv) Point of contact.

3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is.

(A) 12 cm (B) 13 cm

(C) 8.5 cm (D) $\sqrt{119}$ cm

Sol.



O is the centre of the circle. The radius of the circle is 5 cm.

PQ is tangent to the circle at P. Then OP = 5 cm and $\angle OPQ = 90°$.

We are given that OQ = 12 cm.

By Pythagoras Theorem, we have

$PQ^2 = OQ^2 - OP^2$

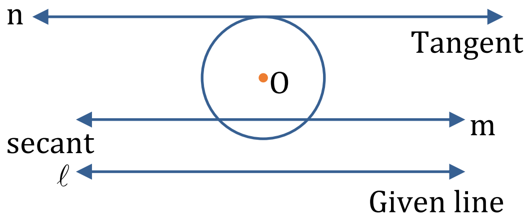
$= (12)^2 - (5)^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

Hence, the correct option is (D).

4. Draw a circle and two lines parallel to a given line such that one is tangent and other a secant to the circle.

Sol. We have the required figure as shown:



Here, ℓ is the given line and a circle with centre O is drawn.

The line n is drawn which is parallel to ℓ and tangent to the circle. Also, m is drawn parallel to line ℓ and is a secant to the circle.

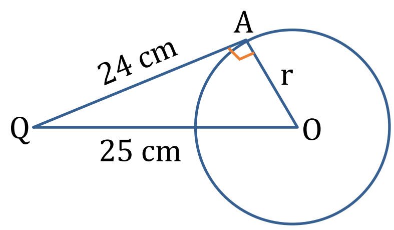
EXERCISE : 10.2

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is -

(A) 7 cm (B) 12 cm

(C) 15 cm (D) 24.5 cm

Sol. From figure,



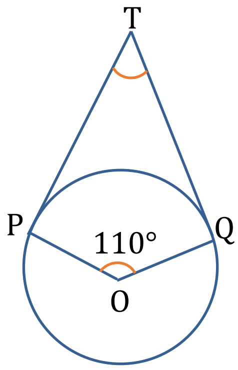
$r^2 = (25)^2 - (24)^2$

$= 625 - 576 = 49$

r = 7 cm

Hence, the correct option is (A).

2. In fig., if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to –



(A) 60° (B) 70°

(C) 80° (D) 90°

Sol. TQ and TP are tangents to a circle with centre O and $\angle POQ = 110°$

$\therefore$ $OP \perp PT$ and $OQ \perp QT$

$\Rightarrow$ $\angle OPT = 90°$ and $\angle OQT = 90°$

Now, in the quadrilateral TPOQ, we get

$\therefore$ $\angle PTQ + 90° + 110° + 90° = 360°$

(Angle sum property of a quadrilateral)

$\angle PTQ + 290° = 360°$

$\angle PTQ = 360° - 290° = 70°$

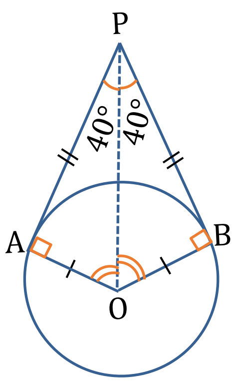
Hence, the correct option is (B)

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle POA$ is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

Sol.



In figure,

$\Delta OAP \cong \Delta OBP$ (SSS congruence)

$\Rightarrow$ $\angle POA = \angle POB$

$= \frac{1}{2}\angle AOB$ ... (i)

Also $\angle AOB + \angle APB = 180°$

$\Rightarrow$ $\angle AOB + 80° = 180°$

$\Rightarrow$ $\angle AOB = 100°$ ... (ii)

Then from (i) and (ii)

$\angle POA = \frac{1}{2} \times 100 = 50°$

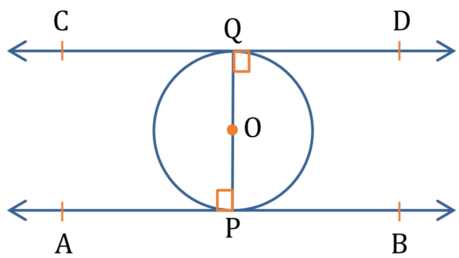
Hence, the correct option is (A)

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol. In the figure, PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ.

Since, the tangents at a point to a circle is perpendicular to the radius through the point.



$\therefore$ $PQ \perp AB$

$\Rightarrow$ $\angle APQ = 90°$ and $PQ \perp CD$

$\Rightarrow$ $\angle PQD = 90°$

$\Rightarrow$ $\angle APQ = \angle PQD$

But they form a pair of alternate angles.

$\therefore$ $AB \parallel CD$.

Hence, the two tangents are parallel.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Sol. Let us assume a circle with centre O and let AB be the tangent intersecting the circle at point P.

Also let us assume a point X such that XP is perpendicular to AB.

We have to prove that XP passes through centre O.

We know that

Tangent of a circle is perpendicular to radius at point of contact.

$\Rightarrow$ OP $\perp$ AB (Therorem-1)

So, $\angle OPB = 90°$ ... (1)

We have already assumed that XP is perpendicular to AB.

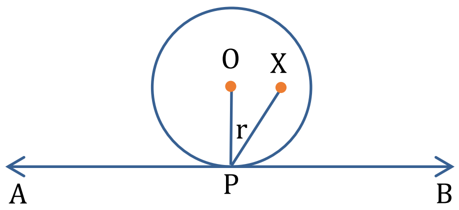
$\angle XPB = 90°$ ... (2)

Now from equation (1) and (2)

$\angle OPB = \angle XPB = 90°$

This condition is possible only if line XP passes through O. Since, XP passes through centre O.

Therefore, it is proved that the the perpendicular at the point of contact of the tangent of a circle passes through the centre.



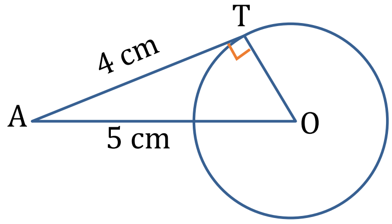
6. The length of a tangent from a point A at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Sol. The tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ $\angle OTA = 90°$

Now, in the right ΔOTA, we have :

$OA^2 = OT^2 + AT^2$ [Pythagoras theorem]



$\Rightarrow$ $5^2 = OT^2 + 4^2$

$\Rightarrow$ $OT^2 = 5^2 - 4^2$

$\Rightarrow$ $OT^2 = (5 - 4)(5 + 4)$

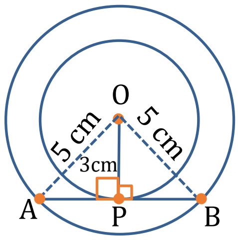
$\Rightarrow$ $OT^2 = 1 \times 9 = 9 = 3^2$

$\Rightarrow$ $OT = 3$

Thus, the radius of the circle is 3 cm.

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Sol. In fig. the two concentric circles have their centre at O. The radius of the larger circle is 5 cm and that of the smaller circle is 3 cm.



AB is a chord of the larger circle and it touches the smaller circle at P.

Join OA, OB and OP.

Now, OA = OB = 5 cm,

OP = 3 cm

And OP $\perp$ AB,

i.e., $\angle OPA = \angle OPB = 90°$

$\Rightarrow \quad \Delta OAP \cong \Delta OBP \quad$ (RHS congruence)

$\Rightarrow \quad AP = BP = \frac{1}{2}AB$ or $AB = 2\ AP$

By Pythagoras theorem,

$OA^2 = AP^2 + OP^2$

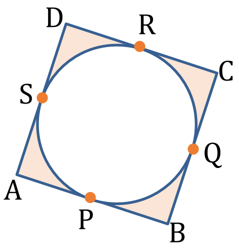
$\Rightarrow \quad (5)^2 = AP^2 + (3)^2$

$\Rightarrow \quad AP^2 = 25 - 9 = 16$

$\Rightarrow \quad AP = 4$ cm

$\Rightarrow \quad AB = 2 \times 4$ cm $= 8$ cm

8. A quadrilateral ABCD is drawn to circumscribe a circle (see fig.). Prove that AB + CD = AD + BC.



Sol. In fig., we observe that

$AP = AS \quad$ ...(i)

($\because$ AP and AS are tangents to the circle drawn from the point A)

Similarly, $BP = BQ \quad$ ...(ii)

$CR = CQ \quad$ ...(iii)

$DR = DS \quad$ ...(iv)

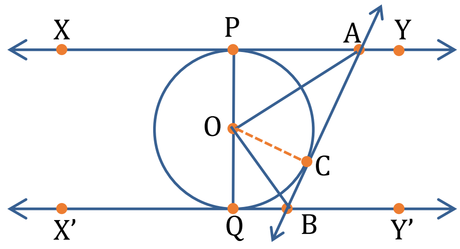
Adding (i), (ii), (iii), (iv), we have

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow \quad AB + CD = AD + BC$

Note: This is a pitot theorem.

9. In fig., XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90°$



Sol. In fig., Join OC and we have ΔAOP and ΔAOC for which

$\Rightarrow \quad AP = AC \quad$ (Both tangents from A)

$\Rightarrow \quad OP = OC \quad$ (Each = radius)

$\Rightarrow \quad OA = OA \quad$ (Common side)

$\Rightarrow \quad \Delta AOP \cong \Delta AOC \quad$ (SSS congruence)

$\Rightarrow \quad \angle PAO = \angle CAO$

$\Rightarrow \quad \angle PAC = 2\angle OAC \quad$ ...(i)

Similarly,

$\angle QBC = 2\angle OBC \quad$ ...(ii)

Adding (i) and (ii),

$\angle PAC + \angle QBC = 2\ (\angle OAC + \angle OBC)$

$\Rightarrow \quad 180° = 2\ (\angle OAC + \angle OBC)$

($\because$ sum of co-interior angles is 180°)

$\Rightarrow \quad \angle OAC + \angle OBC$

$= \frac{1}{2} \times 180° = 90° \quad$ ...(iii)

Now, in ΔAOB we have

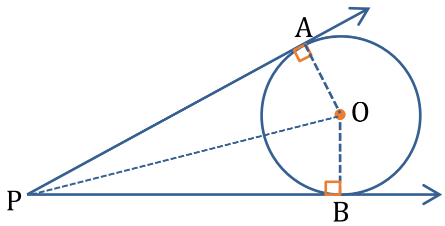
$\angle AOB + \angle OAC + \angle OBC = 180°$

$\Rightarrow \quad \angle AOB + 90° = 180° \quad$ (By (iii))

$\Rightarrow \quad \angle AOB = 90°$

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Sol. Let PA and PB be two tangents drawn from an external point P to a circle with centre O.



Now, in right ΔOAP and right ΔOBP,

we have, PA = PB

[Tangents to circle from an external point]

OA = OB [Radii of the same circle]

OP = OP [Common]

$\Delta OAP \cong \Delta OBP$ [By SSS congruency]

$\therefore$ $\angle OPA = \angle OPB$ [By C.P.C.T.]

and $\angle AOP = \angle BOP$

$\Rightarrow$ $\angle APB = 2\angle OPA$ and $\angle AOB = 2\angle AOP$

But $\angle AOP = 90° - \angle OPA$

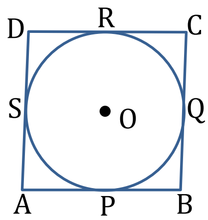
$\Rightarrow$ $2\angle AOP = 180° - 2\angle OPA$

$\Rightarrow$ $\angle AOB = 180° - \angle APB$

$\Rightarrow$ $\angle AOB + \angle APB = 180°$

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. Let ABCD be a parallelogram such that its sides touch a circle with centre O.



$\Rightarrow$ AP = AS [Tangents from an external point are equal]

$\Rightarrow$ BP = BQ [Tangents from an external point are equal]

$\Rightarrow$ CR = CQ [Tangents from an external point are equal]

$\Rightarrow$ DR = DS [Tangents from an external point are equal]

Adding these equations,

$\Rightarrow$ AP + BP + CR + DR = AS + DS + BQ + CQ

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ 2AB = 2BC

$\Rightarrow$ AB = BC

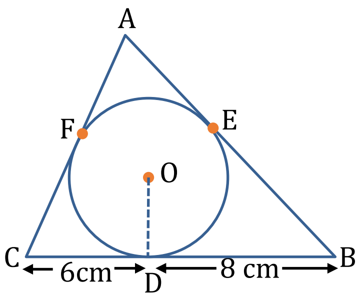
and AB = DC

$\Rightarrow$ AB = BC = CD = DA

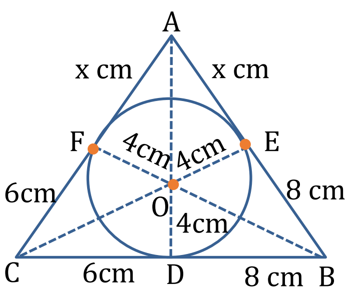
$\Rightarrow$ ABCD is a rhombus.

Hence proved.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see fig.). Find the sides AB and AC.



Sol.



In fig. BD = 8 cm and DC = 6 cm

Then we have BE = 8 cm ($\because$ BE = BD)

and CF = 6 cm ($\because$ CF = CD)

Suppose AE = AF = x cm

In ΔABC,

a = BC = 6 cm + 8 cm = 14 cm

b = CA = (x + 6) cm

c = AB = (x + 8) cm

$s = \frac{a+b+c}{2} = \frac{14+(x+6)+(x+8)}{2}$ cm

$= \frac{2x+28}{2}$ cm

= (x + 14) cm

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)\times x\times 8\times 6}$

$= \sqrt{48x\times(x+14)}\ \text{cm}^2$ ...(i)

Also, area of ΔABC = area of ΔOBC + area of ΔOCA + area of ΔOAB

$= \frac{1}{2}\times 4\times a + \frac{1}{2}\times 4\times b + \frac{1}{2}\times 4\times c$

= 2 (a + b + c) = 2 × 2s = 4s

$= 4\,(x + 14)\ \text{cm}^2$ ...(ii)

From (i) and (ii),

$\sqrt{48x\times(x+14)} = 4\times(x+14)$

On squaring both sides

$\Rightarrow$ $48x\times(x+14) = 16\times(x+14)^2$

$\Rightarrow$ 3x = x + 14

$\Rightarrow$ x = 7 cm

Then AB = c = (x + 8) cm

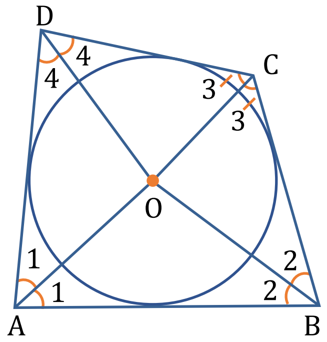
= (7 + 8) cm = 15 cm

and AC = b = (x + 6) cm

= (7 + 6) cm = 13 cm

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol.



Let ABCD be a quadrilateral circumscribing a circle with centre O.

Now join AO, BO, CO, DO.

From the figure, $\angle$DAO = $\angle$BAO

[Since, AB and AD are tangents]

Let $\angle$DAO = $\angle$BAO = $\angle$1

Also $\angle$ABO = $\angle$CBO

[Since, BA and BC are tangents]

Let $\angle$ABO = $\angle$CBO = $\angle$2

Similarly, we take the same way for vertices C and D.

Recall that sum of the angles in quadrilateral, ABCD = 360°

= 2($\angle$1 + $\angle$2 + $\angle$3 + $\angle$4) = 360°

= $\angle$1 + $\angle$2 + $\angle$3 + $\angle$4 = 180°

In ΔAOB, $\angle$BOA = 180° – ($\angle$1 + $\angle$2)

In ΔCOD, $\angle$COD = 180° – ($\angle$3 + $\angle$4)

$\angle BOA + \angle COD = 360° - (\angle 1 + \angle 2 + \angle 3 + \angle 4)$

= 360° – 180° = 180°

$\therefore$ AB and CD subtend supplementary angles at O.

$\because$ Sum of the angles at the centre is 360°.

$\therefore$ AD and BC subtend supplementary angles at O.

Thus, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.