NCERT QUESTIONS WITH SOLUTIONS

EXERCISE : 1.1

1. Is zero a rational number? Can you write it in the form p/q, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number. We can write zero in the form p/q where p and q are integers and $q \neq 0$.

So, 0 can be written as $\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc.

2. Find six rational numbers between 3 and 4.

Sol. First rational number between 3 and 4 is

$$= \frac{3+4}{2} = \frac{7}{2}$$

Similarly, other numbers are

$$\frac{3+\frac{7}{2}}{2} = \frac{13}{4}$$

$$\frac{3+\frac{13}{4}}{2} = \frac{25}{8}$$

$$\frac{3+\frac{25}{8}}{2} = \frac{49}{16}$$

$$\frac{3+\frac{49}{16}}{2} = \frac{97}{32}$$

$$\frac{\frac{97}{32}+3}{2} = \frac{193}{64}$$

So, numbers are $\frac{7}{2}, \frac{13}{4}, \frac{25}{8}, \frac{49}{16}, \frac{97}{32}, \frac{193}{64}$

3. Find five rational numbers between 3/5 and 4/5.

Sol. Let

$$\frac{3}{5}\frac{(n+1)}{n+1} = \frac{3}{5}\times\frac{6}{6} = \frac{18}{30}$$

$$\frac{4}{5}\frac{(n+1)}{n+1} = \frac{4}{5}\times\frac{6}{6} = \frac{24}{30}$$

So, required rational numbers are

$$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$$

4. State whether the following statements are true or false? Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Sol. (i) True, the collection of whole numbers contains all natural numbers.

(ii) False, –2 is not a whole number.

(iii) False, $\frac{1}{2}$ is an integer but a rational number but not a whole number.

EXERCISE : 1.2

1. State whether the following statements are true or false? Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Sol. (i) True, since collection of real numbers consists of rationals and irrationals.

(ii) False, because no negative number can be the square root of any natural number.

(iii) False, 2 is real but not irrational.

2. Are the square roots of all positive integer's irrational? If not, give an example of the square root of a number that is a rational number.

Sol. No, $\sqrt{4} = 2$ is a rational number.

3. Show how $\sqrt{5}$ can be represented on the number line.

Sol. $\sqrt{5}$ on Number line.

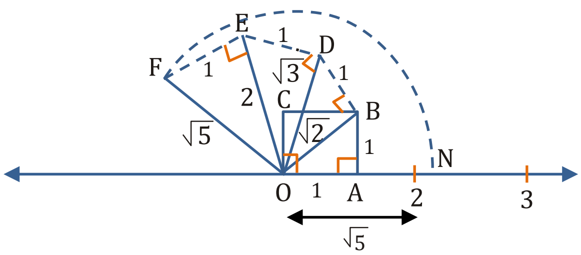
OABC is unit square.

So, OB = $\sqrt{1^2 + 1^2} = \sqrt{2}$

OD = $\sqrt{(\sqrt{2})^2 + 1} = \sqrt{3}$

OE = $\sqrt{(\sqrt{3})^2 + 1} = 2$

OF = $\sqrt{(2)^2 + 1} = \sqrt{5}$



Using compass we can cut arc with centre O and radius = OF on number line. ON is required result.

EXERCISE : 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$

(v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Sol. (i) $\frac{36}{100} = 0.36$

(Terminating)

(ii) $\frac{1}{11} = 0.090909.......$

(Non-Terminating Repeating)

$$\begin{array}{r|l} 11 & 1.00000 \;(0.090909.... \\ & -99 \\ & \overline{\;\;100} \\ & \;\;\;99 \\ & \overline{\;\;\;100} \\ & \;\;\;\;99 \\ & \overline{\;\;\;\;\;1} \end{array}$$

(iii) $4\frac{1}{8} = \frac{33}{8}$ = 4.125

(Terminating decimal)

(iv) $\frac{3}{13}$ = 0.230769230769......

= $0.\overline{230769}$

(Non-Terminating repeating)

(v) $\frac{2}{11} = 0.1818.......$

$= 0.\overline{18}$ (Non-Terminating repeating)

(vi) $\frac{329}{400}$

$$\begin{array}{r|l} 400 & 329.0000\;(0.8225 \\ & \underline{3200} \\ & \;\;900 \\ & \;\;\underline{800} \\ & \;\;1000 \\ & \;\;\;\underline{800} \\ & \;\;\;2000 \\ & \;\;\;\underline{2000} \\ & \;\;\;\;\times \end{array}$$

$\frac{329}{400}$ = 0.8225 (Terminating)

2. You know that $\frac{1}{7}=0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7}$ are, without actually doing the long division? If so, how?

Sol. Yes, we can predict decimal explain without actually doing long division method as

$\frac{2}{7}=2\times\frac{1}{7}$ = 2 × $0.\overline{142857}$ = $0.\overline{285714}$

$\frac{3}{7}=3\times\frac{1}{7}=3\times0.\overline{142857}=0.\overline{428571}$

$\frac{4}{7}=4\times\frac{1}{7}=4\times0.\overline{142857}=0.\overline{571428}$

$\frac{5}{7}=5\times\frac{1}{7}=5\times0.\overline{142857}=0.\overline{714285}$

$\frac{6}{7}=6\times\frac{1}{7}=6\times0.\overline{142857}=0.\overline{857142}$

3. Express the following in the form p/q, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$

(ii) $0.4\overline{7}$

(iii) $0.\overline{001}$

Sol. (i) Let x = 0.6666........ ... (1)

Multiplying both the sides by 10

10x = 6.666.......... ... (2)

Subtract (1) from (2)

10x – x = (6.6666......) – (0.6666.....)

$\Rightarrow 9x = 6 \Rightarrow x = \frac{6}{9}=\frac{2}{3}$

(ii) Let x = $0.4\overline{7}$ = .4777...

Multiply both sides by 10

10x = $4.\overline{7}$ ... (1)

Multiply both sides by 10

100x = $47.\overline{7}$ ... (2)

Subtract (1) from (2)

90x = 43

$x = \frac{43}{90}$

(iii) Let x = $0.\overline{001}=0.001001001......$ (1)

Multiply both sides by 1000

1000x = $1.\overline{001}$ ... (2)

Subtract (1) from (2)

999x = 1

$x = \frac{1}{999}$

4. Express 0.99999..... in the form $\frac{p}{q}$. Are you surprised by your answer ? With your teacher and classmates discuss why the answer makes sense.

Sol. Let x = 0.999.... ... (1)

Multiply both sides by 10 we get

10x = 9.99.... ... (2)

Subtract (1) from (2)

9x = 9

$\Rightarrow$ x = 1

0.9999.... = 1 = $\frac{1}{1}$

$\therefore$ p = 1, q = 1

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol. Maximum number of digits in the repeating block of digits in decimal expansion of $\frac{1}{17}$ can be 16.

$$
\begin{array}{l}
\quad\quad 0.058823529411764705 \\
17\overline{)1.00000000000000000000000000000} \\
\quad 85 \\
\quad 150 \\
\quad 136 \\
\quad\; 140 \\
\quad\; 136 \\
\quad\quad 40 \\
\quad\quad 34 \\
\quad\quad\; 60 \\
\quad\quad\; 51 \\
\quad\quad\quad 90 \\
\quad\quad\quad 85 \\
\quad\quad\quad\; 50 \\
\quad\quad\quad\; 34 \\
\quad\quad\quad\; 160 \\
\quad\quad\quad\; 153 \\
\quad\quad\quad\quad 70 \\
\quad\quad\quad\quad 68 \\
\quad\quad\quad\quad\; 20 \\
\quad\quad\quad\quad\; 17 \\
\quad\quad\quad\quad\quad 30 \\
\quad\quad\quad\quad\quad 17 \\
\quad\quad\quad\quad\quad 130 \\
\quad\quad\quad\quad\quad 119 \\
\quad\quad\quad\quad\quad\; 110 \\
\quad\quad\quad\quad\quad\; 102 \\
\quad\quad\quad\quad\quad\quad 80 \\
\quad\quad\quad\quad\quad\quad 68 \\
\quad\quad\quad\quad\quad\quad 120 \\
\quad\quad\quad\quad\quad\quad 119 \\
\quad\quad\quad\quad\quad\quad\; 100 \\
\quad\quad\quad\quad\quad\quad\quad 85 \\
\quad\quad\quad\quad\quad\quad\quad 150 \\
\quad\quad\quad\quad\quad\quad\quad 136 \\
\quad\quad\quad\quad\quad\quad\quad\quad 4
\end{array}
$$

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

6. Look at several examples of rational numbers in the form p/q ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Sol. There is a property that q must satisfy rational number of form $\frac{p}{q}$ ($q \neq 0$) where p, q are integers with no common factors other than 1 having terminating decimal representation (expansions) is that the prime factorisation of q has only powers of 2 or powers of 5 or both (i.e., q must be of the form $2^m \times 5^n$). Here m, n are whole numbers.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Sol. 0.01001000100001...

0.202002000200002...

0.003000300003...

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol.

$$
\begin{array}{l}
7\overline{)5.000000}(0.714285... \\
\quad 49 \\
\quad\; 10 \\
\quad\quad 7 \\
\quad\quad 30 \\
\quad\quad 28 \\
\quad\quad\; 20 \\
\quad\quad\; 14 \\
\quad\quad\quad 60 \\
\quad\quad\quad 56 \\
\quad\quad\quad\; 40 \\
\quad\quad\quad\; 35 \\
\quad\quad\quad\quad 5
\end{array}
$$

Thus, $\frac{5}{7} = 0.\overline{714285}$

$$
\begin{array}{l}
\frac{9}{11} = 11\overline{)9.0000}(0.8181... \\
\quad\quad\quad 88 \\
\quad\quad\quad\; 20 \\
\quad\quad\quad\; 11 \\
\quad\quad\quad\quad 90 \\
\quad\quad\quad\quad 88 \\
\quad\quad\quad\quad\; 20 \\
\quad\quad\quad\quad\; 11 \\
\quad\quad\quad\quad\quad 9
\end{array}
$$

Thus, $\frac{9}{11} = 0.\overline{81}$

Three different irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$ are taken as

0.75075007500075000...

0.78078007800078000...

0.808008000800008000000...

9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$ (ii) $\sqrt{225}$

(iii) 0.3796 (iv) 7.478478 ......

(v) 1.101001000100001......

Sol. (i) $\sqrt{23}$ = Irrational number

(ii) $\sqrt{225}$ = 15 = Rational number

(iii) 0.3796

decimal expansion is terminating

$\Rightarrow$ .3796 = Rational number

(iv) 7.478478...

$= 7.\overline{478}$ which is non-terminating recurring.

= Rational number

(v) 1.101001000100001.....

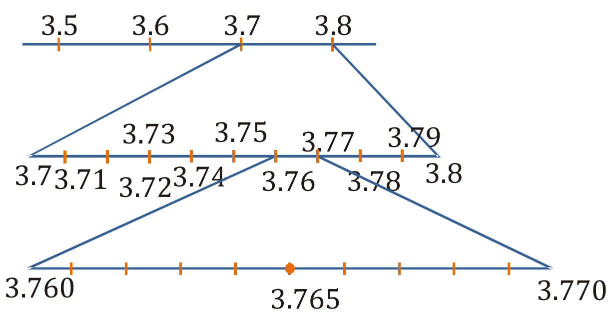
decimal expansion is non terminating and non-repeating.

= Irrational number

EXERCISE : 1.4

1. Visualise 3.765 on the number line, using successive magnification.

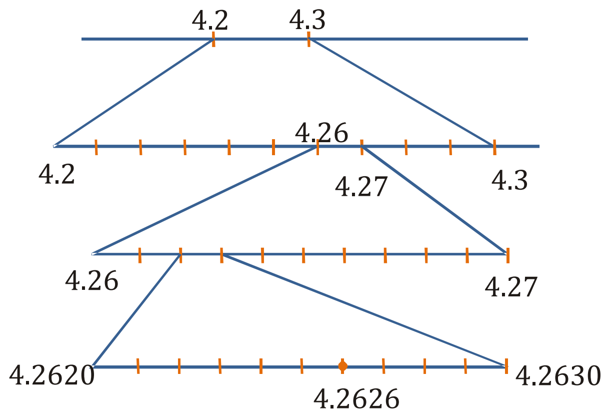
Sol. n = 3.765



2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Sol. n = $4.\overline{26}$

So, n = 4.2626 (upto 4 decimal places)



EXERCISE : 1.5

1. Classify the following numbers as rational or irrational :

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\dfrac{1}{\sqrt{2}}$

(v) 2π

Sol. (i) 2 is a rational number and $\sqrt{5}$ is an irrational number.

$\therefore$ 2 – $\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23} \Rightarrow (3+\sqrt{23})-\sqrt{23}$

= 3 is a rational number.

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}} = \dfrac{2}{7}$ is a rational number.

(iv) $\dfrac{1}{\sqrt{2}}$

$\because$ 1 is a rational number and $\sqrt{2}$ is an irrational number.

So, is an irrational number.

(v) 2π

$\because$ 2 is a rational number and π is an irrational number.

So, 2π is an irrational number.

2. Simplify each of the following expressions :

(i) $(3+\sqrt{3})(2+\sqrt{2})$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Sol. (i) $(3+\sqrt{3})(2+\sqrt{2})=3(2+\sqrt{2})+\sqrt{3}(2+\sqrt{2})$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})=(3)^2-(\sqrt{3})^2$

$= 9 - 3 = 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

$= (\sqrt{5})^2+2\sqrt{10}+(\sqrt{2})^2$

$= 7 + 2\sqrt{10}$

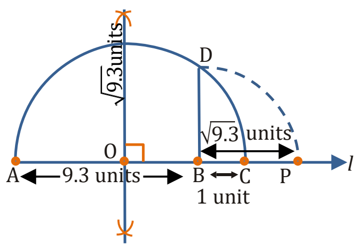
(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = 5 - 2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, π = c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

Sol. There is no contradiction. When we measure a length with a scale or any other device, we only get an approximate rational value. Therefore, we may not realise that c is irrational.

4. Represent $\sqrt{9.3}$ on the number line.

Sol.



Let ℓ be the number line.

Draw a line segment AB = 9.3 units and BC = 1 unit. Find the mid point O of AC.

Draw a semicircle with centre O and radius OA or OC.

Draw BD $\perp$ AC intersecting the semicircle at D. Then, BD = $\sqrt{9.3}$ units. Now, with centre B and radius BD, draw an arc intersecting the number line l at P.

Hence, BD = BP = $\sqrt{9.3}$

5. Rationalise the denominators of the following :

(i) $\dfrac{1}{\sqrt{7}}$ (ii) $\dfrac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\dfrac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\dfrac{1}{\sqrt{7}-2}$

Sol. (i) $\dfrac{1}{\sqrt{7}}=\dfrac{1}{\sqrt{7}}\times\dfrac{\sqrt{7}}{\sqrt{7}}=\dfrac{\sqrt{7}}{7}$

(ii) $\dfrac{1}{\sqrt{7}-\sqrt{6}}=\dfrac{1}{\sqrt{7}-\sqrt{6}}\times\dfrac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$=\dfrac{\sqrt{7}+\sqrt{6}}{7-6}=\dfrac{\sqrt{7}+\sqrt{6}}{1}=\sqrt{7}+\sqrt{6}$

(iii) $\dfrac{1}{\sqrt{5}+\sqrt{2}}$

$\dfrac{1}{\sqrt{5}+\sqrt{2}}\times\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\dfrac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\dfrac{1}{\sqrt{7}-2}=\dfrac{1}{\sqrt{7}-2}\times\dfrac{\sqrt{7}+2}{\sqrt{7}+2}$

$=\dfrac{\sqrt{7}+2}{7-4}=\dfrac{\sqrt{7}+2}{3}$

EXERCISE : 1.6

1. Find :

(i) $(64)^{1/2}$

(ii) $32^{1/5}$

(iii) $125^{1/3}$

Sol. (i) $(64)^{1/2} = (8^2)^{1/2} = (8^{2\times\frac{1}{2}}) = 8^1 = 8$

(ii) $32^{1/5} = (2^5)^{1/5} = (2^{5\times\frac{1}{5}}) = 2^1 = 2$

(iii) $(125)^{1/3} = (5^3)^{1/3} = 5^{3\times\frac{1}{3}} = 5$

2. Find :

(i) $9^{3/2}$ (ii) $32^{2/5}$

(iii) $16^{3/4}$ (iv) $125^{-1/3}$

Sol. (i) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = (3)^3 = 27$

(ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

(iii) $16^{3/4} = (2^4)^{3/4} = 2^3 = 8$

(iv) $125^{-1/3} = (5^3)^{-1/3} = 5^{-1} = 1/5$

3. Simplify :

(i) $2^{2/3}.2^{1/5}$ (ii) $\left(\frac{1}{3^3}\right)^7$

(iii) $\frac{11^{1/2}}{11^{1/4}}$ (iv) $7^{1/2}.8^{1/2}$

Sol. (i) $2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} = \frac{1}{3^{21}} = 3^{-21}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{1}{4}} = \sqrt[4]{11}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times 8)^{1/2} = (56)^{1/2}$