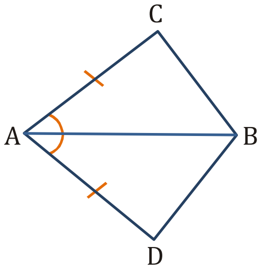
# NCERT QUESTIONS WITH SOLUTIONS

## EXERCISE : 7.1

**1.** In quadrilateral ACBD, AC = AD and AB bisects $\angle A$. Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?




**Sol.** Given : In quadrilateral ACBD, AC = AD and AB bisect $\angle A$.

To prove : $\Delta ABC \cong \Delta ABD$

Proof : In $\Delta ABC$ and $\Delta ABD$

AC = AD (Given)

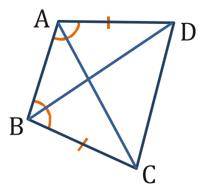
AB = AB (Common)

$\angle CAB = \angle DAB$ (AB bisect $\angle A$)

$\therefore$ $\Delta ABC \cong \Delta ABD$ (by SAS criteria)

BC = BD (by CPCT)

**2.** ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$. Prove that




(i) $\Delta ABD \cong \Delta BAC$

(ii) BD = AC

(iii) $\angle ABD = \angle BAC$

**Sol.** In $\Delta ABD$ and $\Delta BAC$,

AD = BC (Given)

$\angle DAB = \angle CBA$ (Given)

AB = AB (Common side)

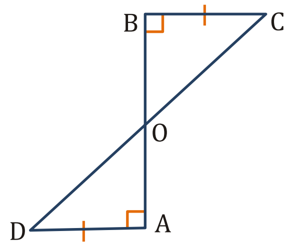
$\therefore$ By SAS congruence rule, we have

$\Delta ABD \cong \Delta BAC$

Also, by CPCT, we have

BD = AC and $\angle ABD = \angle BAC$

**3.** AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.




**Sol.** Given :

AD and BC are equal perpendiculars to line AB.

To prove : CD bisects AB

Proof : In $\Delta OAD$ and $\Delta OBC$

AD = BC (Given)

$\angle OAD = \angle OBC$ (Each 90°)

$\angle AOD = \angle BOC$

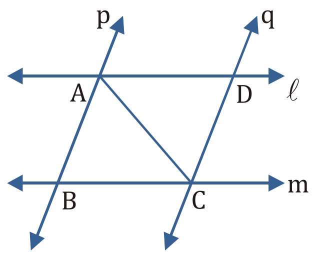
(Vertically opposite angles)

$\Delta OAD \cong \Delta OBC$ (AAS rule)

OA = OB (by CPCT)

$\therefore$ CD bisects AB

**4.** $\ell$ and m are two parallel lines intersected by another pair of parallel lines p and q. Show that $\Delta ABC$ $\Delta CDA$.




**Sol.** In $\Delta ABC$ and $\Delta CDA$

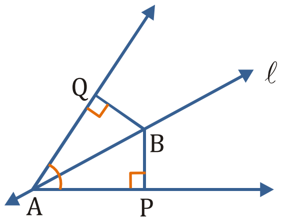
$\angle CAB = \angle ACD$ (Pair of alternate angle)

$\angle BCA = \angle DAC$ (Pair of alternate angle)

AC = AC (Common side)

$\therefore$ $\Delta ABC \cong \Delta CDA$ (ASA criteria)

**5.** Line $\ell$ is the bisector of an angle $\angle A$ and B is any point on $\ell$. BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that :




(i) $\Delta APB \ \Delta AQB$

(ii) BP = BQ or B is equidistant from the arms of $\angle A$

**Sol.** Given : line $\ell$ is bisector of angle A and B is any point on $\ell$. BP and BQ are perpendicular from B to arms of $\angle A$

To prove :

(i) $\Delta APB \cong \Delta AQB$

(ii) BP = BQ

Proof :

(i) In $\Delta APB$ and $\Delta AQB$

$\angle BAP = \angle BAQ$ ($\ell$ is bisector)

AB = AB (common)

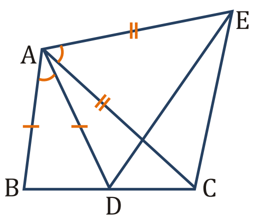
$\angle BPA = \angle BQA$ (Each 90°)

$\therefore$ $\Delta APB \cong \Delta AQB$ (AAS rule)

(ii) $\Delta APB \cong \Delta AQB$

BP = BQ (By CPCT)

**6.** In figure, AC = AE, AB = AD and $\angle BAD = \angle EAC$. Show that BC = DE.




**Sol.** Given : AC = AE

AB = AD,

$\angle BAD = \angle EAC$

To prove : BC = DE

Proof : In $\Delta ABC$ and $\Delta ADE$

AB = AD (Given)

AC = AE (Given)

$\angle BAD = \angle EAC$

Add $\angle DAC$ to both

$\Rightarrow$ $\angle BAD + \angle DAC = \angle DAC + \angle EAC$

$\angle BAC = \angle DAE$

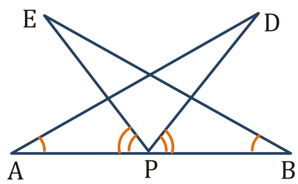
$\Delta ABC \cong \Delta ADE$ (SAS rule)

BC = DE (By CPCT)

**7.** AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that

(i) $\Delta DAP \ \Delta EBP$

(ii) AD = BE




**Sol.** $\angle EPA = \angle DPB$ (Given)

$\Rightarrow$ $\angle EPA + \angle DPE = \angle DPB + \angle DPE$

$\Rightarrow$ $\angle APD = \angle BPE$ ...(1)

Now, in $\Delta DAP$ and $\Delta EBP$, we have

AP = PB ($\because$ P is mid point of AB)

$\angle PAD = \angle PBE$

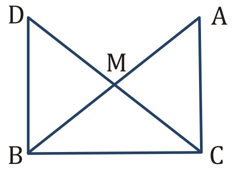
$$\left\{\begin{array}{l}\because \angle PAD = \angle BAD,\ \angle PBE = \angle ABE \\ \text{and we are given that } \angle BAD = \angle ABE\end{array}\right\}$$

Also, $\angle APD = \angle BPE$ (By 1)

$\therefore$ $\Delta DAP \cong \Delta EBP$ (By ASA congruence)

$\Rightarrow$ AD = BE (By CPCT)

**8.** In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that :




(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2}AB$

**Sol.** (i) In $\Delta AMC \cong \Delta BMD$,

AM = BM  (∵ M is mid point of AB)

$\angle AMC = \angle BMD$

(Vertically opposite angles)

CM = DM  (Given)

∴ $\Delta AMC \cong \Delta BMD$  (By SAS congruence)

(ii) $\angle AMC = \angle BMD$,

$\Rightarrow$ $\angle ACM = \angle BDM$  (By CPCT)

$\Rightarrow$ CA || BD

$\Rightarrow$ $\angle BCA + \angle DBC = 180°$

$\Rightarrow$ $\angle DBC = 90°$  (∵ $\angle BCA = 90°$)

(iii) In $\Delta DBC$ and $\Delta ACB$,

DB = AC (∵ $\Delta BMD \cong \Delta AMC$)

$\angle DBC = \angle ACB$  (Each = 90°)

BC = CB  (Common side)

∴ $\Delta DBC \cong \Delta ACB$  (By SAS congruence)

(iv) In $\Delta DBC \cong \Delta ACB$

$\Rightarrow$ CD = AB  ... (1)

$\Rightarrow$ CM = DM  (given)

$\Rightarrow$ $CM = DM = \frac{1}{2}CD$

$\Rightarrow$ CD = 2 CM  ... (2)

From (1) and (2),

2 CM = AB

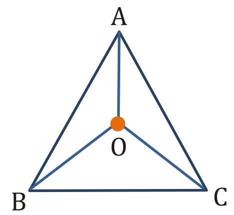
$\Rightarrow$ $CM = \frac{1}{2}AB$

## EXERCISE : 7.2

**1.** In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that :

(i) OB = OC  (ii) AO bisects $\angle A$.

**Sol.**




(i) In $\Delta ABC$, OB and OC are bisectors of $\angle B$ and $\angle C$.

∴ $\angle OBC = \frac{1}{2}\angle B$  ... (1)

$\angle OCB = \frac{1}{2}\angle C$  ... (2)

Also, AB = AC  (Given)

$\Rightarrow$ $\angle B = \angle C$  ... (3)

From (1), (2), (3), we have $\angle OBC = \angle OCB$

Now, in $\Delta OBC$, we have $\angle OBC = \angle OCB$

$\Rightarrow$ OB = OC

(Sides opposite to equal angles are equal)

(ii) $\angle OBA = \frac{1}{2}\angle B$ and $\angle OCA = \frac{1}{2}\angle C$

$\Rightarrow$ $\angle OBA = \angle OCA$ $(\because \angle B = \angle C)$

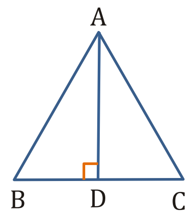
AB = AC and OB = OC

$\therefore$ $\Delta OAB \cong \Delta OAC$ (SAS congruence criteria)

$\Rightarrow$ $\angle OAB = \angle OAC$

$\Rightarrow$ AO bisects $\angle A$.

**2.** In $\Delta ABC$, AD is the perpendicular bisector of BC. Show that $\Delta ABC$ is an isosceles triangle in which AB = AC.




**Sol.** Given : In $\Delta ABC$, AD is perpendicular bisector of BC.

To Prove : $\Delta ABC$ is isosceles $\Delta$ with AB = AC

Proof : In $\Delta ADB$ and $\Delta ADC$

$\angle ADB = \angle ADC$ (Each 90°)

DB = DC (AD is $\perp$ bisector of BC)

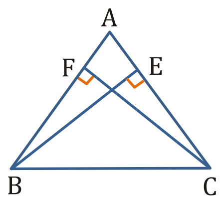
AD = AD (Common)

$\Delta ADB \cong \Delta ADC$ (By SAS rule)

AB = AC (By CPCT)

$\therefore$ $\Delta ABC$ is an isosceles $\Delta$ with AB = AC

**3.** ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.




**Sol.** In $\Delta ABE$ and $\Delta ACF$, we have

$\angle BEA = \angle CFA$ (Each = 90°)

$\angle A = \angle A$ (Common angle)

AB = AC (Given)

$\therefore$ $\Delta ABE \cong \Delta ACF$

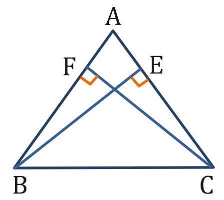
(By AAS congruence criteria)

$\Rightarrow$ BE = CF (By CPCT)

**4.** ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that

(i) $\Delta ABE \cong \Delta ACF$

(ii) AB = AC, i.e., ABC is an isosceles triangle.




**Sol.** (i) In $\Delta ABE$ and $\Delta ACF$,

we have $\angle A = \angle A$ (Common)

$\angle AEB = \angle AFC$ (Each = 90°)

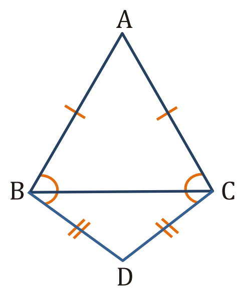
BE = CF (Given)

$\therefore$ $\Delta ABE \cong \Delta ACF$ (By AAS congruence)

(ii) $\Delta ABE \cong \Delta ACF$

$\Rightarrow$ AB = AC (By CPCT)

**5.** ABC and DBC are two isosceles triangles on the same base BC (see figure). Show that $\angle ABD = \angle ACD$.



**Sol.** Given : ABC and BCD are two isosceles triangle on common base BC.

To prove : $\angle ABC = \angle ACD$

Proof : ABC is an isosceles

Triangle on base BC

$\therefore$ $\angle ABC = \angle ACB$ ... (1)

$\because$ DBC is an isosceles $\Delta$ on base BC.

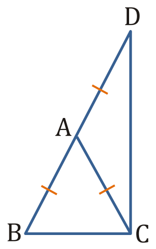
$\angle DBC = \angle DCB$ ... (2)

Adding (1) and (2)

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\Rightarrow$ $\angle ABD = \angle ACD$

**6.** $\Delta ABC$ is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see figure). Show that $\angle BCD$ is a right angle.




**Sol.** In $\Delta ABC$, AB = AC

$\Rightarrow$ $\angle ACB = \angle ABC$ ...(1)

In $\Delta ACD$,

AD = AB (By construction)

$\Rightarrow$ AD = AC

$\Rightarrow$ $\angle ACD = \angle ADC$ ...(2)

Adding (1) and (2),

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow$ $\angle BCD = \angle ABC + \angle ADC$

In $\Delta DBC$,

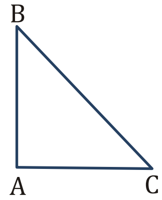
$\angle ABC + \angle BCD + \angle CDB = 180°$

$\Rightarrow$ $2\angle BCD = 180°$

$\Rightarrow$ $\angle BCD = 90°$

**7.** ABC is a right angled triangle in which $\angle A = 90°$ and AB = AC. Find $\angle B$ and $\angle C$.

**Sol.**




In $\Delta ABC$

AB = AC

$\angle B = \angle C$ ... (1)

(angles opposite to equal sides are equal)

In $\Delta ABC$

$\angle A + \angle B + \angle C = 180°$

$90° + \angle B + \angle C = 180°$

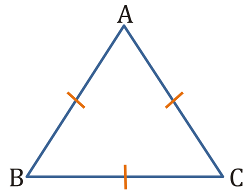
$\angle B + \angle C = 90°$ ... (2)

from (1) and (2)

$\angle B = \angle C = 45°$

**8.** Show that the angles of an equilateral triangle are 60° each.

**Sol.**




$\Delta ABC$ is equilateral triangle.

$\Rightarrow$ AB = BC = CA

Now, AB = BC

$\Rightarrow$ BA = BC

$\Rightarrow$ $\angle C = \angle A$ ... (1)

Similarly, $\angle A = \angle B$ ... (2)

From (1) and (2),

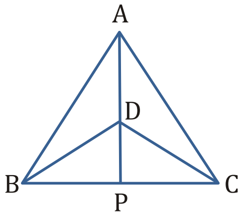
$\angle A = \angle B = \angle C$ ... (3)

Also, $\angle A + \angle B + \angle C = 180°$ ... (4)

$\Rightarrow$ $\angle A = \angle B = \angle C = \frac{1}{3} \times 180° = 60°$

## EXERCISE : 7.3

**1.** $\Delta ABC$ and $\Delta DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see figure). If AD is extended to intersect BC at P, show that




(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC.

**Sol.** (i) In $\Delta ABD$ and $\Delta ACD$,

AB = AC (∵ $\Delta ABC$ is isosceles)

DB = DC (∵ $\Delta DBC$ is isosceles)

AD = AD (Common side)

∴ $\Delta ABD \cong \Delta ACD$ (By SSS congruence rule)

(ii) Now, $\Delta ABD \cong \Delta ACD$

⇒ $\angle BAD = \angle CAD$ (By CPCT) ...(1)

In $\Delta ABP$ and $\Delta ACP$,

AB = AC (∵ $\Delta ABC$ is isosceles)

⇒ $\angle BAP = \angle CAP$ (By 1)

AP = AP (common side)

∴ $\Delta ABP \cong \Delta ACP$ (By SAS congruence rule)

(iii) $\Delta ABD \cong \Delta ACD$ (Proved above)

$\angle BAD = \angle CAD$ (by CPCT)

$\angle ADB = \angle ADC$ (by CPCT)

$180 - \angle ADB = 180 - \angle ADC$

⇒ $\angle BDP = \angle CDP$

AP bisects $\angle A$ as well as $\angle D$

(iv) $\Delta ABP \cong \Delta ACP$ (Proved above)

⇒ BP = CP (By CPCT)

⇒ AP bisects BC

$\angle APB = \angle APC$ (By CPCT)

$\angle APB + \angle APC = 180°$

$2\angle APB = 180°$

$\angle APB = 90°$

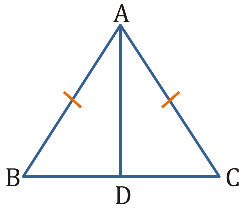
AP is perpendicular bisector of BC

**2.** AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC

(ii) AD bisects $\angle A$

**Sol.** Given : AD is an altitude of an isosceles triangle ABC in which AB = AC.




To Prove :

(i) AD bisect BC.

(ii) AD bisect $\angle A$.

Proof : (i) In right $\Delta ADB$ and right $\Delta ADC$.

Hyp.AB = Hyp. AC

$\angle ADB = \angle ADC$ (Each 90°)

Side AD = side AD (Common)

$\Delta ADB \cong \Delta ADC$ (RHS rule)

⇒ BD = CD (By CPCT)

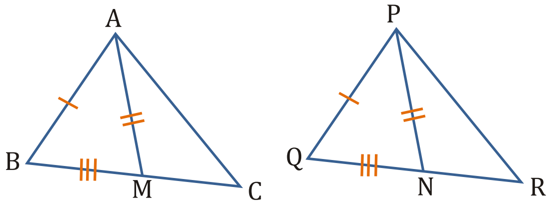
⇒ AD bisect BC

(ii) $\Delta ADB \cong \Delta ADC$

$\angle BAD = \angle CAD$ (By CPCT)

⇒ AD bisect $\angle A$

**3.** Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of $\Delta PQR$ (see figure). Show that :




(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$

**Sol.** (i) $BM = \frac{1}{2} BC$ ($\because$ M is mid-point of BC)

$QN = \frac{1}{2} QR$ ($\because$ N is mid-point of QR)

$\Rightarrow$ $BM = QN$ ($\because$ BC = QR is given)

Now, in $\Delta ABM$ and $\Delta PQN$, we have

$AB = PQ$ (Given)

$BM = QN$ (Proved)

$AM = PN$ (Given)

$\therefore$ $\Delta ABM \cong \Delta PQN$ (SSS congruence criteria)

(ii) $\Delta ABM \cong \Delta PQN$

$\Rightarrow$ $\angle ABM = \angle PQN$

$\angle B = \angle Q$ (By CPCT)

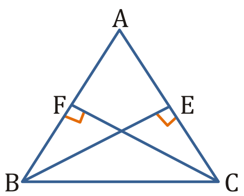
Now, in $\Delta ABC$ and $\Delta PQN$,

$AB = PQ$, $\angle B = \angle Q$ and $BC = QR$

$\Rightarrow$ $\Delta ABC \cong \Delta PQN$ [by SAS congruence]

**4.** BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

**Sol.** Given BE and CF are two altitude of $\Delta ABC$.




To prove : $\Delta ABC$ is isosceles.

Proof : In right $\Delta BEC$ and right $\Delta CFB$

side BE = side CF (Given)

Hyp.BC = Hyp CB (Common)

$\angle BEC = \angle BFC$ (Each 90°)

$\Delta BEC \cong \Delta CFB$ (RHS Rule)

$\therefore$ $\angle BCE = \angle CBF$ (By CPCT)

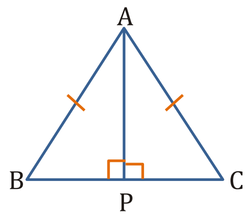
$AB = AC$

(Side opp. to equal angles are equal)

$\Delta ABC$ is isosceles.

**5.** ABC is an isosceles triangle with AB = AC. Draw $AP \perp BC$ to show that $\angle B = \angle C$

**Sol.**




In $\Delta APB$ and $\Delta APC$

$AB = AC$ (Given)

$\angle APB = \angle APC$ (Each = 90°)

$AP = AP$ (common side)

Therefore, by RHS congruence criteria, we have

$\Delta APB \cong \Delta APC$

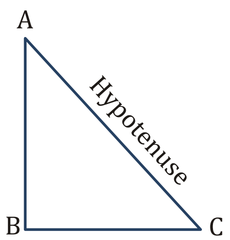
$\Rightarrow$ $\angle ABP = \angle ACP$ (By CPCT)

$\Rightarrow$ $\angle B = \angle C$

## EXERCISE : 7.4

**1.** Show that in a right angled triangle, the hypotenuse is the longest side.

**Sol.**




$\Delta ABC$ is right angled at B.

AC is hypotenuse.

Now, $\angle B = 90°$

and $\angle A + \angle C = 90°$

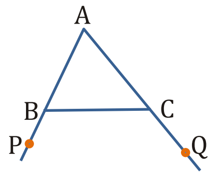
$\Rightarrow$ $\angle A < 90°$ and $\angle C < 90°$

$\Rightarrow$ $\angle B > \angle A$ and $\angle B > \angle C$

$\Rightarrow$ $AC > BC$ and $AC > AB$.

$\therefore$ Hypotenuse AC is the longest side of the right angled $\Delta ABC$.

**2.** In figure, sides AB and AC of $\Delta ABC$ are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$




**Sol.** Sides AB and AC of $\Delta ABC$ are extended to points P and Q

To prove : $AC > AB$

Proof : $\angle PBC < \angle QCB$ (Given )

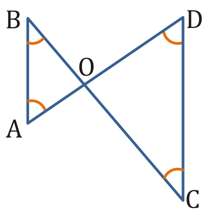
$180° - \angle PBC > 180° - \angle QCB$

$\angle ABC > \angle ACB$

$\Rightarrow$ $AC > AB$

(sides opposite to greater angle is longer)

**3.** In fig, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.




**Sol.** $\angle B < \angle A$ in $\Delta OAB$

$\Rightarrow$ $OA < OB$ ...(1)

Also, $\angle C < \angle D$ in $\Delta OCD$

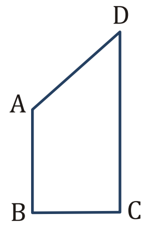
$\Rightarrow$ $OD < OC$ ...(2)

Adding (1) and (2),

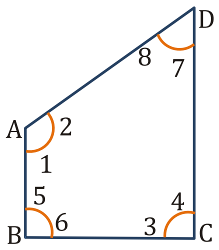
$OA + OD < OB + OC$

$\Rightarrow$ $AD < BC$

**4.** AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see figure). Show that $\angle A > \angle C$ and $\angle B > \angle D$.




**Sol.**




In quadrilateral ABCD, AB is the smallest side and CD is the longest side. Join AC and BD.

In $\Delta ABC$,

$BC > AB$ ($\because$ AB is smallest side)

$\Rightarrow$ $\angle 1 > \angle 3$ ... (1)

In $\Delta ACD$,

$CD > AD$ ($\because$ CD is longest side)

$\Rightarrow$ $\angle 2 > \angle 4$ ... (2)

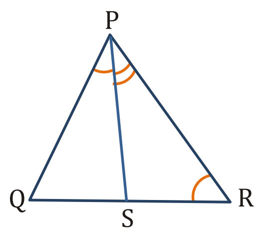
Adding (1) and (2), we have

$\angle 1 + \angle 2 > \angle 3 + \angle 4$

$\Rightarrow$ $\angle A > \angle C$

Similarly, we can prove that $\angle B > \angle D$

**5.** In figure, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.



**Sol.** Given : PR > PQ, PS bisect $\angle QPR$.

To prove : $\angle PSR > \angle PSQ$

Proof : In $\Delta PQR$

PR > PQ (Given)

$\angle PQR > \angle PRQ$

(angle opposite to longer side is greater)

and $\angle PQS > \angle PRS$ ... (1)

PS bisects $\angle QPR$

$\Rightarrow$ $\angle QPS = \angle RPS$ ... (2)

From (1) and (2)

$\angle PQS + \angle QPS > \angle PRS + \angle RPS$

$\angle PSR > \angle PSQ$

(by exterior angle property)

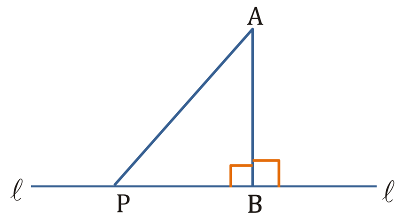
Hence proved.

**6.** Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

**Sol.** Let us have AB as the perpendicular line segment and AP is any other line segment. Now $\Delta ABP$ is right angled and AP is hypotenuse.

Here, $\angle B > \angle P$ $(\because \angle B = 90°)$

$\Rightarrow$ AP > AB




Thus, perpendicular line segment is smallest.