Derivation of Sum of Finite and Infinite Geometric Progression

Geometric Progression, GP

Geometric progression (also known as geometric sequence) is a sequence of numbers where the ratio of any two adjacent terms is constant. The constant ratio is called the common ratio, r of geometric progression. Each term therefore in geometric progression is found by multiplying the previous one by r.

Eaxamples of GP:

- 3, 6, 12, 24, … is a geometric progression with r = 2
- 10, -5, 2.5, -1.25, … is a geometric progression with r = -1/2

The n^{th} term of geometric progression

Given each term of GP as $a_1, a_2, a_3, a_4, \ldots, a_m, \ldots, a_n$, expressing all these terms according to the first term a_1 will give us...

$a_1 = a_1$

$a_2 = a_1 r$

$a_3 = a_2 r = (a_1 r)\, r = a_1 r^2$

$a_4 = a_3 r = (a_1 r^2)\, r = a_1 r^3$

. . .

$a_m = a_1 r^{m-1}$

. . .

$$a_n = a_1 r^{n-1}$$

Where

a_1 = the first term, a_2 = the second term, and so on

a_n = the last term (or the n^{th} term) and

a_m = any term before the last term

Sum of Finite Geometric Progression

The sum in geometric progression (also called geometric series) is given by

$S = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$

$S = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^{n-1}$ → Equation (1)

Multiply both sides of Equation (1) by r will have

$Sr = a_1 r + a_1 r^2 + a_1 r^3 + a_1 r^4 + \ldots + a_1 r^n$ → Equation (2)

Subtract Equation (2) from Equation (1)

$S - Sr = a_1 - a_1 r^n$

$(1 - r)S = a_1(1 - r^n)$

$$S = \frac{a_1(1 - r^n)}{1 - r}$$

The above formula is appropriate for GP with r < 1.0

Subtracting Equation (1) from Equation (2) will give

$Sr - S = a_1 r^n - a_1$

$$(r-1)S = a_1(r^n - 1)$$

$$S = \frac{a_1(r^n - 1)}{r - 1}$$

This formula is appropriate for GP with r > 1.0.

Sum of Infinite Geometric Progression, IGP

The number of terms in infinite geometric progression will approach to infinity (n = ∞). Sum of infinite geometric progression can only be defined at the range of -1.0 < (r ≠ 0) < +1.0 exclusive.

From

$$S = \frac{a_1(1 - r^n)}{1 - r}$$

$$S = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S = \frac{a_1}{1 - r} - \frac{a_1 r^n}{1 - r}$$

For n → ∞, the quantity (a_1 r^n) / (1 - r) → 0 for -1.0 < (r ≠ 0) < +1.0, thus,

$$S = \frac{a_1}{1 - r}$$

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