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Derivation of Sum of Finite and Infinite Geometric Progression



Geometric Progression, GP

Geometric progression (also known as geometric sequence) is a sequence of numbers where the ratio of any two adjacent terms is constant. The constant ratio is called the common ratio, r of geometric progression. Each term therefore in geometric progression is found by multiplying the previous one by r.

Eaxamples of GP:

- 3, 6, 12, 24, ... is a geometric progression with r = 2
- $10, -5, 2.5, -1.25, \dots$ is a geometric progression with r = -1/2

The nth term of geometric progression

Given each term of GP as $a_1, a_2, a_3, a_4, \ldots, a_m, \ldots, a_n$, expressing all these terms according to the first term a_1 will give us...

$$a_1 = a_1$$

$$a_2 = a_1 r$$

$$a_3 = a_2 r = (a_1 r) r = a_1 r^2$$

$$a_4 = a_3 r = (a_1 r^2) r = a_1 r^3$$

. . .

$$a_m=a_1r^{\,m-1}$$

. . .

$$a_n = a_1 r^{n-1}$$

Where

 a_1 = the first term, a_2 = the second term, and so on

 a_n = the last term (or the n^{th} term) and

 $a_m = any term before the last term$

Sum of Finite Geometric Progression

The sum in geometric progression (also called geometric series) is given by

$$S = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$$

$$S = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^{n-1} \rightarrow \text{Equation } (1)$$

Multiply both sides of Equation (1) by r will have

$$Sr = a_1r + a_1r^2 + a_1r^3 + a_1r^4 + \ldots + a_1r^n \Rightarrow \text{Equation (2)}$$

Subtract Equation (2) from Equation (1)

$$S - Sr = a_1 - a_1r^n$$

$$(1-r)S = a_1(1-r^n)$$

$$S = \frac{a_1(1 - r^n)}{1 - r}$$

The above formula is appropriate for GP with r < 1.0

Subtracting Equation (1) from Equation (2) will give

$$Sr - S = a_1r^n - a_1$$

$$(r-1)S = a_1(r^n-1)$$

$$S = \frac{a_1(r^n - 1)}{r - 1}$$

This formula is appropriate for GP with r > 1.0.

Sum of Infinite Geometric Progression, IGP

The number of terms in infinite geometric progression will approach to infinity (n = ∞). Sum of infinite geometric progression can only be defined at the range of -1.0 < (r \neq 0) < +1.0 exclusive.

From

$$S=\frac{a_1(1-r^{\,n})}{1-r}$$

$$S = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S = \frac{a_1}{1 - r} - \frac{a_1 r^n}{1 - r}$$

For $n \to \infty$, the quantity $(a_1 r^n) / (1 - r) \to 0$ for $-1.0 < (r \ne 0) < +1.0$, thus,

$$S = \frac{a_1}{1 - r}$$

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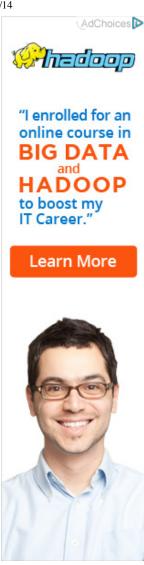
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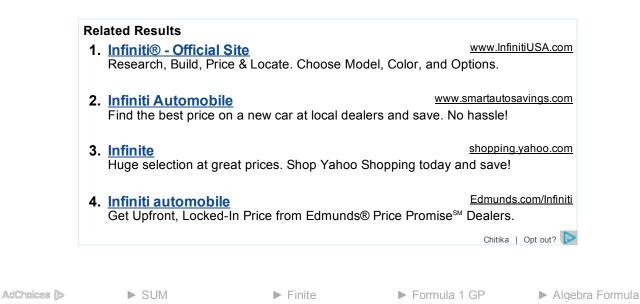
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