Exam 1

Texas A&M University
AERO-430-500 Numerical Simulation
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Contents

1 Abstract

The purpose of this assignment is to formulate the analytical solution to the problem of heat conduction in a bi-material cylindrical pipe, a behavior governed by a second order ODE, as well as finding the total heat loss to the environment. Both analytical and finite difference method (FDM) techniques were used in the analysis of two separate cases. For the first case, the interface between the two materials will be located at $x_2 = x_1 + \frac{x_3 - x_1}{7}$. For the second case, the interface between the two materials will be located at $x_2 = x_1 + \frac{x_3 - x_1}{2\pi}$. In both cases, the conditions were utilized:

- $T(x_1) = 500℃$
- $T_{amb} = 20℃$
- $x_1 = 3\ cm$
- $x_3 = 6.5\ cm$
- $0.4 \leq \beta \leq 0.55 \frac{cal}{sec} * cm^2 * ℃$
- $0.67 \leq \gamma_{ss} \leq 0.90 \frac{cal}{sec} * cm * ℃$
- $1.50 \leq \gamma_{cs} \leq 2.10 \frac{cal}{sec} * cm * ℃$

2 Analytical Solution

2.1 Initialization

The analytical solution can be derived by breaking up the function over the entire bar into a piecewise function, each defining the section of the bar either side of the interface.

$$Case\ 1: \quad T(x) = \begin{cases} T_L(x) & x_1 < x_2 < x_1 + \dfrac{x_3 - x_1}{7} \\ T_R(x) & x_1 + \dfrac{x_3 - x_1}{7} < x_2 < x_3 \end{cases}$$

$$Case\ 2: \quad T(x) = \begin{cases} T_L(x) & x_1 < x_2 < x_1 + \dfrac{x_3 - x_1}{2\pi} \\ T_R(x) & x_1 + \dfrac{x_3 - x_1}{2\pi} < x_2 < x_3 \end{cases}$$

2.2 Boundary Conditions

The equation governing the temperature along the circular cylinder includes several boundary conditions that can be utilized to solve the analytical equation. The distribution of temperature is governed by the following:

$$\begin{cases} \dfrac{d}{dx}\left(x\gamma(x)\dfrac{du}{dx}(x)\right) = 0, \quad x_1 < x < x_2 \\ u = T_0, \quad Q(x_3) = 2\pi x_3 \beta(\mathrm{u}(\mathrm{x}_3) - T_1) \\ \quad [u(x_2)] = 0, \qquad [Q(x_2)] = 0 \end{cases}$$

Here, u(x) is the temperature of the pipe at radius $x, q(x) = -\gamma(x)\frac{du}{dx}(x)$ is the heat flux, $\gamma(x)$is the heat conduction coefficient, T_0 is the temperature at the interior wall, T_1 is the ambient temperature of the air surrounding the bar and β is the heat convection coefficient of the outer wall of the pipe. Additionally, $Q(x) = 2\pi x q(x)$ is the total heat. For this report the heat conduction of the two materials is constrained to the following conditions:

$$\begin{array}{ll} \gamma_{ss}, & x_1 < x < x_2 \\ \gamma_{cs} & x_2 < x < x_3 \end{array}$$

2.3 Exact Solution

By applying these boundary conditions, the exact solution can be obtained. First, we integrate the differential equation

$$u(x) = \begin{cases} \dfrac{C_1}{\gamma_{ss}}\ln(x) + C_2 & x_1 < x < x_2 \\ \dfrac{C_3}{\gamma_{cs}}\ln(x) + C_4 & x_2 < x < x_3 \end{cases}$$

Here, it is clear that there are two unknowns in each case. From the jump condition $[Q(x_2)] = 0$ it can be found that

$$C_1 = C_3 = C$$

From the boundary condition at x_1 it can be found that:

$$C_2 = T_0 - \frac{C}{\gamma_{ss}}\ln(x_1)$$

From the jump condition $[u(x_2)] = 0$ it can be found that:

$$C_4 = \frac{C}{\gamma_{ss}}\ln(x_2) - \frac{C}{\gamma_{cs}}\ln(x_2) + C_2$$

Or using the equation for C_2 the following can be obtained:

$$C_4 = \frac{C}{\gamma_{ss}}\ln\left(\frac{x_2}{x_1}\right) - \frac{C}{\gamma_{cs}}\ln(x_2) + T_0$$

From the boundary condition at x_3 and the previous equation, it can be found that:

$$T_1 - T_0 = C\left(\frac{1}{\beta x_3} + \frac{1}{\gamma_{cs}}\ln\left(\frac{x_3}{x_2}\right) + \frac{1}{\gamma_{ss}}\ln\left(\frac{x_2}{x_1}\right)\right)$$

From these conditions the constants in the equation for u(x) can be obtained and an exact solution of the radial axisymmetric cylindrical rod can be found.

$$C_1 = C_3 = C = \frac{T_1 - T_0}{\frac{1}{\beta x_3} + \frac{1}{\gamma_{cs}}\ln\left(\frac{x_3}{x_2}\right) + \frac{1}{\gamma_{ss}}\ln\left(\frac{x_2}{x_1}\right)}$$

$$C_2 = T_0 - \frac{C}{\gamma_{ss}} * \ln(x_1)$$

$$C_4 = \frac{C}{\gamma_{ss}}\ln(x_2) - \frac{C}{\gamma_{cs}}\ln(x_2) + C_2$$

These constants are now plugged back into the following:

$$u(x) = \begin{cases} \frac{C_1}{\gamma_{ss}}\ln(x) + C_2 & x_1 < x < x_2 \\ \frac{C_3}{\gamma_{cs}}\ln(x) + C_4 & x_2 < x < x_3 \end{cases}$$

2.4 Heat Transfer

In order to derive the equation for the heat transfer through the surface of the bar, one of two methods can be taken. The first involves investigating the heat transfer to the surrounding environment. This process is governed by the following equation:

$$Q(x_3) = 2\pi\beta x_3\left(u_{x_3} - u_{amb}\right)$$

Additionally, the heat transfer through the bar is equivalent to the heat transfer through the derivative of the temperature at the right end of the bar. This process is governed be the following equation:

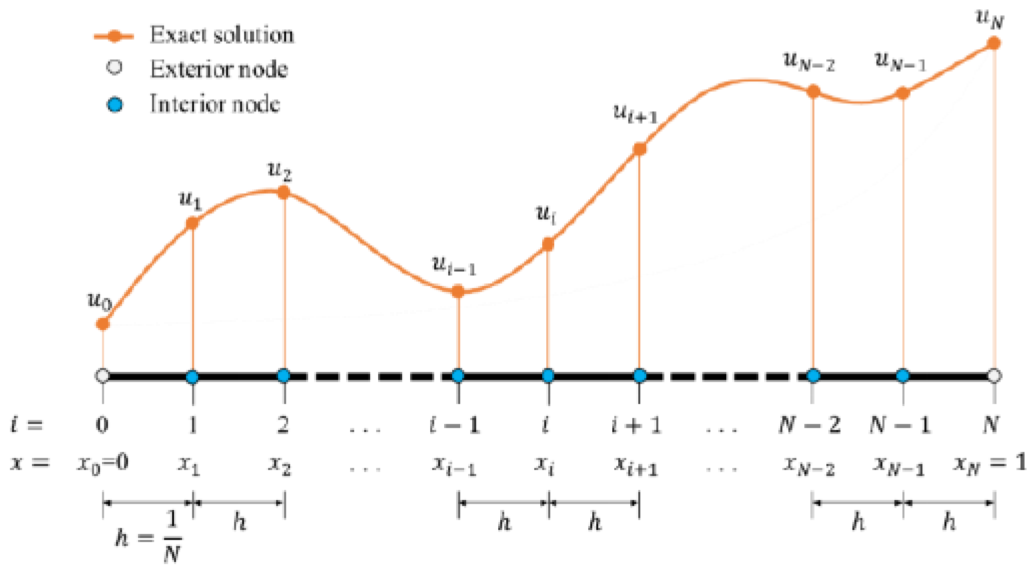
$$Q(x_3) = -2\pi\gamma_{cs}u'(x_3)$$

$$Q(x_3) = -\frac{2\pi\gamma_{cs}\left(U_{x_3} - u_{x_{3-1}}\right)}{\Delta x}$$

3 Finite Difference Method

3.1 Discretization of the Domain

To utilize the FDM, the domain must be discretized into N+1 evenly spaced points. As shown in the figure below, the discretized domain is x_i for $i = 0, 1, \dots, N$ with the spacing between the nodes $\Delta x = h$ and $u_i = u(x_i)$.



3.2 Conservation of Heat Flux

The conservation of heat flux states that the heat flux into the system must be equal to the heat flux leaving the system as shown in the equation below:

$$-\dot{q}(x) + \dot{q}(x + dx) + \dot{q}_c(x) = 0$$

If this concept is applied to the area about a node within the radial direction of the cylindrical bar, the following equation can be derived:

$$\frac{Q_{i+\frac{1}{2}} - Q_{i-\frac{1}{2}}}{\Delta x} = 0$$

Here, Δx is defined by the following:

$$\Delta x = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$$

Additionally, the equations of heat flux are defined by the following:

$$Q_{i+\frac{1}{2}} = 2\pi x_{i+\frac{1}{2}} \gamma_{i+\frac{1}{2}} T'(x_{i+\frac{1}{2}})$$

Expanding the temperature derivative term yields:

$$Q_{i+\frac{1}{2}} = \frac{2\pi x_{i+\frac{1}{2}} \gamma_{i+\frac{1}{2}} (T_{i+1} - T_i)}{\Delta x}$$

A similar equation can be obtained for$Q_{i-\frac{1}{2}}$:

$$Q_{i-\frac{1}{2}} = \frac{2\pi x_{i-\frac{1}{2}}\gamma_{i-\frac{1}{2}}(T_i - T_{i-1})}{\Delta x}$$

Substituting these two equations into the original equation yields the following:

$$\frac{2\pi x_{mid}^R \gamma_{mid}^R (T_i - T_{i-1})}{\Delta x^R} - \frac{2\pi x_{mid}^L \gamma_{mid}^L (T_{i+1} - T_i)}{\Delta x^L} = 0$$

Rearranging terms in this equation in order to isolate the temperature terms produces the following equation:

$$-\frac{x_{mid}^L \gamma_{mid}^L}{\Delta x^L} T_{i-1} + \left(\frac{x_{mid}^R \gamma_{mid}^R}{\Delta x^R} + \frac{x_{mid}^L \gamma_{mid}^L}{\Delta x^L}\right) T_i - \left(\frac{x_{mid}^R \gamma_{mid}^R}{\Delta x^R}\right) T_{i+1} = 0$$

To further simplify this equation, we will represent the following terms with the following variables

$$\kappa^R = \frac{x_{mid}^R \gamma_{mid}^R}{\Delta x^R}$$

$$\kappa^L = \frac{x_{mid}^L \gamma_{mid}^L}{\Delta x^L}$$

This equation will apply to all nodes within our cylindrical rod except for the boundary nodes. For these points, separate equations will need to be used. The equation at $x = x_1$ is trivial and can be represented with:

$$T_0 = 500℃$$

The equation for $x = x_3$ is less trivial and requires some additional calculations. Here, the conservation of heat flux still applies and can be used to derive the following:

$$2\pi \frac{x_{mid}^L \gamma_{mid}^L}{\Delta x^L} (T_N - T_{N-1}) = 2\pi \beta x_3 (T_N - T_{amb})$$

Isolating the temperature terms yields the following:

$$-\frac{x_{mid}^L \gamma_{mid}^L}{\Delta x^L} T_{N-1} + \left(\frac{x_{mid}^L \gamma_{mid}^L}{\beta x_3 \Delta x^L} - 1\right) T_N = T_{amb}$$

Representing the coefficient terms with $\lambda_0 = \frac{x_{mid}^L \gamma_{mid}^L}{\Delta x^L}$ and $\lambda_1 = \frac{x_{mid}^L \gamma_{mid}^L}{\beta x_3 \Delta x^L} - 1$ produces:

$$-\lambda_0 T_{N-1} + \lambda_1 T_N = T_{amb}$$

With these equations, we can now generate a matrix that corresponds to our boundary conditions:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -\kappa^L & \kappa^L + \kappa^R & -\kappa^R & 0 & 0 & 0 \\ 0 & -\kappa^L & \kappa^L + \kappa^R & -\kappa^R & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & -\kappa^L & \kappa^L + \kappa^R & -\kappa^R \\ 0 & 0 & 0 & 0 & \lambda_0 & \lambda_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ u_N \end{Bmatrix} = \begin{Bmatrix} T_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T_{amb} \end{Bmatrix}$$

This matrix will apply to both conformal and non-conformal meshes and will result in temperatures at each node when solved.

3.3 Non-Conformal Mesh

The formulation of the non-conformal mesh can be simply generated with a series of evenly space points between the bounds of the cylindrical rod.

$$\Delta x = \frac{x_3 - x_1}{N}$$

Here, N is the number of elements the rod is divided into.

3.4 Conformal Mesh

In order to avoid the issue of having a node not located on the interface location, we can define are mesh such that it will always fall on the interface location. The conformal mesh can be derived with the following:

$$\begin{cases} \Delta x_1 = \dfrac{x_2 - x_1}{INT\left(\dfrac{N}{2}\right)}, & x_1 < x < x_2, \\ \Delta x_2 = \dfrac{x_3 - x_2}{INT\left(\dfrac{N}{2}\right)}, & x_2 < x < x_3 \end{cases}$$

3.5 Heat Transfer

In order to derive the equation for the heat transfer through the surface of the bar, one of two methods can be taken. The first involves investigating the heat transfer to the surrounding environment. This process is governed by the following equation:

$$Q(x_3) = 2\pi\beta x_3\left(u_{x_3} - u_{amb}\right)$$

Additionally, the heat transfer through the bar is equivalent to the heat transfer through the derivative of the temperature at the right end of the bar. This process is governed be the following equation:

$$Q(x_3) = -2\pi\gamma_{cs}u'(x_3)$$

$$Q(x_3) = -\frac{2\pi\gamma_{cs}\left(U_{x_3} - u_{x_{N-1}}\right)}{\Delta x}$$

4 Performance Analysis

4.1 Error

The errors of the numerical solutions are calculated using the equations below:

$$e_h = u(x) - u_h(x)$$

4.2 Percent Error

The percent error of an estimated quantity $Q_{Estimated}$ (calculated using FDM) against its exact values Q_{Exact} is calculated using the equation below:

$$\%Error = \left| \frac{Q_{Exact} - Q_{Estimated}}{Q_{Exact}} \right| \times 100\%$$

4.3 Extrapolation and Convergence

Richardson's Extrapolation was used to extrapolate an approximate of the exact value from a series of approximated values. In general, error is modeled as:

$$Q_{ex} - Q_h = Ch^{\beta}$$

Where Q is the quantity of interest, Q_h is the approximate value at some mesh size h, C is some constant, and β is the convergence rate. In general, it is rare for the exact value to be known, and it is often difficult or impossible to obtain analytical solutions. In this case it is possible to use Richardson's Extrapolation to obtain reasonably accurate approximate value of the exact solution. If we write this equation at another mesh size, say h/2, the two can be divided and the unknown β can be found.

$$Q_{ex} - Q_h = C(h)^{\beta}$$

$$Q_{ex} - Q_{\frac{h}{2}} = C\left(\frac{h}{2}\right)^{\beta}$$

$$\frac{Q_{ex} - Q_h}{Q_{ex} - Q_{\frac{h}{2}}} = \frac{C(h)^{\beta}}{C(\frac{h}{2})^{\beta}}$$

$$log\left(\frac{Q_{ex} - Q_h}{Q_{ex} - Q_{\frac{h}{2}}}\right) = log\left(\frac{C(h)^{\beta}}{C(\frac{h}{2})^{\beta}}\right)$$

$$log(Q_{ex} - Q_h) - log(Q_{ex} - Q_{\frac{h}{2}}) = \beta log(h) - \beta log\left(\frac{h}{2}\right)$$

$$\boxed{\beta = \frac{log(Q_{ex} - Q_h) - log(Q_{ex} - Q_{\frac{h}{2}})}{log(h) - log(\frac{h}{2})}}$$

Again, Richardson's Extrapolation will be used to derive an expression for an extrapolated value. Here we will have to utilize three mesh sizes rather than the previous two.

$$Q_{ex} - Q_h = C(h)^\beta \approx 2^\beta$$
$$Q_{ex} - Q_{\frac{h}{2}} = C\left(\frac{h}{2}\right)^\beta \approx 2^\beta$$
$$Q_{ex} - Q_{\frac{h}{4}} = C\left(\frac{h}{4}\right)^\beta \approx 2^\beta$$
$$\frac{Q_{ex} - Q_h}{Q_{ex} - Q_{\frac{h}{2}}} \approx 2^\beta \approx \frac{Q_{ex} - Q_{\frac{h}{2}}}{Q_{ex} - Q_{\frac{h}{4}}}$$
$$\frac{Q_{extr} - Q_h}{Q_{extr} - Q_{\frac{h}{2}}} = 2^\beta = \frac{Q_{extr} - Q_{\frac{h}{2}}}{Q_{extr} - Q_{\frac{h}{4}}}$$
$$(Q_{extr} - Q_h)(Q_{extr} - Q_{\frac{h}{4}}) = (Q_{extr} - Q_{\frac{h}{2}})^2$$
$$\boxed{Q_{extr} = \frac{Q_{\frac{h}{2}}^2 - Q_h * Q_{\frac{h}{4}}}{2Q_{\frac{h}{2}} - Q_h - Q_{\frac{h}{4}}}}$$

From here Q_{extra} can be substituted to solve for β, which yields:

$$\boxed{\frac{log\left(\frac{Q_{extr} - Q_h}{Q_{extr} - Q_{\frac{h}{2}}}\right)}{log(2)} = \beta = \frac{log\left(\frac{Q_{extr} - Q_{\frac{h}{2}}}{Q_{extr} - Q_{\frac{h}{4}}}\right)}{log(2)}}$$

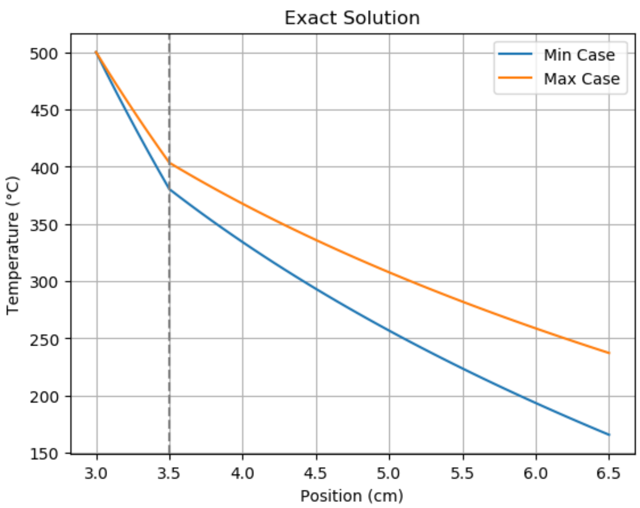
5 Results

In this section, the results of the analytical solution, the non-conformal FDM solution, and the conformal FDM solution for the interface located in the two positions of interest are provided. For each case, the analysis was conducted for mesh sizes varying from a 3-node analysis to a 100-node analysis. Additionally, the analysis was conducted for the maximum heat transfer case, as well as the minimum heat transfer case.

5.1 Analytical Results

The following sections show the results for the utilization of the analytical solution of the distribution of the temperature in the axisymmetric cylindrical rod. The results show both the minimum and the maximum heat transfer cases. The data provided corresponds to a 50-node analysis. Additional data obtained for various mesh sizes can be found in the appropriate appendix section.

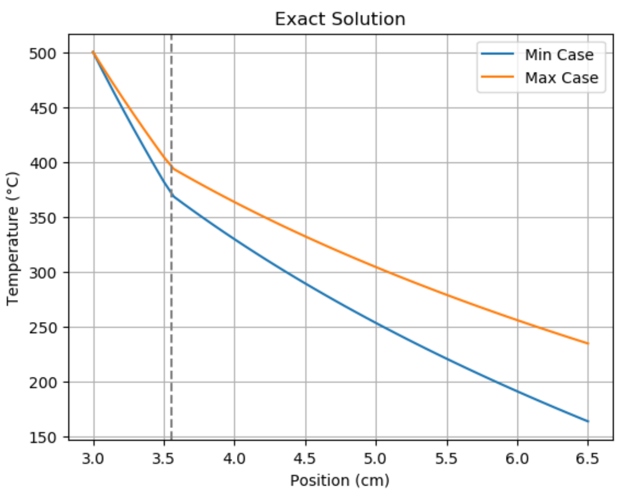
5.1.1 Case 1: Interface Located at $x_1 + \frac{x_3 - x_1}{7}$



Exact Solution
Interface located at x = 3.5 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.07143	481.726	485.247
3.14286	463.872	470.834
3.21429	446.419	456.745
3.28571	429.35	442.965
3.35714	412.648	429.482
3.42857	396.298	416.282
3.5	380.284	403.355
3.57143	373.276	397.926
3.64286	366.407	392.605
3.71429	359.671	387.388
3.78571	353.064	382.27
3.85714	346.579	377.247
3.92857	340.214	372.317
4	333.964	367.476
4.07143	327.824	362.72
4.14286	321.791	358.047
4.21429	315.861	353.454
4.28571	310.031	348.938
4.35714	304.297	344.496
4.42857	298.657	340.127
4.5	293.106	335.828
4.57143	287.644	331.596
4.64286	282.265	327.431
4.71429	276.969	323.328
4.78571	271.753	319.288
4.85714	266.614	315.307
4.92857	261.55	311.384
5	256.558	307.518
5.07143	251.638	303.707
5.14286	246.786	299.949
5.21429	242.001	296.243
5.28571	237.282	292.587
5.35714	232.625	288.98
5.42857	228.031	285.421
5.5	223.496	281.909
5.57143	219.02	278.442
5.64286	214.601	275.019
5.71429	210.238	271.639
5.78571	205.929	268.301
5.85714	201.672	265.004
5.92857	197.468	261.747
6	193.313	258.529
6.07143	189.208	255.35
6.14286	185.151	252.207
6.21429	181.14	249.101
6.28571	177.176	246.03
6.35714	173.256	242.994
6.42857	169.38	239.991
6.5	165.547	237.022

5.1.2 Case 2: Interface Located at $x_1 + \frac{x_3 - x_1}{2\pi}$



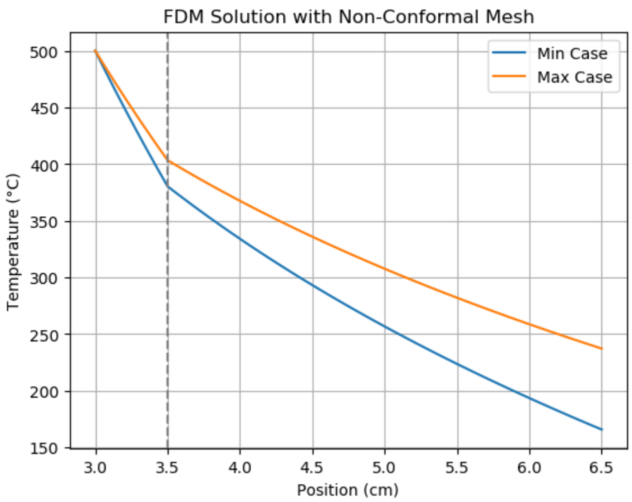
Exact Solution
Interface located at x = 3.557042300821634 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.07143	481.987	485.423
3.14286	464.387	471.182
3.21429	447.184	457.26
3.28571	430.358	443.645
3.35714	413.894	430.322
3.42857	397.777	417.28
3.5	381.992	404.507
3.57143	368.236	393.421
3.64286	361.465	388.163
3.71429	354.825	383.008
3.78571	348.312	377.951
3.85714	341.92	372.988
3.92857	335.646	368.117
4	329.485	363.333
4.07143	323.433	358.634
4.14286	317.486	354.016
4.21429	311.64	349.478
4.28571	305.893	345.016
4.35714	300.241	340.627
4.42857	294.681	336.31
4.5	289.21	332.063
4.57143	283.825	327.881
4.64286	278.524	323.765
4.71429	273.303	319.712
4.78571	268.161	315.719
4.85714	263.095	311.786
4.92857	258.103	307.91
5	253.183	304.09
5.07143	248.333	300.324
5.14286	243.551	296.611
5.21429	238.834	292.949
5.28571	234.182	289.337
5.35714	229.592	285.773
5.42857	225.063	282.257
5.5	220.593	278.786
5.57143	216.181	275.361
5.64286	211.825	271.978
5.71429	207.524	268.639
5.78571	203.276	265.341
5.85714	199.08	262.083
5.92857	194.936	258.865
6	190.841	255.686
6.07143	186.794	252.544
6.14286	182.795	249.438
6.21429	178.841	246.369
6.28571	174.934	243.335
6.35714	171.07	240.335
6.42857	167.249	237.369
6.5	163.471	234.435

5.2 Non-Conformal Mesh Finite Difference Method Results

The following sections show the results for the utilization of the non-conformal mesh FDM solution of the distribution of the temperature in the axisymmetric cylindrical rod. The results show both the minimum and the maximum heat transfer cases. The data provided corresponds to a 50-node analysis. Additional data obtained for various mesh sizes can be found in the appropriate appendix section.

5.2.1 Case 1: Interface Located at $x_1 + \frac{x_3 - x_1}{7}$



Non-Conformal Mesh FDM
Interface located at x = 3.5 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.07143	481.726	485.248
3.14286	463.873	470.835
3.21429	446.42	456.746
3.28571	429.352	442.967
3.35714	412.65	429.483
3.42857	396.3	416.284
3.5	380.287	403.357
3.57143	373.279	397.929
3.64286	366.41	392.608
3.71429	359.674	387.391
3.78571	353.066	382.273
3.85714	346.582	377.25
3.92857	340.217	372.32
4	333.967	367.479
4.07143	327.827	362.723
4.14286	321.794	358.05
4.21429	315.865	353.457
4.28571	310.034	348.941
4.35714	304.301	344.499
4.42857	298.66	340.13
4.5	293.11	335.831
4.57143	287.647	331.6
4.64286	282.269	327.434
4.71429	276.973	323.332
4.78571	271.756	319.291
4.85714	266.617	315.31
4.92857	261.553	311.388
5	256.562	307.522
5.07143	251.641	303.71
5.14286	246.789	299.952
5.21429	242.005	296.246
5.28571	237.285	292.59
5.35714	232.629	288.984
5.42857	228.034	285.425
5.5	223.499	281.912
5.57143	219.023	278.445
5.64286	214.604	275.022
5.71429	210.241	271.642
5.78571	205.932	268.305
5.85714	201.675	265.008
5.92857	197.471	261.751
6	193.316	258.533
6.07143	189.211	255.353
6.14286	185.154	252.21
6.21429	181.143	249.104
6.28571	177.179	246.033
6.35714	173.259	242.997
6.42857	169.383	239.995
6.5	165.55	237.026

Additionally provided below is the conformal mesh FDM solution compared against the exact solution with the percent error. This provides an understanding for the accuracy of the conformal mesh FDM analysis.

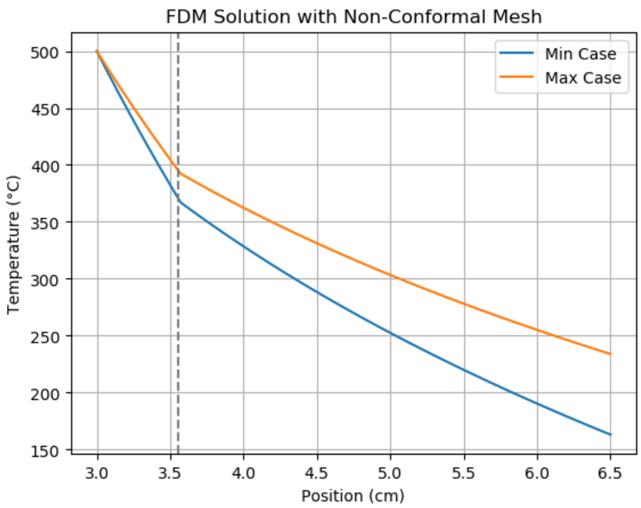
Non-Conformal Mesh FDM Compared to Exact Solution
Max Case, Interface Located at x = 3.5 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	6.73026e-12
3.07143	485.248	485.247	9.44207e-05
3.14286	470.835	470.834	0.000185966
3.21429	456.746	456.745	0.000275
3.28571	442.967	442.965	0.000361864
3.35714	429.483	429.482	0.000446879
3.42857	416.284	416.282	0.000530346
3.5	403.357	403.355	0.000612556
3.57143	397.929	397.926	0.000646737
3.64286	392.608	392.605	0.00067935
3.71429	387.391	387.388	0.000710507
3.78571	382.273	382.27	0.00074031
3.85714	377.25	377.247	0.000768851
3.92857	372.32	372.317	0.000796215
4	367.479	367.476	0.000822481
4.07143	362.723	362.72	0.000847719
4.14286	358.05	358.047	0.000871998
4.21429	353.457	353.454	0.000895376
4.28571	348.941	348.938	0.000917911
4.35714	344.499	344.496	0.000939655
4.42857	340.13	340.127	0.000960656
4.5	335.831	335.828	0.000980959
4.57143	331.6	331.596	0.00100061
4.64286	327.434	327.431	0.00101963
4.71429	323.332	323.328	0.00103808
4.78571	319.291	319.288	0.00105598
4.85714	315.31	315.307	0.00107336
4.92857	311.388	311.384	0.00109026
5	307.522	307.518	0.0011067
5.07143	303.71	303.707	0.0011227
5.14286	299.952	299.949	0.0011383
5.21429	296.246	296.243	0.00115351
5.28571	292.59	292.587	0.00116835
5.35714	288.984	288.98	0.00118285
5.42857	285.425	285.421	0.00119702
5.5	281.912	281.909	0.00121089
5.57143	278.445	278.442	0.00122446
5.64286	275.022	275.019	0.00123776
5.71429	271.642	271.639	0.00125081
5.78571	268.305	268.301	0.0012636
5.85714	265.008	265.004	0.00127617
5.92857	261.751	261.747	0.00128852
6	258.533	258.529	0.00130066
6.07143	255.353	255.35	0.00131261
6.14286	252.21	252.207	0.00132438
6.21429	249.104	249.101	0.00133598
6.28571	246.033	246.03	0.00134742
6.35714	242.997	242.994	0.0013587
6.42857	239.995	239.991	0.00136985
6.5	237.026	237.022	0.00138086

Non-Conformal Mesh FDM Compared to Exact Solution
Min Case, Interface Located at x = 3.5 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	5.33191e-12
3.07143	481.726	481.726	0.00010279
3.14286	463.873	463.872	0.000202968
3.21429	446.42	446.419	0.000300987
3.28571	429.352	429.35	0.000397286
3.35714	412.65	412.648	0.000492293
3.42857	396.3	396.298	0.000586432
3.5	380.287	380.284	0.000680128
3.57143	373.279	373.276	0.000721001
3.64286	366.41	366.407	0.000760083
3.71429	359.674	359.671	0.000797501
3.78571	353.066	353.064	0.000833375
3.85714	346.582	346.579	0.000867814
3.92857	340.217	340.214	0.000900916
4	333.967	333.964	0.000932775
4.07143	327.827	327.824	0.000963474
4.14286	321.794	321.791	0.000993094
4.21429	315.865	315.861	0.00102171
4.28571	310.034	310.031	0.00104938
4.35714	304.301	304.297	0.00107617
4.42857	298.66	298.657	0.00110215
4.5	293.11	293.106	0.00112736
4.57143	287.647	287.644	0.00115185
4.64286	282.269	282.265	0.00117569
4.71429	276.973	276.969	0.0011989
4.78571	271.756	271.753	0.00122153
4.85714	266.617	266.614	0.00124363
4.92857	261.553	261.55	0.00126523
5	256.562	256.558	0.00128636
5.07143	251.641	251.638	0.00130707
5.14286	246.789	246.786	0.00132738
5.21429	242.005	242.001	0.00134732
5.28571	237.285	237.282	0.00136692
5.35714	232.629	232.625	0.00138622
5.42857	228.034	228.031	0.00140524
5.5	223.499	223.496	0.00142401
5.57143	219.023	219.02	0.00144255
5.64286	214.604	214.601	0.0014609
5.71429	210.241	210.238	0.00147906
5.78571	205.932	205.929	0.00149708
5.85714	201.675	201.672	0.00151496
5.92857	197.471	197.468	0.00153275
6	193.316	193.313	0.00155045
6.07143	189.211	189.208	0.0015681
6.14286	185.154	185.151	0.00158571
6.21429	181.143	181.14	0.00160332
6.28571	177.179	177.176	0.00162094
6.35714	173.259	173.256	0.00163861
6.42857	169.383	169.38	0.00165633
6.5	165.55	165.547	0.00167415

5.2.2 Case 2: Interface Located at $x_1 + \frac{x_3 - x_1}{2\pi}$



Non-Conformal Mesh FDM
Interface located at x = 3.557042300821634 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.07143	482.051	485.467
3.14286	464.515	471.268
3.21429	447.372	457.388
3.28571	430.607	443.814
3.35714	414.202	430.531
3.42857	398.142	417.528
3.5	382.414	404.793
3.57143	367.003	392.315
3.64286	360.256	387.073
3.71429	353.64	381.933
3.78571	347.15	376.891
3.85714	340.781	371.943
3.92857	334.529	367.086
4	328.389	362.317
4.07143	322.359	357.632
4.14286	316.433	353.028
4.21429	310.609	348.503
4.28571	304.882	344.055
4.35714	299.25	339.679
4.42857	293.71	335.375
4.5	288.258	331.14
4.57143	282.892	326.971
4.64286	277.609	322.867
4.71429	272.407	318.826
4.78571	267.284	314.845
4.85714	262.236	310.924
4.92857	257.262	307.059
5	252.359	303.251
5.07143	247.526	299.496
5.14286	242.76	295.794
5.21429	238.061	292.143
5.28571	233.425	288.541
5.35714	228.851	284.988
5.42857	224.338	281.482
5.5	219.884	278.022
5.57143	215.488	274.606
5.64286	211.147	271.234
5.71429	206.861	267.904
5.78571	202.629	264.616
5.85714	198.448	261.368
5.92857	194.318	258.16
6	190.237	254.989
6.07143	186.205	251.857
6.14286	182.22	248.761
6.21429	178.281	245.701
6.28571	174.386	242.675
6.35714	170.536	239.684
6.42857	166.729	236.727
6.5	162.964	233.802

Additionally provided below is the conformal mesh FDM solution compared against the exact solution with the percent error. This provides an understanding for the accuracy of the conformal mesh FDM analysis.

Non-Conformal Mesh FDM Compared to Exact Solution
Min Case, Interface Located at x = 3.557042300821634 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	9.66338e-13
3.07143	482.051	481.987	0.013365
3.14286	464.515	464.387	0.0274162
3.21429	447.372	447.184	0.042213
3.28571	430.607	430.358	0.0578218
3.35714	414.202	413.894	0.0743166
3.42857	398.142	397.777	0.0917801
3.5	382.414	381.992	0.110306
3.57143	367.003	368.236	0.334914
3.64286	360.256	361.465	0.334514
3.71429	353.64	354.825	0.33411
3.78571	347.15	348.312	0.3337
3.85714	340.781	341.92	0.333285
3.92857	334.529	335.646	0.332864
4	328.389	329.485	0.332436
4.07143	322.359	323.433	0.332003
4.14286	316.433	317.486	0.331562
4.21429	310.609	311.64	0.331114
4.28571	304.882	305.893	0.330659
4.35714	299.25	300.241	0.330195
4.42857	293.71	294.681	0.329724
4.5	288.258	289.21	0.329243
4.57143	282.892	283.825	0.328753
4.64286	277.609	278.524	0.328253
4.71429	272.407	273.303	0.327743
4.78571	267.284	268.161	0.327223
4.85714	262.236	263.095	0.326691
4.92857	257.262	258.103	0.326148
5	252.359	253.183	0.325592
5.07143	247.526	248.333	0.325024
5.14286	242.76	243.551	0.324443
5.21429	238.061	238.834	0.323847
5.28571	233.425	234.182	0.323237
5.35714	228.851	229.592	0.322612
5.42857	224.338	225.063	0.321971
5.5	219.884	220.593	0.321314
5.57143	215.488	216.181	0.320638
5.64286	211.147	211.825	0.319945
5.71429	206.861	207.524	0.319232
5.78571	202.629	203.276	0.3185
5.85714	198.448	199.08	0.317746
5.92857	194.318	194.936	0.31697
6	190.237	190.841	0.316171
6.07143	186.205	186.794	0.315348
6.14286	182.22	182.795	0.314499
6.21429	178.281	178.841	0.313624
6.28571	174.386	174.934	0.312719
6.35714	170.536	171.07	0.311785
6.42857	166.729	167.249	0.31082
6.5	162.964	163.471	0.309821

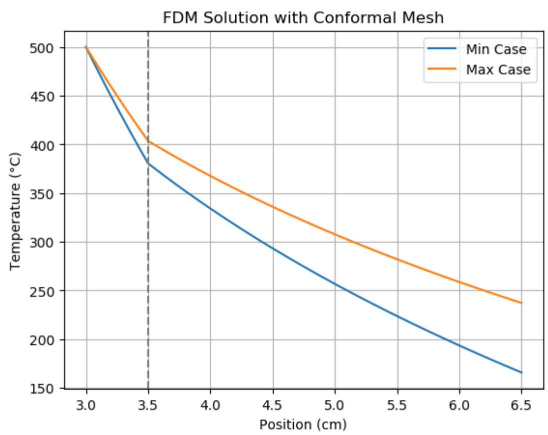
Non-Conformal Mesh FDM Compared to Exact Solution
Max Case, Interface Located at x = 3.557042300821634 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	6.63931e-12
3.07143	485.467	485.423	0.00900415
3.14286	471.268	471.182	0.018333
3.21429	457.388	457.26	0.0280079
3.28571	443.814	443.645	0.0380519
3.35714	430.531	430.322	0.0484899
3.42857	417.528	417.28	0.0593486
3.5	404.793	404.507	0.0706575
3.57143	392.315	393.421	0.281024
3.64286	387.073	388.163	0.280788
3.71429	381.933	383.008	0.280551
3.78571	376.891	377.951	0.280314
3.85714	371.943	372.988	0.280076
3.92857	367.086	368.117	0.279839
4	362.317	363.333	0.2796
4.07143	357.632	358.634	0.279361
4.14286	353.028	354.016	0.279121
4.21429	348.503	349.478	0.27888
4.28571	344.055	345.016	0.278638
4.35714	339.679	340.627	0.278395
4.42857	335.375	336.31	0.27815
4.5	331.14	332.063	0.277904
4.57143	326.971	327.881	0.277657
4.64286	322.867	323.765	0.277407
4.71429	318.826	319.712	0.277157
4.78571	314.845	315.719	0.276904
4.85714	310.924	311.786	0.27665
4.92857	307.059	307.91	0.276393
5	303.251	304.09	0.276135
5.07143	299.496	300.324	0.275874
5.14286	295.794	296.611	0.275611
5.21429	292.143	292.949	0.275346
5.28571	288.541	289.337	0.275078
5.35714	284.988	285.773	0.274808
5.42857	281.482	282.257	0.274535
5.5	278.022	278.786	0.274259
5.57143	274.606	275.361	0.273981
5.64286	271.234	271.978	0.2737
5.71429	267.904	268.639	0.273416
5.78571	264.616	265.341	0.273128
5.85714	261.368	262.083	0.272838
5.92857	258.16	258.865	0.272544
6	254.989	255.686	0.272247
6.07143	251.857	252.544	0.271946
6.14286	248.761	249.438	0.271642
6.21429	245.701	246.369	0.271334
6.28571	242.675	243.335	0.271022
6.35714	239.684	240.335	0.270706
6.42857	236.727	237.369	0.270387
6.5	233.802	234.435	0.270063

5.3 Conformal Mesh Finite Difference Method Results

The following sections show the results for the utilization of the non-conformal mesh FDM solution of the distribution of the temperature in the axisymmetric cylindrical rod. The results show both the minimum and the maximum heat transfer cases. The data provided corresponds to a 50-node analysis. Additional data obtained for various mesh sizes can be found in the appropriate appendix section.

5.3.1 Case 1: Interface Located at $x_1 + \frac{x_3 - x_1}{7}$



Conformal Mesh FDM
Interface located at x = 3.5 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.02083	494.625	495.661
3.04167	489.288	491.352
3.0625	483.986	487.072
3.08333	478.721	482.822
3.10417	473.491	478.6
3.125	468.296	474.406
3.14583	463.136	470.24
3.16667	458.01	466.102
3.1875	452.917	461.99
3.20833	447.857	457.906
3.22917	442.831	453.848
3.25	437.836	449.816
3.27083	432.874	445.81
3.29167	427.943	441.829
3.3125	423.043	437.873
3.33333	418.174	433.943
3.35417	413.335	430.036
3.375	408.526	426.154
3.39583	403.746	422.296
3.41667	398.996	418.461
3.4375	394.275	414.65
3.45833	389.583	410.862
3.47917	384.918	407.096
3.5	380.281	403.353
3.625	368.11	393.925
3.75	356.35	384.816
3.875	344.977	376.007
4	333.964	367.476
4.125	323.29	359.209
4.25	312.935	351.188
4.375	302.88	343.399
4.5	293.108	335.83
4.625	283.604	328.469
4.75	274.354	321.303
4.875	265.343	314.324
5	256.561	307.522
5.125	247.996	300.887
5.25	239.637	294.412
5.375	231.475	288.09
5.5	223.5	281.913
5.625	215.704	275.875
5.75	208.08	269.969
5.875	200.62	264.191
6	193.317	258.534
6.125	186.165	252.994
6.25	179.157	247.565
6.375	172.287	242.245
6.5	165.551	237.027

Additionally provided below is the conformal mesh FDM solution compared against the exact solution with the percent error. This provides an understanding for the accuracy of the conformal mesh FDM analysis.

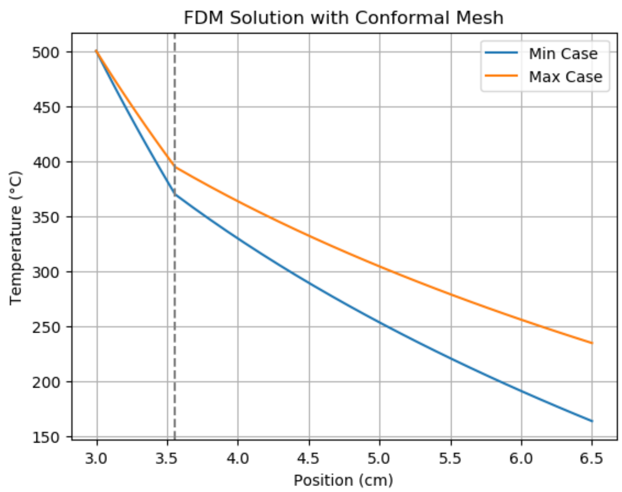
Conformal Mesh FDM Compared to Exact Solution
Max Case, Interface Lccated at x = 3.5 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	3.63798e-13
3.02083	495.661	495.661	1.56023e-05
3.04167	491.352	491.352	3.14181e-05
3.0625	487.072	487.073	4.74499e-05
3.08333	482.822	482.822	6.37005e-05
3.10417	478.6	478.6	8.01729e-05
3.125	474.406	474.407	9.68698e-05
3.14583	470.24	470.241	0.000113794
3.16667	466.102	466.102	0.00013095
3.1875	461.99	461.991	0.000148339
3.20833	457.906	457.907	0.000165966
3.22917	453.848	453.849	0.000183834
3.25	449.816	449.817	0.000201946
3.27083	445.81	445.811	0.000220307
3.29167	441.829	441.83	0.000238919
3.3125	437.873	437.875	0.000257787
3.33333	433.943	433.944	0.000276916
3.35417	430.036	430.038	0.000296309
3.375	426.154	426.156	0.00031597
3.39583	422.296	422.297	0.000335904
3.41667	418.461	418.463	0.000356116
3.4375	414.65	414.651	0.00037661
3.45833	410.862	410.863	0.000397391
3.47917	407.096	407.098	0.000418464
3.5	403.353	403.355	0.000439835
3.625	393.925	393.926	0.000256984
3.75	384.816	384.817	8.8012e-05
3.875	376.007	376.006	6.87334e-05
4	367.476	367.476	0.000214668
4.125	359.209	359.207	0.000351011
4.25	351.188	351.186	0.00047882
4.375	343.399	343.397	0.000599013
4.5	335.83	335.828	0.000712396
4.625	328.469	328.466	0.000819675
4.75	321.303	321.3	0.000921472
4.875	314.324	314.321	0.00101834
5	307.522	307.518	0.00111077
5.125	300.887	300.883	0.00119919
5.25	294.412	294.409	0.00128401
5.375	288.09	288.086	0.00136557
5.5	281.913	281.909	0.0014442
5.625	275.875	275.871	0.00152018
5.75	269.969	269.965	0.00159377
5.875	264.191	264.186	0.00166522
6	258.534	258.529	0.00173474
6.125	252.994	252.989	0.00180254
6.25	247.565	247.561	0.00186881
6.375	242.245	242.24	0.00193372
6.5	237.027	237.022	0.00199743

Conformal Mesh FDM Compared to Exact Solution
Min Case, Interface Lccated at x = 3.5 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	4.43379e-13
3.02083	494.625	494.625	2.6226e-05
3.04167	489.288	489.288	5.29016e-05
3.0625	483.986	483.987	8.00361e-05
3.08333	478.721	478.722	0.000107639
3.10417	473.491	473.492	0.000135721
3.125	468.296	468.297	0.000164292
3.14583	463.136	463.137	0.000193362
3.16667	458.01	458.011	0.000222943
3.1875	452.917	452.918	0.000253047
3.20833	447.857	447.859	0.000283685
3.22917	442.831	442.832	0.00031487
3.25	437.836	437.838	0.000346615
3.27083	432.874	432.875	0.000378933
3.29167	427.943	427.944	0.000411837
3.3125	423.043	423.045	0.000445343
3.33333	418.174	418.176	0.000479464
3.35417	413.335	413.337	0.000514217
3.375	408.526	408.528	0.000549618
3.39583	403.746	403.749	0.000585682
3.41667	398.996	398.999	0.000622428
3.4375	394.275	394.278	0.000659873
3.45833	389.583	389.585	0.000698036
3.47917	384.918	384.921	0.000736936
3.5	380.281	380.284	0.000776593
3.625	368.11	368.112	0.000555983
3.75	356.35	356.352	0.000351109
3.875	344.977	344.977	0.000160032
4	333.964	333.964	1.89175e-05
4.125	323.29	323.29	0.000187191
4.25	312.935	312.934	0.000346055
4.375	302.88	302.879	0.000496623
4.5	293.108	293.106	0.00063988
4.625	283.604	283.602	0.000776704
4.75	274.354	274.351	0.000907883
4.875	265.343	265.341	0.00103413
5	256.561	256.558	0.00115609
5.125	247.996	247.993	0.00127436
5.25	239.637	239.634	0.0013895
5.375	231.475	231.471	0.00150201
5.5	223.5	223.496	0.00161239
5.625	215.704	215.701	0.00172111
5.75	208.08	208.077	0.00182861
5.875	200.62	200.616	0.00193534
6	193.317	193.313	0.00204173
6.125	186.165	186.161	0.00214823
6.25	179.157	179.152	0.00225529
6.375	172.287	172.283	0.00236336
6.5	165.551	165.547	0.00247292

5.3.2 Case 2: Interface Located at $x_1 + \frac{x_3 - x_1}{2\pi}$



Conformal Mesh FDM
Interface located at x = 3.557042300821634 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.02321	494.1	495.226
3.04642	488.245	490.488
3.06963	482.435	485.786
3.09284	476.668	481.119
3.11605	470.944	476.488
3.13926	465.263	471.891
3.16247	459.624	467.327
3.18568	454.026	462.797
3.20889	448.468	458.3
3.2321	442.951	453.836
3.25531	437.473	449.403
3.27852	432.034	445.002
3.30173	426.634	440.632
3.32494	421.271	436.292
3.34815	415.946	431.983
3.37136	410.657	427.703
3.39457	405.404	423.453
3.41778	400.188	419.231
3.44099	395.007	415.039
3.4642	389.86	410.874
3.48741	384.748	406.737
3.51062	379.67	402.628
3.53383	374.625	398.546
3.55704	369.614	394.491
3.67967	358.025	385.493
3.80229	346.817	376.79
3.92491	335.964	368.364
4.04754	325.445	360.197
4.17016	315.24	352.274
4.29278	305.331	344.58
4.4154	295.701	337.103
4.53803	286.334	329.83
4.66065	277.218	322.752
4.78327	268.338	315.857
4.9059	259.682	309.137
5.02852	251.241	302.583
5.15114	243.003	296.187
5.27377	234.958	289.941
5.39639	227.099	283.839
5.51901	219.416	277.874
5.64164	211.902	272.04
5.76426	204.55	266.331
5.88688	197.352	260.743
6.00951	190.303	255.269
6.13213	183.396	249.907
6.25475	176.626	244.65
6.37738	169.987	239.496
6.5	163.475	234.439

Additionally provided below is the conformal mesh FDM solution compared against the exact solution with the percent error. This provides an understanding for the accuracy of the conformal mesh FDM analysis.

Conformal Mesh FDM Compared to Exact Solution
Max Case, Interface Lccated at x = 3.557042300821634 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	1.4893e-12
3.02321	495.226	495.226	1.46161e-05
3.04642	490.488	490.488	2.94745e-05
3.06963	485.786	485.786	4.4578e-05
3.09284	481.119	481.12	5.99295e-05
3.11605	476.488	476.488	7.5532e-05
3.13926	471.891	471.891	9.13887e-05
3.16247	467.327	467.328	0.000107503
3.18568	462.797	462.798	0.000123878
3.20889	458.3	458.301	0.000140518
3.2321	453.836	453.836	0.000157426
3.25531	449.403	449.404	0.000174606
3.27852	445.002	445.003	0.000192063
3.30173	440.632	440.632	0.0002098
3.32494	436.292	436.293	0.000227822
3.34815	431.983	431.984	0.000246134
3.37136	427.703	427.704	0.000264739
3.39457	423.453	423.454	0.000283644
3.41778	419.231	419.233	0.000302854
3.44099	415.039	415.04	0.000322372
3.4642	410.874	410.876	0.000342206
3.48741	406.737	406.739	0.00036236
3.51062	402.628	402.63	0.000382841
3.53383	398.546	398.548	0.000403655
3.55704	394.491	394.492	0.000424807
3.67967	385.493	385.494	0.000258246
3.80229	376.79	376.791	0.000103835
3.92491	368.364	368.364	3.98306e-05
4.04754	360.197	360.196	0.000173959
4.17016	352.274	352.273	0.000299597
4.29278	344.58	344.578	0.000417655
4.4154	337.103	337.101	0.000528931
4.53803	329.83	329.828	0.000634122
4.66065	322.752	322.75	0.000733847
4.78327	315.857	315.855	0.000828649
4.9059	309.137	309.135	0.000919014
5.02852	302.583	302.58	0.00100537
5.15114	296.187	296.184	0.00108812
5.27377	289.941	289.938	0.00116759
5.39639	283.839	283.835	0.00124411
5.51901	277.874	277.87	0.00131796
5.64164	272.04	272.036	0.0013894
5.76426	266.331	266.327	0.00145866
5.88688	260.743	260.739	0.00152596
6.00951	255.269	255.265	0.0015915
6.13213	249.907	249.902	0.00165546
6.25475	244.65	244.646	0.00171801
6.37738	239.496	239.491	0.00177931
6.5	234.439	234.435	0.00183951

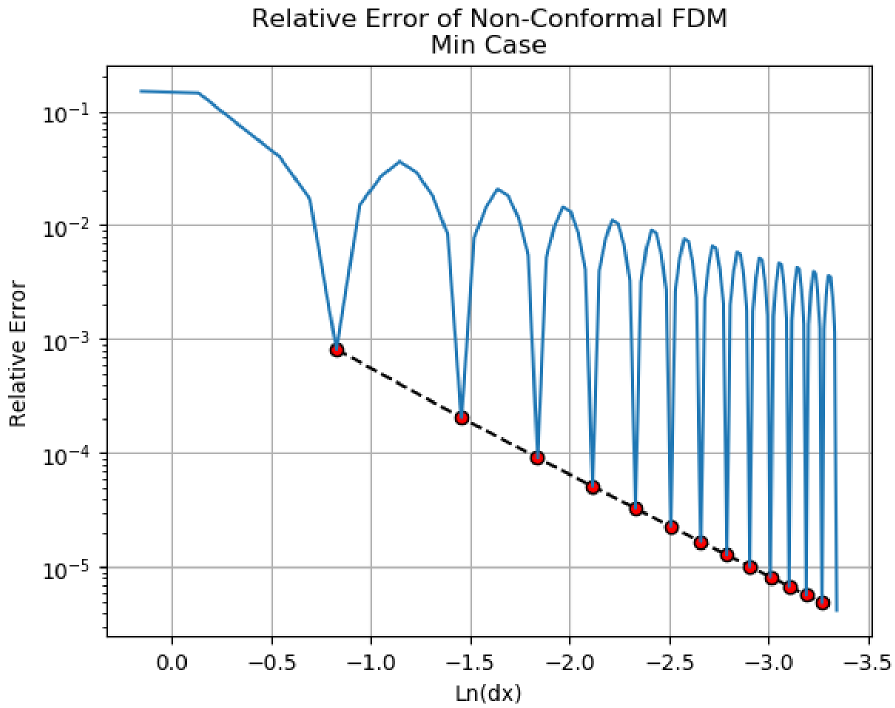
Conformal Mesh FDM Compared to Exact Solution
Min Case, Interface Lccated at x = 3.557042300821634 cm

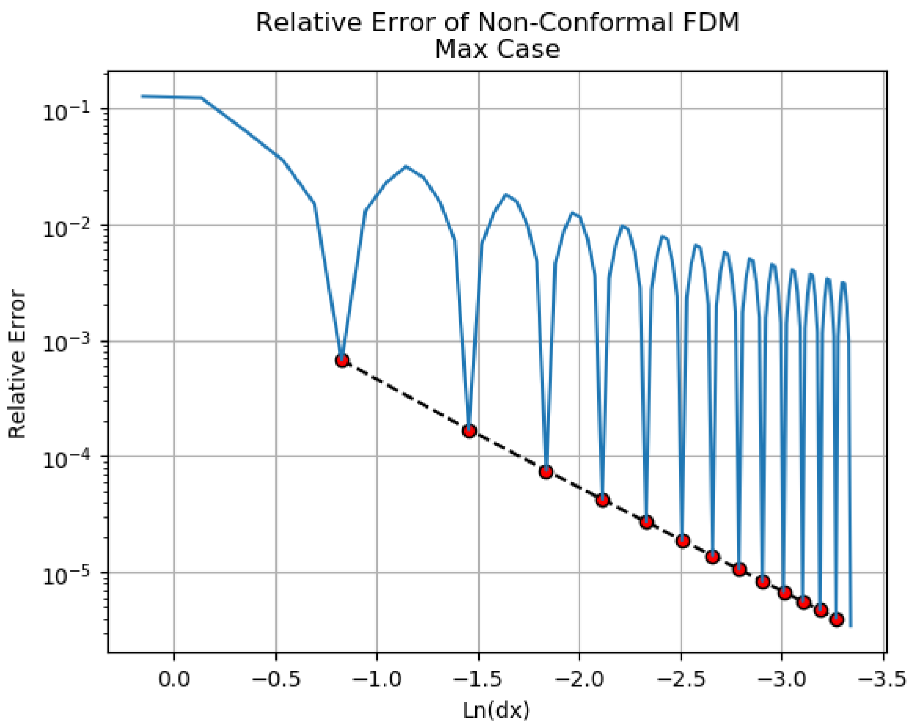
x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	1.13687e-13
3.02321	494.1	494.1	2.49602e-05
3.04642	488.245	488.245	5.04164e-05
3.06963	482.435	482.435	7.63789e-05
3.09284	476.668	476.668	0.000102859
3.11605	470.944	470.945	0.000129868
3.13926	465.263	465.264	0.000157417
3.16247	459.624	459.625	0.00018552
3.18568	454.026	454.027	0.000214189
3.20889	448.468	448.47	0.000243438
3.2321	442.951	442.952	0.00027328
3.25531	437.473	437.475	0.000303731
3.27852	432.034	432.036	0.000334806
3.30173	426.634	426.635	0.00036652
3.32494	421.271	421.273	0.00039889
3.34815	415.946	415.947	0.000431934
3.37136	410.657	410.659	0.000465669
3.39457	405.404	405.407	0.000500114
3.41778	400.188	400.19	0.000535289
3.44099	395.007	395.009	0.000571214
3.4642	389.86	389.863	0.00060791
3.48741	384.748	384.751	0.000645399
3.51062	379.67	379.673	0.000683705
3.53383	374.625	374.628	0.000722852
3.55704	369.614	369.616	0.000762864
3.67967	358.025	358.027	0.000561415
3.80229	346.817	346.818	0.00037371
3.92491	335.964	335.965	0.000198101
4.04754	325.445	325.445	3.31554e-05
4.17016	315.24	315.24	0.000122375
4.29278	305.331	305.33	0.000269585
4.4154	295.701	295.699	0.000409445
4.53803	286.334	286.333	0.000542814
4.66065	277.218	277.216	0.000670463
4.78327	268.338	268.335	0.000793085
4.9059	259.682	259.68	0.000911311
5.02852	251.241	251.238	0.00102572
5.15114	243.003	243	0.00113683
5.27377	234.958	234.956	0.00124515
5.39639	227.099	227.096	0.00135114
5.51901	219.416	219.413	0.00145523
5.64164	211.902	211.899	0.00155786
5.76426	204.55	204.546	0.00165942
5.88688	197.352	197.349	0.00176032
6.00951	190.303	190.299	0.00186096
6.13213	183.396	183.392	0.00196174
6.25475	176.626	176.622	0.00206305
6.37738	169.987	169.983	0.00216534
6.5	163.475	163.471	0.00226902

5.4 Relative Error of the Non-Conformal Mesh

When one of the nodes of the mesh does not fall directly on the material interface. The results that are derived lose accuracy. As such an interesting result can be obtained by observing the convergence of the non-conformal mesh FDM analysis.

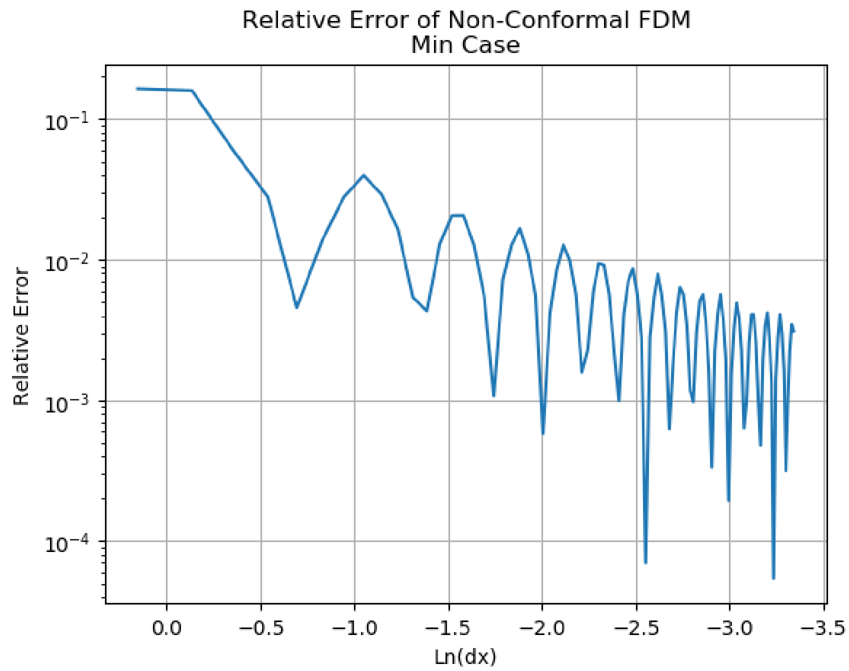
5.4.1 Case 1: Interface Located at $x_1 + \frac{x_3 - x_1}{7}$

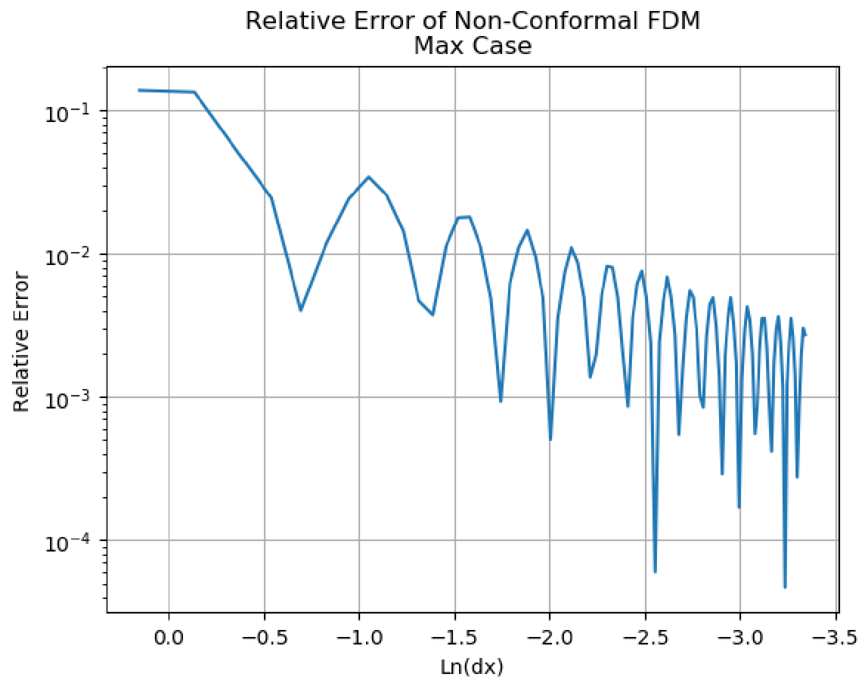




As can be seen in the graph above, the convergence of the does not follow a linear path. Instead, the convergence varies between a slope of -1 and -2. When one of the nodes falls on the interface location, the performance of the FDM non-conformal solution improves greatly, as expected. Because of this inconsistency, a Richardson extrapolation cannot be performed.

5.4.2 Case 2: Interface Located at $x_1 + \frac{x_3 - x_1}{2\pi}$



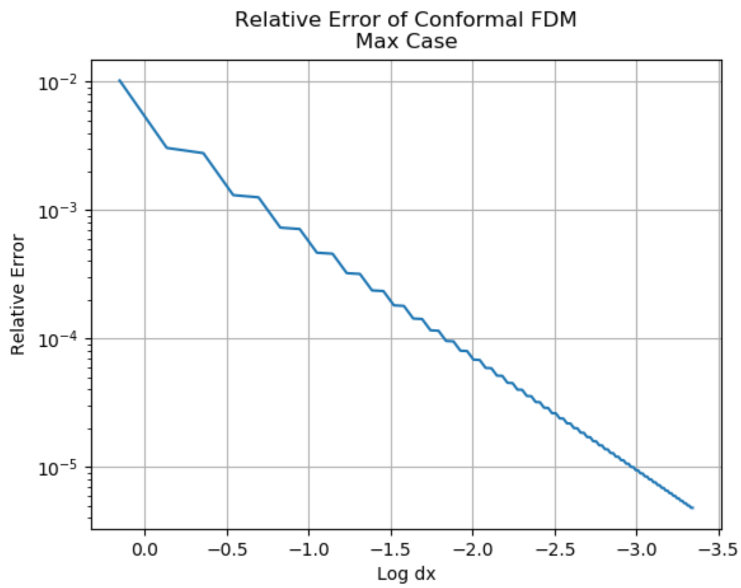


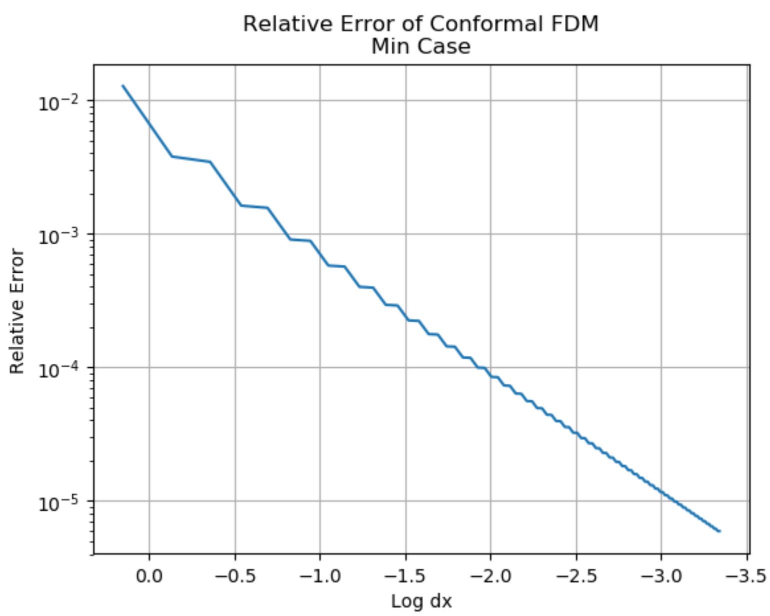
The convergence of these graphs is similarly not linear. It can be observed that when one of the nodes approaches the interface location, the accuracy of the non-conformal mesh FDM analysis improves.

5.5 Relative Error of the Conformal Mesh

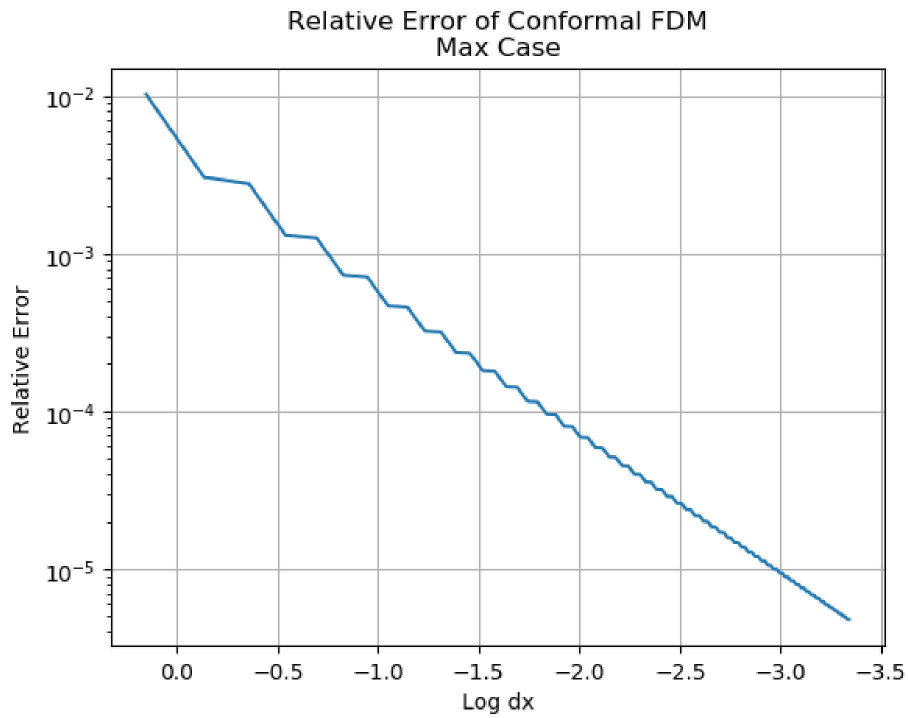
The following section shows the relative error of the conformal mesh FDM analysis. Since the conformal mesh ensures that one of the nodes within the mesh will fall upon the interface point, we can expect the performance of these graphs to be much better than previously seen in the non-conformal case.

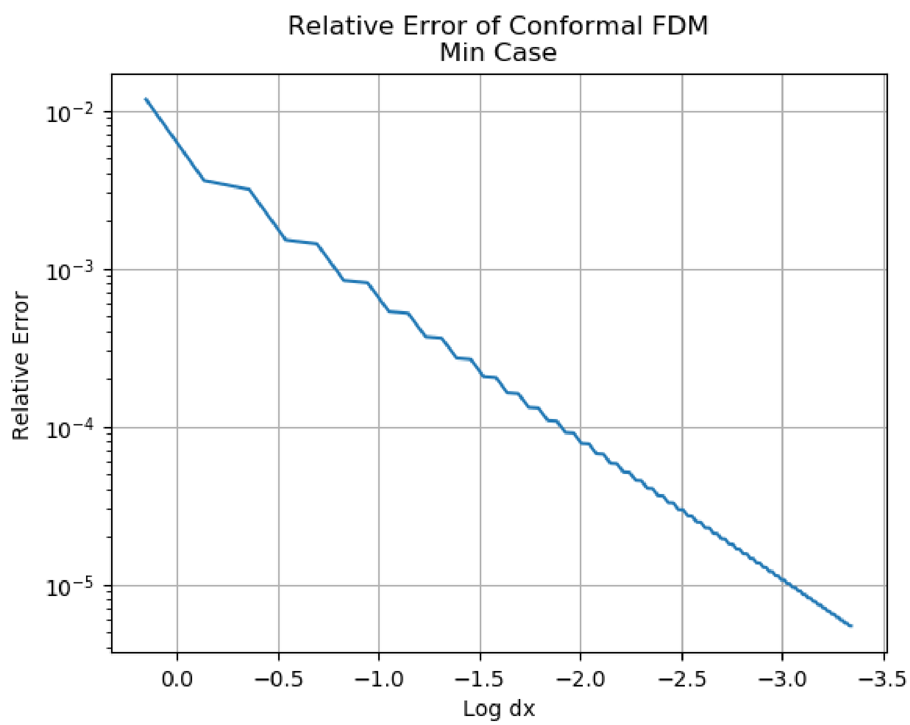
5.5.1 Case 1: Interface Located at $x_1 + \frac{x_3 - x_1}{7}$





5.5.2 Case 2: Interface Located at $x_1 + \frac{x_3 - x_1}{2\pi}$



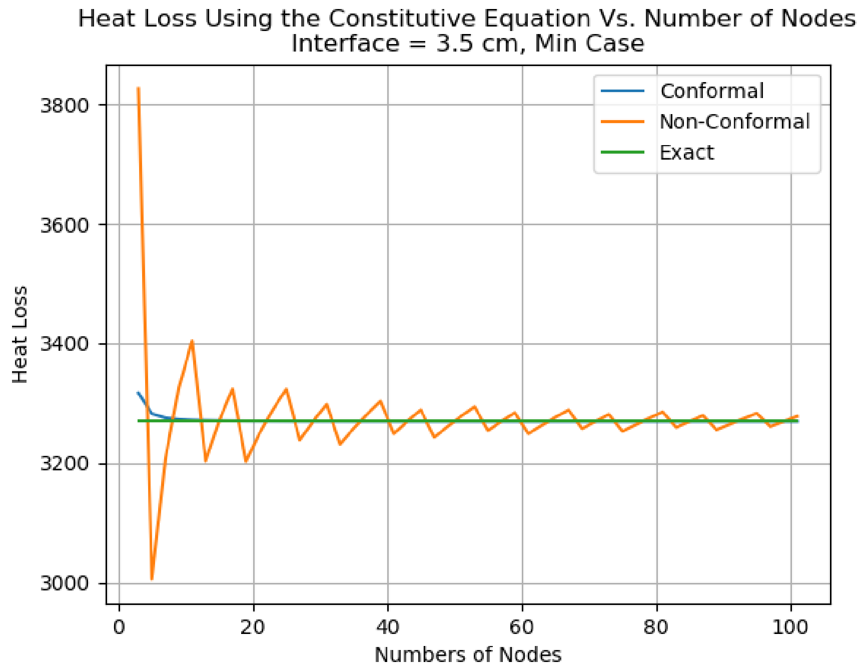


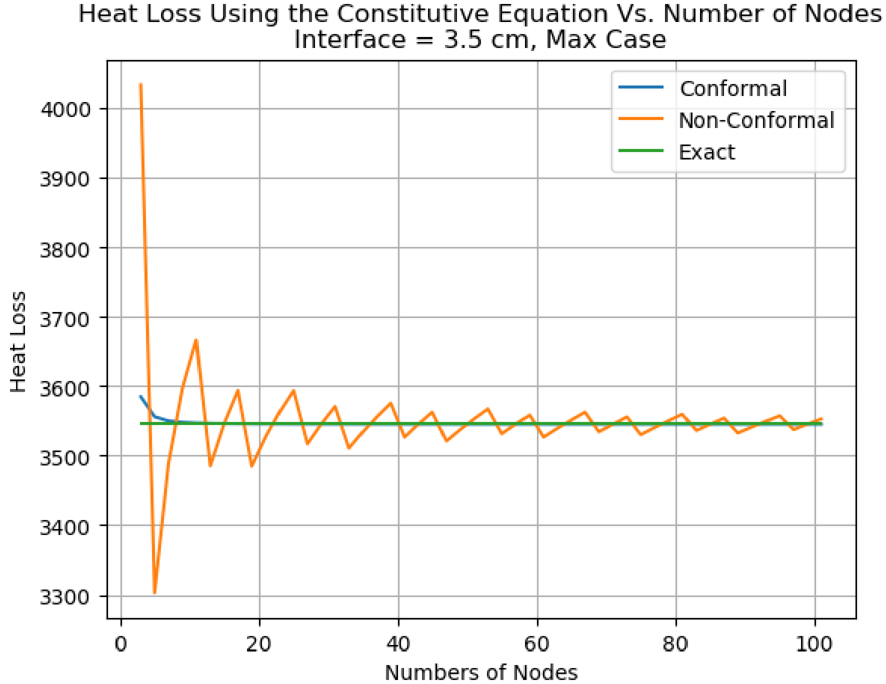
As can be seen above in the graphs, the relative error decreases as a function of the mesh size. Because these graphs follow a monotonic curve, a Richardson extrapolation can be performed on this data.

5.6 Heat Loss to the Environment

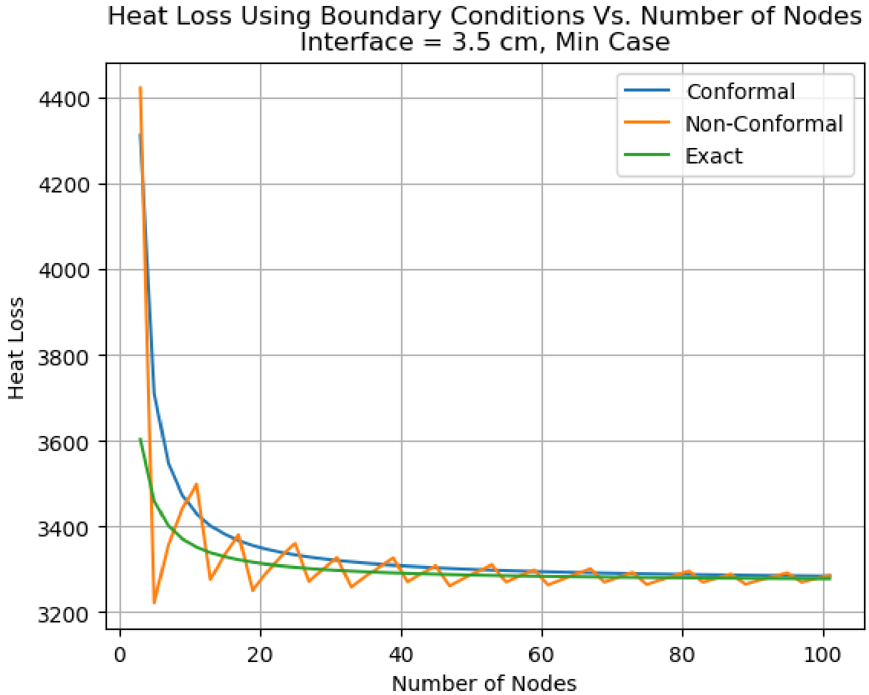
The heat loss to the environment can be computed in two methods. The first method consisted of utilizing the boundary conditions, while the second method involved the utilization of the constitutive equation. The results of these methods are listed in the following sections.

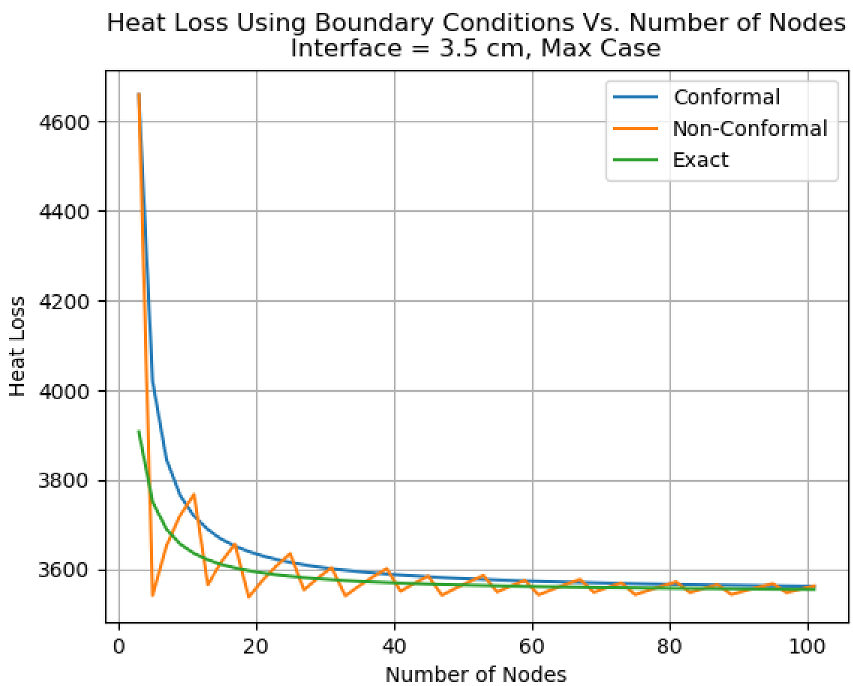
5.6.1 Heat Loss Using the Constitutive Equation



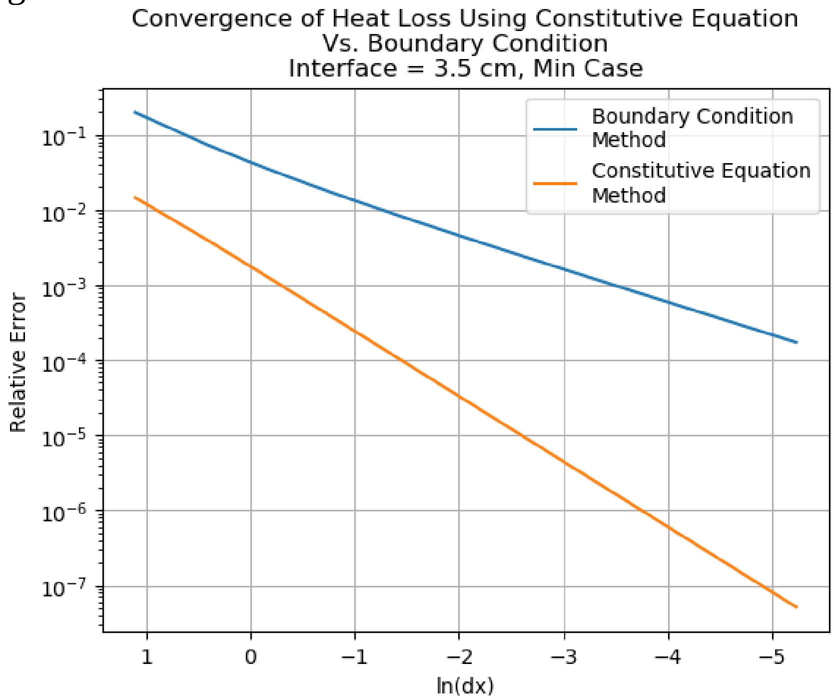


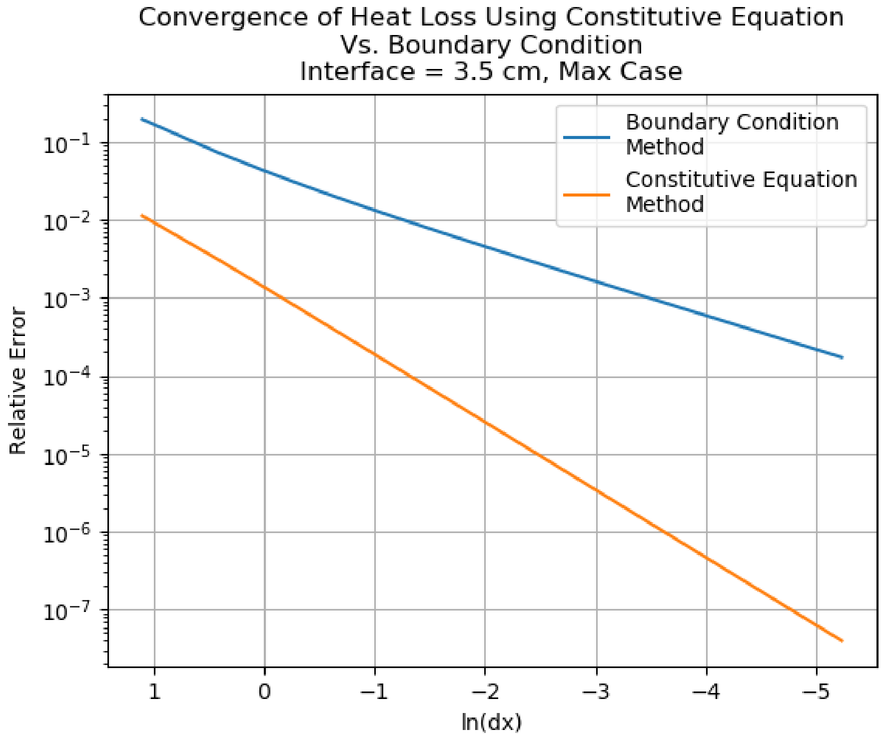
5.6.2 Heat Loss Using the Boundary Conditions

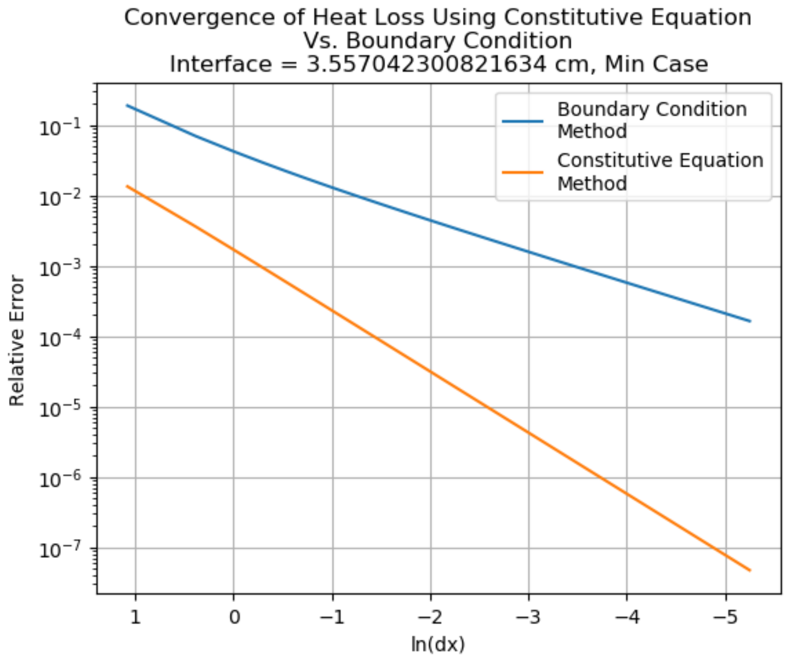


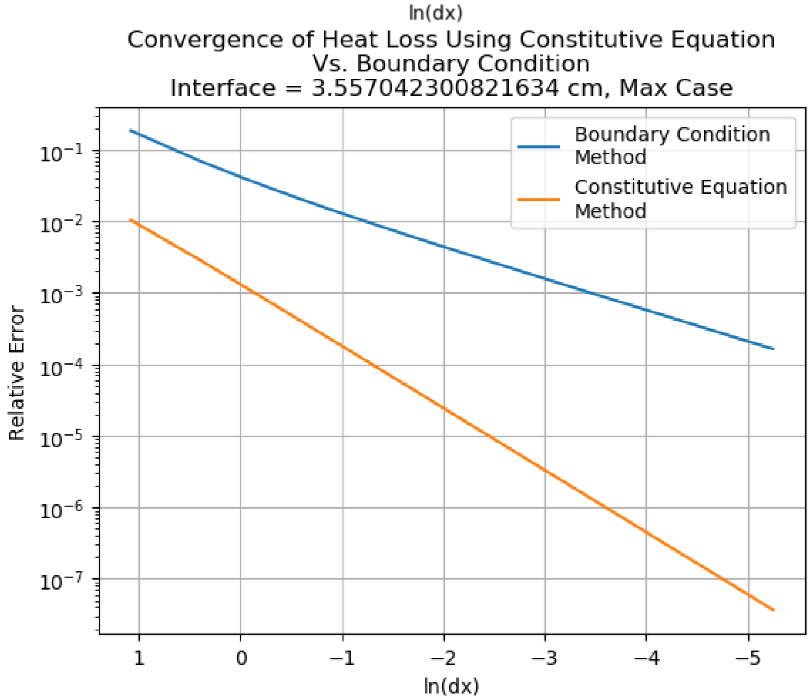


5.6.3 Convergence of the Heat Loss Methods









As can be seen in all cases in the graphs above, the convergence of the heat loss to the environment in the utilization of the constitutive equation performs better than the convergence of the heat loss in the utilization of the boundary conditions. Furthermore, additional tables regarding the beta values of each method is provided on the next page.

Interface located at x = 3.5

dx (cm)	Beta (Constitutive)	Beta (Boundary Condition)
3	2.55174	3.72365
1.5	2.35519	3.09399
1	2.2623	2.87451
0.75	2.20791	2.76123
0.6	2.17218	2.69184
0.5	2.14692	2.64489
0.428571	2.12812	2.61098
0.375	2.11358	2.58535
0.333333	2.102	2.56527
0.3	2.09257	2.54913
0.272727	2.08473	2.53586
0.25	2.07811	2.52476
0.230769	2.07246	2.51534
0.214286	2.06756	2.50725
0.2	2.06329	2.50021
0.1875	2.05952	2.49405
0.176471	2.05618	2.4886
0.166667	2.05319	2.48374
0.157895	2.05051	2.47939
0.15	2.04808	2.47547
0.142857	2.04587	2.47192
0.136364	2.04386	2.46869
0.130435	2.04202	2.46574
0.125	2.04032	2.46303
0.12	2.03876	2.46053
0.115385	2.03731	2.45823
0.111111	2.03597	2.45609
0.107143	2.03472	2.4541
0.103448	2.03356	2.45226
0.1	2.03247	2.45053
0.0967742	2.03145	2.44891
0.09375	2.03049	2.4474
0.0909091	2.02959	2.44597
0.0882353	2.02874	2.44463
0.0857143	2.02793	2.44336
0.0833333	2.02717	2.44217
0.0810811	2.02646	2.44104
0.0789474	2.02577	2.43996
0.0769231	2.02513	2.43895
0.075	2.02451	2.43798
0.0731707	2.02392	2.43706
0.0714286	2.02336	2.43618
0.0697674	2.02283	2.43534
0.0681818	2.02232	2.43455
0.0666667	2.02184	2.43378
0.0652174	2.02137	2.43305
0.0638298	2.02092	2.43235
0.0625	2.02049	2.43168
0.0612245	2.02008	2.43104
0.06	2.01969	2.43042
0.0588235	2.01912	2.46312
0.0566038	2.0181	2.52781
0.0526316	2.01636	2.65475
0.0461538	2.01375	2.90192
0.037037	2.01047	3.38733
0.0265487	2.00712	4.44104
0.0169492	2.00435	10.4912
0.00983607	2.00253	3.9074

Interface located at x = 3.557042300821634

dx (cm)	Beta (Constitutive)	Beta (Boundary Condition)
2.94296	2.56155	3.84292
1.47148	2.35891	3.18189
0.980986	2.26421	2.95445
0.735739	2.20906	2.83759
0.588592	2.17295	2.76615
0.490493	2.14747	2.71789
0.420423	2.12853	2.68307
0.36787	2.1139	2.65676
0.326995	2.10226	2.63617
0.294296	2.09277	2.61961
0.267542	2.0849	2.60601
0.245246	2.07826	2.59464
0.226381	2.07258	2.58498
0.210211	2.06767	2.57669
0.196197	2.06338	2.56949
0.183935	2.0596	2.56317
0.173115	2.05625	2.55759
0.163498	2.05326	2.55262
0.154893	2.05056	2.54816
0.147148	2.04813	2.54415
0.140141	2.04592	2.54051
0.133771	2.0439	2.53721
0.127955	2.04206	2.53418
0.122623	2.04036	2.53141
0.117718	2.0388	2.52886
0.113191	2.03735	2.5265
0.108998	2.036	2.52431
0.105106	2.03475	2.52228
0.101481	2.03358	2.52039
0.0980986	2.03249	2.51862
0.0949341	2.03147	2.51697
0.0919674	2.03051	2.51541
0.0891805	2.02961	2.51396
0.0865576	2.02876	2.51258
0.0840845	2.02795	2.51129
0.0817488	2.02719	2.51007
0.0795394	2.02647	2.50891
0.0774463	2.02579	2.50781
0.0754605	2.02514	2.50677
0.0735739	2.02452	2.50578
0.0717795	2.02394	2.50484
0.0700704	2.02338	2.50394
0.0684409	2.02284	2.50309
0.0668854	2.02233	2.50227
0.0653991	2.02185	2.50149
0.0639773	2.02138	2.50074
0.0626161	2.02093	2.50003
0.0613116	2.0205	2.49934
0.0600604	2.02009	2.49868
0.0588592	2.01969	2.49805
0.0577051	2.01913	2.53337
0.0555275	2.01811	2.60343
0.0516308	2.01637	2.74168
0.0452763	2.01376	3.01406
0.0363328	2.01047	3.56406
0.0260439	2.00712	4.86686
0.0166269	2.00436	6.38881
0.00964904	2.00241	3.44205

There are some notable results that ca be found in these tables. Firstly, the beta values of the constitutive equation method converge towards two, which corresponds to the convergence of this method in the graphs provided earlier in this section. Additionally, the beta values of the boundary condition method converge towards two, however begin to diverge sporadically as the mesh size begins to become too large.

6 Discussion

In this assignment the heat transfer within a bi-material, cylindrical rod with a fluid of 500 ℃ and an ambient, environmental temperature of 20℃ was analyzed using two methods. The first method consisted of deriving the analytical solution with the utilization of piecewise, logarithmic functions. The benefit of this method is that this exact solution will derive the temperature at any place within the cylindrical rod. However, obtaining an exact solution is not always a possibility, thus, a solution derived utilizing the finite difference method can be used instead. This second method can derive a solution for the temperature of the cylindrical bar at several nodes within the bar. However, its accuracy is dependent on the number of nodes used. For this analysis, the properties of the bar were varied between a worst case and best-case heat transfer in order to obtain an understanding of the range of heat transfer and temperature distribution that one might find within this rod. Within the case of this assignment, the location of the interface was varied such that one of the nodes within the FDM analysis does not always fall on the interface location. Because of this, we found that the convergence of an FDM analysis with a non-conformal mesh was sporadic, an only converged quadratically when a node was able to coincide with the location of the interface. However, when a conformal mesh was generated such that on of the nodes of the mesh will always fall on the interface location, it was observed that the convergence of this method was quadratic, as expected.

For this assignment, the heat transfer to the environment was also calculated using two methods. The first consisted of utilizing the boundary conditions in order to obtain the heat transfer of to the environment. The second method consisted of using the constitutive equation in order to find the heat loss to the environment. Both methods yielded the same results in the utilization of the conformal mesh FDM analysis and the exact solution. The results using the non-conformal mesh were, as expected, sporadic and inaccurate. The convergence of these two methods was also calculated and compared. It was found that the convergence of the constitutive equation method performed better than the convergence of the boundary condition method. For the constitutive equation method, the beta values that were obtained for this method converged towards a value of two, which corresponds to the convergence observed in the graphs. The convergence of the boundary condition method did not show a similar result. In the graphs obtained for the convergence of the boundary condition method, the slope of these graphs was shallower. Additionally, the beta values for this method began to converge towards two, then began to sporadically vary once the mesh size grew too large.

7 Appendix A – Code Used in this Assignment

```
"""
AERO 430
Exam 1 Code

Andrew Hollister
127008398
"""

# Imports and constants
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import argrelextrema as min
import tabulate as tbl
import math
pi = np.pi


# Creating a data structure to more easily keep track of all data generated
class DataStructure:
    # Class initialization
    def __init__(self):
        self.data = []

    # Function for adding data to data structure
    def add_data(self, interface, g_ss, g_cs, b, x, t, n_elem):
        self.data.append({"Interface": interface, "Beta": b, "x": x, "Temperature": t, "Nodes": n_elem,
                          "Gamma SS": g_ss, "Gamma CS": g_cs})

    # Function for returning data for graphing
    def return_data(self, interface, g_ss, g_cs, b, n_elem):
        y_data = [item["Temperature"] for item in self.data if item['Interface'] == interface
                  and item['Beta'] == b and item["Nodes"] == n_elem and item['Gamma CS'] == g_cs
                  and item["Gamma SS"] == g_ss]

        x_data = [item["x"] for item in self.data if item['Interface'] == interface
                  and item['Beta'] == b and item["Nodes"] == n_elem and item['Gamma CS'] == g_cs
                  and item["Gamma SS"] == g_ss]

        return x_data, y_data

    def return_temp(self, interface, g_ss, g_cs, b, n_elem, x):
        return [item["Temperature"] for item in self.data if item['Interface'] == interface
                and item['Beta'] == b and item["Nodes"] == n_elem and item['Gamma CS'] == g_cs
                and item["Gamma SS"] == g_ss and item['x'] == x]


class HeatData:
    # Class Initialization
    def __init__(self):
        self.data = []

    def add_data(self, T, vals, dx, inter):
        self.data.append({'T': T, 'vals': vals, 'dx': np.log(dx), 'inter': inter})

    def return_data(self, vals, inter):
        y_data = [item['T'] for item in self.data if item['vals'] == vals and item['inter'] == inter]
        x_data = [item['dx'] for item in self.data if item['vals'] == vals and item['inter'] == inter]

        return x_data, y_data


# Post processing
def get_error(exact: float, approx: float):
    return abs((exact-approx)/exact)


# Case 1: Analytical Solution
# Bar Properties
x1 = 3  # cm
x3 = 6.5  # cm
```

```
Beta = [0.4, 0.55]
gamma_ss = [0.67, 0.90]
gamma_cs = [1.50, 2.10]

min_case = [Beta[1], gamma_ss[0], gamma_cs[0]]
max_case = [Beta[0], gamma_ss[1], gamma_cs[1]]

# Interface Location
interface = np.array([x1 + (x3 - x1) / 7, x1 + (x3 - x1) / (2 * pi)])

# Temperatures
T0 = 500  # deg C
T1 = 20  # deg C


def get_beta(exact, approx, approx_2, h, h_2):
    A = np.log(abs(exact - approx))
    B = np.log(abs(exact - approx_2))
    C = np.log(h) - np.log(h_2)
    return -(A - B) / C


def rich_extra(q, q2, q4):
    q_extr = (q2**2 - q*q4)/(2*q2-q-q4)
    beta = np.log((q_extr - q2)/(q_extr-q4))/np.log(2)
    return q_extr, beta

# Heat Loss to the environment
def heat_loss_bc(T3, B):
    return 2*pi*B*x3*(T3-T1)


# Heat Loss to the environment
def heat_loss_der(T3, Tn_1, g, dx):
    return -2*pi*g*x3*(T3-Tn_1)/dx


def temp_exact(x, x2, g_ss, g_cs, B):
        del_t = T1-T0
        c_coefficient = (1/B/x3) + (1/g_cs*np.log(x3/x2)) + (1/g_ss*np.log(x2/x1))
        c = del_t/c_coefficient

        c1 = c
        c2 = T0-(c/g_ss*np.log(x1))
        c3 = c1
        c4 = c / g_ss * np.log(x2) - c / g_cs * np.log(x2) + c2

        if x < x2:
            return c1/g_ss*np.log(x) + c2
        elif x >= x2:
            return c3/g_cs*np.log(x) + c4


def temp_fdm(x2, mesh, g_ss, g_cs, B):
    n_nodes = len(mesh)
    for node in range(n_nodes):

        # Creating A matrix
        A = np.zeros((n_nodes, n_nodes))

        for node in range(1, len(mesh)-1):
            dx_l = (mesh[node]-mesh[node-1])
            dx_r = (mesh[node+1]-mesh[node])

            x_mid_l = (mesh[node]+mesh[node-1])/2
            x_mid_r = (mesh[node+1]+mesh[node])/2

            if x_mid_l < x2:
                g_mid_l = g_ss
            else:
                g_mid_l = g_cs

            if x_mid_r < x2:
                g_mid_r = g_ss
            else:
                g_mid_r = g_cs

            k_l = (x_mid_l*g_mid_l/dx_l)
            k_r = (x_mid_r*g_mid_r/dx_r)
```

```
            A[node][node - 1] = -k_l

            A[node][node] = k_l+k_r

            A[node][node + 1] = -k_r

        dx = mesh[-1] - mesh[-2]
        x_mid = (mesh[-1]+mesh[-2])/2
        A[-1][-2] = -x_mid*g_cs/(dx*B*x3)
        A[-1][-1] = x_mid*g_cs/(dx*B*x3) + 1
        A[0][0] = 1

        # Creating B matrix
        B_mat = np.zeros(n_nodes)
        B_mat[0] = T0  # deg C
        B_mat[-1] = T1

        # Solving
        T = np.linalg.solve(A, B_mat)

        return T, mesh


Q_der_con = HeatData()
Q_bc_con = HeatData()
Q_der_nonc = HeatData()
Q_bc_nonc = HeatData()
Q_bc_exact = HeatData()
Q_der_exact = HeatData()

# data collection loop
nodes = [3 + interval for interval in range(97)]+[99 + 2**interval for interval in range(11)]
exact_con_data = DataStructure()
exact_nonc_data = DataStructure()
con_fdm_data = DataStructure()
nonc_fdm_data = DataStructure()
for inter in interface:
    for vals in [max_case, min_case]:

        B = vals[0]
        g_ss = vals[1]
        g_cs = vals[2]

        for num_elem in nodes:

            # Building non-conformal mesh
            nonc_mesh = np.linspace(x1, x3, num_elem)

            # Building conformal mesh
            con_mesh_A = np.linspace(inter, x3, 1 + math.trunc(num_elem / 2))
            con_mesh_B = np.linspace(x1, inter, math.ceil(num_elem / 2))[:-1]
            con_mesh = np.append(con_mesh_B, con_mesh_A)

            # Deriving data for non conformal and conformal meshes
            con_fdm_temp, con_fdm_mesh = temp_fdm(inter, con_mesh, g_ss, g_cs, B)
            nonc_fdm_temp, nonc_fdm_mesh = temp_fdm(inter, nonc_mesh, g_ss, g_cs, B)

            # Saving conformal mesh data to data structure and creating corresponding exact data
            for i in range(len(con_fdm_mesh)):
                x = con_fdm_mesh[i]
                exact_temp = temp_exact(x, inter, g_ss, g_cs, B)
                exact_con_data.add_data(inter, g_ss, g_cs, B, x, exact_temp, num_elem)
                con_fdm_data.add_data(inter, g_ss, g_cs, B, x, con_fdm_temp[i], num_elem)

            # Saving non-conformal mesh data to data structure and creating corresponding exact data
            for i in range(len(nonc_fdm_mesh)):
                x = nonc_fdm_mesh[i]
                exact_temp = temp_exact(x, inter, g_ss, g_cs, B)
                exact_nonc_data.add_data(inter, g_ss, g_cs, B, x, exact_temp, num_elem)
                nonc_fdm_data.add_data(inter, g_ss, g_cs, B, x, nonc_fdm_temp[i], num_elem)


# Creating Table
if input('Create Temperature Tables? (y/n): ') == 'y':
    for inter in interface:
        for vals in [min_case, max_case]:
            B = vals[0]
            g_ss = vals[1]
```

```
            g_cs = vals[2]

            if vals == min_case:
                title = 'Min Case'
            else:
                title = 'Max Case'

            table = []
            print('\nConformal Mesh FDM Compared to Exact Solution')
            print(title+', Interface Lccated at x = '+str(inter)+' cm')
            headers = ['x (cm)', 'FDM Temp (\u00B0C)', 'Exact Temp (\u00B0C)', 'Percent Error']
            x_data, y_data = con_fdm_data.return_data(inter, g_ss, g_cs, B, 50)
            exact_y_data = exact_con_data.return_data(inter, g_ss, g_cs, B, 50)[1]

            err = []
            for i in range(len(exact_y_data)):
                err.append(get_error(exact_y_data[i], y_data[i])*100)

            table.append(x_data)
            table.append(y_data)
            table.append(exact_y_data)
            table.append(err)
            print(tbl.tabulate(np.transpose(np.array(table)), headers=headers))

            table = []
            print('\nNon-Conformal Mesh FDM Compared to Exact Solution')
            print(title+', Interface Located at x = '+str(inter)+' cm')
            x_data, y_data = nonc_fdm_data.return_data(inter, g_ss, g_cs, B, 50)
            exact_y_data = exact_nonc_data.return_data(inter, g_ss, g_cs, B, 50)[1]

            err = []
            for i in range(len(exact_y_data)):
                err.append(get_error(exact_y_data[i], y_data[i])*100)

            table.append(x_data)
            table.append(y_data)
            table.append(exact_y_data)
            table.append(err)
            print(tbl.tabulate(np.transpose(np.array(table)), headers=headers))

            input('\nProceed?')

# Computing the heat loss
hl_nodes = [3 + interval for interval in range(97)][::2]+[99 + 2**interval for interval in range(1, 11)]
if input('Heat Transfer and Convergence? (y/n): ') == 'y':
    for inter in interface:
        for vals in [min_case, max_case]:
            B = vals[0]
            g_ss = vals[1]
            g_cs = vals[2]

            for num_elem in hl_nodes:

                # Heat Transfer Calculations
                T = con_fdm_data.return_data(inter, g_ss, g_cs, B, num_elem)[1]
                dx = con_fdm_data.return_data(inter, g_ss, g_cs, B, num_elem)[0][-1] -
con_fdm_data.return_data(inter, g_ss, g_cs, B, num_elem)[0][-2]
                Q_bc_con.add_data(heat_loss_der(T[-1], T[-2], g_cs, dx), vals, dx, inter)
                Q_der_con.add_data(heat_loss_bc(T[-1], B), vals, dx, inter)

                T = nonc_fdm_data.return_data(inter, g_ss, g_cs, B, num_elem)[1]
                dx = nonc_fdm_data.return_data(inter, g_ss, g_cs, B, num_elem)[0][-1] -
nonc_fdm_data.return_data(inter, g_ss, g_cs, B, num_elem)[0][-2]
                Q_bc_nonc.add_data(heat_loss_der(T[-1], T[-2], g_cs, dx), vals, dx, inter)
                Q_der_nonc.add_data(heat_loss_bc(T[-1], B), vals, dx, inter)

                T3 = temp_exact(x3, inter, g_ss, g_cs, B)
                dx = (x3-x1)/num_elem
                TN_1 = temp_exact(x3 - dx, inter, g_ss, g_cs, B)
                Q_bc_exact.add_data(heat_loss_der(T3, TN_1, g_cs, dx), vals, dx, inter)
                Q_der_exact.add_data(heat_loss_bc(T3, B), vals, dx, inter)

            Q_con_der_err = []
            Q_con_bc_err = []
            Q_nonc_der_err = []
            Q_nonc_bc_err = []

            for i in range(len(Q_bc_exact.return_data(vals, inter)[1])):
                Q_con_der_err.append(get_error(Q_der_exact.return_data(vals, inter)[1][i],
```

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Q_der_con.return_data(vals, inter)[1][i]))
                Q_con_bc_err.append(get_error(Q_bc_exact.return_data(vals, inter)[1][i],
Q_bc_con.return_data(vals, inter)[1][i]))
                Q_nonc_der_err.append(get_error(Q_der_exact.return_data(vals, inter)[1][i],
Q_der_nonc.return_data(vals, inter)[1][i]))
                Q_nonc_bc_err.append(get_error(Q_bc_exact.return_data(vals, inter)[1][i],
Q_bc_nonc.return_data(vals, inter)[1][i]))

            title = 'Interface = '+str(inter)+' cm'
            if vals == min_case:
                title += ', Min Case'
            else:
                title += ', Max Case'
            plt.plot(hl_nodes[:50], Q_bc_con.return_data(vals, inter)[1][:50], label='Conformal')
            plt.plot(hl_nodes[:50], Q_bc_nonc.return_data(vals, inter)[1][:50], label='Non-Conformal')
            plt.plot(hl_nodes[:50], Q_bc_exact.return_data(vals, inter)[1][:50], label='Exact')
            plt.title('Heat Loss Using Boundary Conditions Vs. Number of Nodes\n'+title)
            plt.xlabel('Number of Nodes')
            plt.ylabel('Heat Loss')
            plt.grid()
            plt.legend()
            plt.show()

            plt.plot(hl_nodes[:50], Q_der_con.return_data(vals, inter)[1][:50], label='Conformal')
            plt.plot(hl_nodes[:50], Q_der_nonc.return_data(vals, inter)[1][:50], label='Non-Conformal')
            plt.plot(hl_nodes[:50], Q_der_exact.return_data(vals, inter)[1][:50], label='Exact')
            plt.title('Heat Loss Using the Constitutive Equation Vs. Number of Nodes\n'+title)
            plt.xlabel('Numbers of Nodes')
            plt.ylabel('Heat Loss')
            plt.grid()
            plt.legend()
            plt.show()

            plt.plot(Q_bc_con.return_data(vals, inter)[0], Q_con_bc_err, label='Boundary Condition\nMethod')
            plt.plot(Q_der_con.return_data(vals, inter)[0], Q_con_der_err, label='Constitutive
Equation\nMethod')
            plt.yscale('log')
            plt.title('Convergence of Heat Loss Using Constitutive Equation\nVs. Boundary Condition\n' + title)
            plt.xlabel('ln(dx)')
            plt.ylabel('Relative Error')
            plt.grid()
            plt.legend()
            plt.gca().invert_xaxis()
            plt.show()

    # Beta convergence
    if input(Obtain Beta Values? (y/n): ') == 'y':
        for inter in interface:
            for vals in [min_case, max_case]:

                B = vals[0]
                g_ss = vals[1]
                g_cs = vals[2]

                Q_bc_con_data = Q_bc_con.return_data(vals, inter)[1]
                Q_bc_exact_data = Q_bc_exact.return_data(vals, inter)[1]
                Q_der_con_data = Q_der_con.return_data(vals, inter)[1]
                Q_der_exact_data = Q_der_exact.return_data(vals, inter)[1]

                beta_der = []
                beta_bc = []
                lg_dx = []

                for i in range(len(hl_nodes)-1):
                    beta_bc.append(get_beta(Q_bc_exact_data[i], Q_bc_con_data[i], Q_bc_con_data[i+1],
hl_nodes[i], hl_nodes[i+1]))
                    beta_der.append(get_beta(Q_der_exact_data[i], Q_der_con_data[i], Q_der_con_data[i+1],
hl_nodes[i], hl_nodes[i+1]))
                    lg_dx.append(con_fdm_data.return_data(inter, g_ss, g_cs, B, hl_nodes[i])[0][-1] -
                                 con_fdm_data.return_data(inter, g_ss, g_cs, B, hl_nodes[i])[0][-2])

            print('Interface located at x = '+str(inter))
            print(tbl.tabulate(np.transpose(np.array([lg_dx, beta_der, beta_bc])),
                               headers=['dx (cm)', 'Beta (Constitutive)', 'Beta (Boundary Condition)']))

if input('Temperature Tables without analytical comparison? (y/n): ') == 'y':
    for inter in interface:
        table1 = []
        table2 = []
```

```
        table3 = []

        headers = ['x (cm)', 'Min Case Temp (\u00B0C)', 'Max Case Temp (\u00B0C)']

        for vals in [min_case, max_case]:
            B = vals[0]
            g_ss = vals[1]
            g_cs = vals[2]

            x_data1, y_data1 = con_fdm_data.return_data(inter, g_ss, g_cs, B, 50)
            x_data2, y_data2 = nonc_fdm_data.return_data(inter, g_ss, g_cs, B, 50)
            x_data3, y_data3 = exact_nonc_data.return_data(inter, g_ss, g_cs, B, 50)

            if vals == min_case:
                label = 'Min Case'
                table1.append(x_data1)
                table2.append(x_data2)
                table3.append(x_data3)
            else:
                label = 'Max Case'

            table1.append(y_data1)
            table2.append(y_data2)
            table3.append(y_data3)

        print('Conformal Mesh FDM')
        print('Interface located at x = '+str(inter)+' cm')
        print(tbl.tabulate(np.transpose(np.array(table1)), headers=headers))

        print('Non-Conformal Mesh FDM')
        print('Interface located at x = ' + str(inter) + ' cm')
        print(tbl.tabulate(np.transpose(np.array(table2)), headers=headers))

        print('Exact Solution')
        print('Interface located at x = ' + str(inter) + ' cm')
        print(tbl.tabulate(np.transpose(np.array(table3)), headers=headers))

# Plotting Temperature Graphs for 50 nodes
if input('Plot Temperature Distribution? (y/n): ') == 'y':
    for inter in interface:
        for vals in [min_case, max_case]:
            B = vals[0]
            g_ss = vals[1]
            g_cs = vals[2]

            if vals == min_case:
                label = 'Min Case'
            else:
                label = 'Max Case'

            plt.figure(2)
            x_data, y_data = con_fdm_data.return_data(inter, g_ss, g_cs, B, 50)
            plt.plot(x_data, y_data, label=label)

            plt.figure(3)
            x_data, y_data = nonc_fdm_data.return_data(inter, g_ss, g_cs, B, 50)
            plt.plot(x_data, y_data, label=label)

            plt.figure(1)
            x_data, y_data = exact_nonc_data.return_data(inter, g_ss, g_cs, B, 50)
            plt.plot(x_data, y_data, label=label)

        for i in range(1, 4):
            plt.figure(i)
            plt.grid()
            plt.legend()
            plt.xlabel('Position (cm)')
            plt.ylabel('Temperature (\u00B0C)')
            plt.axvline(x=inter, color='grey', linestyle='--')

        plt.figure(1)
        plt.title('Exact Solution')

        plt.figure(2)
        plt.title('FDM Solution with Conformal Mesh')

        plt.figure(3)
        plt.title('FDM Solution with Non-Conformal Mesh')
```

```
        plt.show()

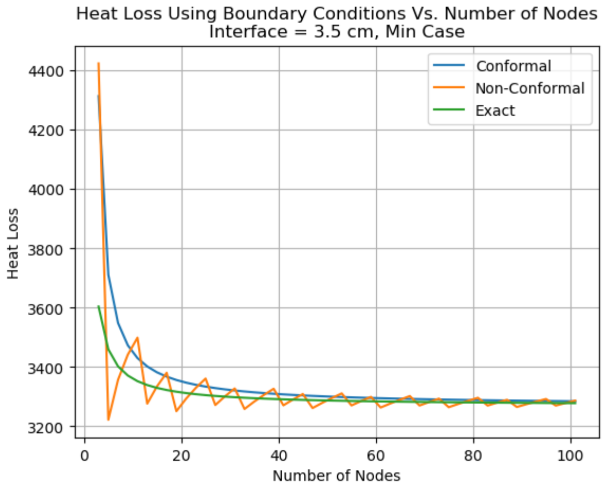

# Plotting Convergence Graphs of the Temperature
if input('Plot Convergence Graphs of the Temperature Distribution? (y/n): ') == 'y':
    for inter in interface:
        for vals in [min_case, max_case]:
            err_data = []
            dx_data = []
            for num_elem in nodes:
                    B = vals[0]
                    g_ss = vals[1]
                    g_cs = vals[2]
                    exact = exact_nonc_data.return_data(inter, g_ss, g_cs, B, num_elem)[1][-1]
                    approx = nonc_fdm_data.return_data(inter, g_ss, g_cs, B, num_elem)[1][-1]
                    test = get_error(exact, approx)
                    err_data.append(get_error(exact, approx))
                    dx_data.append(np.log((x3-x1)/num_elem))

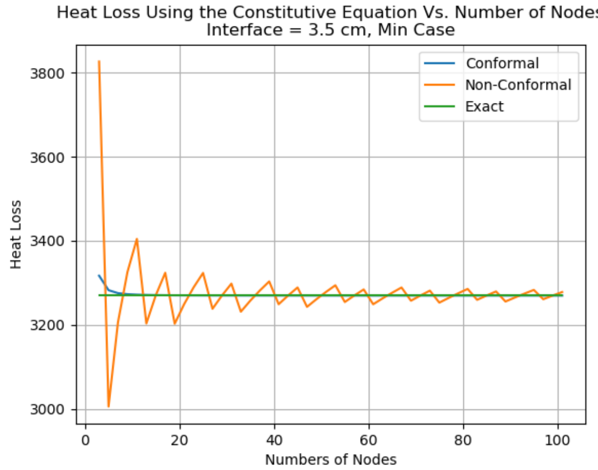
            local_mins = (min(np.array(err_data[:100]), np.less))
            mins_points = [[], []]
            for i in local_mins[0]:
                mins_points[0].append(dx_data[i])
                mins_points[1].append(err_data[i])

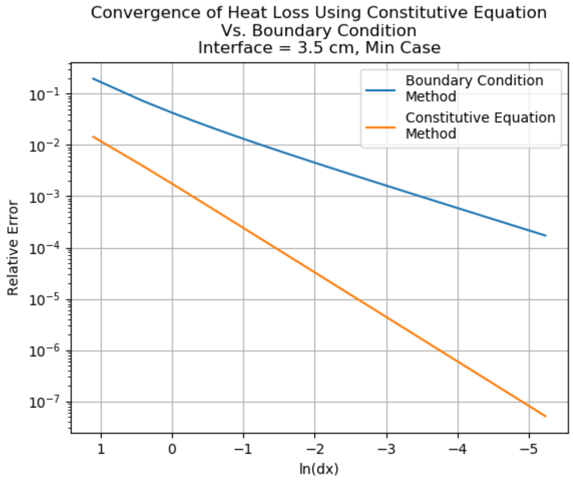
            plt.ylabel('Relative Error')
            plt.xlabel('Ln(dx)')
            plt.yscale('log')
            plt.gca().invert_xaxis()
            if inter == interface[0]:
                plt.plot(mins_points[0], mins_points[1], color='black', linestyle='dashed', marker='o',
                         markerfacecolor='red')
            plt.plot(dx_data[:100], err_data[:100])
            if vals == min_case:
                title = 'Min Case'
            else:
                title = 'Max Case'
            plt.title('Relative Error of Non-Conformal FDM\n'+title)
            plt.grid()
            plt.show()

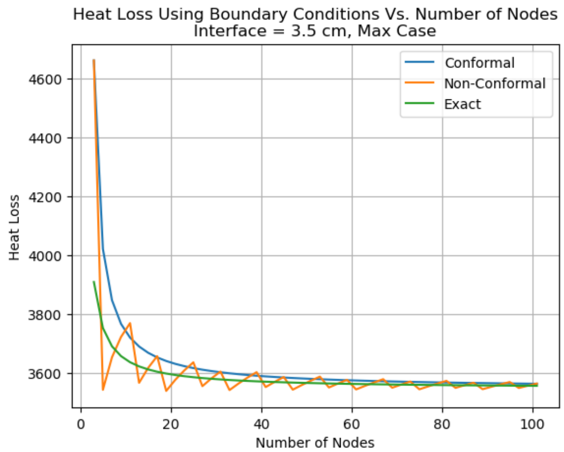
    for inter in interface:
        for vals in [min_case, max_case]:
            err_data = []
            dx_data = []
            for num_elem in nodes:
                    B = vals[0]
                    g_ss = vals[1]
                    g_cs = vals[2]
                    exact = exact_con_data.return_data(inter, g_ss, g_cs, B, num_elem)[1]
                    approx = con_fdm_data.return_data(inter, g_ss, g_cs, B, num_elem)[1]
                    test = get_error(exact[-1], approx[-1])
                    err_data.append(get_error(exact[-1], approx[-1]))
                    dx_data.append(np.log((x3-x1)/num_elem))
            plt.ylabel('Relative Error')
            plt.xlabel('Log dx')
            plt.yscale('log')
            plt.gca().invert_xaxis()
            plt.grid()
            if vals == min_case:
                title = 'Min Case'
            else:
                title = 'Max Case'
            plt.title('Relative Error of Conformal FDM\n' + title)
            plt.plot(dx_data[:100], err_data[:100])
            plt.show()
```

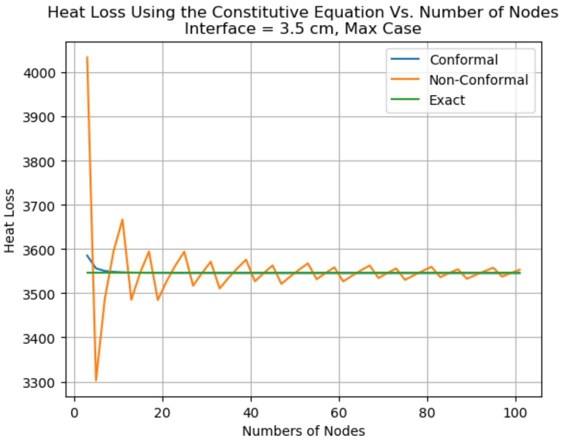
8 Appendix B – Code Graph Output

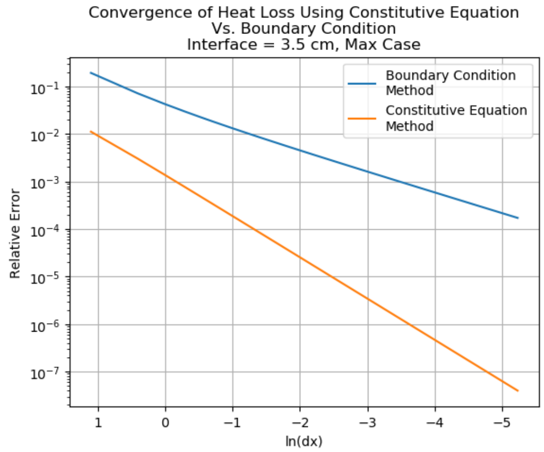


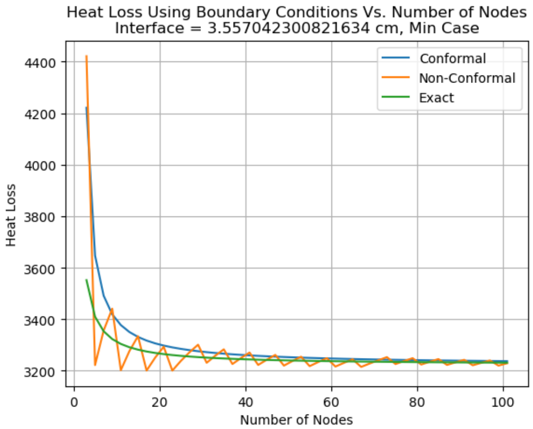


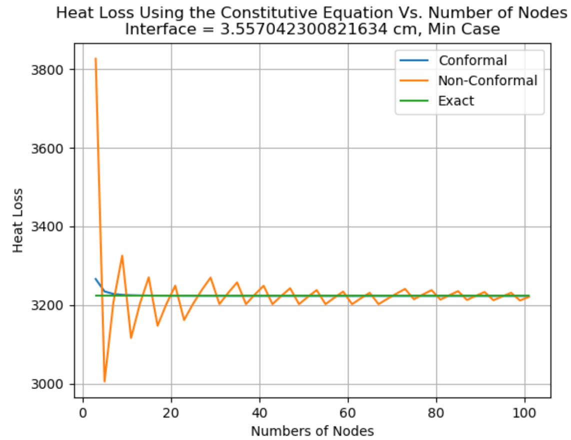


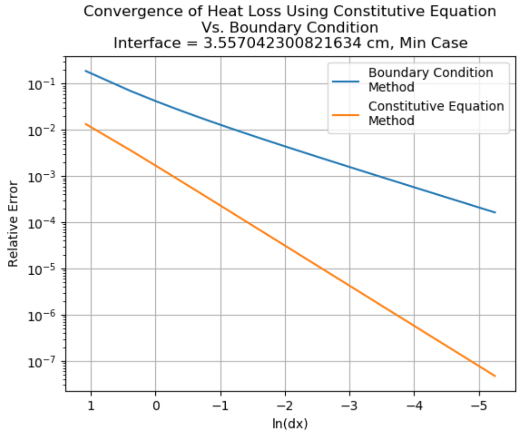


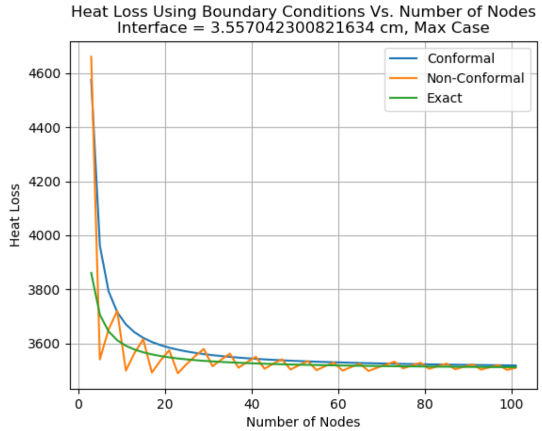


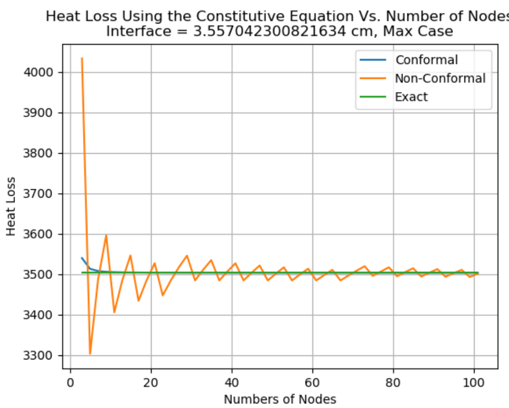


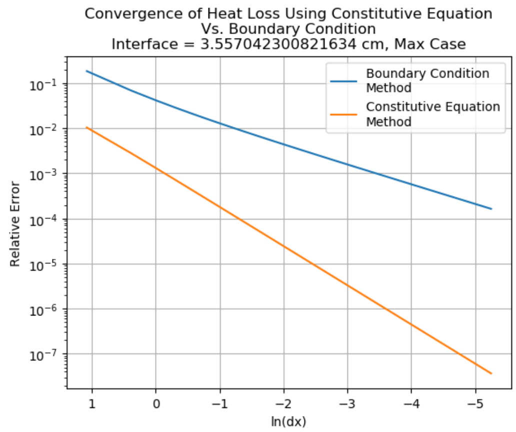

Heat Loss Using Boundary Conditions Vs. Number of Nodes
Interface = 3.557042300821634 cm, Min Case
Conformal
Non-Conformal
Exact
Heat Loss
Number of Nodes

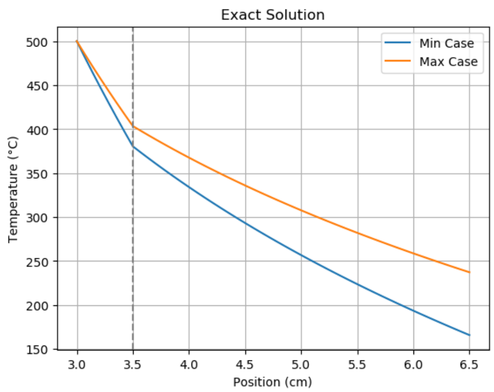

Heat Loss Using the Constitutive Equation Vs. Number of Nodes
Interface = 3.557042300821634 cm, Min Case
Conformal
Non-Conformal
Exact
Heat Loss
Numbers of Nodes

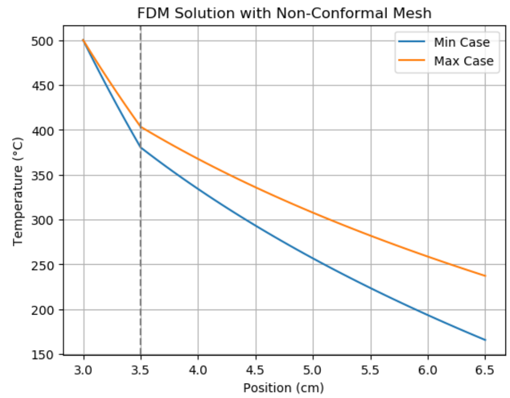

Convergence of Heat Loss Using Constitutive Equation
Vs. Boundary Condition
Interface = 3.557042300821634 cm, Min Case
Boundary Condition Method
Constitutive Equation Method
Relative Error
ln(dx)

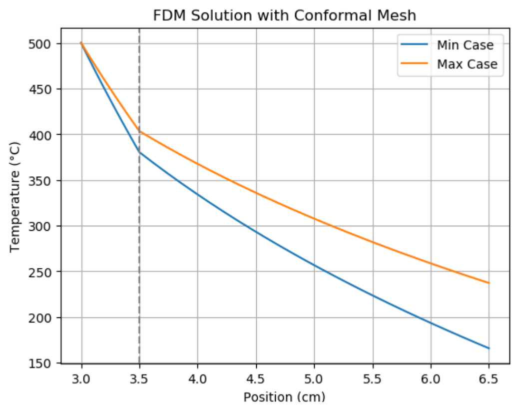

Heat Loss Using Boundary Conditions Vs. Number of Nodes
Interface = 3.557042300821634 cm, Max Case
Conformal
Non-Conformal
Exact
Heat Loss
Number of Nodes

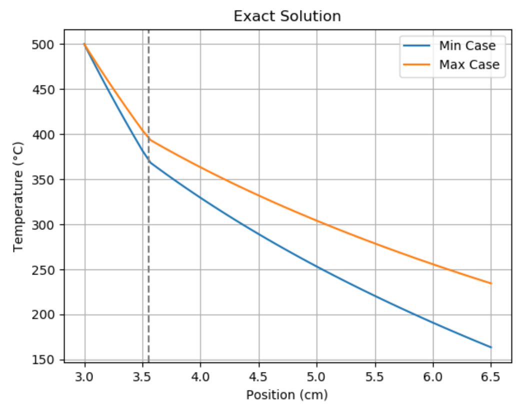

Heat Loss Using the Constitutive Equation Vs. Number of Nodes
Interface = 3.557042300821634 cm, Max Case
Conformal
Non-Conformal
Exact
Heat Loss
Numbers of Nodes

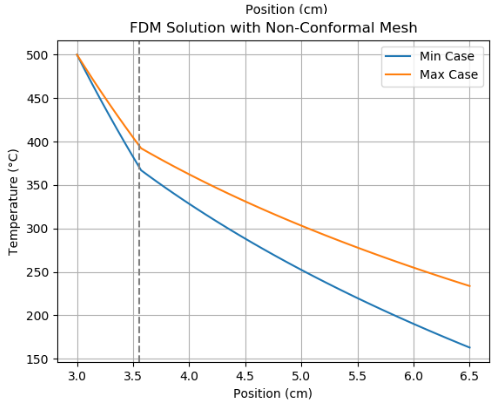

Convergence of Heat Loss Using Constitutive Equation
Vs. Boundary Condition
Interface = 3.557042300821634 cm, Max Case
Boundary Condition Method
Constitutive Equation Method
Relative Error
ln(dx)

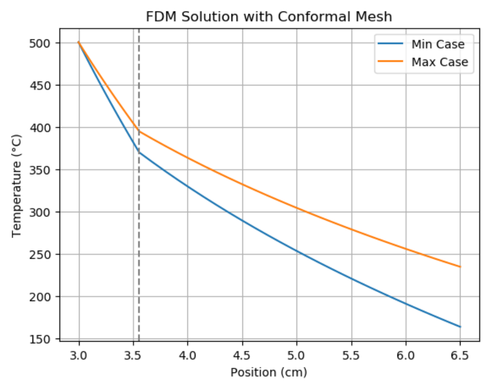

Exact Solution
Temperature (°C)
Position (cm)
Min Case
Max Case

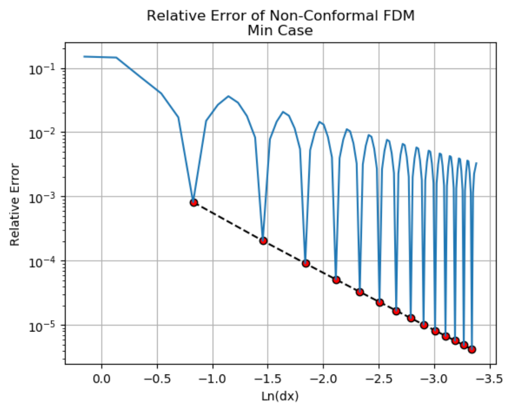

FDM Solution with Non-Conformal Mesh
Temperature (°C)
Position (cm)
Min Case
Max Case

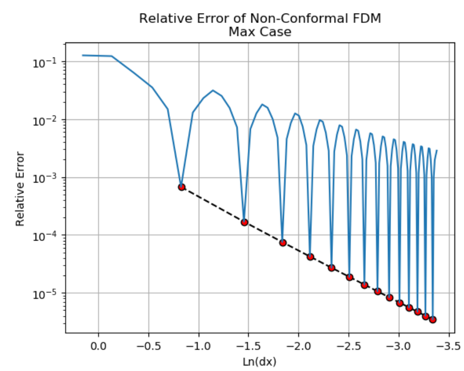

FDM Solution with Conformal Mesh
Temperature (°C)
Position (cm)
Min Case
Max Case

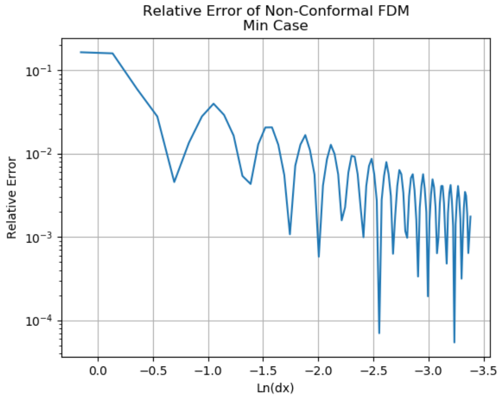

Exact Solution
Temperature (°C)
Position (cm)
Min Case
Max Case

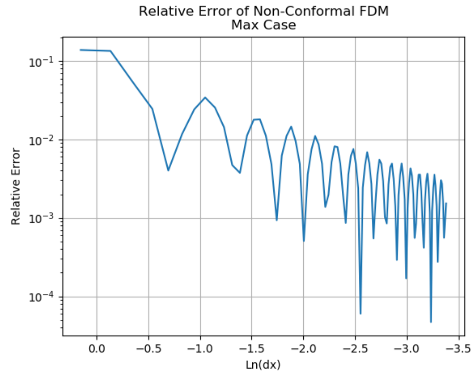

FDM Solution with Non-Conformal Mesh
Temperature (°C)
Position (cm)
Min Case
Max Case

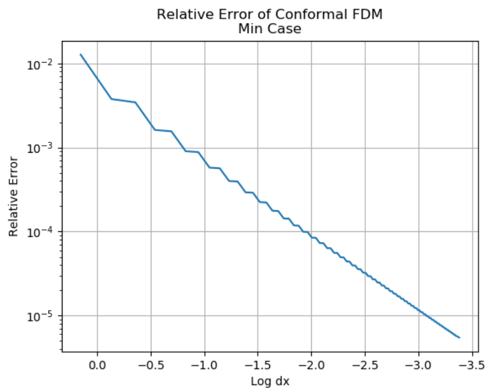

FDM Solution with Conformal Mesh
Temperature (°C)
Position (cm)
Min Case
Max Case

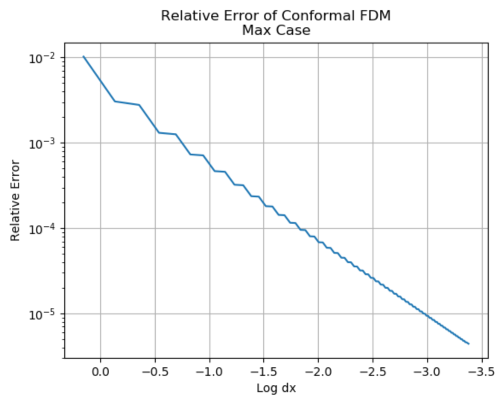

Relative Error of Non-Conformal FDM
Min Case
Relative Error
Ln(dx)

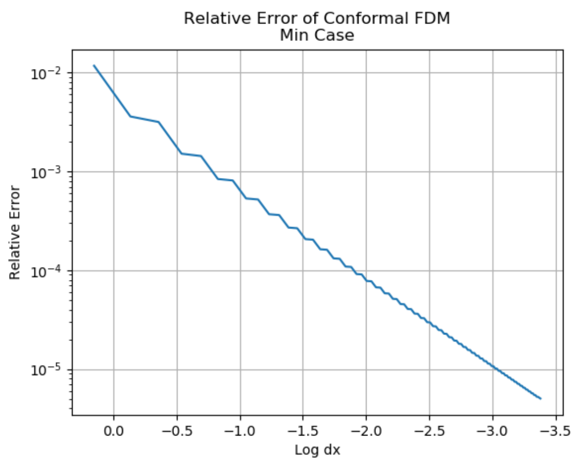

Relative Error of Non-Conformal FDM
Max Case
Relative Error
Ln(dx)

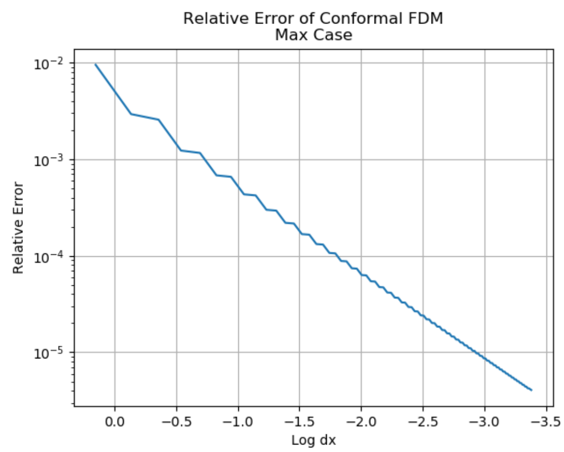

Relative Error of Non-Conformal FDM
Min Case
Relative Error
Ln(dx)
10⁻¹
10⁻²
10⁻³
10⁻⁴
0.0 −0.5 −1.0 −1.5 −2.0 −2.5 −3.0 −3.5


Relative Error of Non-Conformal FDM
Max Case
Relative Error
Ln(dx)
10⁻¹
10⁻²
10⁻³
10⁻⁴
0.0 −0.5 −1.0 −1.5 −2.0 −2.5 −3.0 −3.5


Relative Error of Conformal FDM
Min Case
Relative Error
Log dx
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0.0 −0.5 −1.0 −1.5 −2.0 −2.5 −3.0 −3.5


Relative Error of Conformal FDM
Max Case
Relative Error
Log dx
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0.0 −0.5 −1.0 −1.5 −2.0 −2.5 −3.0 −3.5


Relative Error of Conformal FDM
Min Case
Relative Error
Log dx
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0.0 −0.5 −1.0 −1.5 −2.0 −2.5 −3.0 −3.5


Relative Error of Conformal FDM
Max Case
Relative Error
Log dx
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0.0 −0.5 −1.0 −1.5 −2.0 −2.5 −3.0 −3.5

9 Appendix C – Code Console Output

C:\Users\Andrew\Anaconda3\python.exe
C:/Users/Andrew/PycharmProjects/JSC_Internship_Practice/AERO_430_Exam_1.py
Create Temperature Tables? (y/n): y

Conformal Mesh FDM Compared to Exact Solution
Min Case, Interface Lccated at x = 3.5 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	9.66338e-13
3.02083	494.625	494.625	2.39039e-05
3.04167	489.288	489.288	4.82229e-05
3.0625	483.986	483.987	7.29655e-05
3.08333	478.721	478.722	9.81404e-05
3.10417	473.491	473.492	0.000123757
3.125	468.296	468.297	0.000149824
3.14583	463.136	463.137	0.000176352
3.16667	458.01	458.011	0.000203352
3.1875	452.917	452.918	0.000230832
3.20833	447.858	447.859	0.000258805
3.22917	442.831	442.832	0.000287282
3.25	437.836	437.838	0.000316274
3.27083	432.874	432.875	0.000345794
3.29167	427.943	427.944	0.000375854
3.3125	423.043	423.045	0.000406468
3.33333	418.174	418.176	0.000437649
3.35417	413.335	413.337	0.00046941
3.375	408.526	408.528	0.000501768
3.39583	403.747	403.749	0.000534737
3.41667	398.997	398.999	0.000568332
3.4375	394.275	394.278	0.00060257
3.45833	389.583	389.585	0.000637469
3.47917	384.918	384.921	0.000673045
3.5	380.282	380.284	0.000709317
3.62	368.589	368.59	0.000513849
3.74	357.277	357.278	0.000331816
3.86	346.322	346.323	0.000161624
3.98	335.703	335.703	1.89188e-06
4.1	325.399	325.398	0.000148587
4.22	315.392	315.391	0.000290869
4.34	305.666	305.665	0.000425889
4.46	296.205	296.204	0.000554475
4.58	286.996	286.994	0.000677367
4.7	278.024	278.022	0.000795231

4.82	269.279	269.277	0.00090867
4.94	260.749	260.746	0.00101823
5.06	252.423	252.42	0.00112442
5.18	244.293	244.29	0.00122771
5.3	236.349	236.345	0.00132853
5.42	228.582	228.579	0.00142728
5.54	220.986	220.983	0.00152438
5.66	213.552	213.549	0.00162018
5.78	206.275	206.271	0.00171505
5.9	199.147	199.143	0.00180935
6.02	192.162	192.159	0.00190345
6.14	185.316	185.312	0.00199769
6.26	178.602	178.598	0.00209245
6.38	172.015	172.011	0.0021881
6.5	165.551	165.547	0.00228504

Non-Conformal Mesh FDM Compared to Exact Solution
Min Case, Interface Located at x = 3.5 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	5.33191e-12
3.07143	481.726	481.726	0.00010279
3.14286	463.873	463.872	0.000202968
3.21429	446.42	446.419	0.000300987
3.28571	429.352	429.35	0.000397286
3.35714	412.65	412.648	0.000492293
3.42857	396.3	396.298	0.000586432
3.5	380.287	380.284	0.000680128
3.57143	373.279	373.276	0.000721001
3.64286	366.41	366.407	0.000760083
3.71429	359.674	359.671	0.000797501
3.78571	353.066	353.064	0.000833375
3.85714	346.582	346.579	0.000867814
3.92857	340.217	340.214	0.000900916
4	333.967	333.964	0.000932775
4.07143	327.827	327.824	0.000963474
4.14286	321.794	321.791	0.000993094
4.21429	315.865	315.861	0.00102171
4.28571	310.034	310.031	0.00104938
4.35714	304.301	304.297	0.00107617
4.42857	298.66	298.657	0.00110215
4.5	293.11	293.106	0.00112736
4.57143	287.647	287.644	0.00115185
4.64286	282.269	282.265	0.00117569
4.71429	276.973	276.969	0.0011989

4.78571	271.756	271.753	0.00122153
4.85714	266.617	266.614	0.00124363
4.92857	261.553	261.55	0.00126523
5	256.562	256.558	0.00128636
5.07143	251.641	251.638	0.00130707
5.14286	246.789	246.786	0.00132738
5.21429	242.005	242.001	0.00134732
5.28571	237.285	237.282	0.00136692
5.35714	232.629	232.625	0.00138622
5.42857	228.034	228.031	0.00140524
5.5	223.499	223.496	0.00142401
5.57143	219.023	219.02	0.00144255
5.64286	214.604	214.601	0.0014609
5.71429	210.241	210.238	0.00147906
5.78571	205.932	205.929	0.00149708
5.85714	201.675	201.672	0.00151496
5.92857	197.471	197.468	0.00153275
6	193.316	193.313	0.00155045
6.07143	189.211	189.208	0.0015681
6.14286	185.154	185.151	0.00158571
6.21429	181.143	181.14	0.00160332
6.28571	177.179	177.176	0.00162094
6.35714	173.259	173.256	0.00163861
6.42857	169.383	169.38	0.00165633
6.5	165.55	165.547	0.00167415

Proceed?y

Conformal Mesh FDM Compared to Exact Solution
Max Case, Interface Lccated at x = 3.5 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	1.73941e-12
3.02083	495.661	495.661	1.41534e-05
3.04167	491.352	491.352	2.85048e-05
3.0625	487.072	487.073	4.30566e-05
3.08333	482.822	482.822	5.78113e-05
3.10417	478.6	478.6	7.27715e-05
3.125	474.406	474.407	8.79397e-05
3.14583	470.24	470.241	0.000103319
3.16667	466.102	466.102	0.000118911
3.1875	461.991	461.991	0.000134721
3.20833	457.906	457.907	0.00015075
3.22917	453.848	453.849	0.000167001
3.25	449.816	449.817	0.000183479

3.27083	445.81	445.811	0.000200186
3.29167	441.829	441.83	0.000217126
3.3125	437.874	437.875	0.000234302
3.33333	433.943	433.944	0.000251718
3.35417	430.036	430.038	0.000269379
3.375	426.154	426.156	0.000287287
3.39583	422.296	422.297	0.000305446
3.41667	418.461	418.463	0.000323862
3.4375	414.65	414.651	0.000342538
3.45833	410.862	410.863	0.000361479
3.47917	407.096	407.098	0.000380689
3.5	403.353	403.355	0.000400173
3.62	394.296	394.297	0.000238143
3.74	385.534	385.534	8.79595e-05
3.86	377.049	377.048	5.17349e-05
3.98	368.823	368.822	0.000182112
4.1	360.842	360.841	0.000304188
4.22	353.091	353.089	0.000418846
4.34	345.557	345.555	0.000526859
4.46	338.229	338.227	0.000628905
4.58	331.096	331.093	0.000725582
4.7	324.146	324.144	0.000817421
4.82	317.372	317.37	0.000904892
4.94	310.765	310.762	0.000988414
5.06	304.316	304.313	0.00106837
5.18	298.019	298.015	0.00114508
5.3	291.865	291.862	0.00121887
5.42	285.85	285.846	0.00129
5.54	279.966	279.962	0.00135873
5.66	274.208	274.204	0.00142529
5.78	268.571	268.567	0.00148988
5.9	263.049	263.045	0.00155269
6.02	257.639	257.635	0.00161391
6.14	252.336	252.332	0.00167369
6.26	247.135	247.131	0.00173218
6.38	242.034	242.029	0.00178953
6.5	237.027	237.022	0.00184587

Non-Conformal Mesh FDM Compared to Exact Solution
Max Case, Interface Located at x = 3.5 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	6.73026e-12
3.07143	485.248	485.247	9.44207e-05
3.14286	470.835	470.834	0.000185966

3.21429	456.746	456.745	0.000275
3.28571	442.967	442.965	0.000361864
3.35714	429.483	429.482	0.000446879
3.42857	416.284	416.282	0.000530346
3.5	403.357	403.355	0.000612556
3.57143	397.929	397.926	0.000646737
3.64286	392.608	392.605	0.00067935
3.71429	387.391	387.388	0.000710507
3.78571	382.273	382.27	0.00074031
3.85714	377.25	377.247	0.000768851
3.92857	372.32	372.317	0.000796215
4	367.479	367.476	0.000822481
4.07143	362.723	362.72	0.000847719
4.14286	358.05	358.047	0.000871998
4.21429	353.457	353.454	0.000895376
4.28571	348.941	348.938	0.000917911
4.35714	344.499	344.496	0.000939655
4.42857	340.13	340.127	0.000960656
4.5	335.831	335.828	0.000980959
4.57143	331.6	331.596	0.00100061
4.64286	327.434	327.431	0.00101963
4.71429	323.332	323.328	0.00103808
4.78571	319.291	319.288	0.00105598
4.85714	315.31	315.307	0.00107336
4.92857	311.388	311.384	0.00109026
5	307.522	307.518	0.0011067
5.07143	303.71	303.707	0.0011227
5.14286	299.952	299.949	0.0011383
5.21429	296.246	296.243	0.00115351
5.28571	292.59	292.587	0.00116835
5.35714	288.984	288.98	0.00118285
5.42857	285.425	285.421	0.00119702
5.5	281.912	281.909	0.00121089
5.57143	278.445	278.442	0.00122446
5.64286	275.022	275.019	0.00123776
5.71429	271.642	271.639	0.00125081
5.78571	268.305	268.301	0.0012636
5.85714	265.008	265.004	0.00127617
5.92857	261.751	261.747	0.00128852
6	258.533	258.529	0.00130066
6.07143	255.353	255.35	0.00131261
6.14286	252.21	252.207	0.00132438
6.21429	249.104	249.101	0.00133598
6.28571	246.033	246.03	0.00134742
6.35714	242.997	242.994	0.0013587

6.42857	239.995	239.991	0.00136985
6.5	237.026	237.022	0.00138086

Proceed?y

Conformal Mesh FDM Compared to Exact Solution
Min Case, Interface Lccated at x = 3.557042300821634 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	1.8531e-12
3.02321	494.1	494.1	2.2648e-05
3.04642	488.245	488.245	4.57543e-05
3.06963	482.435	482.435	6.93285e-05
3.09284	476.668	476.668	9.33806e-05
3.11605	470.944	470.945	0.000117921
3.13926	465.263	465.264	0.00014296
3.16247	459.624	459.625	0.00016851
3.18568	454.026	454.027	0.000194582
3.20889	448.469	448.47	0.000221188
3.2321	442.951	442.952	0.000248341
3.25531	437.473	437.475	0.000276055
3.27852	432.034	432.036	0.000304344
3.30173	426.634	426.635	0.000333221
3.32494	421.271	421.273	0.000362702
3.34815	415.946	415.947	0.000392803
3.37136	410.657	410.659	0.000423541
3.39457	405.405	405.407	0.000454931
3.41778	400.188	400.19	0.000486993
3.44099	395.007	395.009	0.000519744
3.4642	389.86	389.863	0.000553205
3.48741	384.748	384.751	0.000587395
3.51062	379.67	379.673	0.000622335
3.53383	374.625	374.628	0.000658048
3.55704	369.614	369.616	0.000694556
3.67476	358.481	358.483	0.000516081
3.79248	347.7	347.701	0.000349341
3.9102	337.248	337.249	0.000192978
4.02792	327.107	327.107	4.58099e-05
4.14563	317.257	317.257	9.32008e-05
4.26335	307.683	307.682	0.000224968
4.38107	298.37	298.369	0.000350303
4.49879	289.304	289.302	0.000469929
4.61651	280.472	280.47	0.000584494
4.73423	271.862	271.86	0.000694583
4.85194	263.464	263.461	0.00080073

4.96966	255.267	255.264	0.000903421
5.08738	247.262	247.259	0.0010031
5.2051	239.44	239.437	0.0011002
5.32282	231.793	231.79	0.00119509
5.44054	224.313	224.31	0.00128816
5.55825	216.993	216.99	0.00137974
5.67597	209.827	209.824	0.00147019
5.79369	202.808	202.805	0.00155983
5.91141	195.93	195.927	0.00164899
6.02913	189.188	189.185	0.00173799
6.14685	182.576	182.573	0.00182716
6.26456	176.089	176.086	0.00191683
6.38228	169.724	169.72	0.00200735
6.5	163.474	163.471	0.00209907

Non-Conformal Mesh FDM Compared to Exact Solution
Min Case, Interface Located at x = 3.557042300821634 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	9.66338e-13
3.07143	482.051	481.987	0.013365
3.14286	464.515	464.387	0.0274162
3.21429	447.372	447.184	0.042213
3.28571	430.607	430.358	0.0578218
3.35714	414.202	413.894	0.0743166
3.42857	398.142	397.777	0.0917801
3.5	382.414	381.992	0.110306
3.57143	367.003	368.236	0.334914
3.64286	360.256	361.465	0.334514
3.71429	353.64	354.825	0.33411
3.78571	347.15	348.312	0.3337
3.85714	340.781	341.92	0.333285
3.92857	334.529	335.646	0.332864
4	328.389	329.485	0.332436
4.07143	322.359	323.433	0.332003
4.14286	316.433	317.486	0.331562
4.21429	310.609	311.64	0.331114
4.28571	304.882	305.893	0.330659
4.35714	299.25	300.241	0.330195
4.42857	293.71	294.681	0.329724
4.5	288.258	289.21	0.329243
4.57143	282.892	283.825	0.328753
4.64286	277.609	278.524	0.328253
4.71429	272.407	273.303	0.327743
4.78571	267.284	268.161	0.327223

4.85714	262.236	263.095	0.326691
4.92857	257.262	258.103	0.326148
5	252.359	253.183	0.325592
5.07143	247.526	248.333	0.325024
5.14286	242.76	243.551	0.324443
5.21429	238.061	238.834	0.323847
5.28571	233.425	234.182	0.323237
5.35714	228.851	229.592	0.322612
5.42857	224.338	225.063	0.321971
5.5	219.884	220.593	0.321314
5.57143	215.488	216.181	0.320638
5.64286	211.147	211.825	0.319945
5.71429	206.861	207.524	0.319232
5.78571	202.629	203.276	0.3185
5.85714	198.448	199.08	0.317746
5.92857	194.318	194.936	0.31697
6	190.237	190.841	0.316171
6.07143	186.205	186.794	0.315348
6.14286	182.22	182.795	0.314499
6.21429	178.281	178.841	0.313624
6.28571	174.386	174.934	0.312719
6.35714	170.536	171.07	0.311785
6.42857	166.729	167.249	0.31082
6.5	162.964	163.471	0.309821

Proceed?y

Conformal Mesh FDM Compared to Exact Solution
Max Case, Interface Lccated at x = 3.557042300821634 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	6.59384e-13
3.02321	495.226	495.226	1.31666e-05
3.04642	490.488	490.488	2.65588e-05
3.06963	485.786	485.786	4.01789e-05
3.09284	481.12	481.12	5.40295e-05
3.11605	476.488	476.488	6.81133e-05
3.13926	471.891	471.891	8.2433e-05
3.16247	467.327	467.328	9.69917e-05
3.18568	462.797	462.798	0.000111792
3.20889	458.3	458.301	0.000126838
3.2321	453.836	453.836	0.000142133
3.25531	449.403	449.404	0.000157679
3.27852	445.002	445.003	0.000173481
3.30173	440.632	440.632	0.000189543

3.32494	436.292	436.293	0.000205868
3.34815	431.983	431.984	0.000222461
3.37136	427.703	427.704	0.000239326
3.39457	423.453	423.454	0.000256467
3.41778	419.232	419.233	0.000273888
3.44099	415.039	415.04	0.000291596
3.4642	410.874	410.876	0.000309594
3.48741	406.738	406.739	0.000327887
3.51062	402.628	402.63	0.000346482
3.53383	398.546	398.548	0.000365383
3.55704	394.491	394.492	0.000384596
3.67476	385.847	385.848	0.000237013
3.79248	377.476	377.477	9.97994e-05
3.9102	369.361	369.361	2.81982e-05
4.02792	361.487	361.486	0.00014798
4.14563	353.839	353.839	0.000260417
4.26335	346.406	346.405	0.000366271
4.38107	339.175	339.174	0.000466209
4.49879	332.136	332.134	0.000560822
4.61651	325.278	325.276	0.00065063
4.73423	318.594	318.591	0.000736097
4.85194	312.073	312.071	0.000817636
4.96966	305.709	305.706	0.000895618
5.08738	299.494	299.491	0.000970373
5.2051	293.42	293.417	0.0010422
5.32282	287.483	287.48	0.00111138
5.44054	281.676	281.672	0.00117814
5.55825	275.993	275.989	0.00124272
5.67597	270.429	270.425	0.00130531
5.79369	264.979	264.975	0.00136611
5.91141	259.639	259.635	0.00142528
6.02913	254.404	254.4	0.00148299
6.14685	249.27	249.266	0.00153939
6.26456	244.234	244.23	0.0015946
6.38228	239.291	239.287	0.00164876
6.5	234.439	234.435	0.00170198

Non-Conformal Mesh FDM Compared to Exact Solution
Max Case, Interface Located at x = 3.557042300821634 cm

x (cm)	FDM Temp (°C)	Exact Temp (°C)	Percent Error
3	500	500	6.63931e-12
3.07143	485.467	485.423	0.00900415
3.14286	471.268	471.182	0.018333
3.21429	457.388	457.26	0.0280079

3.28571	443.814	443.645	0.0380519
3.35714	430.531	430.322	0.0484899
3.42857	417.528	417.28	0.0593486
3.5	404.793	404.507	0.0706575
3.57143	392.315	393.421	0.281024
3.64286	387.073	388.163	0.280788
3.71429	381.933	383.008	0.280551
3.78571	376.891	377.951	0.280314
3.85714	371.943	372.988	0.280076
3.92857	367.086	368.117	0.279839
4	362.317	363.333	0.2796
4.07143	357.632	358.634	0.279361
4.14286	353.028	354.016	0.279121
4.21429	348.503	349.478	0.27888
4.28571	344.055	345.016	0.278638
4.35714	339.679	340.627	0.278395
4.42857	335.375	336.31	0.27815
4.5	331.14	332.063	0.277904
4.57143	326.971	327.881	0.277657
4.64286	322.867	323.765	0.277407
4.71429	318.826	319.712	0.277157
4.78571	314.845	315.719	0.276904
4.85714	310.924	311.786	0.27665
4.92857	307.059	307.91	0.276393
5	303.251	304.09	0.276135
5.07143	299.496	300.324	0.275874
5.14286	295.794	296.611	0.275611
5.21429	292.143	292.949	0.275346
5.28571	288.541	289.337	0.275078
5.35714	284.988	285.773	0.274808
5.42857	281.482	282.257	0.274535
5.5	278.022	278.786	0.274259
5.57143	274.606	275.361	0.273981
5.64286	271.234	271.978	0.2737
5.71429	267.904	268.639	0.273416
5.78571	264.616	265.341	0.273128
5.85714	261.368	262.083	0.272838
5.92857	258.16	258.865	0.272544
6	254.989	255.686	0.272247
6.07143	251.857	252.544	0.271946
6.14286	248.761	249.438	0.271642
6.21429	245.701	246.369	0.271334
6.28571	242.675	243.335	0.271022
6.35714	239.684	240.335	0.270706
6.42857	236.727	237.369	0.270387

6.5	233.802	234.435	0.270063

Proceed?y
Heat Transfer and Convergence? (y/n): y
Perform Richardson Extrapolation? (y/n): y
Interface located at x = 3.5

dx (cm)	Beta (Constitutive)	Beta (Boundary Condition)
3	2.55174	3.72365
1.5	2.35519	3.09399
1	2.2623	2.87451
0.75	2.20791	2.76123
0.6	2.17218	2.69184
0.5	2.14692	2.64489
0.428571	2.12812	2.61098
0.375	2.11358	2.58535
0.333333	2.102	2.56527
0.3	2.09257	2.54913
0.272727	2.08473	2.53586
0.25	2.07811	2.52476
0.230769	2.07246	2.51534
0.214286	2.06756	2.50725
0.2	2.06329	2.50021
0.1875	2.05952	2.49405
0.176471	2.05618	2.4886
0.166667	2.05319	2.48374
0.157895	2.05051	2.47939
0.15	2.04808	2.47547
0.142857	2.04587	2.47192
0.136364	2.04386	2.46869
0.130435	2.04202	2.46574
0.125	2.04032	2.46303
0.12	2.03876	2.46053
0.115385	2.03731	2.45823
0.111111	2.03597	2.45609
0.107143	2.03472	2.4541
0.103448	2.03356	2.45226
0.1	2.03247	2.45053
0.0967742	2.03145	2.44891
0.09375	2.03049	2.4474
0.0909091	2.02959	2.44597
0.0882353	2.02874	2.44463
0.0857143	2.02793	2.44336
0.0833333	2.02717	2.44217
0.0810811	2.02646	2.44104

0.0789474	2.02577	2.43996
0.0769231	2.02513	2.43895
0.075	2.02451	2.43798
0.0731707	2.02392	2.43706
0.0714286	2.02336	2.43618
0.0697674	2.02283	2.43534
0.0681818	2.02232	2.43455
0.0666667	2.02184	2.43378
0.0652174	2.02137	2.43305
0.0638298	2.02092	2.43235
0.0625	2.02049	2.43168
0.0612245	2.02008	2.43104
0.06	2.01969	2.43042
0.0588235	2.01912	2.46312
0.0566038	2.0181	2.52781
0.0526316	2.01636	2.65475
0.0461538	2.01375	2.90192
0.037037	2.01047	3.38733
0.0265487	2.00712	4.44104
0.0169492	2.00435	10.4912
0.00983607	2.00253	3.9074

Interface located at x = 3.557042300821634

dx (cm)	Beta (Constitutive)	Beta (Boundary Condition)
2.94296	2.56155	3.84292
1.47148	2.35891	3.18189
0.980986	2.26421	2.95445
0.735739	2.20906	2.83759
0.588592	2.17295	2.76615
0.490493	2.14747	2.71789
0.420423	2.12853	2.68307
0.36787	2.1139	2.65676
0.326995	2.10226	2.63617
0.294296	2.09277	2.61961
0.267542	2.0849	2.60601
0.245246	2.07826	2.59464
0.226381	2.07258	2.58498
0.210211	2.06767	2.57669
0.196197	2.06338	2.56949
0.183935	2.0596	2.56317
0.173115	2.05625	2.55759
0.163498	2.05326	2.55262
0.154893	2.05056	2.54816
0.147148	2.04813	2.54415
0.140141	2.04592	2.54051

0.133771	2.0439	2.53721
0.127955	2.04206	2.53418
0.122623	2.04036	2.53141
0.117718	2.0388	2.52886
0.113191	2.03735	2.5265
0.108998	2.036	2.52431
0.105106	2.03475	2.52228
0.101481	2.03358	2.52039
0.0980986	2.03249	2.51862
0.0949341	2.03147	2.51697
0.0919674	2.03051	2.51541
0.0891805	2.02961	2.51396
0.0865576	2.02876	2.51258
0.0840845	2.02795	2.51129
0.0817488	2.02719	2.51007
0.0795394	2.02647	2.50891
0.0774463	2.02579	2.50781
0.0754605	2.02514	2.50677
0.0735739	2.02452	2.50578
0.0717795	2.02394	2.50484
0.0700704	2.02338	2.50394
0.0684409	2.02284	2.50309
0.0668854	2.02233	2.50227
0.0653991	2.02185	2.50149
0.0639773	2.02138	2.50074
0.0626161	2.02093	2.50003
0.0613116	2.0205	2.49934
0.0600604	2.02009	2.49868
0.0588592	2.01969	2.49805
0.0577051	2.01913	2.53337
0.0555275	2.01811	2.60343
0.0516308	2.01637	2.74168
0.0452763	2.01376	3.01406
0.0363328	2.01047	3.56406
0.0260439	2.00712	4.86686
0.0166269	2.00436	6.38881
0.00964904	2.00241	3.44205

Temperature Tables without analytical comparison? (y/n): y
Conformal Mesh FDM
Interface located at x = 3.5 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.02083	494.625	495.661
3.04167	489.288	491.352

3.0625	483.986	487.072
3.08333	478.721	482.822
3.10417	473.491	478.6
3.125	468.296	474.406
3.14583	463.136	470.24
3.16667	458.01	466.102
3.1875	452.917	461.991
3.20833	447.858	457.906
3.22917	442.831	453.848
3.25	437.836	449.816
3.27083	432.874	445.81
3.29167	427.943	441.829
3.3125	423.043	437.874
3.33333	418.174	433.943
3.35417	413.335	430.036
3.375	408.526	426.154
3.39583	403.747	422.296
3.41667	398.997	418.461
3.4375	394.275	414.65
3.45833	389.583	410.862
3.47917	384.918	407.096
3.5	380.282	403.353
3.62	368.589	394.296
3.74	357.277	385.534
3.86	346.322	377.049
3.98	335.703	368.823
4.1	325.399	360.842
4.22	315.392	353.091
4.34	305.666	345.557
4.46	296.205	338.229
4.58	286.996	331.096
4.7	278.024	324.146
4.82	269.279	317.372
4.94	260.749	310.765
5.06	252.423	304.316
5.18	244.293	298.019
5.3	236.349	291.865
5.42	228.582	285.85
5.54	220.986	279.966
5.66	213.552	274.208
5.78	206.275	268.571
5.9	199.147	263.049
6.02	192.162	257.639
6.14	185.316	252.336
6.26	178.602	247.135

6.38	172.015	242.034
6.5	165.551	237.027

Non-Conformal Mesh FDM
Interface located at x = 3.5 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.07143	481.726	485.248
3.14286	463.873	470.835
3.21429	446.42	456.746
3.28571	429.352	442.967
3.35714	412.65	429.483
3.42857	396.3	416.284
3.5	380.287	403.357
3.57143	373.279	397.929
3.64286	366.41	392.608
3.71429	359.674	387.391
3.78571	353.066	382.273
3.85714	346.582	377.25
3.92857	340.217	372.32
4	333.967	367.479
4.07143	327.827	362.723
4.14286	321.794	358.05
4.21429	315.865	353.457
4.28571	310.034	348.941
4.35714	304.301	344.499
4.42857	298.66	340.13
4.5	293.11	335.831
4.57143	287.647	331.6
4.64286	282.269	327.434
4.71429	276.973	323.332
4.78571	271.756	319.291
4.85714	266.617	315.31
4.92857	261.553	311.388
5	256.562	307.522
5.07143	251.641	303.71
5.14286	246.789	299.952
5.21429	242.005	296.246
5.28571	237.285	292.59
5.35714	232.629	288.984
5.42857	228.034	285.425
5.5	223.499	281.912
5.57143	219.023	278.445
5.64286	214.604	275.022
5.71429	210.241	271.642

5.78571	205.932	268.305
5.85714	201.675	265.008
5.92857	197.471	261.751
6	193.316	258.533
6.07143	189.211	255.353
6.14286	185.154	252.21
6.21429	181.143	249.104
6.28571	177.179	246.033
6.35714	173.259	242.997
6.42857	169.383	239.995
6.5	165.55	237.026

Exact Solution
Interface located at x = 3.5 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.07143	481.726	485.247
3.14286	463.872	470.834
3.21429	446.419	456.745
3.28571	429.35	442.965
3.35714	412.648	429.482
3.42857	396.298	416.282
3.5	380.284	403.355
3.57143	373.276	397.926
3.64286	366.407	392.605
3.71429	359.671	387.388
3.78571	353.064	382.27
3.85714	346.579	377.247
3.92857	340.214	372.317
4	333.964	367.476
4.07143	327.824	362.72
4.14286	321.791	358.047
4.21429	315.861	353.454
4.28571	310.031	348.938
4.35714	304.297	344.496
4.42857	298.657	340.127
4.5	293.106	335.828
4.57143	287.644	331.596
4.64286	282.265	327.431
4.71429	276.969	323.328
4.78571	271.753	319.288
4.85714	266.614	315.307
4.92857	261.55	311.384
5	256.558	307.518
5.07143	251.638	303.707

5.14286	246.786	299.949
5.21429	242.001	296.243
5.28571	237.282	292.587
5.35714	232.625	288.98
5.42857	228.031	285.421
5.5	223.496	281.909
5.57143	219.02	278.442
5.64286	214.601	275.019
5.71429	210.238	271.639
5.78571	205.929	268.301
5.85714	201.672	265.004
5.92857	197.468	261.747
6	193.313	258.529
6.07143	189.208	255.35
6.14286	185.151	252.207
6.21429	181.14	249.101
6.28571	177.176	246.03
6.35714	173.256	242.994
6.42857	169.38	239.991
6.5	165.547	237.022

Conformal Mesh FDM
Interface located at x = 3.557042300821634 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.02321	494.1	495.226
3.04642	488.245	490.488
3.06963	482.435	485.786
3.09284	476.668	481.12
3.11605	470.944	476.488
3.13926	465.263	471.891
3.16247	459.624	467.327
3.18568	454.026	462.797
3.20889	448.469	458.3
3.2321	442.951	453.836
3.25531	437.473	449.403
3.27852	432.034	445.002
3.30173	426.634	440.632
3.32494	421.271	436.292
3.34815	415.946	431.983
3.37136	410.657	427.703
3.39457	405.405	423.453
3.41778	400.188	419.232
3.44099	395.007	415.039
3.4642	389.86	410.874

3.48741	384.748	406.738
3.51062	379.67	402.628
3.53383	374.625	398.546
3.55704	369.614	394.491
3.67476	358.481	385.847
3.79248	347.7	377.476
3.9102	337.248	369.361
4.02792	327.107	361.487
4.14563	317.257	353.839
4.26335	307.683	346.406
4.38107	298.37	339.175
4.49879	289.304	332.136
4.61651	280.472	325.278
4.73423	271.862	318.594
4.85194	263.464	312.073
4.96966	255.267	305.709
5.08738	247.262	299.494
5.2051	239.44	293.42
5.32282	231.793	287.483
5.44054	224.313	281.676
5.55825	216.993	275.993
5.67597	209.827	270.429
5.79369	202.808	264.979
5.91141	195.93	259.639
6.02913	189.188	254.404
6.14685	182.576	249.27
6.26456	176.089	244.234
6.38228	169.724	239.291
6.5	163.474	234.439

Non-Conformal Mesh FDM
Interface located at x = 3.557042300821634 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.07143	482.051	485.467
3.14286	464.515	471.268
3.21429	447.372	457.388
3.28571	430.607	443.814
3.35714	414.202	430.531
3.42857	398.142	417.528
3.5	382.414	404.793
3.57143	367.003	392.315
3.64286	360.256	387.073
3.71429	353.64	381.933
3.78571	347.15	376.891

3.85714	340.781	371.943
3.92857	334.529	367.086
4	328.389	362.317
4.07143	322.359	357.632
4.14286	316.433	353.028
4.21429	310.609	348.503
4.28571	304.882	344.055
4.35714	299.25	339.679
4.42857	293.71	335.375
4.5	288.258	331.14
4.57143	282.892	326.971
4.64286	277.609	322.867
4.71429	272.407	318.826
4.78571	267.284	314.845
4.85714	262.236	310.924
4.92857	257.262	307.059
5	252.359	303.251
5.07143	247.526	299.496
5.14286	242.76	295.794
5.21429	238.061	292.143
5.28571	233.425	288.541
5.35714	228.851	284.988
5.42857	224.338	281.482
5.5	219.884	278.022
5.57143	215.488	274.606
5.64286	211.147	271.234
5.71429	206.861	267.904
5.78571	202.629	264.616
5.85714	198.448	261.368
5.92857	194.318	258.16
6	190.237	254.989
6.07143	186.205	251.857
6.14286	182.22	248.761
6.21429	178.281	245.701
6.28571	174.386	242.675
6.35714	170.536	239.684
6.42857	166.729	236.727
6.5	162.964	233.802

Exact Solution

Interface located at x = 3.557042300821634 cm

x (cm)	Min Case Temp (°C)	Max Case Temp (°C)
3	500	500
3.07143	481.987	485.423
3.14286	464.387	471.182

3.21429	447.184	457.26
3.28571	430.358	443.645
3.35714	413.894	430.322
3.42857	397.777	417.28
3.5	381.992	404.507
3.57143	368.236	393.421
3.64286	361.465	388.163
3.71429	354.825	383.008
3.78571	348.312	377.951
3.85714	341.92	372.988
3.92857	335.646	368.117
4	329.485	363.333
4.07143	323.433	358.634
4.14286	317.486	354.016
4.21429	311.64	349.478
4.28571	305.893	345.016
4.35714	300.241	340.627
4.42857	294.681	336.31
4.5	289.21	332.063
4.57143	283.825	327.881
4.64286	278.524	323.765
4.71429	273.303	319.712
4.78571	268.161	315.719
4.85714	263.095	311.786
4.92857	258.103	307.91
5	253.183	304.09
5.07143	248.333	300.324
5.14286	243.551	296.611
5.21429	238.834	292.949
5.28571	234.182	289.337
5.35714	229.592	285.773
5.42857	225.063	282.257
5.5	220.593	278.786
5.57143	216.181	275.361
5.64286	211.825	271.978
5.71429	207.524	268.639
5.78571	203.276	265.341
5.85714	199.08	262.083
5.92857	194.936	258.865
6	190.841	255.686
6.07143	186.794	252.544
6.14286	182.795	249.438
6.21429	178.841	246.369
6.28571	174.934	243.335
6.35714	171.07	240.335

```
 6.42857            167.249            237.369
 6.5            163.471            234.435
Plot Temperature Distribution? (y/n): y
Plot Convergence Graphs of the Temperature Distribution? (y/n): y

Process finished with exit code 0
```