University of Geneva GSEM

Numerical Methods (S210007) NM – 2022/23

I. Roko

Assignment #2 *The answers should be submitted until Wednesday October 19, 16:00*

Exercise 1

This exercise tests some properties of the normal distribution by using `for` and `while` loops, as well as some R functions.

1. Generate a random vector v of 10'000 elements following a standard normal distribution.
2. Count the number of observations lying beyond 3 standard deviations (i.e. outside the interval[1] $[-3, 3]$), with and without using a `for` loop.
3. Express the result as a percentage of the generated points. Is the percentage near to what you expected ?
4. Find the indices of the elements found beyond 3 standard deviations, with and without using a `for` loop. Compare the results.
5. Display the values of the elements found beyond 3 standard deviations. Display the largest and the smallest observed value (see the R commands `max` and `min`). How many standard deviations are they far from the mean ?
6. By using a `while` loop, count how many simulations (1 "simulation" $==$ 10'000 samples) are needed to obtain a first observation beyond 5 standard deviations.

Exercise 2

This exercise consists of counting the number of times that we have to generate a random number following the standard uniform distribution, such that we obtain a value less than 0.1.

1. By using the command `runif` generate a number `u` and verify if it is less than 0.1.
2. Generate a vector `U` with 50 random standard uniform numbers without calling a `for` loop.
3. Using a loop `for(i in ...)` go through each element of the vector `U`. Introduce a counter that calculates the number of times that `U(i)<0.1`. Outside the loop show the value of this counter.

[1] Consider the interval closed. Note that the result will be the same for an open interval. In fact the probability to obtain exactly 3 or -3 is 0.

4. Considering again the initial problem, introduce in the `for` loop an instruction `break`, executed when the element `U(i)<0.1`. Which is the first element of `U` less than 0.1? Indicate its position in the vector `U` and its value.

5. Let's consider the problem in a different way: by using a `while` loop generate a uniform random variable and repeat the execution as long as its value is greater or equal to 0.1. In order to obtain the number of iterations needed to get a value less than 0.1, introduce a counter `j`, initialized to 1 before starting the loop, and increased by 1 inside the loop. Show the value of `j`.

6. The previous result will (most likely) not be equal to the one with the `for` loop. The reason is that the series of numbers generated by R between the two procedures is different. In order to verify if for the same series of generated numbers the result is the same, go through the vector `U` by using a `while` loop. Use a counter `j` similar to the previous point. Is the result the same as the one at point (4) ?

7. The last `while` loop does not consider the (unlikely) situation where all the values are greater or equal to 0.1. For this purpose introduce a procedure of control that shows a message and stop the execution of the loop if the number of iterations reaches 50. To verify your code replace the threshold 0.1 with 0.00001.