Fourier Series

1. Basic s (I.B.th integration)
2. Types of limits → type 1 $(0, 2\pi)$
   type 2 $(-\pi, \pi)$
   type 3 $(0, 2l)$
   type 4 $(-l, l)$
3. Formula.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

4. half range series
   - H. R. cosine series
   - H. R. sine series
5. Even odd fxn
6. complicated Ques. (CA, Mid term, end term)
   practise

(Easy)

Basics

① $\sin(n\pi) = 0 \quad n = 1, 2, 3, \ldots$

② $\cos(n\pi) \to \cos(2n\pi) = 1$ (even)

$\downarrow$

$(-1)^n \to \cos(2n\pi \pm 1) = -1$ (odd)

③ • $\sin(2n+1)\left(\frac{\pi}{2}\right) = (-1)^n$

• $\cos(2n+1)\left(\frac{\pi}{2}\right) = 0$

④ • $\sin(-\theta) = -\sin\theta$

• $\cos(-\theta) = \cos\theta$

⑤ Imp formulas.

① $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

② $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

③ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

④ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(yaad kar ligiye.)

Integration (simple basics)

1. $\int x^n \, dx = \frac{x^{n+1}}{n+1}$
2. $\int \sin ax \, dx = \frac{-\cos ax}{a}$
3. $\int \cos ax \, dx = \frac{\sin ax}{a}$
4. $\int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$
5. $\int e^{ax} \cos bx = \frac{e^{ax}}{a^2+b^2}(a \cos bx - b \sin bx)$

Integration by parts (12th)

$$\int \text{I} \cdot \text{II} \, dx = \text{I} \int \text{II} - \int \left[\frac{d(\text{I})}{dx} \int \text{II} \, dx \right] dx$$

ILATE

Integration (used in this chapter)

$$\int (u \cdot v) = u v_1 - u' v_2 + u'' v_3 - \ldots\ldots$$

$u' \rightarrow$ diff (1)

$v' \rightarrow$ integration (1)

$u'' \rightarrow$ diff (2)

$v'' \rightarrow$ integration (2)

Integration practise

$$\int u \cdot v = uv_1 - u'v_2 + u''v_3 \ldots$$

Ques. $\int x \sin x$ $= x(-\cos x) - (1)(-\sin x) + 0 \ldots$

$= -x\cos x + \sin x$

Ques. $\int x^2 \sin x \, dx \Rightarrow x^2(-\cos x) - (2x)(-\sin x) + (2)(+\cos x) - 0 \ldots$

$= -x^2\cos x + 2x\sin x + 2\cos x$

Ques. $\int x^2 \cos nx \, dx \Rightarrow x^2\left(\frac{\sin nx}{n}\right) - (2x)\left(-\frac{\cos nx}{n^2}\right) + (2)\left(-\frac{\sin nx}{n^3}\right)$

$= x^2\frac{\sin nx}{n} + 2x\frac{\cos nx}{n^2} - 2\frac{\sin nx}{n^3}$

Ques. practise.

(1) $\int (\pi - x)\cos nx \, dx$

(3) $\int x \sin x \cos nx \, dx$

(2) $\int \left(\frac{\pi - x}{2}\right)^2 \sin nx \, dx$

(4) $\int x^2 \cos\left(\frac{n\pi x}{l}\right) dx.$

Q1. $\int (\pi - x) \cos nx \, dx.$

Soln. $\Rightarrow (\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(-\frac{\cos nx}{n^2}\right)$

$= (\pi - x)\left(\frac{\sin nx}{n}\right) - \frac{\cos nx}{n^2}$

Q2. $\int \left(\frac{\pi - x}{2}\right)^2 \sin nx \, dx$

Soln. $\left(\frac{\pi - x}{2}\right)^2 \left(-\frac{\cos nx}{n}\right) - \left(2\left(\frac{\pi - x}{2}\right)(-1)\left(-\frac{\sin nx}{n^2}\right)\right)$

$+ \left(\frac{1}{2} \frac{\cos nx}{n^3}\right)$

$= -\left(\frac{\pi - x}{2}\right)^2 \frac{\cos nx}{n} - \frac{1}{2}(\pi - x)\frac{\sin nx}{n^2} + \frac{1}{2}\frac{\cos nx}{n^2}$

Q4. $\int x^2 \cos\left(\frac{n\pi}{l}\right)x \, dx.$

Soln. $x^2 \frac{\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)} - (2x)\left[-\frac{\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^2}\right] + 2\left[-\frac{\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^3}\right]$

$\Rightarrow \frac{x^2 \sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)} + 2x\frac{\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^2} - 2\frac{\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^3}$

Q4. $\int x \sin x \cos nx \, dx.$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

Soln. $\int \frac{2}{2} x \sin x \cos nx \, dx$

$= \int \frac{x}{2} [\sin(1+n)x] + \int \frac{x}{2} [\sin(1-n)x]$

$\Rightarrow \frac{1}{2}\left[-x \frac{\cos(1+n)x}{(1+n)} - \frac{(1)(-\sin(1+n)x)}{(1+n)^2}\right] + \frac{1}{2}\left[x\left(-\frac{\cos(1-n)x}{(1-n)}\right) - (1)\left(-\frac{\sin(1-n)x}{(1-n)^2}\right)\right]$

$\Rightarrow \frac{1}{2}\left[-\frac{x\cos(1+n)x}{(1+n)} + \frac{\sin(1+n)x}{(1+n)^2} - \frac{x\cos(1-n)x}{(1-n)} + \frac{\sin(1-n)}{(1-n)^2}\right]$

H.W

Ans.

Q5. $\int x^2 \cos\left(\frac{n\pi}{l}\right)x \sin nx$

Q6. $\int x \sin(n\pi)x \cos(\pi)x$

Even odd fxn

(*) Even function

$$\boxed{f(x) = f(-x)}$$

Eg → x^2, x^4, x^6 ——

→ x^{2n} ;

Eg. (1) $f(x) = x^2$

Soln. put $x \to -x$

$f(-x) = (-x)^2$

$= x^2$

$= f(x)$

⇒ $\boxed{f(-x) = f(x)}$

∴ $f(x)$ is even fxn.

(2) $f(x) = \cos x$

put $x = -x$

$f(-x) = \cos(-x)$

$= \cos(x)$

$= f(x)$

⇒ $\boxed{f(-x) = f(x)}$

∴ $f(x) = \cos x$ is even fxn

(*) odd function

$$\boxed{f(-x) = -f(x)}$$

Eg → $x^3, x^5,$ ——

→ $x^{2n \pm 1}$

Eg. $f(x) = \sin x$.

put $x = -x$

⇒ $f(-x) = \sin(-x)$

$= -\sin x$

$= -f(x)$

⇒ $\boxed{f(-x) = -f(x)}$

∴ $f(x) = \sin x$ is odd fxn.

Note :-

- $f(x) = 0$ is only function which is both even and odd fxn.

- $f(x) = e^x$ }
- $f(x) = \log(x)$ } are neither even nor odd fxn.

End term.

Q. $\cos 2n\pi = ??$

a.) -1 b.) 0 c.) 1 d.) π

① way -> $\cos 2n\pi = 1$

Que. $\sin nx \cdot \cos nx$.

a.) odd fxn b.) even fxn
c.) can'nt det. d.) NOT

Soln.

$f(x) = \sin(nx) \cdot \cos(nx)$

$x \to (-x)$

$f(-x) = \sin(-nx) \cdot \cos(-nx)$

$= -\sin(nx) \cdot \cos(nx)$

$= -f(x)$

$\Rightarrow \boxed{f(-x) = -f(x)}$

$\Rightarrow$ odd. fxn

Ques. which of the following is an odd f/xn.

a.) t^2 ✗

b.) $t^2 - 4t$ ✗

c.) $\sin(2t) + (3t)$ ✓

d.) $t^3 + 6$ ✗

Soln.

all option check.

① t^2. $(t \to -t) \Rightarrow t^2$ ✗ (even)

② $t^2 - 4t \Rightarrow (-t)^2 - 4(-t) = t^2 + 4t$ (✗) (not even/odd)

③ $\sin 2t + 3t$

$f(t) \to$

① $t \to (-t) \Rightarrow \sin(-2t) - 3t$

$f(-t) \Rightarrow -\sin 2t - 3t$

$= -(\sin 2t + 3t)$ $\Rightarrow \boxed{f(-t) = -f(t)}$ ✓

$\Rightarrow -f(t)$

d.) $t^3 + 6$

$-t^3 + 6$

not odd/even. ✗ ✗

types of limits $\rightarrow$ Type ① $(0, 2\pi)$

※ Fourier series $\rightarrow$ $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx\,dx$

$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \sin nx\,dx.$

Ques. find fourier series of $f(x) = x$ in $(0, 2\pi)$

Soln. $a_0 = \frac{1}{2\pi}\int_0^{2\pi} x\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi}$

$= \frac{1}{4\pi}\left[4\pi^2 - 0\right]$

$= \frac{4\pi^2}{4\pi} = \pi$

$a_n = \frac{1}{\pi}\int_0^{2\pi} x \cos nx\,dx$

$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} + (1)\frac{\cos nx}{n^2}\right]_0^{2\pi}$

$= \frac{1}{\pi}\left(\left[(2\pi)\frac{\sin 2n\pi}{n} + \frac{\cos 2n\pi}{n^2}\right] - \left[0 + \frac{1}{n^2}\right]\right)$

$\frac{1}{\pi}\left[0 + \frac{1}{n^2} - 0 - \frac{1}{n^2}\right]$

$\Rightarrow 0$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x \sin nx \, dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left(\left[-2\pi\frac{\cos 2n\pi}{n} + \frac{\sin 2n\pi}{n^2}\right] - \left[0 + \frac{\sin(0)}{n^2}\right]\right)$$

$$= \frac{1}{\pi}\left[\frac{-2\pi}{n}\right] = \frac{-2}{n}$$

∴ final fourier series

$$f(x) = x = \pi + \boxed{\sum_{n=1}^{\infty}\left(\frac{2}{\cancel{\pi n^2}}\right)\cos nx} + \sum_{n=1}^{\infty}\left(\frac{-2}{n}\right)\sin nx$$

0

Ans

$$\boxed{f(x) = x = \pi + \sum_{n=1}^{\infty}\left(\frac{-2}{n}\right)\sin nx}$$

Ans

Ques: find fourier series of $f(x) = x^2$ in $(0, 2\pi)$

Soln.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\therefore a_0 = \frac{1}{2\pi}\int_0^{2\pi} x^2\,dx = \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_0^{2\pi} = \frac{1}{6\pi}\left[8\pi^3\right] = \frac{4}{3}\pi^2$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \cos nx\,dx = \frac{1}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$\Rightarrow \frac{1}{\pi}\left[\left(4\pi^2 \frac{\sin 2n\pi}{n} + 4\pi\frac{\cos 2n\pi}{n^2} - 2\frac{\sin 2n\pi}{n^3}\right) - \left(0 - 0 - \frac{2\sin(0)}{n^3}\right)\right]$$

$$= \frac{1}{\pi}\left[0 + \frac{4\pi}{n^2} - 0\right] = 4/n^2$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \sin nx\,dx$$

$$= \frac{1}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) - (2x)\left(-\frac{\sin nx}{n^2}\right) + (2)\frac{\cos nx}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\left(-\frac{4\pi^2}{n}\cos 2n\pi + 4\pi\frac{\sin 2n\pi}{n^2} + \frac{2\cos 2n\pi}{n^3}\right) - \left(0 - 0 + \frac{2\cos(0)}{n^3}\right)\right]$$

$$= \frac{1}{\pi}\left[-\frac{4\pi^2}{n} + 0 + \frac{2}{n^3} - \frac{2}{n^3}\right] = -4\pi/n$$

$\therefore\ f(x) = x^2 = \frac{4}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4}{n^2}\cos nx + \sum_{n=1}^{\infty} \frac{-4\pi}{n}\sin nx$

Ans.

Q3. $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval of $[0, 2\pi]$

Soln.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{--- (1)}$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 = \frac{1}{8\pi}\int_0^{2\pi}(\pi - x)^2\,dx = \frac{1}{8\pi}\left[\frac{(\pi - x)^3}{-3}\right]_0^{2\pi} = \frac{1}{8\pi}\left[\frac{\pi^3}{3} + \frac{\pi^3}{3}\right]$$

$$= \frac{2\pi^3}{8\pi \cdot 3} = \frac{\pi^2}{12}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 \cos nx\,dx$$

$$= \frac{1}{4\pi}\left[(\pi - x)^2 \frac{\sin nx}{n} - 2(\pi - x)(-1)\left(-\frac{\cos nx}{n^2}\right) + 2(-1)(-1)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[(\pi - x)^2 \frac{\sin nx}{n} - 2(\pi - x)\frac{\cos nx}{n^2} - 2\frac{\sin nx}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\left(0 + \frac{2\pi}{n^2} - 0\right) - \left(0 - \frac{2\pi}{n^2} - 0\right)\right] = \frac{1}{4\pi}\left[\frac{2\pi}{n^2} + \frac{2\pi}{n^2}\right] = \frac{4\pi}{4\pi \cdot n^2}$$

$$= 1/n^2$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \left(\frac{\pi-x}{2}\right)^2 \sin nx \, dx.$$

$$\Rightarrow \frac{1}{4\pi}\int_0^{2\pi} (\pi-x)^2 \sin nx \, dx$$

$$\Rightarrow \frac{1}{4\pi}\left[(\pi-x)^2\left(-\frac{\cos nx}{n}\right) - \left(2(\pi-x)(-1)\right)\left(-\frac{\sin nx}{n^2}\right) + 2(-1)(-1)\left(+\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$\Rightarrow \frac{1}{4\pi}\left[-(\pi-x)^2\frac{\cos nx}{n} - 2(\pi-x)\frac{\sin nx}{n^2} + \frac{2\cos nx}{n^3}\right]_0^{2\pi}$$

$$\Rightarrow \frac{1}{4\pi}\left[\left(\frac{-\pi^2}{n} + \cdot 0 + \frac{2}{n^3}\right) - \left(\frac{-\pi^2}{n} - 0 + \frac{2}{n^3}\right)\right]$$

$$= \frac{1}{4\pi}\left[\frac{-\pi^2}{n} + \frac{2}{n^3} + \frac{\pi^2}{n} - \frac{2}{n^3}\right] = 0$$

Now, put the values of a_0, a_n, b_n in Eq^n (1) we get.

$$\boxed{f(x) = \left(\frac{\pi-x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2}\cos nx}$$

Ans.

Types of limits Type 2 limit $(-\pi$ to $\pi)$

Q. $f(x) = (\pi + x)$ in the interval $(-\pi, \text{to}, \pi)$ =

Sol^n. $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ — ①

$\therefore a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (x+\pi)\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2} + \pi x\right]_{-\pi}^{\pi} = \frac{1}{2\pi}\left[\left(\frac{\pi^2}{2} + \pi^2\right) - \left(\frac{\pi^2}{2} - \pi^2\right)\right]$

$= \frac{1}{2\pi}\left[2\pi^2\right] = \pi$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (\pi + x)\cos nx\,dx$

$= \frac{1}{\pi}\left[(\pi + x)\left(\frac{\sin nx}{n}\right) - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{\pi}$

$= \frac{1}{\pi}\left[\left(0 + \frac{\cos n\pi}{n^2}\right) - \left(0 + \frac{\cos n\pi}{n^2}\right)\right] = \Rightarrow 0$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (\pi + x)\sin nx\,dx = \frac{1}{\pi}\left[(\pi + x)\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{\pi}$

$= \frac{1}{\pi}\left[\left(-2\pi\frac{\cos n\pi}{n} + 0\right) - (0 + 0)\right] = \frac{-2\cos n\pi}{n} = \frac{-2}{n}(-1)^n = \frac{2}{n}(-1)^{n+1}$

put the values of a_0 a_n & b_n in Eq^n (1) to get final fourier series.

$$\therefore \boxed{f(x) = (\pi + x) = \pi + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx}$$

Ans.

Ques. find the fourier series to represents $x - x^2$ from $x = -\pi$ to $x = \pi$

Sol^n.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (x - x^2)\,dx = -\pi^2/3 \quad -(1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\cos nx\,dx = \frac{-4}{n^2}(-1)^n$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\sin nx\,dx = \frac{-2}{n}(-1)^n$$

(D I B Y)

$$\therefore f(x) = x - x^2 = \frac{-\pi^2}{3} - 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx + \sum_{n=1}^{\infty} \frac{-2}{n}(-1)^n \sin nx\,dx$$

Ques. Expand the fourier series for the fxn

$$f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

Soln.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{0} -x\,dx + \frac{1}{2\pi}\int_{0}^{\pi} x\,dx = \frac{-\pi}{2\pi}[x]_{-\pi}^{0} + \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{-\pi^2}{2\pi} + \frac{1}{2\pi}\left(\frac{\pi^2}{2}\right)$$

$$= \frac{-\pi}{2} + \frac{\pi}{4} = \frac{-\pi}{4} \quad -(1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -x \cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x \cos nx\,dx$$

$$= \frac{1}{\pi}(-x)\left[\frac{\sin nx}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[x\frac{\sin nx}{n} - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= (-1)[0+0] + \frac{1}{\pi}\left[\pi\cdot 0 + \frac{\cos n\pi}{n^2}\right] - \left(0 + \frac{1}{n^2}\right)]$$

$$= \frac{1}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right]$$

$$= \boxed{\frac{(-1)^n - 1}{\pi n^2}}$$

$$\frac{1}{\pi}\left(\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} -\pi \sin nx \, dx + \frac{1}{\pi}\int_{0}^{\pi} x \sin nx \, dx$$

$$(-1)\left[-\frac{\cos nx}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$\Rightarrow \left[\frac{1}{n} - \frac{\cos n\pi}{n}\right] + \frac{1}{\pi}\left[-\pi\frac{\cos n\pi}{n} + 0\right] - [0 + 0]$$

$$= \frac{1}{n} - \frac{\cos n\pi}{n} - \frac{1\cancel{\pi}}{\cancel{\pi}}\frac{\cos n\pi}{n} \qquad \frac{1 - 2\cos n\pi}{n} = \frac{1 - 2(-1)^n}{n}$$

$$=$$

$$\boxed{f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2\pi}\cos nx + \sum_{n=1}^{\infty} \frac{1 - 2(-1)^n}{n}\sin nx.}$$

Ques. find fourier series. $f(x) = x + \frac{\pi}{2}$ $-\pi < x < 0$

$\frac{\pi}{2} - x$ $0 < x < \pi$

Soln.

$a_0 = 0$

$b_n = 0$

$a_n = \frac{2(1-(-1)^n)}{\pi n^2}$

H.w.

PYQ(5). Q. $f(x) = x \sin x$ $-\pi \le x \le \pi$ $a_0 = ??$

Soln.

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x \sin x \, dx$

$\frac{1}{2\pi}\left[x(-\cos x) - (1)(-\sin x)\right]_{-\pi}^{\pi}$

$= \frac{1}{2\pi}\left(\left[-\pi \cos \pi + \sin \pi\right] - \left[+\pi \cos(-\pi) + \sin(-\pi)\right]\right)$

$= \frac{1}{2\pi}(\pi + \pi) = \frac{2\pi}{2\pi} = 1$ Ans.

a.) 0

b.) 2π

c.) $2/\pi$

d.) 2

e.) None of the above

PYQ. (3)

$$f(x) = \begin{cases} 1+x & -\pi \le x \le 0 \\ 1-x & 0 \le x \le \pi \end{cases}$$

$a_0 = ??$

Soln.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{0}(1+x)\,dx + \frac{1}{2}\int_{0}^{\pi}(1-x)\,dx$$

$$\Rightarrow \frac{1}{2\pi}\left[x+\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{1}{2}\left[x-\frac{x^2}{2}\right]_{0}^{\pi}$$

$$\Rightarrow \frac{1}{2\pi}\left[(0+0)-\left(-\pi+\frac{\pi^2}{2}\right)\right] + \frac{1}{2}\left[\pi-\frac{\pi^2}{2}\right]$$

$$\Rightarrow \frac{1}{2\pi}\left[\pi-\frac{\pi^2}{2}+\pi-\frac{\pi^2}{2}\right]$$

$$= \frac{1}{2\pi}\left[2\pi-\pi^2\right]$$

$$= 1-\frac{\pi}{2} \quad \text{Ans.}$$

a.) 2
b.) π
c.) $\pi/2$
d.) $2-\pi$
e.) NOT.

Type ③ limit $\underline{(0,2l)}$ $\rightarrow$ c to $c+2l$ | $(0,2l)$

$\rightarrow$ c to $c+2a$ | $(0,a)$ . . .

$$l = \text{Interval} = \frac{\text{final Limit} - \text{initial Limit}}{2}$$

$$\therefore f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{1}{2l}\int_c^{c+2l} f(x)\,dx \quad \rightarrow \int_0^{2l}$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx \quad \int_0^{2l}$$

$$\left\{\begin{array}{l} c \Rightarrow \text{initial Limit} \\ c+2l \Rightarrow \text{final Limit} \end{array}\right\}$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx. \quad \int_0^{2l}$$

$c=0$

Que. find the fourier series expansion of x in $[0, a]$

Solⁿ. $l = \frac{U.L - L.L}{2} = \frac{a-0}{2} = a/2$

$$a_0 = \frac{1}{2l}\int_0^a x\,dx = \frac{1}{2\cdot\left(\frac{a}{2}\right)}\left[\frac{x^2}{2}\right]_0^a = \frac{1}{2a}\left[a^2 - 0\right] = a/2$$

$$a_n = \frac{1}{l}\int_0^a x\cos\left(\frac{n\pi x}{l}\right)dx = \frac{1}{a/2}\int_0^a x\cos\left(\frac{2n\pi x}{a}\right)dx$$

$$= \frac{2}{a}\left[\frac{x\sin\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)} - (1)\frac{\left(-\cos\left(\frac{2n\pi x}{a}\right)\right)}{\left(\frac{2n\pi}{a}\right)^2}\right]_0^a$$

$$= \frac{2}{a}\left(\left[a\cdot 0 + \frac{1}{\left(\frac{2n\pi}{a}\right)^2}\right] - \left[0 + \frac{1}{\left(\frac{2n\pi}{a}\right)^2}\right]\right) = 0$$

$$b_n = \frac{1}{l}\int_0^a x\sin\left(\frac{n\pi x}{l}\right)dx = \frac{2}{a}\left[\frac{x\left(-\cos\left(\frac{2n\pi x}{a}\right)\right)}{\left(\frac{2n\pi}{a}\right)} - (1)\frac{\left(-\sin\left(\frac{2n\pi x}{a}\right)\right)}{\left(\frac{2n\pi}{a}\right)^2}\right]_0^a$$

$$\Rightarrow \frac{2}{a}\left[\left(\frac{-a\cdot a}{2n\pi} - 0\right) - (0 - 0)\right]$$

$$= -a/n\pi$$

$$\therefore\ \boxed{f(x) = \frac{a}{2} + \sum_{n=1}^{\infty} \left(\frac{-a}{2n\pi}\right) \sin\left(\frac{n\pi x}{l}\right)} \quad \text{Ans.}$$

Eg ② $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$

Solⁿ. $l = \frac{2-0}{2} = 1$

$$a_0 = \frac{1}{2l}\int_0^2 f(x)\,dx = \frac{1}{2}\int_0^1 \pi x\,dx + \frac{1}{2}\int_1^2 0\,dx$$

$$= \frac{\pi}{2}\left[\frac{x^2}{2}\right]_0^1 = \pi/4$$

$$a_n = \frac{1}{l}\int_0^1 (\pi x)\cos\left(\frac{n\pi x}{l}\right)dx + \frac{1}{l}\int_1^2 (0)\cos\left(\frac{n\pi x}{l}\right)dx = \int_0^1 (\pi x)\cos(n\pi x)\,dx$$

$$= \pi\left[x\frac{\sin(n\pi x)}{(n\pi)} - (1)\left(-\frac{\cos n\pi x}{(n\pi)^2}\right)\right]_0^1$$

$$= \pi\left(\left[0 + \frac{\cos(n\pi)}{(n\pi)^2}\right] - \left[0 + \frac{1}{(n\pi)^2}\right]\right)$$

$$= \pi\left[\frac{(-1)^n - 1}{(n\pi)^2}\right]$$

$$b_n = \frac{1}{1}\int_0^1 \pi x \sin(n\pi x)\,dx = \left[-\pi x \frac{\cos n\pi x}{n\pi} - \pi \frac{\sin n\pi x}{(n\pi)^2}\right]_0^1$$

$$= \left(-\pi \frac{\cos n\pi}{n\pi} - 0\right) - (0 - 0)$$

$$= \frac{(-1)(-1)^n}{\pi}$$

$$\therefore \boxed{f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \pi\left[\frac{(-1)^n - 1}{(n\pi)^2}\right]\cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} \pi\,\frac{(-1)(-1)^n}{\pi}\sin\left(\frac{n\pi x}{l}\right)}$$

$f(x) = x^2 \quad [0, a]$

An.

Type (4) $[-l \text{ to } l]$

Ques: $1-x^2$ in $(-1 \text{ to } 1)$

Sol^n. $f(x) = 1-x^2$ $l = \frac{F-I}{2} = \frac{1-(-1)}{2} = 1$

$a_0 = ??$ $\rightarrow$ $2/3$

$a_n = ??$ $\rightarrow$ $\frac{-4(-1)^n}{n^2\pi^2}$

$b_n = ??$ $\rightarrow$ 0

$\therefore$ $$\boxed{f(x) = 1-x^2 = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{-4(-1)^n}{n^2\pi^2} \cos\left(\frac{n\pi}{l}\right)x}$$

An.

H.W.

(1) (2) (3) (4)

H.

PYQ ① $f(t) = \begin{cases} t^2 & 0 \le t \le 2 \\ -t+6 & 2 \le t \le 6 \end{cases}$ $a_0 = ??$ a.) 8/9 b.) 16/9 c.) 24/9 d.) 32/9

Solⁿ. $l = \frac{6-0}{2} = 3$

$a_0 = \frac{1}{2l} \int_0^6 f(t)\, dt$

$= \frac{1}{6}\left[\int_0^2 t^2 + \int_2^6 -t+6\right]$

$= \frac{1}{6}\left[\frac{t^3}{3}\right]_0^2 + \frac{1}{6}\left[-\frac{t^2}{2} + 6t\right]_2^6$

$= \frac{1}{3\times6}[8] + \frac{1}{6}\left[\left(\frac{-36}{2} + 36\right) - \left(-\frac{4}{2} + 12\right)\right]$

$= \frac{4}{9} + \frac{1}{6}\left[\frac{36}{2} + \frac{4}{2} - 12\right]$

$= \frac{4}{9} + \frac{1}{6}[18 + 2 - 12]$

$= \frac{4}{9} + \frac{1}{6} \times 8$

$= \frac{4}{9} + \frac{4}{3}$

$= \frac{4+12}{9} = \frac{16}{9}$ (b) Ans.

PYQ ② $f(t) = \begin{cases} 1 & -1 \le t < 0 \\ -2 & 0 \le t < 1 \end{cases}$

$a_0 = ??$ $l = \frac{1-(-1)}{2} = 1$

Solⁿ. $a_0 = \frac{1}{2l} \int_{-1}^{1} f(t)\, dt$

$= \frac{1}{2} \int_{-1}^{0} (1)\, dt + \frac{1}{2} \int_0^1 (-2)\, dt$

$= \frac{1}{2}[x]_{-1}^{0} + \frac{1}{2}(-2)[x]_0^1$

$= \frac{1}{2}[0-(-1)] + (-1)(1-0)$

$= \frac{1}{2} - 1 = \boxed{-1/2}$ Ans.

a.) 0 c.) -1

b.) 1 d.) -2

e.) Not

Half Range Series

1. Half range cosine series
2. Half range sine series

(0, 2l)

for Limit (−l to l) :-

$\boxed{\text{value of } l = \text{Upper Limit} - \text{Lower Limit}}$

$$a_0 = \frac{1}{l}\int_0^l f(x)\,dx$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx.$$

Note

1. In Half range sine series

$a_0 = 0$

$a_n = 0$

$b_n \neq 0$

⇓

only find b_n

2. In Half range cosine series

$a_0 \neq 0$??

$a_n \neq 0$??

$b_n = 0$

only find a_0 & a_n

Ques. Express $f(x) = x$ as a half range sine series in $0 < x < 2$

Soln. ∴ Half sine series ⇒ $\boxed{a_0 = a_n = 0}$ $b_n = ??$

$\boxed{l = 2 - 0 = 2}$

$$b_n = \frac{2}{l}\int_0^l x \sin\left(\frac{n\pi x}{l}\right) dx = \frac{2}{2}\int_0^2 x \sin\left(\frac{n\pi x}{2}\right) dx$$

$$= \left[\frac{x\left(-\cos\left(\frac{n\pi x}{2}\right)\right)}{\left(\frac{n\pi}{2}\right)} - (1)\frac{\left(-\sin\left(\frac{n\pi x}{2}\right)\right)}{\left(\frac{n\pi}{2}\right)^2} \right]_0^2$$

$$= \left[-\frac{2\cos(n\pi)}{(n\pi)} \cdot 2 + 0 \right] - [0 - 0]$$

$$\boxed{b_n = \frac{-4}{n\pi}(-1)^n}$$

$b_n =$

$\boxed{a_0 = a_n = 0}$

thus.

$$\boxed{f(x) = x = \frac{-4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi x}{2}\right)}$$

Ans.

Ques: Express $f(x) = x$ as a half range cosine series in $0 < x < 2$

Sol^n: half range cosine series $\Rightarrow$ $\boxed{b_n = 0}$ $a_0 = ??$ & $a_n = ??$

$\boxed{l = 2 - 0 = 2}$

$\therefore a_0 = \frac{1}{l}\int_0^l f(x)\,dx = \frac{1}{2}\int_0^2 x\,dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^2 = \frac{1}{2} \times \frac{4}{2} = \boxed{1}$

$a_n = \frac{2}{l}\int_0^2 x \cos\left(\frac{n\pi}{l}\right)x\,dx = \frac{\cancel{2}}{\cancel{2}}\int_0^2 x \cos\left(\frac{n\pi}{2}\right)x\,dx$

$= \left[\frac{x \sin\left(\frac{n\pi}{2}\right)x}{\left(\frac{n\pi}{2}\right)} - \frac{(1)\left(-\cos\left(\frac{n\pi}{2}\right)x\right)}{\left(\frac{n\pi}{2}\right)^2}\right]_0^2$

$= \left[0 + \frac{\cos n\pi}{(n\pi)^2}\cdot 4\right] - \left[0 + \frac{4}{(n\pi)^2}\right]$

$= \frac{4(-1)^n}{(n\pi)^2} - \frac{4}{(n\pi)^2} = \frac{4\left((-1)^n - 1\right)}{(n\pi)^2}$

a_n

$\boxed{b_n = 0}$

$\therefore f(x) = x$

$= \boxed{1 + \frac{4}{(\pi)^2}\sum_{n=1}^{\infty} \frac{1}{n^2}\left((-1)^n - 1\right)\cos\left(\frac{n\pi x}{l}\right)}$

Ans.

$a_0 +$

a) Find the Fourier series expansion for $f(x) = x$ in the interval $[-\Pi \ \ \Pi]$.

Find the Fourier series expansion for $f(x) = \frac{(\Pi - x)^2}{2}$ in the interval $[0 \quad 2\Pi]$.

C

Find the Fourier series expansion for $f(x) = x^2$ in the interval $[0 \;\; a]$.

Find the Fourier series expansion for $f(x) = 1 - x^2$ in the interval $[0 \;\; a]$.

Q1. CA PYQ $f(x) = x \quad (-\pi, \pi)$

PYQ

Sol^n. $f(x) = x = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$\therefore a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{\pi} = \frac{1}{2\pi}\left[\frac{\pi^2}{2} - \frac{\pi^2}{2}\right] = 0$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cos nx\,dx = \frac{1}{\pi}\left[x\frac{\sin nx}{n} - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{\pi}$

$= \frac{1}{\pi}\left[\left(\pi \cdot 0 + \frac{\cos n\pi}{n^2}\right) - \left(-\pi \cdot 0 + \frac{\cos n\pi}{n^2}\right)\right]$

$= \frac{1}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos n\pi}{n^2}\right] = 0$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx\,dx$

$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left[\left(-\pi\frac{\cos n\pi}{n} + 0\right) - \left(\pi\frac{\cos n\pi}{n} + 0\right)\right]$

$= \frac{1}{\pi}\left[-2\pi\frac{\cos n\pi}{n}\right] = \frac{-2}{n}(-1)^n$

$$\boxed{f(x) \Rightarrow x = \sum_{n=1}^{\infty} \frac{-2}{n}(-1)^n \sin nx}$$

Ans.

Que② $f(x) = \frac{(\pi - x)^2}{2}$ in the interval $[0, 2\pi]$.

Soln.

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{(\pi - x)^2}{2}\,dx = \frac{1}{4\pi}\left[\frac{[\pi - x]^3}{-3}\right]_0^{2\pi} = \frac{-1}{12\pi}\left[-\pi^3 - \pi^3\right] = \frac{2\pi^3}{12\pi}$$

$$= \pi^2/6$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{(\pi - x)^2}{2}\cos nx\,dx = \frac{1}{2\pi}\left[(\pi - x)^2\left(\frac{\sin nx}{n}\right) - 2(\pi - x)(-1)\left(-\frac{\cos nx}{n^2}\right) + 2(-1)(-1)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$\Rightarrow \frac{1}{2\pi}\left[\left(0 - 2\cdot(-\pi)\cdot\frac{1}{n^2} + 0\right) - \left(0 - \frac{2\pi}{n^2} + 0\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{2\pi}{n^2} + \frac{2\pi}{n^2}\right] = \frac{2}{n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{(\pi - x)^2}{2}\sin nx\,dx = \frac{1}{2\pi}\left[(\pi - x)^2\left(-\frac{\cos nx}{n}\right) - (2)(\pi - x)(-1)\left(-\frac{\sin nx}{n^2}\right) + 2(-1)(-1)\left(-\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$\Rightarrow \frac{1}{2\pi}\left(\left[\frac{-\pi^2}{n} - 0 - \frac{2}{n^3}\right] - \left[\frac{-\pi^2}{n} + 0 - \frac{2}{n^3}\right]\right)$$

$$= \frac{1}{2\pi}\left[\frac{-\pi^2}{n} - \frac{2}{n^3} + \frac{\pi^2}{n} + \frac{2}{n^3}\right] = 0$$

$$\therefore \boxed{f(x) = \frac{(\pi - x)^2}{2} = \frac{\pi^2}{6} + \sum_{n=1}^{\infty}\frac{2}{n^2}\cos nx}$$

Ans.

$$f(x) = \frac{\cdots}{4} + \sum_{n=1} \left[\frac{(-1)^n - 1}{(n\pi)^2}\right] \cos\left(\frac{n\pi x}{l}\right) \quad \cdots \sum_{n=1} \quad \pi \quad \cdots \quad l$$

CA

Py 08.

$f(x) = x^2 \qquad [0, a]$

Soln.

$$l = \frac{a-0}{2} = a/2$$

$$a_0 = \frac{1}{2l}\int_0^a f(x)\,dx = \frac{1}{2\left(\frac{a}{2}\right)}\int_0^a x^2\,dx = \frac{1}{3a}\left[x^3\right]_0^a = \frac{1}{3a}a^3 = \boxed{a^2/3} \quad \underline{\underline{a_0}}$$

$$a_n = \frac{1}{l}\int_0^a x^2 \cos\left(\frac{n\pi x}{l}\right)dx = \frac{2}{a}\int_0^a x^2 \cos\left(\frac{2n\pi}{a}\right)x\,dx$$

$$= \frac{2}{a}\left[x^2\frac{\sin\left(\frac{2n\pi}{a}\right)x}{\left(\frac{2n\pi}{a}\right)} - (2x)\frac{\left(-\cos\left(\frac{2n\pi}{a}\right)x\right)}{\left(\frac{2n\pi}{a}\right)^2} + (2)\frac{\left(-\sin\left(\frac{2n\pi}{a}\right)x\right)}{\left(\frac{2n\pi}{a}\right)^3}\right]_0^a$$

$$= \frac{2}{a}\left(\left[0 + \frac{2a\cdot a^2}{(2n\pi)^2} + 0\right] - \left[0 - 0 - 0\right]\right) = \frac{2}{a}\cdot\frac{2a^3}{4n^2\pi^2} = \boxed{a^2/n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_0^a x^2 \sin\left(\frac{2n\pi}{a}\right) dx$$

$$= \frac{1}{(a/2)}\int_0^a x^2 \sin\left(\frac{2n\pi}{a}\right) dx$$

$$= \frac{2}{a}\left[\frac{x^2\left(-\cos\left(\frac{2n\pi}{a}\right)x\right)}{\left(\frac{2n\pi}{a}\right)} - \frac{(2x)\left(-\sin\left(\frac{2n\pi}{a}\right)x\right)}{\left(\frac{2n\pi}{a}\right)^2} + \frac{2\left(-\cos\left(\frac{2n\pi}{a}\right)x\right)}{\left(\frac{2n\pi}{a}\right)^3}\right]_0^a$$

$$= \frac{2}{a}\left(\left[\frac{-a^2 \cdot a}{2n\pi} + \cancel{0} - \frac{2}{(2n\pi)^3} \cdot a^3\right] - \left[0 + 0 - \frac{2}{(2n\pi)^3} \cdot a^3\right]\right)$$

$$= \frac{2}{a}\left[\frac{-a^3}{2n\pi} - \frac{2a^3}{(2n\pi)^3} + \frac{2a^3}{(2n\pi)^3}\right] = \frac{2}{a}\left[\frac{-a^3}{2n\pi}\right] = \boxed{\frac{-a^2}{n\pi}}$$

$$\therefore \boxed{f(x) = x^2 = \frac{a^2}{3} + \sum_{n=1}^{\infty}\left(\frac{a^2}{n^2\pi^2}\right)\cos\left(\frac{n\pi}{l}\right)x + \sum_{n=1}^{\infty}\left(\frac{-a^2}{n\pi}\right)\sin\left(\frac{n\pi}{l}\right)x}$$

Ans.

Ans.

CA ⊛ PYQ.

$f(x) = 1 - x^2$ in $[0, a]$

$\boxed{l = a/2}$

Soln.

$$a_0 = \frac{1}{2l}\int_0^a f(x)\,dx = \frac{1}{2\cdot\left(\frac{a}{2}\right)}\int_0^a (1-x^2)\,dx = \frac{1}{a}\left[x - \frac{x^3}{3}\right]_0^a = \frac{1}{a}\left[a - \frac{a^3}{3}\right]$$

$$= \boxed{1 - a^2/3}$$

$$a_n = \frac{1}{l}\int_0^a (1-x^2)\cos\left(\frac{n\pi}{l}\right)x\,dx = \frac{2}{a}\left[\frac{(1-x^2)\sin\left(\frac{2n\pi}{a}\right)}{\left(\frac{2n\pi}{a}\right)} - \frac{(-2x)\left(-\cos\frac{2n\pi}{a}\right)}{\left(\frac{2n\pi}{a}\right)^2} + \frac{(-2)\left(-\sin\frac{2n\pi}{a}\right)}{\left(\frac{2n\pi}{a}\right)^3}\right]_0^a$$

$$= \frac{2}{a}\left(\left[0 - \frac{2\cdot a^3}{(2n\pi)^2} + 0\right] - \left[0 - 0 + 0\right]\right) = \frac{-2\times 2}{a}\cdot\frac{a^3}{4n^2\pi^2} = \boxed{\frac{-a^2}{n^2\pi^2}}$$

a_n

$$b_n = \frac{1}{l}\int_0^a (1-x^2)\sin\left(\frac{2n\pi}{a}\right)x\,dx$$

$$= \frac{2}{a}\left[\frac{(1-x^2)\left(-\cos\left(\frac{2n\pi}{a}\right)x\right)}{\left(\frac{2n\pi}{a}\right)} - \frac{(-2x)\left(-\sin\left(\frac{2n\pi}{a}\right)x\right)}{\left(\frac{2n\pi}{a}\right)^2} + \frac{(-2)\left(\cos\left(\frac{2n\pi}{a}\right)x\right)}{\left(\frac{2n\pi}{a}\right)^3}\right]_0^a$$

$$= \frac{2}{a}\left(\left[\frac{-(1-a^2)\cdot a}{(2n\pi)} + 0 - \frac{2\cdot a^3}{(2n\pi)^3}\right] - \left[\frac{-1\cdot a}{2n\pi} + 0 \; \frac{-2\cdot a^3}{(2n\pi)^3}\right]\right)$$

$$= \frac{2}{a}\left[\frac{-a}{2n\pi} + \frac{a^3}{2n\pi} - \frac{2a^3}{(2n\pi)^3} + \frac{a}{2n\pi} + \frac{2a^3}{(2n\pi)^3}\right] = \frac{2}{a}\cdot\frac{a^3}{2n\pi} = \boxed{\frac{a^2}{n\pi}}$$

$$\therefore f(x) = 1 - x^2 = 1 - \frac{a^2}{3} + \sum_{n=1}^{\infty} \left(\frac{-a}{2n^2\pi^2}\right) \cos\left(\frac{n\pi}{l}\right)x + \sum_{n=1}^{\infty} \left(\frac{a^2}{n\pi}\right) \sin\left(\frac{n\pi}{l}\right)x$$

(D) 0 to $\frac{\pi}{4}$ CO5, L1__

Q 46 For $f(x) = x$ in $(0, 2\pi)$, value of a_0 is

(A) π (B) 4π (C) $4\pi^2$ (D) 2π CO6_, L3___

Q 47 For $f(x) = 1 + |x|, -3 < x < 3$, $b_n =$

(A) $\frac{2(1-(-1)^n)}{n\pi}$ (B) $\frac{2(-1)^n}{n\pi}$ (C) $\frac{2}{n\pi}$ (D) 0 CO6_, L2___

Q 48 For $f(x) = \begin{cases} 0 & -1 < x < 0 \\ -2x, & 0 \le x < 1 \end{cases}$, $b_2 =$

(A) 0 (B) $\frac{2(-1)^n}{n\pi}$ (C) $\frac{1}{\pi}$ (D) $\frac{-1}{\pi}$ CO_6, L3___

Q 49 For $f(x) = 2, 0 \le x \le 5$, a_0 in Fourier cosine series is

(A) 1 (B) 4 (C) 2 (D) 5 CO6, L1___

Q 50 Let $\left[-\frac{1}{2} + \sum_{n=1}^{\infty} 2\left(\frac{1-\cos n\pi}{n^2\pi^2}\right)\cos n\pi x + \left(\frac{\cos n\pi}{n\pi}\right)\sin n\pi x\right]$ be the Fourier series representation of $f(x)$ in $(-1, 1)$, then value of a_0 is

(A) $-\frac{1}{2}$ (B) -1 (C) $-\frac{1}{4}$ (D) Cannot be determined CO6, L4___

Reg No.:

Q 51 Let $\left[\frac{1}{4}+\sum_{n=1}^{\infty}\left(\frac{1-\cos n\pi}{n^2\pi^2}\right)\cos n\pi x+\left(\frac{-1}{n\pi}\right)\sin n\pi x\right]$ be the Fourier series representation of $f(x)$ in $(-1, 1)$, then value of a_3 is

(A) $\frac{2}{3\pi^2}$ (B) 0 (C) $\frac{2}{9\pi^2}$ (D) cannot be determined CO6, L4

Q 52 For $f(x)=\begin{cases}0, & 0<x<l\\ 6, & l<x<2l\end{cases}$ value the Fourier coefficient C_1 in complex form of Fourier series is

(A) 0 (B) $-\frac{12}{i\pi}$ (C) $-\frac{6}{i\pi}$ (D) $-\frac{18}{i\pi}$ CO 6, L1

Q 53 Let $f(x)$ be a 2π periodic function, defined as $f(x)=\begin{cases}-1, & -\pi<x<0\\ 1, & 0<x<\pi\end{cases}$, having Fourier coefficients as $a_0=a_n=0, b_n=\frac{2}{n\pi}(1-\cos n\pi)$, then using Parseval's identity, the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ converges to

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{6}$ (D) $\frac{\pi^2}{8}$ CO 6, L3

Q 54 If $f(x)=\sin x,\ 0\le x\le\pi$, then coefficient of $\cos 4x$ in Fourier sine series of $f(x)$ is

(A) $-\frac{4}{35\pi}$ (B) $-\frac{4}{15\pi}$ (C) 0 (D) $-\frac{4}{3\pi}$ CO6, L1

35π 15π 3π

Q 55 Which of the following is an even function in the given interval?

(A) $f(x) = x^3 - x, \quad -\pi \le x \le \pi$ (B) $f(x) = \begin{cases} 1, & -\pi \le x < 0 \\ 2, & 0 \le x \le \pi \end{cases}$

(C) $f(x) = \begin{cases} 2 - x, & -\pi \le x \le 0 \\ 2 + x, & 0 \le x \le \pi \end{cases}$ (D) $f(x) = e^{2x}, \quad -\pi \le x \le \pi$ CO6, L4__

Q 56 For $f(x) = \cosh\left(\frac{x}{2}\right), 0 \le x \le 2, a_0$ in Fourier cosine series =

(A) $\frac{e^2 - 1}{e}$ (B) $\frac{e^2 - 2e + 1}{e}$ (C) $\frac{1 - e^2 - e}{e}$ (D) $-\left(\frac{e^2 + 2e + 1}{e}\right)$ CO6, L3___

Q 57 For $f(x) = x - x^3$ in $(-\pi, \pi)$, value of a_5 is

(A) $-\frac{2}{25}$ (B) $\frac{2}{25\pi^2}$ (C) $\frac{4}{125}$ (D) 0 CO6, L1___

Q 58 If $f(x) = \sin x, \ 0 \le x \le \pi$, then coefficient of $\cos 4x$ in Fourier sine series of $f(x)$ is

(A) $-\frac{4}{35\pi}$ (B) $-\frac{4}{15\pi}$ (C) 0 (D) $-\frac{4}{3\pi}$ CO6. L1___

Reg No:______

Q 59 For $f(x) = \pi + x$ in $(0, 2\pi)$, Constant term in Fourier series of $f(x)$ is

(A) π (B) 4π (C) $4\pi^2$ (D) 2π CO6, L3___

Q 60 Let $\left[\frac{1}{4} + \frac{1}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{1-\cos n\pi}{n^2}\right)\cos n\pi x - \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)\sin n\pi x\right]$ be the Fourier series representation of $f(x)$ in $(-1, 1)$, then value of b_4 is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4\pi}$ (C) 0 (d) Cannot be determined CO6, L4___

END TERM (1)

Q 46 $f(x) = x$ $(0, 2\pi)$ $a_0 = ??$ a.) π b.) 4π c.) $4\pi^2$ d.) 2π

Sol^n $a_0 = \frac{1}{2\pi}\int_0^{2\pi} x\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} = \frac{1}{4\pi}(4\pi^2) = \underline{\pi}$

47 $f(x) = 1 + |x|$ $-3 < x < 3$ $b_n = ??$ a.) $\frac{2(1-(-1)^n)}{n\pi}$ B.) $\frac{2(-1)^n}{n\pi}$ c.) $\frac{2}{n\pi}$ d.) 0

Sol' $f(x) = \begin{cases} 1+x & 0 < x < 3 \\ 1-x & -3 < x < 0 \end{cases}$

$l = \frac{3-(-3)}{2} = 3$.

$b_n = \frac{1}{l}\int_{-3}^{3} f(x)\sin\frac{n\pi x}{l}\,dx = \frac{1}{3}\int_{-3}^{0}(1-x)\sin\left(\frac{n\pi x}{3}\right) + \int_0^3 (1+x)\sin\left(\frac{n\pi x}{3}\right)dx$

$\Rightarrow \frac{1}{3}\left[\frac{(1-x)\left(-\cos\left(\frac{n\pi x}{3}\right)\right)}{\left(\frac{n\pi}{3}\right)} - \frac{(-1)\left(-\sin\left(\frac{n\pi x}{3}\right)\right)}{\left(\frac{n\pi}{3}\right)^2}\right]_{-3}^{0} + \frac{1}{3}\left[\frac{(1+x)\left(-\cos\left(\frac{n\pi x}{3}\right)\right)}{\left(\frac{n\pi}{3}\right)} - \frac{(1)\left(-\sin\frac{n\pi x}{3}\right)}{\left(\frac{n\pi}{3}\right)^2}\right]_0^3$

$= \frac{1}{3}\left[\left(\frac{-3}{n\pi}\right) - \left(\frac{-4(-1)^n}{\frac{n\pi}{3}}\right)\right] + \frac{1}{3}\left[\frac{-4(-1)^n}{\left(\frac{n\pi}{3}\right)} + \frac{3}{n\pi}\right] = 0 + 0$

$= 0$

Q48. $f(x) = \begin{cases} 0 & -1 < x < 0 \\ -2x & 0 \le x < 1 \end{cases}$ $b_2 = ??$ A.) 0 B.) $\frac{2(-1)^n}{n\pi}$ C.) $\frac{1}{\pi}$ d.) $\frac{-1}{\pi}$

Sol^n. $\boxed{l = 1}$

$b_n = \frac{1}{l}\int_{-1}^{0} 0 \cdot \sin\frac{n\pi x}{l}\,dx + \frac{1}{l}\int_0^1 -2x \cdot \sin\frac{n\pi x}{l}\,dx.$

$\downarrow$

$0 + -2\left[x\left(-\frac{\cos n\pi x}{n\pi}\right) - (1)\left(-\frac{\sin n\pi x}{(n\pi)^2}\right)\right]_0^1$

$= -2\left[-\frac{(-1)^n}{n\pi}\right] = \frac{2(-1)^n}{n\pi}$

$b_2 \rightarrow \frac{2(-1)^2}{2\pi} = 1/\pi$

Q49. $f(x) = 2$ $0 \le x \le 5$ $a_0 = ??$ cosine series. a.) 1 B.) 4 c.) 2 d.) 5

Sol^n. $l = 5 - 0 = 5$

$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$= \frac{2}{5}\int_0^5 2\,dx = \frac{4}{5}(x)_0^5 = \frac{4}{5}(5-0) = 4$

Q50. $\frac{-1}{2} + \sum_{n=1}^{\infty} 2\left(\frac{1-\cos n\pi}{n^2\pi^2}\right)\cos n\pi x + \frac{\cos n\pi}{n\pi^2}\sin n\pi x$ $f(x)\ (-1, 1)$ $a_0 = ??$ a.) $-1/2$ b.) -1 c.) $\frac{-1}{4}$ d.) can't det.

a_n b_n

$a_0 = ??$

Sol^n. $\boxed{a_0 = \frac{-1}{2}}$

Q51. $\frac{1}{4} + \sum_{n=1}^{\infty} \left(\frac{1-\cos n\pi}{n^2\pi^2} \right) \cos n\pi x + \left(\frac{-1}{n\pi} \right) \sin n\pi x$

$a_3 = ??$

a.) $\frac{2}{3\pi^2}$ b.) 0 c.) $\frac{2}{9\pi^2}$ d.) cannot det.

Soln. $a_n = \frac{1-\cos n\pi}{n^2\pi^2}$ $\boxed{n=3}$

$a_3 = \frac{1-\cos 3\pi}{9\pi^2} = \frac{1-(-1)}{9\pi^2} = \frac{2}{9\pi^2}$

52 → complex form of F. series

53 → parseval's identity.

(x) (✓)

Q54. $f(x) = \sin x$ $0 \le x \le \pi$ coeff. of $\cos 4x = ??$

fourier sine series.

a.) $\frac{-1}{35\pi}$ b.) $\frac{-4}{15\pi}$ c.) 0 d.) $\frac{-4}{3\pi}$

Soln. In fourier sine series → only sin term is there

NO cos term

$\therefore$ coeff. of $\cos 4x = 0$.

55. even fxn.

a) $x^3 - x$ (odd) (e/o)

b) $f(x) = \begin{cases} 1 & -\pi \le x < 0 \\ 2 & 0 \le x \le \pi \end{cases}$ (Both)

c) $f(x) = \begin{cases} 2-x & -\pi \le x \le 0 \\ 2+x & 0 \le x \le \pi \end{cases}$

$\boxed{f(-x) = f(x)}$

d) $f(x) = e^{2x}$ $-\pi \le x \le \pi$ → (Neither o/e)

(56) $f(x) = \cosh\left(\frac{x}{2}\right)$ $\quad 0 \le x \le 2$ $\quad a_0 = ??$ cosine series.

a.) $\frac{e^2-1}{e}$ (b) $\frac{e^2-2e+1}{e}$ (c) $\frac{1-e^2-e}{e}$ (d.) $-\left(\frac{e^2+2e+1}{e}\right)$

Soln.

$\cosh(x) = \frac{e^x + e^{-x}}{2}$

$\Rightarrow \cosh\left(\frac{x}{2}\right) = \frac{e^{x/2} + e^{-x/2}}{2}$ $\quad l = 2-0$

$\therefore a_0 = \frac{1}{l}\int_0^2 \frac{e^{x/2}+e^{-x/2}}{2} = \frac{1}{2}\int_0^2 \frac{e^{x/2}+e^{-x/2}}{2} = \frac{1}{4}\left[2 e^{x/2} - 2e^{-x/2}\right]_0^2$

$= \frac{1}{2}\left[(e - e^{-1}) - (0-0)\right]$

$= \frac{1}{2}\left(\frac{e^2-1}{e}\right) = \frac{e^2-1}{2e}$

$\boxed{\text{(e) } \frac{e^2-1}{2e}}$

(option not matched.)

(57) $f(x) = x - x^3$ $\quad (-\pi, \pi)$ $\quad a_5$

Soln. for type (2) & (4) i.e $(-\pi, \pi)$ & $(-l, l)$

if $f(x)$ is odd fxn $\rightarrow$ $\boxed{a_n = 0}$

~~a)~~ $\frac{-2}{25}$ ~~b.)~~ $\frac{2}{25\pi^2}$ ~~c)~~ $\frac{4}{125}$ d.) 0

(58) $f(x) = \sin x$ $\quad 0 \le x \le \pi$ $\quad$ coeff. of $\cos 4x$ sine series $f(x)$

a.) $\frac{-4}{35\pi}$ b.) $\frac{-4}{15\pi}$ c.) 0 d.) $\frac{-4}{3\pi}$

Soln.

(same Q.54).

Q 59. $f(x) = \pi + x \quad (0, 2\pi)$ $\quad a_0 = ??$ $f(x)$

constant ↑ term

a.) π b.) 4π

c.) $4\pi^2$ d.) 2π

Soln

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi}(\pi + x)\,dx$$

$$= \frac{1}{2\pi}\left[\pi x + \frac{x^2}{2}\right]_0^{2\pi}$$

$$\Rightarrow \frac{1}{2\pi}\left[\pi(2\pi - 0) + \frac{1}{2}(4\pi^2 - 0)\right] = \frac{1}{2\pi}\left[2\pi^2 + 2\pi^2\right]$$

$$\Rightarrow \frac{4\pi^2}{2\pi} = 2\pi$$

Q 60. $\frac{1}{4} + \frac{1}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{1 - \cos n\pi}{n^2}\right)\cos n\pi x - \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)\sin n\pi x$

$b_4 = ??$

a.) 1/4 b.) $\frac{-1}{4\pi}$

c.) 0 d. —

Soln.

$$b_n = \frac{-1}{\pi}\cdot\frac{1}{n}$$

$$b_4 \Rightarrow (n = 4) \Rightarrow \frac{-1}{4\pi}$$

Part 1 ✓

2 15 Que

Q1) Which of the following condition is necessary for Fourier series expansion of $f(x)$ in $(c, c+2l)$.
(a) $f(x)$ should be continuous in $(c, c+2l)$
(b) $f(x)$ should be periodic
(c) $f(x)$ should be even function
(d) $f(x)$ should be odd function.

CO3, L3

Q2) Given the periodic function $f(t) = \begin{cases} 1 \; for \; -1 \le t < 0 \\ -2 \; for \; 0 \le t < 1 \end{cases}$

The coefficient a_0 of the continuous Fourier series associated with the given function $f(t)$ can be computed as

(a) 0 (b) 1 (c) -1 (d) -2

CO3, L3

Q3) Given the periodic function $f(x) = \begin{cases} 1 + x \; for \; -\pi \le x \le 0 \\ 1 - x \; for \; 0 \le x \le \pi \end{cases}$

The coefficient a_0 of the continuous Fourier series associated with the given function $f(x)$ can be computed as

(a) 2 (b) π (c) $\frac{\pi}{2}$ (d) $2 - \pi$

CO3, L3

Q4) The value of $\cos 2n\pi$ is
(a) -1 (b) 0 (c) 1 (d) π

CO3, L3

Q5) Given the periodic function $f(x) = x \sin x, -\pi \le x \le \pi$ with period 2π The coefficient a_0 of the continuous Fourier series associated with the given function $f(x)$ can be computed as

(a) 0 (b) 2π (c) $\frac{2}{\pi}$ (d) 2

CO3, L3

Q6) The half range Fourier sine series of $f(x) = 1$ in $(0, \pi)$ is
(a) 0 (b) $\frac{4}{\pi}(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots)$
(c) $\frac{4}{\pi}(\sin x - \frac{\sin 3x}{3} + \frac{\sin 5x}{5} - \cdots)$
(d) $\frac{4}{\pi}(\sin 2x + \frac{\sin 4x}{2} + \frac{\sin 6x}{3} + \cdots)$

CO3, L3

Q7) The function sin nx cosnx is.
(a) Odd fuction (b) even function (c) cannot determined (d) none of these

CO3, L3

Q8) Given the periodic function $f(t) = \begin{cases} t^2 \; for \; 0 \le t \le 2 \\ -t+6 \; for \; 2 \le t \le 6 \end{cases}$

The coefficient a_0 of the continuous Fourier series associated with the given function $f(t)$ can be computed as

(a) $\frac{8}{9}$ (b) $\frac{16}{9}$ (c) $\frac{24}{9}$ (d) $\frac{32}{9}$

Q9) The period of the $f(x) = \cos 2x$ is

(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 4π

Q10) Which of the following is an "odd" function of t?

(a) t^2 (b) $t^2 - 4t$ (c) $\sin 2t + 3t$ (d) $t^3 + 6$

Q36) Which of the following is an "even" function of t?

(a) t^2 (b) $t^2 - 4t$ (c) $\sin 2t + 3t$ (d) $t^3 + 6$

Q37) Given the periodic function $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. then the value of the fourier coefficient b_n can be computed as

(a) $\frac{(-1)^n}{n\pi}$ (b) $\frac{1}{n\pi}$ (c) 0 (d) none of these

Q38) In the Fourier series of function $f(x) = \sin x$, $0 < x < 2\pi$, the value of the Fourier. coefficient b_n is

(a) $b_n = 0 \, \forall \, n$ (b) $b_n = \frac{(-1)^n}{n\pi}$ (c) $b_n = 0, n \neq 1 \; and \; b_1 = 1$ (d) none of these

Q39) For Fourier series expansion of periodic function $f(x)$ defined in $(-1,1)$ if $f(x)$ is an even function then.

(a) $a_n = 0$ (b) $b_n = 0$ (c) $a_0 = 0$ (d) both a_0 and a_n is zero

Q40) Fourier series of the periodic function with period 2π defined by
$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ is $\frac{\pi}{4} + \sum \left[\frac{1}{\pi n^2} (\cos n\pi - 1) \cos nx - \frac{1}{n} \cos n\pi \sin nx \right]$.
Then the value of the sum of the series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{12}$

END TERM PAPER 2

(ch-4)

Q1. $f(x)$ in $(c, c+2l)$

~~a.)~~ $f(x)$ should be contineous in $(c, c+2l)$

~~b.)~~ periodic ~~c.)~~ even fxn ~~d.)~~ odd fxn.

Q2. $f(t) = \begin{cases} 1 & -1 \le t < 0 \\ -2 & 0 \le t < 1 \end{cases}$ $a_0 = ??$ a.) 0 b.) 1 c.) -1 d.) -2 ~~e.)~~ NOT

Soln.

$l = \frac{1-(-1)}{2} = 1$ $a_0 = \frac{1}{2l}\int_{-1}^{0} 1\, dt + \frac{1}{2l}\int_0^1 -2\, dt$

D

$\Rightarrow \frac{1}{2}(x)_{-1}^{0} + \frac{1}{2}\left(-2(x)_0^1\right)$

$= \frac{1}{2} + -1 = -1/2$

Q3. $f(x) = \begin{cases} 1+x & -\pi \le x \le 0 \\ 1-x & 0 \le x < \pi \end{cases}$ $a_0 = ??$ a.) 2 b.) π c.) $\pi/2$ d.) $2-\pi$ ~~e.)~~ NOT

D

Soln.

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0}(1+x)\,dx + \frac{1}{2\pi}\int_0^{\pi}(1-x)\,dx$

$1 - \frac{\pi}{2}$ $\frac{2-\pi}{2}$

$\Rightarrow \frac{1}{2\pi}\left(x + \frac{x^2}{2}\right)_{-\pi}^{0} + \frac{1}{2\pi}\left[x - \frac{x^2}{2}\right]_0^{\pi} = \frac{1}{2\pi}\left[(0) - \left(-\pi + \frac{\pi^2}{2}\right)\right] + \frac{1}{2}\left[\pi - \frac{\pi^2}{2} - 0\right]$

$\Rightarrow \frac{1}{2\pi}\left[\pi - \frac{\pi^2}{2} + \pi - \frac{\pi^2}{2}\right] = \frac{1}{2\pi}\left[2\pi - \pi^2\right] = \boxed{1 - \frac{\pi}{2}}$

Q4. $\cos 2n\pi \rightarrow$ a.) -1 b.) 0 c.) 1 d.) π.

done

Q5. $f(x) = x\sin x \quad -\pi < x < \pi \quad a_0 = ??$ a.) 0 b.) 2π c.) $2/\pi$ d.) 2

Sol $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\sin x = \frac{1}{2\pi}\left[x(-\cos x) - (1)(-\sin x)\right]_{-\pi}^{\pi}$ D

$= \frac{1}{2\pi}\left[(-\pi\cos\pi + \sin\pi) - ((-\pi)(-\cos(-\pi)) + \sin(-\pi)\right]$ (-1)

$= \frac{1}{2\pi}\left[\pi + \pi\right] = \frac{2\pi}{2\pi} = 1$

Q6. $f(x) = 1 \quad (0, \pi)$

a.) 0 $\boxed{l = \pi}$ b.) $\frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right)$ c.) $\frac{4}{\pi}\left(\sin x - \frac{\sin 3x}{3} + \frac{\sin 5x}{5} \dots\right)$

Sol $b_n = \frac{2}{l}\int_0^{\pi} 1\cdot\sin\left(\frac{n\pi}{l}\right)x\,dx.$ d.) $\frac{4}{\pi}\left[\sin 2x + \frac{\sin 4x}{2} + \frac{\sin 6x}{3} + \dots\right]$

$\Rightarrow \frac{2}{\pi}\int_0^{\pi}\sin nx \Rightarrow \frac{2}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi} \Rightarrow \frac{-2}{n\pi}\left[\cos n\pi - \cos 0\right] = \frac{-2}{n\pi}\left[(-1)^n - 1\right]$ b_n

$f(x) = \sum_{n=1}^{\infty}\frac{-2}{n\pi}\left((-1)^n - 1\right)\sin\left(\frac{n\pi}{l}\right)x = \frac{-2}{\pi}\sum_{n=1}^{\infty}\frac{\left((-1)^n - 1\right)}{n}\sin nx$

$n=1$ $\frac{-2}{\pi}\left[-2\sin x\right] = \frac{4}{\pi}\sin x.$

$\boxed{n=2 \rightarrow 0}$ $n=3 \Rightarrow \frac{-2}{\pi}\left(-2\frac{\sin 3x}{3}\right)$ similarly. $\boxed{n=4 \Rightarrow 0}$ $\& \; n_5 = \frac{-2}{\pi}\left(-2\frac{\sin 5x}{5}\right)$

87 $\sin nx \cos nx$

(done? (previous lecture) (odd)

a.) odd b.) even c.) cannot d.) NOT.

(88) $f(t) = \begin{cases} t^2 & 0 \le t \le 2 \\ -t+6 & 2 \le t \le 6 \end{cases}$ $a_0 = ??$

a.) 8/9 b.) 16/9 c.) $\frac{24}{9}$ d.) 32/9

Sol^n: $l = \frac{6-0}{2} = 3$

$$a_0 = \frac{1}{2l}\int_0^l f(t)\,dt \Rightarrow \frac{1}{6}\int_0^2 t^2 + \frac{1}{6}\int_2^6 -t+6$$

$$= \frac{1}{6}\left[\frac{t^3}{3}\right]_0^2 + \frac{1}{6}\left[-\left(\frac{t^2}{2}\right) + 6(t)\right]_2^6$$

$$= \frac{1}{18}(8) + \frac{1}{6}\left[-\left(\frac{36}{2} - \frac{4}{2}\right) + 6(6-2)\right]$$

$$= \frac{8}{18} + \frac{1}{6}[-16 + 24]$$

$$= \frac{4}{9} + \frac{1}{6}[8] \Rightarrow \frac{4}{9} + \frac{4}{3} = \frac{4+12}{9} = \frac{16}{9}$$

Q9 → period of $f(x) = \cos 2x$. a.) π b.) $\pi/2$ c.) 2π d.) 4π

10 odd fxn (done) → a.) t^2 b.) t^2-4t c.) $\sin 2t + 3t$ d.) t^3+6

36 even fxn a.) t^2 b.) t^2-4t c.) $\sin 2t + 3t$ d.) t^3+6

$f(t) = t^2$
$t \to -t$

$f(-t) = (-t)^2 = t^2 = f(t)$

$\Rightarrow f(-t) = f(t) \Rightarrow$ even fxn

37 $f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$ $b_n = ?$ a.) $\frac{(-1)^n}{n\pi}$ b) $\frac{1}{n\pi}$ c) 0 d) NOT

Sol^n

$b_n = \frac{1}{\pi}\int_{-\pi}^{0} -x \sin nx + \frac{1}{\pi}\int_0^{\pi} x \sin nx$

$\Rightarrow \frac{1}{\pi}\left[-\left(\frac{x^2}{2}\right)_{-\pi}^{0}\right] + \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$

$\Rightarrow \frac{-1}{2\pi}\left[0 - \pi^2\right] + \frac{1}{2\pi}\left(\pi^2\right]$

$= \frac{\pi^2}{2\pi} + \frac{\pi^2}{2\pi} =$

$\Rightarrow \frac{-1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_{-\pi}^{0}$

$+ \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi}$

$\Rightarrow \frac{-1}{\cancel{\pi}}\left[0 - \left(\frac{\cancel{\pi}(-1)^n}{n}\right)\right] + \frac{1}{\cancel{\pi}}\left[\frac{-\cancel{\pi}(-1)^n}{n}\right]$

$\Rightarrow \frac{(-1)^n}{n} - \frac{(-1)^n}{n} = 0$

(38) $f(x) = \sin x \quad 0 < x < 2\pi \quad b_n = ?$

a.) $b_n = 0 \quad \forall n$

b.) $b_n = \frac{(-1)^n}{n\pi}$

c.) $b_n = 0 \quad n \neq 1$ & $b_1 = 1$

d.) NOT

Soln.

$b_n = \frac{1}{\pi} \int_0^{2\pi} \sin x \cdot \sin nx \, dx$

$\Rightarrow \frac{1}{\pi} \int_0^{2\pi} \frac{2}{2} \sin x \cdot \sin nx \, dx$

$\frac{1}{2\pi} \int_0^{2\pi} \cos(1-n)x - \cos(1+n)x \, dx$

$\Rightarrow \frac{1}{2\pi} \left[\frac{\sin(1-n)x}{1-n} - \frac{\sin(1+n)x}{1+n} \right]_0^{2\pi}$

$\Rightarrow \frac{1}{2\pi} \left[(0-0) - (0-0) \right] = 0$

(39) $f(x)$ even $(-1 \text{ to } 1)$ (4)

a.) $a_n = 0$

b.) $\boxed{b_n = 0}$

c.) $a_0 = 0$ ✗

d.) both a_0 & a_n is zero

$\boxed{\text{Let } \cancel{f(x) = x^2} \text{ (even)} \quad a_0 = \int_{-1}^{1}}$

** concept $(-\pi, \pi)$ (2) | $(-l, l)$ (4)

if $f(x)$ is even

$\Rightarrow \boxed{b_n = 0}$

$\boxed{f(x) = \text{even}}$

$\downarrow$

$b_n = 0$

$\boxed{a_n = 0}$

$f(x) \to$ even

$b_n = 0$

(*) $f(x) = \text{odd}$

$a_n = 0$

Que. $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$

$$\frac{\pi}{4} + \sum \left(\frac{1}{\pi n^2} (\cos n\pi - 1) \cos nx - \frac{1}{n} \cos n\pi \sin nx \right)$$

a.) $\frac{\pi^2}{4}$ b.) $\frac{\pi^2}{6}$ c.) $\frac{\pi^2}{8}$ d.) $\pi^2/12$

$1 + \frac{1}{3^2} + \frac{1}{5^2} + \text{---}$

Sol^n.

$$f(x) = \frac{\pi}{4} + \left[\frac{1}{\pi} (-1-1)(-1) + 0 \quad \frac{2}{9\pi} \quad 0 \quad \frac{2}{25\pi} \right]$$

$n = 1$ ($n=2$) $n=3$ $n=4$ $n=5$

$$\frac{\pi}{4} + \left[\frac{2}{\pi} + \frac{2}{9\pi} + \frac{2}{25\pi} + \text{---} \right]$$

$$\frac{\pi}{4} + \frac{2}{\pi}\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + - \right]$$

$1 + \frac{1}{3^2} + \frac{1}{5^2} + \text{---}$

$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

$= \frac{\pi^2}{8}$

Let $f(x) = 0$

$0 \quad \frac{-\pi}{4} \times \frac{\pi}{2} = \frac{-\pi^2}{8}$