MTH - 174

TOTAL MARKS → 100

Chapters.

1. Matrix Algebra
2. Linear differential Eq^n ①
3. Linear differential Eq^n ②
4. Fourier series
5. Multivariate calculus
6. Integral Calculus.

Exams

1. CA → ① ② ③ (Best two).
   count.
2. MID TERM
3. END TERM

MARKS

① Attendance	→	5	5
② CA	→	100	25
③ MID TERM	→	30	20
④ END TERM	→	60	50

$\boxed{100}$

MATRICES

Ch→1.

Def :- A matrix is a rectangular arrangement of numbers.

→ Horizontal lines forms Rows, and vertical lines forms columns.

→ matrix with m rows and n columns is said to be an m×n matrix

→ General representation of matrix. :-

$$A = [a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

→ m×n → Shows total number of elements in the matrix

Eg a_{31} → 3rd row & 1st column

Types of matrices

① Empty matrix $\rightarrow$ matrix with no elements (ϕ)

② zero / Null matrix $\rightarrow$ All entries becomes zero. represented by 0

eg $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ or $O_{2\times2}$

③ Row matrix $\rightarrow$ matrix which has only 1 row

eg $A_{1\times n}$, $[a_{11} \ a_{12} \ \cdots \ a_{1n}]$

④ column matrix $\rightarrow$ only 1 column

$A_{m\times1}$ $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix}$

⑤ unitary / Identity $\rightarrow$ All diagonal entries are 1 and other are zero

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

⑥ Square matrix $\rightarrow$ No. of rows equals to No. of columns.

$A_{2\times2} = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$ $A_{3\times3}$; $A_{4\times4}$ ——

⑦ Diagonal matrix $\rightarrow$ If all diagonal ~~matrix~~ elements are simultaneously Not zero & other elements are zero.

eg $A_{3\times3} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ $a, b, c \neq 0$

Represented by $Diag[a_{11} \ a_{22} \cdots a_{nn}]$

(8) Scalar matrix. A diagonal matrix with all diagonal elements same.

$$A_{3\times 3} = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}. \quad a \neq 0$$

(9) Upper triangular matrix. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 7 \end{bmatrix}$

(10) Lower triangular $\begin{bmatrix} 1 & 0 & 0 \\ 6 & 5 & 0 \\ 8 & 7 & 9 \end{bmatrix}$

***

(11) Idempotent matrix. $\boxed{A^2 = A} \rightarrow A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

(12) Involutary matrix. $\boxed{A^2 = I} \rightarrow \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

(13) Nilpotent matrix matrix is said to be nilpotent if $\boxed{A^k = 0}$ where k is a (+)ve integer; k is called index of matrix

eg

$$A = \begin{bmatrix} 1 & -3 & 4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$$

$\boxed{k = 2}$

(14) Orthogonal matrix → square matrix A is said to be orthogonal if $\boxed{AA^T = I}$ ⇒ $\boxed{A^T = A^{-1}}$

(15) Symmetric matrix A square matrix A is said to be symmetric if $\boxed{A^T = A}$

eg. $\begin{bmatrix} -7 & 4 \\ 4 & 8 \end{bmatrix}$ $\begin{bmatrix} 0 & 3 & -2 \\ 3 & 8 & 4 \\ -2 & 4 & 9 \end{bmatrix}$

✓ * Transpose of a matrix.-

matrix obtained by interchanging the row into column and vice-versa ; represented by A^T or A'

eg → $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

* Properties

(1) $(A+B)^T = A^T + B^T$

(2) $(AB)^T = B^T A^T$

(3) $(kA)^T = kA^T$

(4) $(A^T)^T = A$

(5) AA^T is a symmetric matrix

(6) $A + A^T$ is a " "

(7) $(A^{-1})^T = (A^T)^{-1}$

* ~~orthogonal~~ matrix.
Skew Symmetric $\boxed{A^T = -A}$

result ① $A - A'$ is a skew symmetric matrix

result ② diagonal elements of skew symmetric matrix are zero

i.e → $a_{ii} = -a_{ii} \Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii} = 0$

*** result ③ Any square matrix A can be written as sum of symmetric & skew symmetric matrix.

i.e $$\boxed{A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')}$$

eg. $$A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$$

$$\frac{1}{2}(A + A') = \begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 5 & 9/2 \\ 3/2 & 9/2 & 3 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \begin{bmatrix} 0 & 2 & 5/2 \\ -2 & 0 & 3/2 \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

* Conjugate of a matrix. ($\bar{A}$)

matrix A obtained by taking conjugate of each entry of matrix is said to be conjugate of matrix, represented by $\bar{A}$

i.e $\bar{A} = [\bar{a}_{ij}]$

Eg. $A = \begin{bmatrix} 2+3i & i \\ -6i & -2+4i \end{bmatrix} \Rightarrow \bar{A} = \begin{bmatrix} 2-3i & -i \\ 6i & -2-4i \end{bmatrix}$

Properties.

1. $\overline{A+B} = \bar{A} + \bar{B}$
2. $\overline{kA} = \bar{k}\bar{A}$
3. $\overline{AB} = \bar{A}\cdot\bar{B}$
4. $\bar{\bar{A}} = A$

* Transposed Conjugate of a matrix (A^θ)

transpose of the conjugate of a matrix represented by A^θ or A^* or $A^\#$

Eg $A = \begin{bmatrix} 1+i & 2-i & -3-i \\ 4 & 6-i & -7-8i \\ 0 & 12i & -7 \end{bmatrix}_{3\times3}$ $A^\theta = \begin{bmatrix} 1-i & 4 & 0 \\ 2+i & 6+i & -12i \\ -3+i & -7+8i & -7 \end{bmatrix}_{3\times3}$

Properties

① $(A+B)^\theta \Rightarrow A^\theta + B^\theta$

② $(AB)^\theta \Rightarrow B^\theta \cdot A^\theta$

③ $(KA)^\theta \Rightarrow \bar{K} A^\theta$

④ $(A^\theta)^\theta \Rightarrow A$

** Hermitian matrix $\boxed{A^\theta = A}$ (CA) *

** → All diagonal elements of Hermitian matrix are real.

* Skew Hermitian matrix. $\boxed{A^\theta = -A}$

** diagonal elements are either 0 or purely imaginary

$$\boxed{A = \frac{1}{2}[A + A^\theta] + \frac{1}{2}[A - A^\theta]}$$

* Unitary matrix

$$\boxed{A A^\theta = A^\theta A = I}$$

Let matrix A= $B = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 5 & 4 \\ -2 & 4 & 9 \end{bmatrix}$. find 1) symmetric matrix, 2) skew symmetric matrix, 3) Hermitian matrix, 4) skew Hermitian matrix.	CO1

Let matrix A= $B = \begin{bmatrix} 4 & 3 & -2 \\ 3 & -2 & 4 \\ -2 & 4 & 6 \end{bmatrix}$. find 1) symmetric matrix, 2) skew symmetric matrix, 3) Hermitian matrix, 4) skew Hermitian matrix.

CA PYQ.s

Que. $B = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 5 & 4 \\ -2 & 4 & 9 \end{bmatrix}$?

Solⁿ.

① Symmetric matrix $(B^T) = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 5 & 4 \\ -2 & 4 & 9 \end{bmatrix}$

② Skew symmetric matrix $(-B^T) = \begin{bmatrix} -1 & -3 & 2 \\ -3 & -5 & -4 \\ 2 & -4 & -9 \end{bmatrix}$

③ Hermitian matrix $(B^\theta) = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 5 & 4 \\ -2 & 4 & 9 \end{bmatrix}$

④ Skew Hermitian matrix $(-B^\theta) = \begin{bmatrix} -1 & -3 & 2 \\ -3 & -5 & -4 \\ 2 & -4 & -9 \end{bmatrix}$

Que. $\begin{bmatrix} 0 & 3 & -2 \\ 3 & 8 & 4 \\ -2 & 4 & 9 \end{bmatrix}$ ※

Que $\begin{bmatrix} 4 & 3 & -2 \\ 3 & -2 & 4 \\ -2 & 4 & 6 \end{bmatrix}$ ※

DIBy
[Same approach]

Minors :- Determinant of every square sub matrix of matrix A is called minor of matrix Denoted by M_{11}

Eg →
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 2 & 3 \end{bmatrix}$$

Soln
$$M_{11} = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = 12 - 2 = 10$$

$$M_{12} = \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} = 6 - 3 = 3$$

$$M_{13} = \begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} = 4 - 12 = -8$$

$$M_{21} = \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = 6 - 6 = 0$$

$$M_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 3 \end{vmatrix} = 3 - 9 = -6$$

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 2 - 6 = -4$$

$$M_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = 2 - 12 = -10$$

$$M_{32} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1 - 6 = -5$$

$$M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4 - 4 = 0$$

Cofactor ⇒ $(-1)^{i+j} M_{ij} = C_{ij}$

∴ Cofactor matrix ⇒
$$\begin{bmatrix} 10 & -3 & -8 \\ 0 & -6 & +4 \\ -10 & +5 & 0 \end{bmatrix}$$

Adjoint of a matrix. [adj(A)]

Transpose of the cofactor matrix.

Result → for any square matrix of order n

$$\boxed{A(\text{adj}A) = |A| I_n = (\text{adj}A)A}$$

⊛ Determinants.

① order 1 ⇒ $[a] = a$

② order 2 ⇒ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

③ order 3 ⇒ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ ⇒ $1(45-48)$ $-2[36-42)$ $3[32-35]$ ⇒ $-3 + 12 - 9$

$= -12 + 12$

$= 0$

Ans.

Properties of Determinants

① $|kA| = k^n |A|$ $n \to$ order.

② $|A^T| = |A|$

③ $|AB| = |A||B|$

* Singular matrix. $|A| = 0$
* Non singular matrix. $|A| \neq 0$

* Invertible matrices (Inverse)

$$\boxed{A^{-1} = \frac{adj(A)}{|A|}}$$

Eg: $A = \begin{bmatrix} 2 & -5 & -3 \\ -1 & 3 & 2 \\ 3 & 1 & 3 \end{bmatrix}$

Soln $|A| = 2(9-2) + 5(-3-6) - 3(-1-9)$

$= 14 - 45 + 30$

$= -1$

$adj A = \begin{bmatrix} 7 & 12 & -1 \\ 9 & 15 & -1 \\ -10 & -17 & 1 \end{bmatrix}$

$$\boxed{A^{-1} = \frac{1}{-1} adj(A)}$$

L O s

b. Find inverse of matrix $B = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

b. Find inverse of matrix $B = \begin{bmatrix} 2 & 4 & 2 \\ 6 & 4 & 6 \\ 2 & 4 & 4 \end{bmatrix}$.

b. Find inverse of matrix $B = \begin{bmatrix} 2 & -5 & -3 \\ -1 & 3 & 2 \\ 3 & 1 & 3 \end{bmatrix}$.

Q 1 If $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$, then $|9A| =$

(A) -9 2 (B) -729 (C) 81 (D) None of these CO_1_, L1__

CA pyQs

Q1. $B = \begin{bmatrix} 2 & -5 & -3 \\ -1 & 3 & 2 \\ 3 & 1 & 3 \end{bmatrix}$

Ques $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ $|9A| = ??$ $9^n |A|$ $|A| = -1$

Sol. $= 9^3 |A|$

$= 729 \times (-1) = -729$ Ans.

(B)

A

Soln. We can find B^{-1} as:- $B^{-1} = \frac{adj(B)}{|B|}$

$\therefore |B| = \begin{vmatrix} 2 & -5 & -3 \\ -1 & 3 & 2 \\ 3 & 1 & 3 \end{vmatrix} = 2[9-2] + 5[-3-6] - 3[-1-9] = 14 - 45 + 30$

$= -1$

$C_{ij} = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix} \Rightarrow Adj(B) = (C_{ij})^T = \begin{bmatrix} 7 & 12 & -1 \\ 9 & 15 & -1 \\ -10 & -17 & 1 \end{bmatrix}$

$\therefore B^{-1} = \frac{adj(B)}{|B|} = \frac{\begin{bmatrix} 7 & 12 & -1 \\ 9 & 15 & -1 \\ -10 & -17 & 1 \end{bmatrix}}{-1} = \begin{bmatrix} -7 & -12 & 1 \\ -9 & -15 & 1 \\ 10 & 17 & -1 \end{bmatrix}$

Ans.

$\cancel{A}$

By the same approach solve next 3 CA PYQs.

Q2. $\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ ✓

Q3. $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$ -

Q4. $\begin{bmatrix} 2 & 4 & 2 \\ 6 & 4 & 6 \\ 2 & 4 & 4 \end{bmatrix}$ ✓

CA PyQs.

*****

mere yaaro avi ke avi esko solve krye baad ke Liye please kuch mt chordiye

"Thank you"

Rank of a matrix [$\rho(A)$] $\rho(A)$.

① No. of Linearly independent rows in a matrix

$\boxed{OR}$

② No. of non-zero rows in a matrix of Echelon form.

Ques.

$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \longrightarrow 2R_1 - R_2 \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = 1$

Ques.

$A = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \\ 3 & 3 & 3 \end{bmatrix} \longrightarrow R_1 \rightarrow R_1/2 \quad \begin{bmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \\ 3 & 3 & 3 \end{bmatrix} \rightarrow \begin{matrix} R_2 \rightarrow R_2 - 4R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \rho(A) = 1$

Ques.

$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3} \rightarrow \rho(A) = 3$

Ques.

$\begin{bmatrix} a_t & 0 & 0 \\ 0 & a_t & 0 \\ 0 & 0 & a_t \end{bmatrix} \rightarrow \rho(A) = 3$

Note.

① $|A| \neq 0 \Rightarrow \rho(A) = \text{order of } A$

② Rank of null matrix is 0

③ eg $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$. $|A| = 1 \Rightarrow |A| \neq 0 \quad \therefore \rho(A) = 3$

(2.) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}_{3\times3}$. $|A| = 2 \Rightarrow |A| \neq 0 \quad \therefore \rho(A) = 3$

Echelon form of a matrix

A matrix 'A' is said to be in Echelon form if :-

(1.) Every row of 'A' which has all its entries zero occurs below every row which has a non-zero entry.

(2.) No. of zeroes before the 1st non-zero element in a row is less than the No. of such zeroes in the next row.

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3\times4}$$

* NOTE ✓

(1) Interchange of rows not change the rank of matrix

(2) Rank of matrix in echlon form is the No. of Non-zero rows.

(3) Rank of A = Rank of A^T

... rough sheet(s) along with the OMR sheet to the invigilator before l...
examination hall.

Q(1) Let $A_{3\times 3}$ be a non-singular matrix. Then the rank of the matrix A is

(a) 2 (b) 3 (c) 1 (d) 0

Q(2) Consider the following statements f...

Q(2) Consider the following statements for the matrix $M = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 1 \\ 0 & 3 & 0 \end{pmatrix}$. CO1,L1

(A) Assertion: The rank of the matrix is 2.

(B) Reason: The determinant of the matrix is 0

Choose the correct option.

(a) Both (A) and (B) are true. (b) (A) is false but (B) is true. (c) (A) is true but (B) is false. (d) Both (A) and (B) are false.

CO1,L1

Que. for what values of x Rank of matrix is 3

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 2 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}_{3\times 3}$$

Solⁿ. order = Rank.

$\Rightarrow |A| \neq 0 \Rightarrow 2(x-0) - 4(2x-2) + 2(0-1) \neq 0$

$\Rightarrow 2x - 8x + 8 - 2 \neq 0$

$\Rightarrow -6x + 6 \neq 0 \Rightarrow \boxed{x \neq 1}$

Elementary operations

1. interchanging of two rows or columns
2. multiply a row/c by some scalar k
3. Adding a row with some other row after applying with some scalar $(k \neq 0)$

$R_1 \leftrightarrow R_2$, $R_2 \to R_1 - R_2$

$$M = \begin{vmatrix} -1 & 1 & 1 \\ 1 & 2 & -1 \\ 0 & 3 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 2 & -1 \\ 0 & 3 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 0 & -3 & 0 \\ 0 & 3 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$\therefore \boxed{\rho(A) = 2}$

Assertion ✓

Reason ✓ [redacted]

$\Rightarrow$ a.) both correct ✓

END TERM $A_{3\times 3}$

Q1— a.) 2 ✓b.) 3 c.) 1 d.) 0

Solⁿ. ✓Non singular $\Rightarrow |A| \neq 0$

singular $\Rightarrow |A| = 0$

$\boxed{\rho(A) = \text{order} = 3}$

Que. $M = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 1 \\ 0 & 3 & 0 \end{pmatrix}_{3\times 3}$

~~a.)~~ —
b.) —
c.) —
d.) —

Solⁿ. $|M| = 1(0-3) - 2(0-0) - 1(-3)$

$= -3 + 3 = 0$

$\boxed{\rho(M) \neq 3}$

~~Assertion → false~~
~~Reason → false~~

CO1

Q(4) Let A be a matrix of order $m \times n$. Then which of the following is correct.

(a) $\rho(A) = m$ (b) $\rho(A) < min\{m,n\}$ (c) $\rho(A) = min\{m,n\}$ (d) $\rho(A) \leq min\{m,n\}$

CO1

Q(5) The system of equations given by $AX = O$ can never have

Q 2 The rank of matrix $A = \begin{bmatrix} 2 & -4 & 6 \\ -1 & 2 & -3 \\ 3 & -6 & 9 \end{bmatrix}$ is

(A) 4 (B) 2 (C) 0 (D) 1

Q) $A = m \times n$

~~a)~~ $\rho(A) = m$ ~~b)~~ $\rho(A) < \min(m,n)$ ~~c)~~ $\rho(A) = \min(m,n)$ ~~d)~~ $\rho(A) \le \min(m,n)$

Soln.

eg. a:) $\begin{bmatrix} 1 & 1 & 4 & 6 \\ 6 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 3×4

$\Downarrow$

$\rho(A) = 2$ $A/B \to 3$

b:) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 7 \end{bmatrix}_{2 \times 3}$

$\Downarrow$

$\rho(A) = 2$
$A/B \to$ Less than 2

$\rho(A) = 2$
$A/B \to 3$

Not matched any option.

$\begin{bmatrix} 1 & 2 \\ 4 & 6 \\ 5 & 3 \end{bmatrix}_{3 \times 2}$

$A/B \to$ 2 or less

$\rho(A) = 3$ $\to$ ×.

d.

Que. $\begin{bmatrix} 2 & -2 & 1 \\ -1 & 0 & 1 \\ 3 & -2 & 3 \end{bmatrix}$

Que. $\begin{bmatrix} 2 & -4 & 6 \\ -1 & 2 & -3 \\ 3 & -6 & 9 \end{bmatrix}$ Rank (A) = ??

A:) 4 C:) 0
B:) 2 ✓d:) 1

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

Soln. $\begin{bmatrix} -1 & 2 & -3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{bmatrix} \longleftrightarrow \begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{bmatrix} \longleftrightarrow \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 1

$\therefore$ $\boxed{\rho(A) = 1}$

Question Statement
a. Find the rank of the matrix $A = \begin{bmatrix} 1 & w & r & 0 \\ 0 & s & t & 1 \\ -1 & -w & -r & 0 \\ 0 & s & t & 1 \end{bmatrix}, (w, r, s, t \neq 0)$.

a. Find the rank of the matrix $A = \begin{bmatrix} k & m & r & 0 \\ 0 & s & t & 1 \\ -k & m & r & 0 \\ 0 & s & t & 1 \end{bmatrix}, (k, m, r, s, t \neq 0).$

a. Find the rank of the matrix $A = \begin{bmatrix} k & m & -r & 0 \\ 0 & s & t & 1 \\ -k & -m & r & 0 \\ 0 & s & t & 1 \end{bmatrix}, (k, m, r, s, t \neq 0)$.

C

CA_PYQs

Ques. $A = \begin{bmatrix} k & m & -r & 0 \\ 0 & s & t & 1 \\ -k & -m & r & 0 \\ 0 & s & t & 1 \end{bmatrix}$

Sol^n. $R_3 \to R_1 + R_3$ & $R_4 \to R_2 - R_4$

$\Rightarrow A = \begin{bmatrix} k & m & -r & 0 \\ 0 & s & t & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore \boxed{\rho(A) = 2}$ Ans.

Ques. $A = \begin{bmatrix} 1 & w & r & 0 \\ 0 & s & -t & 1 \\ -1 & w & -r & 0 \\ 0 & s & t & 1 \end{bmatrix}$

Sol^n. Since, number of non zero rows $= 4$

$\therefore \boxed{\rho(A) = 4}$ Ans.

Ques. $A = \begin{bmatrix} k & m & r & 0 \\ 0 & s & t & 1 \\ -k & m & r & 0 \\ 0 & s & t & 1 \end{bmatrix}$

Sol^n. $R_4 \to R_2 - R_4$

$\begin{bmatrix} k & m & r & 0 \\ 0 & s & t & 1 \\ -k & m & r & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore \boxed{\rho(A) = 3}$

Ques. $A = \begin{bmatrix} 1 & w & r & 0 \\ 0 & s & t & 1 \\ -1 & -w & -r & 0 \\ 0 & s & t & 1 \end{bmatrix}$

Sol^n. $R_3 \to R_1 + R_3$ & $R_4 \to R_2 - R_4$

$\Rightarrow A = \begin{bmatrix} 1 & w & r & 0 \\ 0 & s & t & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore \boxed{\rho(A) = 2}$

Ans.

System of Linear Equations

Linear eqⁿ → A Linear eqⁿ is a eqⁿ that can be written in the form

$$a_1 x_1 + a_2 x_2 + \dots + a_n x_n = c$$

System of Linear Eqⁿ → two or more Linear eqⁿ contains the same variables.

Note → Every Linear system of eqⁿ has exactly one solution, no solution or infinite solutions.

Consistent Linear System → A Linear system is consistent if it has a solution.

Inconsistent Linear System → If system has no solution.

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}}_{b} \Rightarrow \boxed{Ax = b}$$

- if $\vec{b} = \vec{0}$ homogeneous
- if $\vec{b} \neq \vec{0}$ Not homogeneous.

Eg.

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_2 &= 3 \\ x_1 + x_3 &= 5 \end{aligned} \Rightarrow \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}}_{b}$$

$\therefore$ $\vec{b} \neq 0 \Rightarrow$ System is non - homogeneous.

$\boxed{Ax = 0}$ → homogeneous ✓

$\boxed{Ax = B}$ → Non- homogeneous ✓

Homogeneous

* **Note** * • if Rank of coefficient matrix A = No. of variables.

⇒ System has unique solution (zero solution)

• if rank < No. of variables

⇒ System has infinite (∞) solutions.

** → 1st convert matrix in echelon form

** $|A| \neq 0$ ⇒ unique solution *

$|A| = 0$ ⇒ ∞ solution. *

eg ① $x + y = 0$
$2x + 2y = 0$ find sol^n

sol^n. $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ homogeneous system.

Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$\therefore \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\therefore \boxed{Rank(A) = 1}$

$\therefore$ Rank(A) < No. of variables [(2) (x, y)]

Now $x + y = 0$
Let us take $\boxed{y = c}$ $c \in R$
then $\boxed{x = -c}$

Now write general sol^n as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -c \\ c \end{bmatrix} = c\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\therefore$ System has infinite solution.

eg ②
$x + 2y + 3z = 0$
$3x + 4y + 4z = 0$
$7x + 10y + 12z = 0$

find sol^n.

$$Ax = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, reduce matrix A to Echelon. form by elementary row operations.

$R_2 \rightarrow R_2 - 3R_1$ & $R_3 \rightarrow R_3 - 7R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & -4 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ Rank of A = no. of variables = order of matrix = $\underline{3}$

therefore. System has unique solution

i.e $\longrightarrow$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Ques. $x + 3y - 2z = 0$
$2x - y + 4z = 0$ find solⁿ.
$x - 11y + 14z = 0$

Ques. $x + y + z = 0$ $3x - 5y + 4z = 0$
$2x - y - 3z = 0$ $x + 17y + 4z = 0$

Solⁿ → system has unique solⁿ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solⁿ.
$$Ax = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} x + 3y - 2z &= 0 \\ -7y + 8z &= 0 \\ 0z &= 0 \end{aligned}$$

⇒ Let $\boxed{z = c}$ (any arbitrary constant) ($c \in \mathbb{R}$)

then, $y = \frac{8}{7}c$ & $x = 2c - \frac{24c}{7} = \frac{14c - 24c}{7} = \frac{-10c}{7}$

therefore. General solution is given as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -10c/7 \\ 8c/7 \\ c \end{bmatrix} = c \begin{bmatrix} -10/7 \\ 8/7 \\ 1 \end{bmatrix}$$

∴ Rank < no. of variable
⇒ infinite solⁿ
or
⇒ directly from this solⁿ.

Q(5) The system of equations given by $AX = O$ can never have

(a) a unique solution.

(b) infinite number of solutions

(c) no solution

(d) None of above.

(c) no solution

(d) None of above.

CO1,L1

Q(6) Consider the system of equations $AX = O$. Then which of the following is correct.

(a) If $|A| \neq 0$, then $X = O$ is the only solution.

(b) If $|A| = 0$, then the system has infinite number of solutions.

(c) The system is always consistent.

(d) All of the above.

CO1,L1

END TERM

Que. 5 $Ax = 0$ can never have. ⇒ soln.

a.) unique soln c.) ∞ soln

b.) No. soln d.) NOT

$Ax = 0$ → homogeneous.
→ Unique soln (zero soln)
→ ∞ soln.

Que 6. $Ax = 0$ → unique → zero.

a.) $|A| \neq 0$, then $x = 0$ is the only soln

b.) if $|A| = 0$ then system has ∞ No. of soln.

c.) always consistent

d.) All.

soln. — $|A| \neq 0$ unique soln (zero sol) ⇒ a.)

$|A| = 0$ ∞ soln b.)

$\boxed{Ax = 0}$ ⇒ consistent. c.)

⇒ d

NON - homogeneous Equation -

Eg. $a_1x + b_1y = c_1$ $\quad c_1 \neq 0$ $\quad$ or $\quad 3x + 4y = 5$

$a_2x + b_2y = c_2$ $\quad c_2 \neq 0$ $\quad\quad 6x + 8y = 13$

Imp. points

* Augmented matrix → Let $Ax = B$ is system of Linear eq^n then matrix of the form $[A : B]$ known as Augmented matrix.

1. if $\rho(A) = \rho(A:B)$ → System is consistent.
2. $\rho(A) = \rho(A:B)$ = (equals to) No. of variables ⇒ unique sol^n.
3. $\rho(A) = \rho(A:B)$ < No. of variable ⇒ ∞ sol^n
4. System is inconsistent if $\rho(A) \neq \rho(A:B)$ or System has no sol^n.

$\boxed{\text{Steps to find the sol}^n}$

1. Convert the system into matrix form $\boxed{Ax = B}$
2. Write $[A:B]$ matrix, then convert $[A:B]$ matrix into echelon form
3. find rank of A & Rank of $[A:B]$ matrix.

Ques. $x + y + z = -3$ $\quad 3x + y - 2z = -2$ $\quad 2x + 4y + 7z = 7$

Soln

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}$$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & -3 \\ 3 & 1 & -2 & : & -2 \\ 2 & 4 & 7 & : & 7 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$ & $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 & : & -3 \\ 0 & -2 & -5 & : & 7 \\ 0 & 2 & 5 & : & 13 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & : & -3 \\ 0 & -2 & -5 & : & 7 \\ 0 & 0 & 0 & : & 20 \end{bmatrix} \Rightarrow \rho(A) = 2 \text{ \& } \rho(A:B) = 3$$

Since $\rho(A) \neq \rho(A:B)$

Inconsistent

therefore system of Eq^n has no soln.

Ques. find system is consistent or inconsistent also find soln

$2x - y + 3z = 8$ $\quad -x + 2y + z = 4$ $\quad 3x + y - 4z = 0$

$$Ax = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} = B$$

$$[A:B] = \left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\sim \left[\begin{array}{ccc:c} -1 & 2 & 1 & 4 \\ 2 & -1 & 3 & 8 \\ 3 & 1 & -4 & 0 \end{array}\right]$$

$R_2 \to R_2 + 2R_1$ & $R_3 \to R_3 + 3R_1$

$$\sim \left[\begin{array}{ccc:c} -1 & 2 & 1 & 4 \\ 0 & 3 & 5 & 16 \\ 0 & 7 & -1 & 12 \end{array}\right]$$

$R_3 \to 3R_3$

$$\sim \left[\begin{array}{ccc:c} -1 & 2 & 1 & 4 \\ 0 & 3 & 5 & 16 \\ 0 & 21 & -3 & 36 \end{array}\right]$$

$R_3 \to R_3 - 7R_2$

$$\sim \left[\begin{array}{ccc:c} -1 & 2 & 1 & 4 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & -38 & -76 \end{array}\right]$$

$R_3 \to \frac{-R_3}{38}$

$$\sim \left[\begin{array}{ccc:c} -1 & 2 & 1 & 4 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\therefore \rho(A) = 3$ & $\rho(A:B) = 3$
= No. of variables.

$\therefore$ System is unique soln.

$$\begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \\ 2 \end{bmatrix}$$

$-x + 2y + z = 4$

$3y + 5z = 16$

& $\boxed{z = 2}$.

$\Rightarrow 3y = 16 - 10 = 6$

$\boxed{y = 2}$

& $\boxed{x = 2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$ unique soln

Ans

Que. ① find soln of system of eqn.

$x + 2y - z = 3$ $3x - y + 2z = 1$

$2x - 2y + 3z = 2$ $x - y + z = -1$

Hint

$$A \sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

unique soln.

$\Rightarrow$ $x = -1$, $y = 4$, $z = 4$

② $x + y + z = 9$ $2x + 5y + 7z = 52$

$2x + y - z = 0$

Hint.

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

unique soln

$x = 1$ $y = 3$ $z = 5$

Que. Investigate for what values of λ, μ the simultaneous eqn $x + y + z = 6$ $x + 2y + 3 = 10$ $x + 2y + \lambda z = \mu$

have ① no solution

② a unique soln

③ an infinite number of soln.

Soln. matrix form of the given system of equation is

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix} = B$$

Augmented matrix $[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

NOTE.

① for No. soln. $[\,\rho(A) \neq \rho(A:B)\,]$

$\therefore$ If $\lambda = 3$ & $\mu \neq 10$ then $\rho(A) \neq \rho(A:B)$

thus in this case $\rho(A) = 2$ & $\rho(A:B) = 3$

posses no soln.

② Unique soln. $[\,\rho(A) = \rho(A:B) = \text{No. of variables}\,]$

$\therefore$ if $\lambda \neq 3$, we have $\rho(A) = \rho(A:B) = 3 = \text{No. of variables}$.

System is consistent & has unique soln for any value of μ.

③ infinite soln

$\rho(A) = \rho(A:B) < \text{No. of variables}$

$\therefore$ if $\lambda = 3$ & $\mu = 10$

we have $\rho(A) = 2 = \rho[A:B] < \text{No. of variable}$.

system is consistent & posses infinite No. of solutions.

Q. For what value of n the eqⁿˢ $x+y+z=1$, $x+2y+4z=n$, $x+4y+10z=n^2$ have a solution and solve system in each case.

Sol.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ n \\ n^2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$ $\quad R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ n-1 \\ n^2-1 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ n-1 \\ n^2-3n+2 \end{bmatrix}$$

System is consistent iff

$n^2 - 3n + 2 = 0$

i.e $(n-2)(n-1) = 0$

or $n = 2$ or $n = 1$

Now.

Case ① if $n = 2$ then

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $y + 3z = 1$ $\Rightarrow y = 1-3z$

or $x + y + z = 1$

Let $z = k$ then. $y = 1-3k$

& $x = 2k$

$k \in R$

Case ② if $n = 1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $y + 3z = 0$

$x + y + z = 1$

Let $z = c$ then $y = -3c$ & $x = 1+2c$

$c \in R$

$\lfloor 1 \quad 1 \quad 3 \rfloor$

Find out for what value of λ and μ system of linear equations. $2\mathbf{x} + \mathbf{2y} + \mathbf{2z} = 8$, $4\mathbf{x} + \mathbf{2\,y} + \mathbf{3\,z} = 10$, $4\mathbf{x} + \mathbf{2\,y} + \boldsymbol{\lambda}\,\mathbf{z} = \mu$.

have (1) unique solution (2) no solution (3) infinite solution.

Find out for what value of λ and μ system of linear equations. $2\mathbf{x} + \mathbf{2y} + \mathbf{2z} = 8$, $4\mathbf{x} + \mathbf{2}\,\mathbf{y} + \mathbf{3}\,\mathbf{z} = 10$, $4\mathbf{x} + \mathbf{2}\,\mathbf{y} + \boldsymbol{\lambda}\,\mathbf{z} = \mu$.

have (1) unique solution (2) no solution (3) infinite solution.

Find out for what value of λ and μ system of linear equations $\mathbf{x}+\mathbf{y}+\mathbf{z}=6$, $\mathbf{x}+2\mathbf{y}+3\mathbf{z}=10$, $\mathbf{x}+2\mathbf{y}+\boldsymbol{\lambda}\mathbf{z}=\mu$. Have (1) unique solution

(2) no solution (3) infinite solution .

Ques.

$$2x + 2y + 2z = 10$$
$$4x + 2y + 3z = 10$$
$$4x + 2y + \lambda z = \mu$$

① unique soln ② no soln ③ Infinite soln.

Soln.)

$$AX = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 2 & 3 \\ 4 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ \mu \end{bmatrix}$$

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 2 & 2 & 10 \\ 4 & 2 & 3 & 10 \\ 4 & 2 & \lambda & \mu \end{array}\right]$$

$R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - 2R_1$

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 2 & 2 & 10 \\ 0 & -2 & -1 & -10 \\ 0 & -2 & \lambda-4 & \mu-20 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$[A:B] \left[\begin{array}{ccc:c} 2 & 2 & 2 & 10 \\ 0 & -2 & -1 & -10 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

NOW.

① For unique solution.
[$\rho(A) = \rho(A:B) =$ No. of variables $= 3$]

if $\lambda \neq 3$, we have $\rho(A) = \rho(A:B) = 3 =$ No. of variables
System is consistent & has unique soln for any value of μ.

② No soln [$\rho(A) \neq \rho(A:B)$]

if $\lambda = 3$ & $\mu \neq 10$ then $\rho(A) \neq \rho(A:B)$
thus in this case $\rho(A) = 2$ & $\rho(A:B) = 3$ posses No solution.

③ Infinite soln. [$\rho(A) = \rho(A:B) <$ No. of var]

if $\lambda = 3$ & $\mu = 10$ we have $\rho(A) = 2 = \rho(A:B)$ Less than No. of variables

$\Rightarrow$ System is consistent & posses infinite No. of solution.

Ques.

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

① unique soln ✓
② no soln ✓
③ infinite soln.

Soln.

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda - 1 & \mu - 6 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{array}\right]$$

Now:

Solve by yourself

(same as prev. Ques).

Ques. $2x + 2y + 2z = 8$

$4x + 2y + 3z = 10$

$4x + 2y + \lambda z = \mu$

① unique soln

② no soln

③ ∞ soln

Soln.

$$Ax = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 2 & 3 \\ 4 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ \mu \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$[A:B] = \begin{bmatrix} 2 & 2 & 2 & : & 8 \\ 4 & 2 & 3 & : & 10 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$

{ Now, same as previous, solve by yourself. }

*** END TERM

$$\begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 7 & 3 & -2 & : & 8 \\ 2 & 3 & \lambda & : & 9 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 7 & 3 & -2 & : & 8 \\ 0 & 0 & \lambda-5 & : & 0 \end{bmatrix}$$

$\lambda \neq 5$

for unique soln.

$\rho(A) = \rho(A:B) = \text{No. of var.} = 3$

$\lambda - 5 \neq 0$

$\Rightarrow \boxed{\lambda \neq 5}$

a.) $\lambda = 15$

b.) $\lambda = 5$

c.) $\lambda \neq 15$

d.) $\lambda \neq 5$

Eigen-values and Eigen-vectors

* $(A - \lambda I)$ → characteristic matrix
* $|A - \lambda I|$ → characteristic eqⁿ of A
* Roots of $|A - \lambda I| = 0$ is called as Eigen values or characteristic root or characteristic values or Latent root or proper value of matrix A
* Set of Eigen values of matrix A called as Spectrum of A

* ## How to find Eigen-values.

Step ① write characteristic eqⁿ $|A - \lambda I| = 0$
Step ② solve for value of λ.

Eg $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find Eigen value.

Solⁿ $A - \lambda I = \begin{bmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} = 0$

$\Rightarrow (1-\lambda)^2 = 0 \Rightarrow \boxed{\lambda = 1, 1}$ ∴ Eigen values are 1, 1

Q. $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

Sol^n. $A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix} = 0$

$\Rightarrow (1-\lambda)(3-\lambda) = 0$

$\Rightarrow \lambda = 1$ & $\lambda = 3$ are the Eigen values.

*** Result: For a Diagonal matrix, upper Δ matrix ; Scalar matrix, Lower Δ matrix, Eigen values are their corresponding diagonal elements.

Q. Find Eigen values of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$.

Sol^n characteristic matrix of A is $A - \lambda I$

$\therefore A - \lambda I = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} -\lambda & 1 & 2 \\ 1 & -\lambda & -1 \\ 2 & -1 & -\lambda \end{bmatrix} = 0$

$\Rightarrow -\lambda(\lambda^2 - 1) - 1(-\lambda + 2) + 2(-1 + 2\lambda) = 0$

$\Rightarrow -\lambda^3 + \lambda + \lambda - 2 - 2 + 4\lambda = 0$

$\Rightarrow \underbrace{-\lambda^3 + 6\lambda}_{\text{characteristic polynomial}} - 4 = 0$ $\Rightarrow (\lambda - 2)(\lambda^2 - 2\lambda - 2) = 0$

roots are $\lambda = 2$, $\lambda = -1 \pm \sqrt{3}$ are the Eigen values.

Que. find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

Soln. characteristic matrix $A - \lambda I = \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$

characteristic eqn $|A - \lambda I| = 0$

$\Rightarrow (5-\lambda)(2-\lambda) - 4 = 0$

$\Rightarrow 10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$

$\Rightarrow \lambda^2 - 7\lambda + 6 = 0 \quad \Rightarrow \boxed{\lambda_1 = 6 \ \& \ \lambda_2 = 1} \rightarrow$ Eigen values.

Now find eigen vectors.

Let $\underline{x_1} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigen vector corresponding to eigen value $\underline{6}$ are given by

$(A - 6I)x = 0$

$R_2 \to R_2 + R_1$

$\Rightarrow \left[\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -x_1 + 4x_2 = 0$

Let $x_2 = c_1 \Rightarrow x_1 = 4c_1$

$\therefore \begin{bmatrix} X_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4c_1 \\ c_1 \end{bmatrix} = c_1 \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

where c_1 is any arbitrary constant

** Set of all eigen vectors corresponding to eigen value 6 are $c_1 x_1$

Eigen vector corresponding to eigen value 1 is given by $(A-1I)X=0$

$\Rightarrow \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \xrightarrow{R_2 \to R_1 - 4R_2} \begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\therefore \quad 4x_1 + 4x_2 = 0 \qquad$ Let $x_2 = c_2 \Rightarrow x_1 = -c_2$

$$\therefore \; [X_2] = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -c_2 \\ c_2 \end{bmatrix} = c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\therefore$ Set of all eigen vectors corresponding to eigen value 1 is $c_2 X_2$ —

Ques ① Find eigen values and eigen vector of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Solⁿ. $\boxed{\lambda = 0, 3, 15}$ * E. values.

$X_1 = c_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad X_2 = c_2 \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix} \quad X_3 = c_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

E. vectors.

Ques ② $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Solⁿ $\lambda = 2, 2, 8$

$X_1 = c_1 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \quad X_2 = c_2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad X_3 = c_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

** $c_1 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ also eigen vectors corresponding to eigen value 2.

Q(7) if A=$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$then its eigen values are .

a) 1,2,3 b)0,2,1 c)0,0,3 d) None of these

CO1,L

Q(8) If two eigen values of a matrix of order 3x3, whose trase is 11 are 2&4, then the third eigen value is

a) 1,2,3

Q(8) If two eigen values of a matrix of order 3x3, whose trase is 11 are 2&4, then the third eigen value is

a) 5 b) 4 c) 3 d) None of these

Q(9) Let A be an invertible matrix of order n. Then choose the correct option.

(b) 0 can never be an eigen value of A.

Q5) If two of the eigen values of a matrix of order 3 x 3, whose determinant is 36 are 2 & 3 than the third eigen value is.

(a) 2 (b) 3 (c) 4 (d) 6

Results

1. Product of Eigen values = determinant of matrix. $|A| = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdots \lambda_n$
2. Sum of Eigen values = sum of diagonal elements of matrix.
3. No. of Non-zero Eigen value of matrix = Rank of matrix.
4. zero is Eigen value of matrix if and only if the matrix is singular i.e $|A| = 0$
5. If Eigen values of matrix A are $\lambda_1, \lambda_2 \cdots \lambda_n$ then Eigen values of KA are $K\lambda_1, K\lambda_2, \cdots K\lambda_n$
6. $A \to \lambda_1, \lambda_2, \lambda_3 \cdots \lambda_n$

$\Rightarrow A^2 \to \lambda_1^2, \lambda_2^2, \lambda_3^2 \cdots \lambda_n^2$

PYQ

7. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

a.) 1 2 3
b.) 0 2 1
c.) 0 0 3
d.) NOT

Q5. $|A| = 36$ Eigen value 2 8 3

a.) 2 b.) 3
c.) 4 d.) 6

Sol^n

R1 $2 \times 3 \times x = 36$

$x = 36/6$

$\boxed{x = 6}$

Diagonal sum = 11

2 8 4. (x)

8. trace = 11

Eigen value 2 8 4.

a.) 5
b.) 4
c.) 3
d.) NOT

R2 $2 + 4 + x = 11$

$x = 11 - 6$

$= 5$

$11 = 2 + 4 + x$

$11 = 6 + x$

$x = 11 - 6$

$= 5$

CO_1_, L4_

Q 3 $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ is an Eigen vector of $\begin{bmatrix} 4 & -2 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{bmatrix}$ corresponding to Eigen value

(A) 3 (B) 2 (C) 5 (D) -3

CO_1_, L4__

Ques. $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 4 & -2 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{bmatrix}$

a.) 3 ✗
b.) 2 ✓
c.) 5 ✗
d.) -3 ✗

Soln

$$\begin{bmatrix} 4-\lambda & -2 & 1 \\ 2 & -\lambda & 1 \\ 2 & -2 & 3-\lambda \end{bmatrix} = 0$$

$\Rightarrow \lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

$\Rightarrow (\lambda-2)(\lambda-2)(\lambda-3)$

$\boxed{\lambda = 2,\ 2,\ 3}$

a.) 3.

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & 1 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 - 2x_2 + x_3 = 0$

$2x_1 - 3x_2 + x_3 = 0$

$2x_1 - 2x_2 = 0$

$\boxed{x_1 = c \Rightarrow x_2 = c \Rightarrow x_3 = c}$ $\Rightarrow c\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

✗

b.) 2.

$$\begin{bmatrix} 2 & -2 & 1 \\ 2 & -2 & 1 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\boxed{2x_1 - 2x_2 + x_3 = 0}$

$\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$2 \times 1 - 2 \times 2 + 2$

$\Rightarrow 2 - 4 + 2$

$\Rightarrow 0 = 0$ ✓ ✓

Cayley Hamilton theorem

Every sq. matrix A satisfy its characteristic polynomial.

$|A - \lambda I| = A_0 + \lambda A_1 + A_2 \lambda^2 + \dots A_n \lambda^n$

$|A - \lambda I| = A_0 + A A_1 + A^2 A_2 + \dots A^n A_n$

Ques. find the characteristic eq^n of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfy by A & hence obtain A^{-1}

Sol^n. $|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = -\lambda^3 + 6\lambda^2 - 9\lambda + 4.$

Characteristic eq^n of matrix A is $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

replace λ by A. & verify it.

$\Rightarrow A^3 - 6A^2 + 9A - 4 = 0 \quad \text{—(1)}$

Now find A^{-1}; multiply eq^n (1) by A^{-1}.

$A^2 - 6A + 9 - 4A^{-1} = 0$

$\Rightarrow 4A^{-1} = A^2 - 6A + 9$

$\Rightarrow A^{-1} = \frac{A^2}{4} - \frac{6A}{4} + 9 = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Que: Find the characteristic eq^n of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ & find A^{-1}.

Sol^n. $|A - \lambda I| = -\lambda^3 - \lambda^2 + 5\lambda + 5$

& $A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix}$

Que. Using Cayley Hamilton theorem express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A, when $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$(3-\lambda)(2-\lambda) + 1$

$6 - 3\lambda - 2\lambda + \lambda^2 + 1$

$\lambda^2 - 5\lambda + 7.$

Sol^n. $|A - \lambda I| = \lambda^2 - 5\lambda + 7$

By Cayley Hamilton theorem. $(\lambda \to A]$

$A^2 - 5A + 7I = 0$

$\Rightarrow A^2 = 5A - 7I$ — ①

we can write $2A^5 - 3A^4 + A^2 - 4I$

$\Rightarrow$ $A^3 = 5A^2 - 7A$) A

$A^4 = 5A^3 - 7A^2$) A

$A^5 = 5A^4 - 7A^3$

$\therefore 2A^5 - 3A^4 + A^2 - 4I.$

$= 2(5A^4 - 7A^3) - 3(5A^3 - 7A^2) + 5A - 7I - 4I$

$= 7A^4 - 14A^3 + A^2 - 4I$

$= 21A^3 - 48A^2 - 4I.$

$= 57A^2 - 147A - 4I.$

$\Rightarrow 138A - 403I$

Linear polynomial in A.

Q 4 Let A be matrix of order 3×3 with characteristic equation $\lambda^3 + \lambda^2 + 2\lambda + 1 = 0$, then $A^{-1} =$

(A) $A^2 + A + I$

(B) $-(A^2 + A + 2I)$

(C) $-(A^2 + 2A + I)$

(D) cannot be determined

PYQ.

Q4. $\lambda^3 + \lambda^2 + 2\lambda + 1 = 0$ $A^{-1} = ??$

a) $A^2 + A + I$ ×

~~b)~~ $-(A^2 + A + 2I)$

c.) $-(A^2 + 2A + I)$

d.) cannot be det.

Solⁿ. $\lambda \to A.$

$A^3 + A^2 + 2A + 1 = 0$

multiply A^{-1} by both side.

$A^2 + A + 2I + A^{-1} = 0$

$\Rightarrow A^{-1} = -A^2 - A - 2I$

$= -(A^2 + A + 2I)$

1 → 1 ✓

2 → 2|7 ✓

3 → 4|5|6

Q1) If $\begin{bmatrix} a+b & 3 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 5 & 3 \end{bmatrix}$, then what are the values of a and b?

(a) (2, 1) or (1, 2) (b) (2, 4) or (4, 2) (c) (0, 3) or (3, 0) (d) (1, 3) or (3, 1)

Q3) If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then what is the value of k for which $A^2 = 8A + kI$? CO1, L1

(a) 7 (b) -7 (c) 10 (d) 8

Q4) CO1, L1

Q2) If $B = \begin{bmatrix} 1 & 7 \\ 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 \\ 6 & 8 \end{bmatrix}$, and $2A + 3B - 6C = 0$, then what is the value of A? CO1, L1

(a) $\begin{bmatrix} 21/2 & 27/2 \\ -15/2 & 45/2 \end{bmatrix}$

(b) $\begin{bmatrix} 21/4 & 27/4 \\ -15/4 & 45/4 \end{bmatrix}$

(c) $\begin{bmatrix} 21/4 & -15/4 \\ 27/4 & 45/4 \end{bmatrix}$

(d) $\begin{bmatrix} 21/2 & -15/2 \\ 27/2 & 45/2 \end{bmatrix}$

Q 12. $B = \begin{bmatrix} 1 & 7 \\ 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 \\ 6 & 8 \end{bmatrix}$

$2A + 3B - 6C = 0$ $\quad A = ??$

Sol Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow$ A/C.

$$\begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} + \begin{bmatrix} 3 & 21 \\ 9 & 3 \end{bmatrix} - \begin{bmatrix} 24 & 6 \\ 36 & 48 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a+3-24 & 2b+21-6 \\ 2c+9-36 & 2d+3-48 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$2a + 3 - 24 = 0$

$\Rightarrow 2a = 21$

$\Rightarrow \boxed{a = \frac{21}{2}}$

b c d
× × ✓

$\therefore$ Correct option $\downarrow$ (d.)

$2d + 3 - 48 = 0$

$2d = 45$

$\boxed{d = 45/2}$

$2b + 21 - 6 = 0$

$2b = -15$

$\boxed{b = \frac{-15}{2}}$

$$\begin{bmatrix} 21/2 & -15/2 \\ 27/2 & 45/2 \end{bmatrix}$$

Q 11 $\begin{bmatrix} a+b & 3 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 5 & 3 \end{bmatrix}$ $\quad a \,\&\, b = ??$

Sol $a + b = 4$ $\quad ab = 3$

* 1st way $\rightarrow$ option check
* 2nd way (solve)

a.) × ✗
b.) × ✗
c.) × ✗
d.) ✓

Correct option $\rightarrow$ (d.)

$a+b=4$ $\quad ab=3$

(3 1)
(1 3)

$\cancel{(a+b)^2 = a^2 + b^2 + 2ab}$

(2 4) (0 3) (1 3) (3 1)

8 13 $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A^2 = 8A + kI$?

$k = ??$

Soln. $A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0 \\ -1-7 & 49 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$

$8A = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix}$

∵ A/q. $A^2 = 8A + kI$.

$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1-8 & 0-0 \\ -8+8 & 49-56 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\Rightarrow \boxed{k = -7}$ Ans.

Option (b.)

Ans.