DIFFERENTIAL EQ^n (Ch → 2 | 3)

Definition :- An eq^n involving derivatives or differentials of one or more dependent variables w.r.t one or more independent variables is called a differential eq^n.

Eg: ① $\frac{\partial y}{\partial x} = x + \cos x$ ② $\frac{\partial^2 x}{\partial t^2} + \left(\frac{\partial x}{\partial t}\right)^2 = \log t$ (CA) (M E)

Note Representation of differentials or derivatives as:

- $\frac{dy}{dx} = y'$, $\frac{\partial^2 y}{\partial x^2} = y''$, $\frac{\partial^3 y}{\partial x^3} = y'''$ .... $\frac{\partial^n y}{\partial x^n} = y^n$

Wronskian [W(x)]

Ques. ① $\sin x, \cos x$

Soln. $W(x) = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}$

$= -\sin^2 x - \cos^2 x$

$= -(\sin^2 x + \cos^2 x) = -1$ Ans.

Ques ② $y_1 = 2x$, $y_2 = x$

Soln. $W(x) = \begin{vmatrix} 2x & x \\ x & 1 \end{vmatrix}$

$= 2x - x^2$

- If $W(x) = 0 \Rightarrow$ Linearly dependent ✓
- If $W(x) \neq 0 \Rightarrow$ Linearly independent ✓

Eg: (1) If $y_1(x) = \sin 3x$, $y_2(x) = \cos 3x$ are two arbitrary solutions of $y'' + 4y = 0$, show that y_1 & y_2 are Linearly independent

$\boxed{W(x) \neq 0}$

Soln. $\because y_1(x) = \sin 3x$ & $y_2(x) = \cos 3x$

$$\Rightarrow W(x) = \begin{vmatrix} \sin 3x & \cos 3x \\ 3\cos 3x & -3\sin 3x \end{vmatrix} \Rightarrow -3\sin^2 3x - 3\cos 3x$$

$$\Rightarrow -3(\sin^2 3x + \cos^2 3x)$$

$$\Rightarrow -3(1) = -3 \neq 0$$

Since, $W(x) \neq 0$.

$\Rightarrow y_1$ & y_2 are Linearly independent

Que. $y_1 \rightarrow e^x \sin x$, $y_2 = e^x \cos x$ check L.D or L.I.D.

Sln.

$$W(x) = \begin{vmatrix} e^x \sin x & e^x \cos x \\ e^x \cos x + \sin x \, e^x & -e^x \sin x + \cos x \, e^x \end{vmatrix} \Rightarrow \begin{vmatrix} e^x \sin x & e^x \cos x \\ e^x(\cos x + \sin x) & e^x(\cos x - \sin x) \end{vmatrix}$$

$$\Rightarrow e^{2x}[\sin x \cos x - \sin^2 x] - e^{2x}[\cos^2 x + \cos x \sin x]$$

$$\Rightarrow e^{2x}[\cancel{\sin x \cos x} - \cancel{\cos x \sin x} - (\sin^2 x + \cos^2 x)]$$

$$\Rightarrow e^{2x}[-1] \Rightarrow -e^{2x} \neq 0$$

$\therefore W(x) = -e^{2x} \neq 0$

$\Rightarrow y_1(x)$ & $y_2(x)$ are Linearly independent

Que. $y_1(x) = \sin x$. $y_2(x) = \sin x - \cos x$

Soln.

$$W(x) = \begin{vmatrix} \sin x & \sin x - \cos x \\ \cos x & \cos x + \sin x \end{vmatrix} \Rightarrow \sin x \cos x + \sin^2 x - \sin x \cos x + \cos^2 x$$

$$\Rightarrow \sin^2 x + \cos^2 x \Rightarrow 1 \neq 0$$

$\therefore$ Linearly independent

Ques 20 fxn. $f_1\ f_2\ f_3 \cdots f_n \to$ Linearly dependent $\quad W(x) = 0$

$W(x) = ??$

End term

$W(x) \to = 0$ Linearly dependent

$W(x) \to \neq 0$ Linearly independent.

a.) 0 ✓
b.) 1 ✗
c.) Non zero ✗
d.) NOT

$\therefore W(x) = 0$

Ques. Find followings are Linearly dependent or independent

	$y_1(x)$	$y_2(x)$	$y_3(x)$	Soln
①	x	$x e^x$	–	LI
②	x^2	$x^2 \log x$	–	LI
③	e^x	e^{-x}	e^{2x}	LI
④	e^{-x}	e^{3x}	e^{4x}	LI

⑤ $1 \quad x \quad x^2$ LI

⑥ $x \quad x^2 \quad x^3$ LI

⑦ $\sin x, \cos x, \sin x - \cos x$ LD

⑧ $e^{m_1 x}, e^{m_2 x}, e^{m_3 x}$ LI

⑨ $e^x \quad e^{-x} \quad \cos x$ LI

⑩ $1+x, 1+2x, x^2$ LI

⑪ $\sin x \quad \cos x \quad \sin 2x$ LI

⑫ $e^x \sin x, e^x \cos x$ LI.

ORDER.

① $\frac{\partial y}{\partial x} = \sin x \rightarrow 1$

② $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2 y}{\partial x^2} = 0 \rightarrow 2$

③ $k\left(\frac{\partial^2 y}{\partial x^2}\right) = \left\{1 + \left(\frac{\partial y}{\partial x}\right)^2\right\}^{3/2} \rightarrow 2$

Degree

① $\left(\frac{\partial y}{\partial x}\right)^1 = x + \sin x \rightarrow 1$

② $y = \sqrt{x}\left(\frac{dy}{dx}\right)^1 + \frac{k}{\left(\frac{dy}{dx}\right)} \rightarrow 1$

③ $y = x\left(\frac{dy}{dx}\right) + a\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/2} \rightarrow 2$

④ $\left(\left(\frac{\partial^2 y}{\partial x^2}\right)^{1/3}\right)^6 = \left(\left(y + \frac{dy}{dx}\right)^{1/2}\right)^6 \rightarrow 2$

$\Rightarrow \left(\frac{\partial^2 y}{\partial x^2}\right)^2 = \left(y + \frac{d^1 y}{dx}\right)^3$

* Methods of Solutions of Linear differential eq^n *

① Operator Method

$D \leftrightarrow$ operator $\leftrightarrow \frac{\partial}{\partial x}$

$Dy \longleftrightarrow \frac{\partial y}{\partial x}$

$\frac{\partial^2 y}{\partial x} \Rightarrow D^2 y$

$\Rightarrow D^2 y \longleftrightarrow \frac{\partial^2 y}{\partial x^2}$

$\Rightarrow \vdots \quad \vdots \quad \vdots$

$\Rightarrow D^n y \longleftrightarrow \frac{\partial^n y}{\partial x^n}$

Structures

Eg $\frac{\partial^2 y}{\partial x^2} + \frac{\partial y}{\partial x} + y = 0$ (simple Que.)

$\rightarrow D^2 y + Dy + y = 0$

$\rightarrow (D^2 + D + 1) y = 0$ $(D \rightarrow M) \Rightarrow \boxed{m^2 + m + 1 = 0}$

auxillary eq^n

Now solve

* Eq^n in terms of D $\rightarrow$ operator eq^n

* operator eq^n in terms of m $\rightarrow$ auxillary eq^n

Q. $\frac{\partial^3 y}{\partial x^3} + \frac{x\,\partial^2 y}{\partial x^2} + y = \sin x$

$\rightarrow$ $D^3y + xD^2y + y = \sin x$

$\rightarrow$ $(D^3 + xD^2 + 1)y = \sin x$

$\rightarrow$ $\boxed{m^3 + xm^2 + 1 = 0}$ $(D \rightarrow m)$

TYPES

1. homogeneous (RHS = 0)
2. Non homogeneous (RHS ≠ 0)

Homogeneous

* Nature of roots. $(m_1, m_2, m_3 \ldots)$

1. Both roots are diff. $\boxed{m_1 \neq m_2}$

2. Both roots are same $\boxed{m_1 = m_2}$

3. Roots are imaginary

$m_1 = a + ib$

$m_2 = a - ib$

$\rightarrow$ $\boxed{a + ib}$

TYPE ① $(m_1 \neq m_2)$

(when both roots are diff) Solⁿ → $y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \cdots$

Ques $\frac{\partial^2 y}{\partial x^2} + \frac{5\,\partial y}{\partial x} + 6y = 0$

Solⁿ $D^2 y + 5Dy + 6y = 0$

$\Rightarrow (D^2 + 5D + 6)y = 0$

$\Rightarrow D^2 + 5D + 6 = 0$

$\Rightarrow m^2 + 5m + 6 = 0$

$\Rightarrow m^2 + 2m + 3m + 6 = 0$

$\Rightarrow m(m+2) + 3(m+2) = 0$

$\Rightarrow (m+2)(m+3) = 0$

$\Rightarrow \boxed{m_1 = -2}$ & $\boxed{m_2 = -3}$

Since, both roots are diff.

$\therefore \boxed{y = c_1 e^{-2x} + c_2 e^{-3x}}$

Ques. $\frac{\partial^3 y}{\partial x^3} + 6\left(\frac{\partial^2 y}{\partial x^2}\right) + 11\frac{\partial y}{\partial x} + 6y = 0$

Solⁿ. $D^3 y + 6D^2 y + 11Dy + 6y = 0$

$\Rightarrow m^3 + 6m^2 + 11m + 6 = 0$

$\Rightarrow (m+1)(m+2)(m+3) = 0$

$\Rightarrow m_1 = -1 \; ; \; m_2 = -2 \; ; \; m_3 = -3$

Since, All roots are diff.

$\therefore y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{-3x}$.

Ans.

TYPE ② ($m_1 = m_2$) when both roots are same

Ques. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

Soln. $\therefore D^2y + 4Dy + 4y = 0$

$(D^2 + 4D + 4)y = 0$

$\Rightarrow m^2 + 4m + 4 = 0$

$\Rightarrow (m+2)^2 = 0$

$\therefore \boxed{m_1 = m_2 = -2}$

Since, both roots are equal

$\therefore y = (c_1 + c_2 x)e^{-2x}$

Ans.

Ques $4y'' + 4y' + y = 0$

Soln. $(4D^2 + 4D + 1)y = 0$

$4m^2 + 4m + 1 = 0$

$\Rightarrow (2m+1)^2 = 0$

$(2m+1)(2m+1) = 0$

$\therefore \boxed{m_1 = m_2 = -1/2}$

Since, both roots are equal,

$\therefore y = (c_1 + c_2 x)e^{-1/2 x}$

Ans.

$$y = (c_1 + c_2 x + c_3 x^2 + c_4 x^3 + \dots)e^{mx}$$

TYPE ③ roots are imaginary ($m_1 = a + ib$; $m_2 = a - ib$)

$$y = e^{ax}\left[c_1 \cos bx + c_2 \sin bx\right]$$

Ques. $y'' + 2y' + 2y = 0$

Sol^n $D^2y + 2Dy + 2y = 0$

$\Rightarrow m^2 + 2m + 2 = 0$

$\Rightarrow m = \dfrac{-2 \pm \sqrt{4-8}}{2}$

$\Rightarrow m = \dfrac{-2 \pm \sqrt{-4}}{2}$

$\Rightarrow m = \dfrac{-2 \pm 2i}{2}$

$\Rightarrow \boxed{m = -1 \pm i}$

Since, roots are imaginary.

$\therefore y = e^{-x}\left[c_1 \cos x + c_2 \sin(x)\right]$

Ans.

Ques. $y'' + 4y' + 13y = 0$

Sol^n $D^2y + 4Dy + 13y = 0$

$\Rightarrow m^2 + 4m + 13 = 0$

$4 \times 1 \times 13 = 52$

$m = \dfrac{-4 \pm \sqrt{16 - 52}}{2}$

$= \dfrac{-4 \pm \sqrt{-36}}{2}$

$= \dfrac{-4 \pm 6i}{2}$

$= \boxed{-2 \pm 3i}$

$\sqrt{-1} = i$

$\boxed{m = -2 \pm 3i}$

Since, roots are imaginary =

$y = e^{-2x}\left[c_1 \cos 3x + c_2 \sin 3x\right]$

Ans.

Ques $4y'' - 8y' + 3y = 0$, $y(0) = 1$; $y'(0) = 3$.

Soln. $4D^2y - 8Dy + 3y = 0$

$\Rightarrow 4m^2 - 8m + 3 = 0$

$\Rightarrow 4m^2 - 6m - 2m + 3 = 0$

$\Rightarrow 2m(2m-3) - 1(2m-3) = 0$

$\Rightarrow (2m-3)(2m-1) = 0$

$\Rightarrow \boxed{m_1 = 3/2 \quad \& \quad m_2 = 1/2}$

Since, All roots are diff.

$\therefore \quad y = c_1 e^{3/2 x} + c_2 e^{1/2 x} \quad -①$

Now, A/q

- $y(0) = 1 \Rightarrow c_1 + c_2 \quad -(ii)$
- $y'(0) = 3 \Rightarrow c_1 \frac{3}{2} e^{3/2 x} + c_2 \frac{1e}{2}^{1/2 x} \quad -(iii)$

$\Rightarrow \frac{3}{2} c_1 + \frac{c_2}{2}$.

$\therefore$ from eqn (ii) & (iii) we get.

$$c_1 + c_2 = 1$$
$$\pm 3c_1 \pm c_2 = \_6$$
$$-2c_1 + 0 = -5$$

$\Rightarrow \boxed{c_1 = 5/2}$

$\frac{5}{2} + c_2 = 1$

$\Rightarrow c_2 = 1 - \frac{5}{2}$

$\boxed{c_2 = \frac{-3}{2}}$

put the values of c_1 & c_2 in eqn ①

$$\boxed{y = \frac{5}{2} e^{\frac{3}{2}x} + \left(\frac{-3}{2}\right) e^{\frac{1}{2}x}}$$

Ans.

CO1, L1

Q16) Find the solution to $9y'' + 6y' + y = 0$ for $y(0) = 4$ and $y'(0) = -1/3$.

(a) $y = (4 + x)e^{-x/3}$ (b) $y = (4 - x)e^{-x/3}$ (c) $y = (8 - 2x)e^{x/3}$ (d) $y = (1 - x)e^{-x/3}$

CO2, L2

Q17) Find the solution to $y'' - y = 0$.

(a) $y = c_1e^{x} - c_2e^{x}$ (b) $y = c_1(e^{x} + e^{-x})$ (c) $y = c_1e^{x} + c_2e^{-x}$ (d) $y = c_1e^{x} - c_2e^{-x}$

CO2, L2

Q18) Complementary Function of differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ is

(a) $y = e^{-x}(\cos x + \sin x)$ (b) $y = c_1e^{x}\cos(x + c_2)$ (c) $y = c_1\cos x + c_2\sin x$

(d) $y = e^{-x}(c_1\cos x + c_2\sin x)$

CO2, L2

If one root of the auxiliary equation is in the form $\alpha + i\beta$, where α, β *are real and* $\beta \neq 0$ then complementary part of solution of differential equation is

Q19) If one root of the auxiliary equation is in the form $\alpha + i\beta$, where α, β *are real and* $\beta \neq 0$ then complementary part of solution of differential equation is

(a) $e^{\alpha x}(c_1 \cos \alpha x + c_2 \sin \alpha x)$ (b) $e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$ (c) $e^{\alpha x}(c_1 \cos \alpha x + c_2 \sin \beta x)$

(d) $e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \alpha x)$ CO2, L2

Q20) The functions $f_1, f_2, f_3, \dots . f_n$ are said to be linearly dependent if Wronskian of the functions $W(f_1, f_2, f_3, \dots . f_n) =$

END TERM (PYQ.s)

Q16. $9y'' + 6y' + y = 0$ $y(0) = 4$, $y'(0) = -1/3$

Sol^n.

$9m^2 + 6m + 1 = 0$

$\Rightarrow (3m+1)^2 = 0$

$\Rightarrow \boxed{m_1 = m_2 = -1/3}$

$y = (c_1 + c_2 x) e^{-1/3 x}$ — ①

- $y(0) = 4 \Rightarrow c_1 \Rightarrow \boxed{c_1 = 4}$
- $y'(0) = -1/3 \Rightarrow c_1\left(\frac{-1}{3}\right) e^{-1/3x} + c_2 e^{-1/3x} \cdot 1 + c_2 x \left(\frac{-1}{3}\right) e^{-1/3x}$

$\Rightarrow \frac{-c_1}{3} + c_2 + 0 = -\frac{1}{3}$

$\Rightarrow \frac{-4}{3} + c_2 = -1/3$

$\Rightarrow c_2 = \frac{4}{3} - \frac{1}{3} = 1$ $\boxed{c_2 = 1}$

$\therefore \boxed{y = (4 + x) e^{-1/3x}}$ **(a)** ✓

Q17 $y'' - y = 0$

Sol^n. $m^2 - 1 = 0 \Rightarrow \boxed{m = \pm 1}$.

$\boxed{y = c_1 e^{x} + c_2 e^{-x}}$ **(c)**

Q18. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ $y = ??$

Sol^n $m^2 + 2m + 2 = 0$ $m = \frac{-2 \pm \sqrt{4-8}}{2}$

$\boxed{m = -1 \pm i}$

$y = e^{-x}[c_1 \cos x + c_2 \sin x]$ Ans.

(d).

Q19. $\alpha + i\beta$. α $\boxed{a \pm ib}$

Sol^n. $\Rightarrow \alpha - i\beta$

$y = e^{\alpha x}[c_1 \cos \beta x + c_2 \sin \beta x]$

(b)

* Some more Questions practise *

Ques. $y^{IV} - y''' - 9y'' - 11y' - 4y = 0$

Sol^n. $m^4 - m^3 - 9m^2 - 11m - 4 = 0$

$\Rightarrow (m+1)(m^3 - 2m^2 - 7m - 4) = 0$

$\Rightarrow (m+1)(m+1)(m^2 - 3m - 4) = 0$

$\Rightarrow (m+1)(m+1)(m+1)(m-4) = 0$

$\therefore \boxed{m_1 = m_2 = m_3 = -1 \ \& \ m_4 = 4}$

$\therefore y = (c_1 + c_2 x + c_3 x^2)e^{-x} + c_4 e^{4x}$ **Ans.**

(*) $m+1 \,)\, m^4 - m^3 - 9m^2 - 11m - 4 = 0 \,(\, m^3 - 2m^2 - 7m - 4$

$\pm m^4 \pm m^3$

$-2m^3 - 9m^2 - 11m - 4$

$\mp 2m^3 \mp 2m^2$

$-7m^2 - 11m - 4$

$\mp 7m^2 \mp 7m$

$-4m - 4$

$\mp 4m \mp 4$

0

(*) $m+1 \,)\, m^3 - 2m^2 - 7m - 4 \,(\, m^2 - 3m - 4$

$\pm m^3 \pm m^2$

$-3m^2 - 7m - 4$

$\mp 3m^2 \mp 3m$

$-4m - 4$

$\mp 4m \mp 4$

0

(*) $m+1 \,)\, m^2 - 3m - 4 \,(\, m - 4$

$\pm m^2 \pm m$

$-4m - 4$

$\mp 4m \mp 4$

0

* Questions Practise.

1. $y'' + 6y' + 9y = 0 \rightarrow (c_1 + c_2 x) e^{-3x}$
2. $y''' - 3y'' + 3y' + y = 0 \rightarrow (c_1 + c_2 x + c_3 x^2) e^{-x}$
3. $9y'' + 12y' + 4y = 0 \rightarrow (c_1 + c_2 x) e^{-2/3 x}$
4. $y'' - y' - 6y = 0 \rightarrow c_1 e^{-2x} + c_2 e^{-3x}$
5. $4y'' - 8y' + 3y = 0 \rightarrow c_1 e^{1/2 x} + c_2 e^{3/2 x}$
6. $y'' - 5y' + 6y = 0 \rightarrow c_1 e^{2x} + c_2 e^{3x}$
7. $y''' + 2y'' - 15y' = 0 \rightarrow c_1 + c_2 e^{3x} + c_3 e^{-5x}$
8. $y^{IV} - y''' - 9y'' - 11y' - 4y = 0 \rightarrow c_1 e^{4x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$
9. $y^{IV} + 2y''' - 3y'' - 4y' + 4y = 0 \rightarrow (c_1 + c_2 x) e^{x} + (c_3 + c_4 x) e^{-2x}$
10. $y'' + 2y' + 2y = 0 \rightarrow e^{-x} [c_1 \cos x + c_2 \sin x]$
11. $y'' + 4y' + 13y = 0 \rightarrow e^{-2x} [c_1 \cos 3x + c_2 \sin 3x]$
12. $y^{IV} - 2y''' + 2y'' - 2y' + y = 0 \rightarrow (c_1 + c_2 x) e^{x} + e^{0x} [c_3 \cos x + c_4 \sin x]$

(13) $4y''' - 4y'' - 9y' + 9y = 0$, $y(0) = 1$, $y'(0) = 0$ $y''(0) = 0$

(14) $y''' - 3y' - 2y = 0$ $(c_1 + c_2 x)e^{-x} + c_3 e^{2x}$ $\frac{9}{5} e^{x} - e^{\frac{3}{2}x} + \frac{1}{5} e^{-3/2 x}$

(15) $8y''' - 12y'' + 6y' - y = 0$ $(c_1 + c_2 x + c_3 x^2) e^{x/2}$

(16) $y''' + 3y'' - 4y = 0$, $y(0) = 1$ $y'(0) = 0$ $y''(0) = 1/2$ $\frac{1}{2} e^{x} + (\frac{1}{2} + \frac{x}{2}) e^{-2x}$

(17) $y^{iv} + 32y'' + 256y = 0$ $(c_1 + x c_2) \cos 4x + (c_3 + x c_4) \sin 4x$

(18) $y^{iv} + y''' = 0$ $y(0) = 1$, $y'(0) = 2$, $y''(0) = -1$

$y'''(0) = -1$ $(x + \cos x + \sin x)$

(19) $y''' + y = 0$ $c_1 e^{-x} + e^{-x/2} [c_3 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x]$

(20) $y'' + y' + 5y = 0$ $e^{-x/2} [c_1 \cos \sqrt{19} x + c_2 \sin \sqrt{19} x]$

mber	Question Statement
	a) Solve by operator method $$y''' + \ y = 0$$

ic Task Number: _02 Course code: MTH174 MARKS 30 SE

r	Question Statement
	a) Solve by operator method $$3y''' - 2y'' - 3y' + 2y = 0$$

Q(14) The differential equation of the form $y'' + a(x)y' + b(x)y = 0$ for which the functions e^{3x}, e^{-2x} are solutions is

(a) $y'' + 5y' + 6y = 0$ (b) $y'' + y - 6y = 0$ (c) $y'' + y' + 6y = 0$ (d) $y'' - y' - 6y = 0$

Q(15) The general solution of the differential equation $y'' + 25y = 0$ is

(a) $\cos 5x + \sin 5x$

(b) $2\cos 5x + 3\sin 5x$

(c) $\sin 5x - \cos 5x$

(d) $A\cos 5x + B\sin 5x$, A and B are arbitrary constants

Q(18) The general solution of the differential equation $y^{iv} - 3y''' + 3y'' - y' = 0$ is

(a) $(c_1 + c_2x + c_3x^2 + c_4x^3)e^x$

(b) $(c_1 + c_2x)e^x + c_3e^{3x} + c_4$

(c) $c_1 + c_2e^x + c_3e^{2x} + c_4e^{3x}$

(d) $(c_1 + c_2x + c_3x^2)e^x + c_4$

CA PYQs.

Ques. $y''' + y = 0$

Sol^n. We can write given questions in operator method as :-

$D^3 y + y = 0$

$\Rightarrow \quad m^3 + 1 = 0$

$\Rightarrow \quad m_1 = -1 \ \& \ m_2 = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} i$

$\therefore$ All roots are different & imaginary

$\therefore \quad y = c_1 e^{-x} + e^{\frac{-1}{2}x}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right]$

Ans.

Ques. $3y''' - 2y'' - 3y' + 2y = 0$

Sol^n. $3D^3 y - 2D^2 y - 3Dy + 2y = 0$

$\Rightarrow \quad (D-1)(D+1)(3D-2) = 0$

$D = 1$, $D = -1$ & $D = 2/3$

$\therefore$ All roots are different

$\therefore \quad \boxed{y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2/3 x}}$

Q14. e^{3x} e^{-2x} $\Rightarrow$ roots $\to 3 \,|\, -2$.

a.) $D^2 + 5D + 6 = 0$ c.) $D^2 + D + 6 = 0$

b.) $D^2 + D - 6 = 0$ d.) $D^2 - D - 6 = 0$

(d.)

(18) $y^{iv} - 3y''' + 3y'' - y' = 0$

Sol^n $D^4 - 3D^3 + 3D^2 - D = 0$

$D(D^3 - 3D^2 + 3D - 1) = 0$

$D = 0, 1, 1, 1$

$\Rightarrow \therefore \quad (c_1 + c_2 x + c_3 x^2) e^{x} + c_4 e^{0}$

a.) $(c_1 + c_2 x + c_3 x^2 + c_4 x^3) e^{x}$

b.) $(c_1 + c_2 x) e^{x} + c_3 e^{3x} + c_4$

c.) $c_1 + c_2 e^{x} + c_3 e^{2x} + c_4 e^{3x}$

d.) $(c_1 + c_2 x + c_3 x^2) e^{x} + c_4$

Q15. $D^2 + 25D = 0$

a.) $\cos 5x + \sin 5x$ c.) $\sin 5x - \cos 5x$

b.) $2\cos 5x + 3\sin 5x$ d.) $A\cos 5x + B\sin 5x$.

$A, B \to$ const.

What is wronskian of 1, e^x?

(b) $1+e^x$ (c) e^x (d) None of the above [redacted]) The solution $y_1(x)$ & $y_2(x)$ are said to be L.I. if ......

[redacted] $W(y_1, y_2) = 1$ (b) $W(y_1, y_2) = 0$ (c) $W(y_1, y_2) \neq 0$ (d) None of the above CO2,L2

[redacted] Solution of $x^2y'' - 2.5xy' - 2y = 0$ is .....

Q(20) If $\{e^x, e^{4x}\}$ form the basis of the equation $y'' - 5y' + 4y = 0,\ y(0) = 2,\ y'(0) = 1,$ then the solution is CO2,L2

(a) $\frac{e^x - 7e^{4x}}{3}$ (b) $\frac{4e^{4x} - e^x}{3}$ (c) $\frac{e^{4x} + 7e^x}{3}$ (d) $\frac{7e^x - e^{4x}}{3}$

Q.25. $1, e^x$ a.) b.) $1+e^x$ c.) e^x d.) NOT

$\begin{vmatrix} 1 & e^x \\ 0 & e^x \end{vmatrix}$ $e^x - 0 = e^x$.

Q.26 $y_1(x)$ & $y_2(x)$ L.I ?? a.) $W(y_1, y_2) = 1$ b.) $W(y_1, y_2) = 0$

=> c.) $W(y_1, y_2) \neq 0$ d.) NOT

(20) $y'' - 5y' + 4y = 0$ $y(0) = 2$, $y'(0) = 1$

a.) $\frac{e^x - 7e^{4x}}{3}$ b.) $\frac{4e^{4x} - e^x}{3}$

c.) $\frac{e^{4x} + 7e^x}{3}$ d.) $\frac{7e^x - e^{4x}}{3}$

Soln. $D^2 - 5D + 4 = 0$

$D^2 - 4D - D + 4 = 0$

$(D-4)(D-1) = 0$ $D = 4 | 1$

$\boxed{y_c \to c_1 e^{4x} + c_2 e^x.}$

A/Q. $2 = c_1 + c_2$

$1 = 4c_1 + c_2$

$1 = -3c_1$ => $\boxed{c_1 = -1/3}$ $\boxed{c_2 = 7/3}$

$\therefore y_c = \frac{-1}{3} e^{4x} + \frac{7}{3} e^x$

$\frac{7e^x - e^{4x}}{3}$ <= $\frac{-e^{4x} + 7e^x}{3}$ (d)