Chapter 9
Morphological Image Processing

- Used to extract image components that are useful in the representation and description of region shape, such as
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning

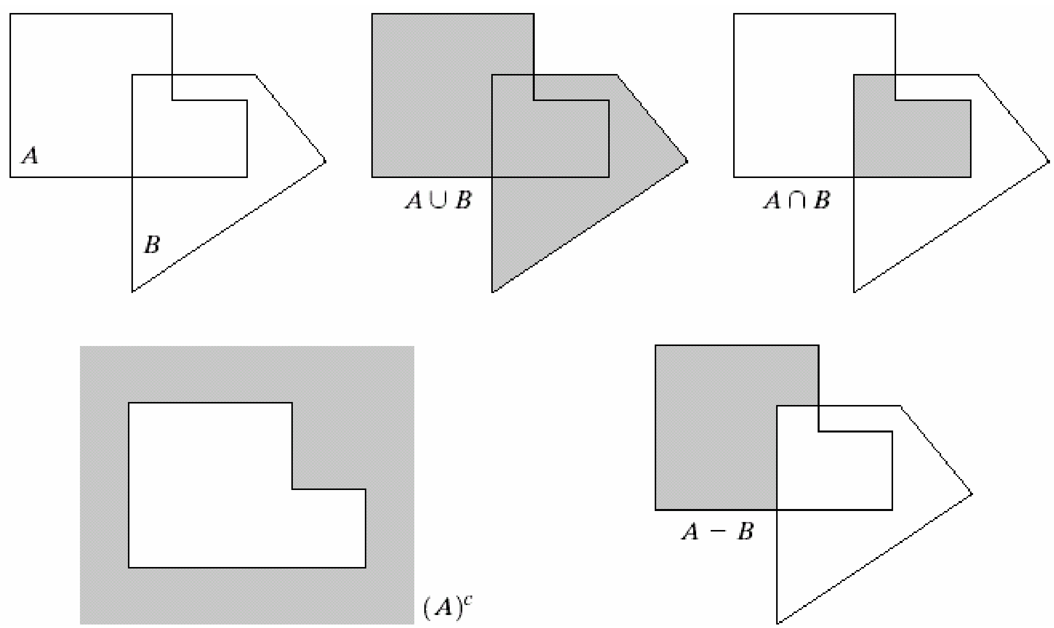
- Sets in mathematic morphology represent objects in an image:
  - binary image (0 = white, 1 = black) : the element of the set is the coordinates (x,y) of pixel belong to the object Z^2
  - gray-scaled image : the element of the set is the coordinates (x,y) of pixel belong to the object and the gray levels ⇨ Z^3

Chapter 9
Morphological Image Processing

- Preliminary:
  - A: A set in Z^2 with elements of $\mathbf{a}=(a_1,a_2)$
  - $a \in A$
  - $a \notin A$
  - $A \subseteq B$
  - $C = A \bigcup B$
  - $D = A \bigcup B$
  - $A \bigcap B = \varnothing$
  - $A^c = \{ w | w \notin A \}$
  - $A - B = \{ w | w \in A, w \notin B \} = A \bigcap B^c$

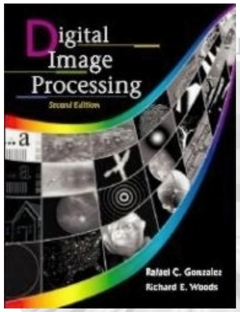
Chapter 9
Morphological Image Processing

- Operators by examples:



a b c
d e

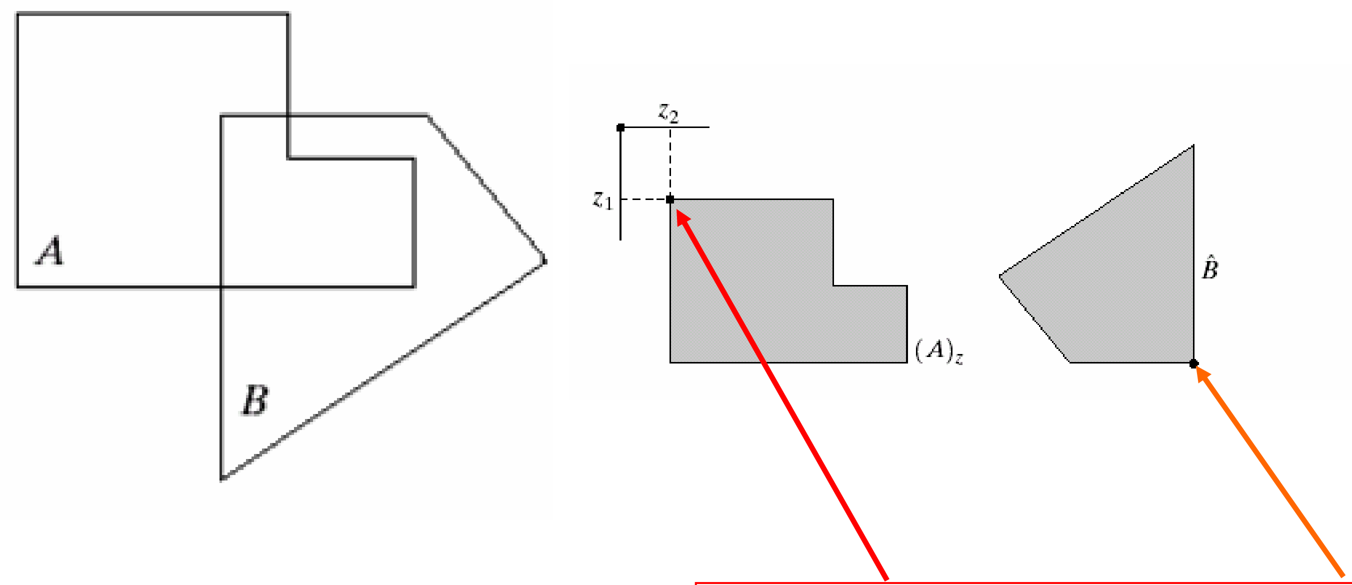
FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.



- Preliminary:
  - A: A set in Z^2 with elements of $\mathbf{a}=(a_1,a_2)$
  - $\hat{B} \in \{ w \mid w = -b, \quad \text{for} \quad b \in B \}, \quad \text{Reflection}$
  - $(A)_z \in \{ c \mid c = a + z, \quad \text{for} \quad a \in A \}, \quad \text{Translation}$

Chapter 9
Morphological Image Processing

- Reflection and Translation by examples:
  - Need for a reference point.



a b

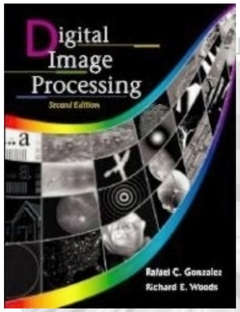
FIGURE 9.2 (a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.

Reference point

- Basic Logic Operations:

TABLE 9.1 The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0



Chapter 9
Morphological Image Processing

- Examples

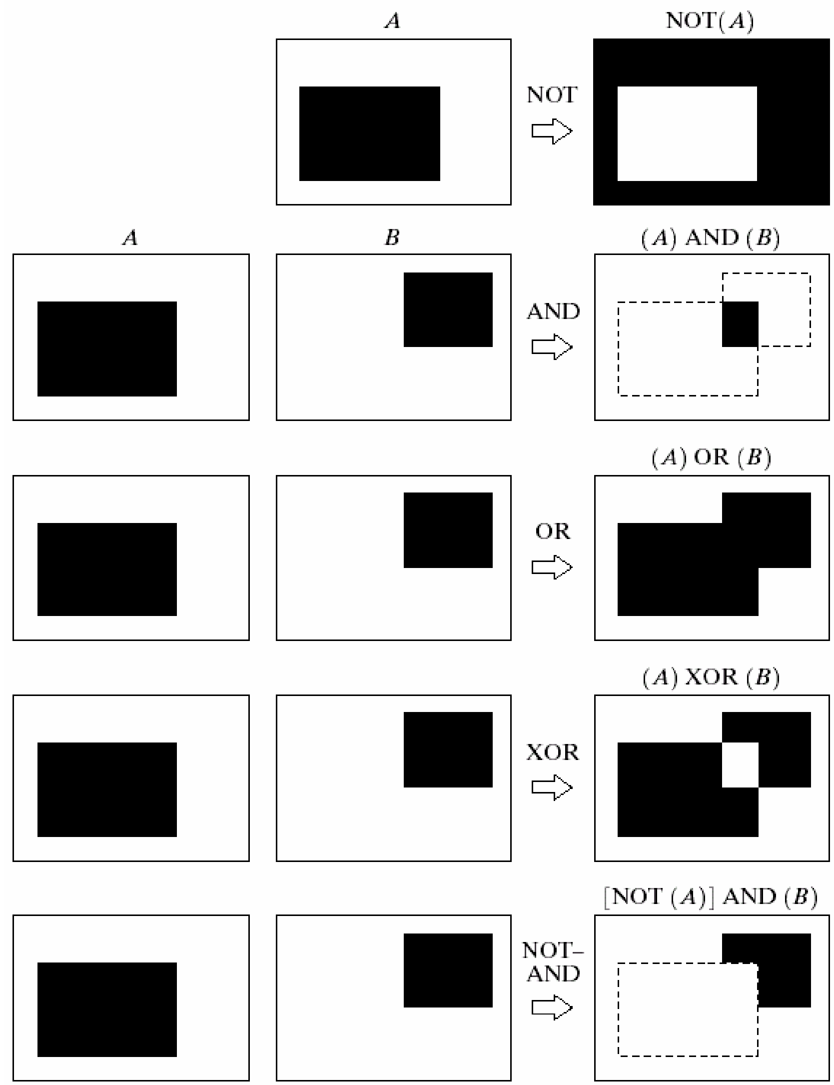


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

- Two Fundamental Morphological Operators:
  - Dilation:

$$A \oplus B = \left\{ z \middle| \left(\hat{B}\right)_z \cap A \neq \varnothing \right\} = \left\{ z \middle| \left(\hat{B}\right)_z \cap A \subseteq A \right\}$$

    - Set B: A structural elements.
    - Relation to Convolution mask:
      - Flipping
      - Overlapping
    - Other names: Grow, Expand

- Dilation by example:

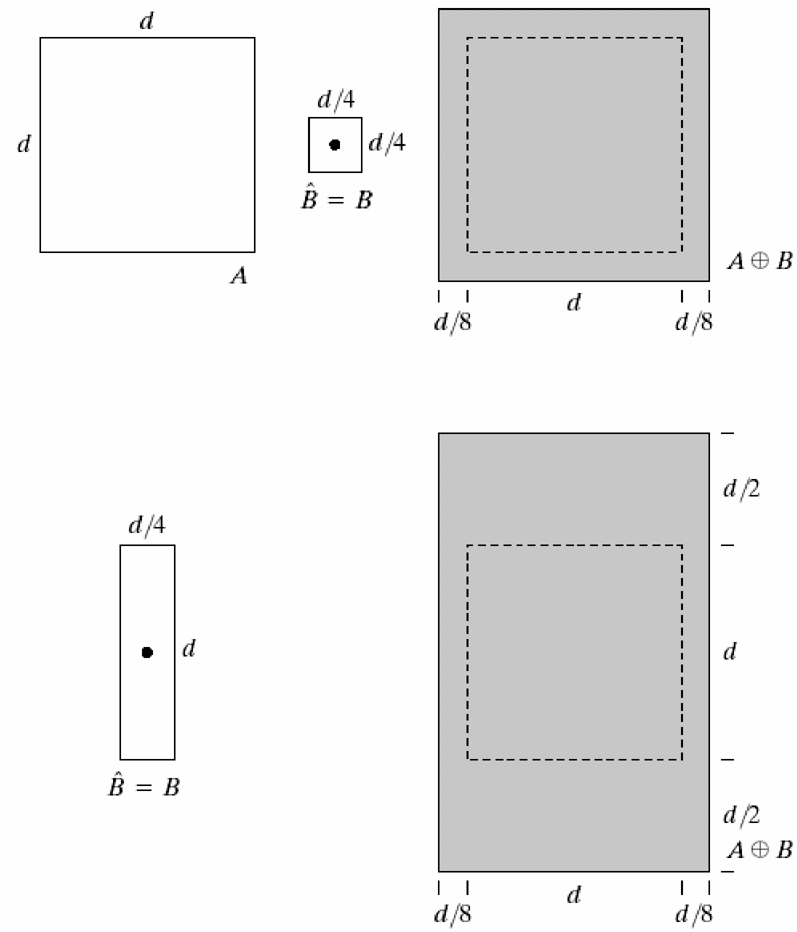
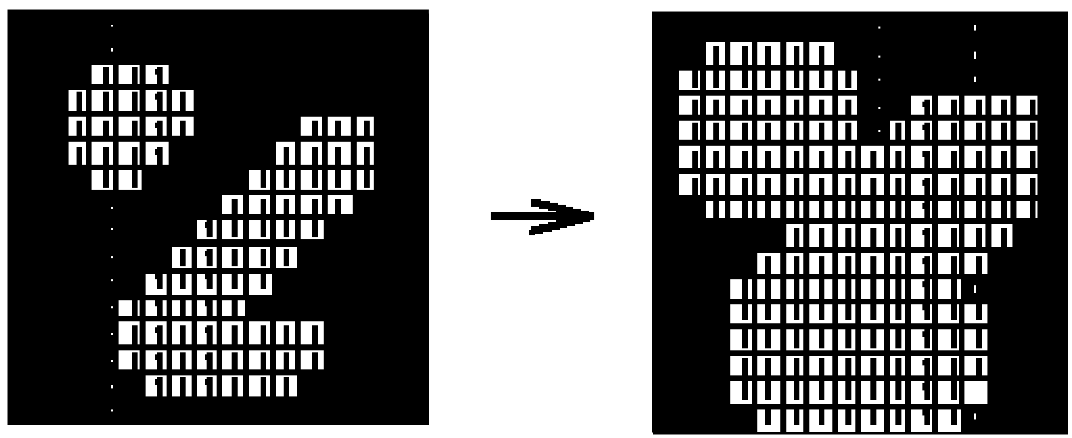


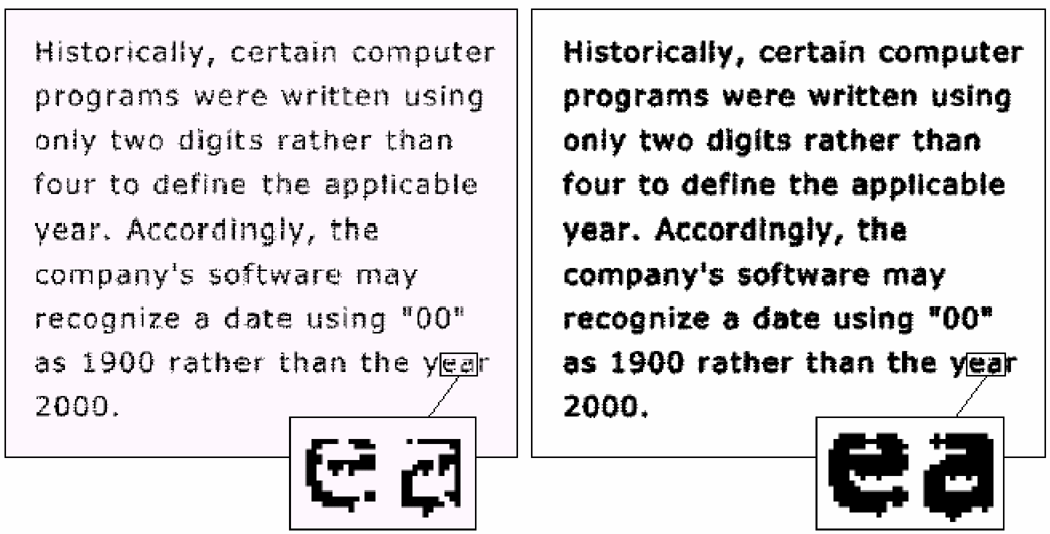
FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.

- Dilation by example:
  - B: a 3×3 mask.



Chapter 9
Morphological Image Processing

- Application:
  - Gap filling


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

0	1	0
1	1	1
0	1	0

a c
b

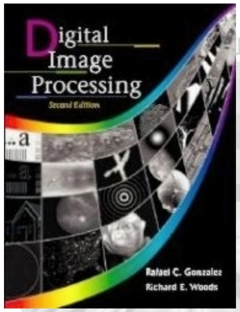
FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Chapter 9
Morphological Image Processing

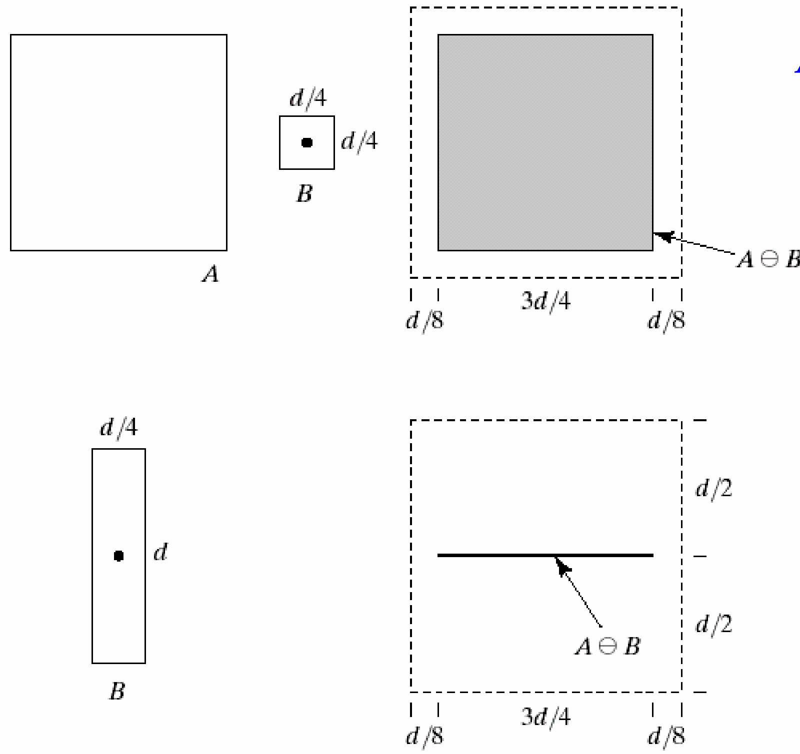
- Two Fundamental Morphological Operators:
  - Erosion:

$$A \odot B = \left\{ z \middle| \left(B \right)_z \subseteq A \right\}$$

    - Set B: A structural elements.
    - Other banes: Shrink, Reduce



Chapter 9
Morphological Image Processing

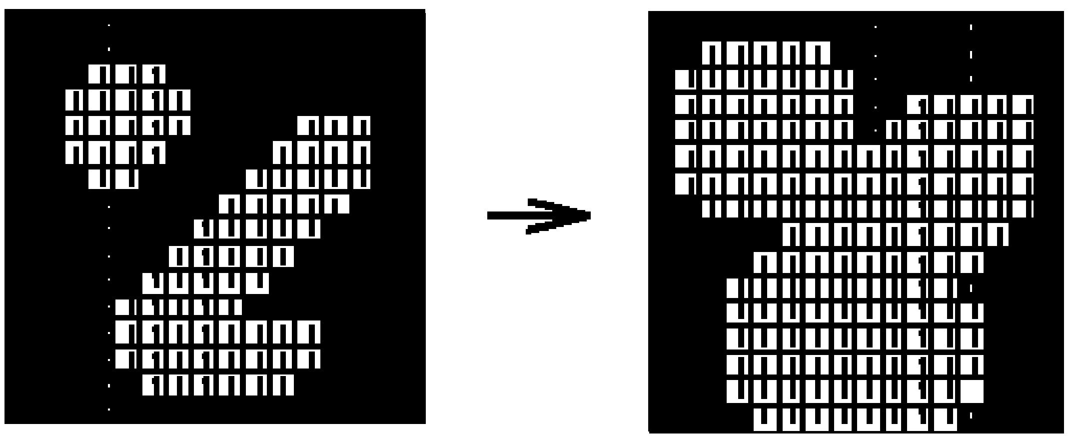


$$A \odot B = \left\{ z \middle| (B)_z \subseteq A \right\}$$

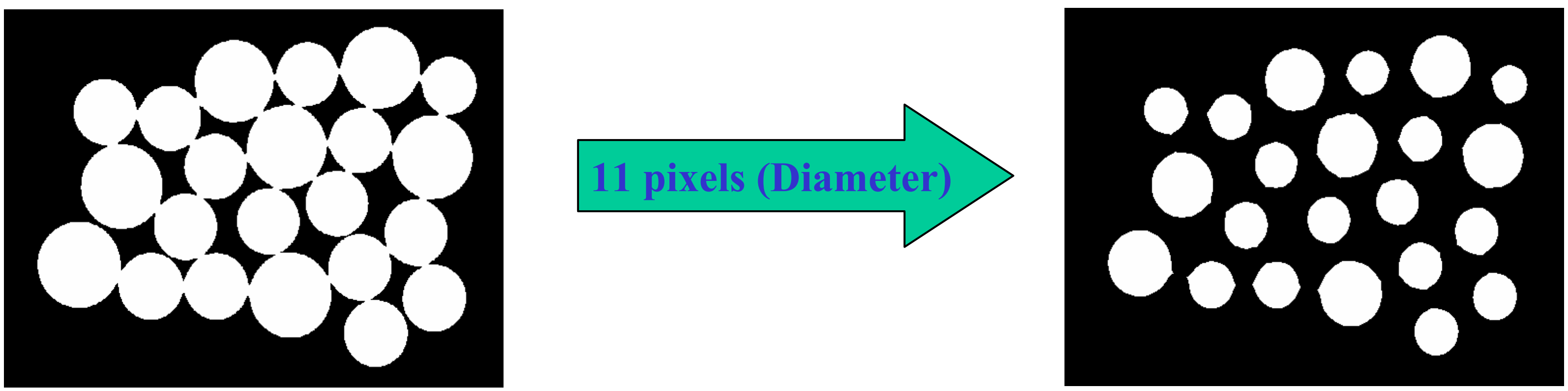
a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

- Erosion by example:
  - B: a 3×3 mask.



- Erosion by example:



Chapter 9
Morphological Image Processing

- Dilation-Erosion Duality:

$$(A \ominus B)^c = \left\{ z \middle| (B)_z \subseteq A \right\}^c = \left\{ z \middle| (B)_z \cap A^c = \varnothing \right\}^c$$

$$= \left\{ z \middle| (B)_z \cap A^c \neq \varnothing \right\} = A^c \oplus \hat{B}$$

Remember: $A \oplus B = \left\{ z \middle| \left(\hat{B}\right)_z \cap A \neq \varnothing \right\}$

- Erosion Application:
  - Remove details

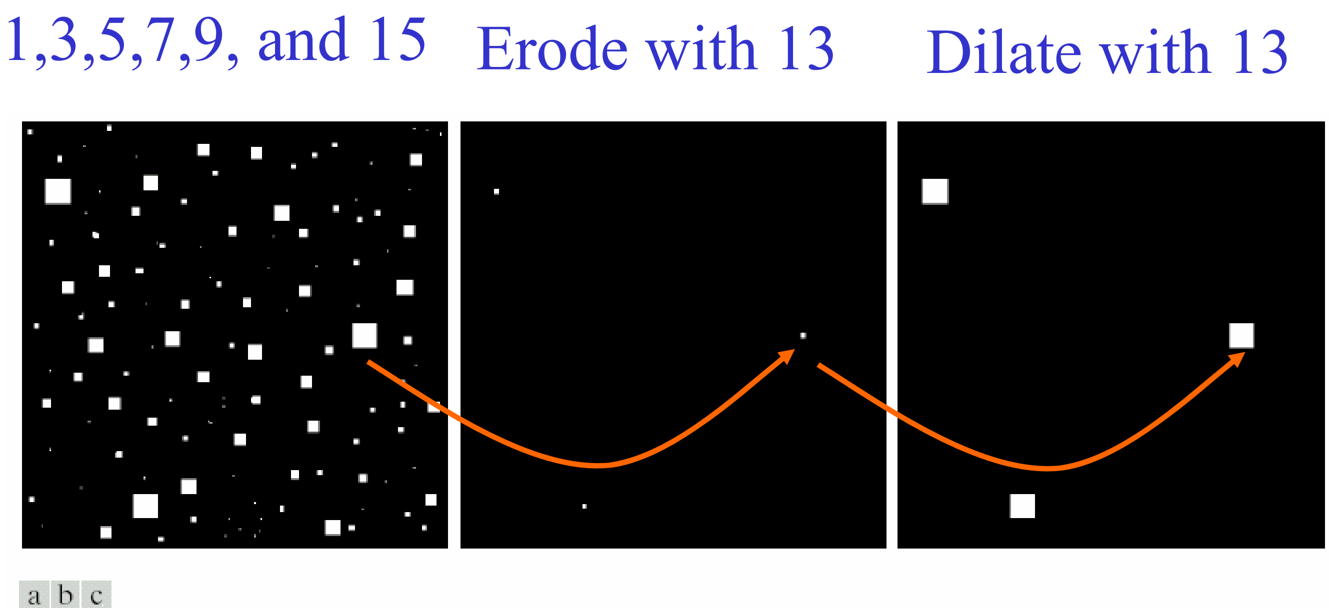


FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

- Opening and Closing:
  - Dilation expands and Erosion shrinks.
  - Opening:
    - Smooth contour
    - Break narrow isthmuses (Means: تنگه)
    - Remove thin protrusion
  - Closing:
    - Smooth contour
    - Fuse narrow breaks, and long thin gulfs.
    - Remove small holes, and fill gaps.

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Morphological Image Processing

- Opening and Closing:
  - Dilation expands and Erosion shrinks.
  - Opening:
    - A erosion followed by a dilation using the ***same structuring element*** for both operations.

$$A \circ B = (A \Theta B) \oplus B = \bigcup \{ (B_z) \mid (B_z) \subseteq A \}$$

  - Closing:
    - A Dilation followed by a erosion using the ***same structuring element*** for both operations.

$$A \circ B = (A \oplus B) \Theta B$$

- Opening Illustration:

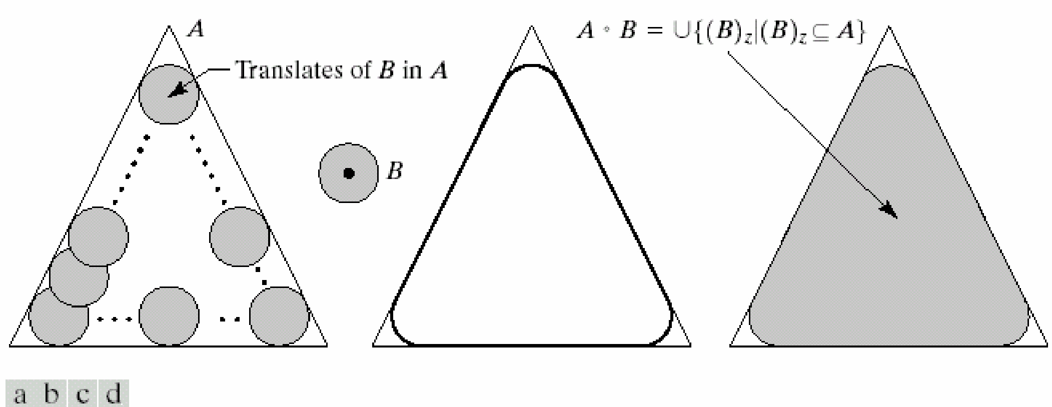

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

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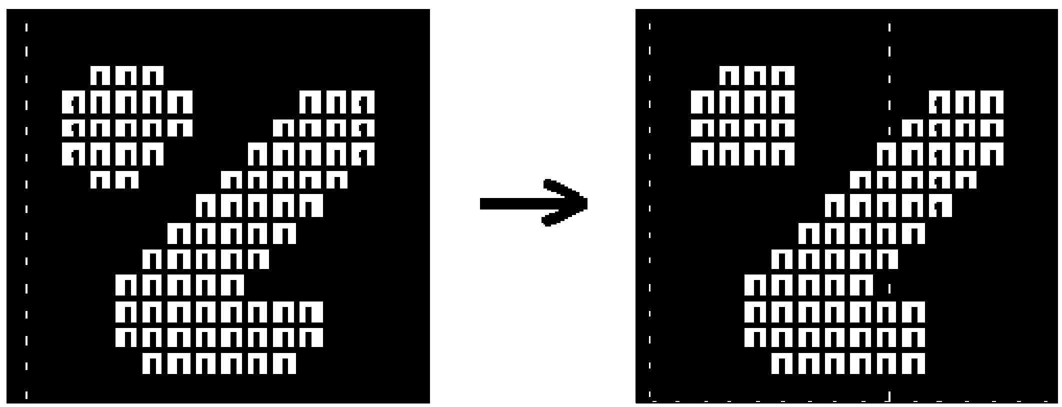
Morphological Image Processing

- Opening Illustration:



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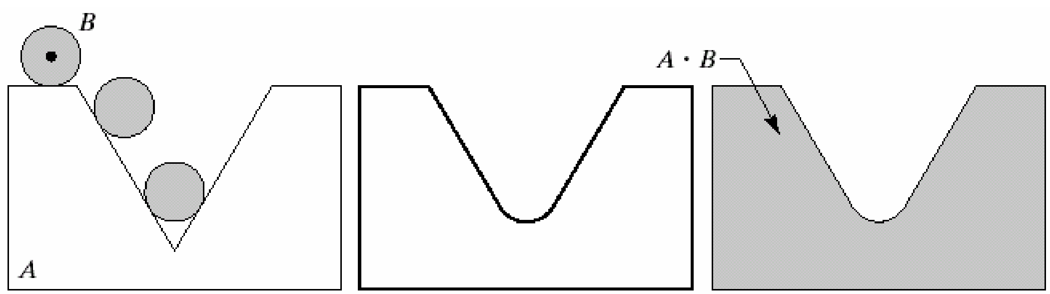
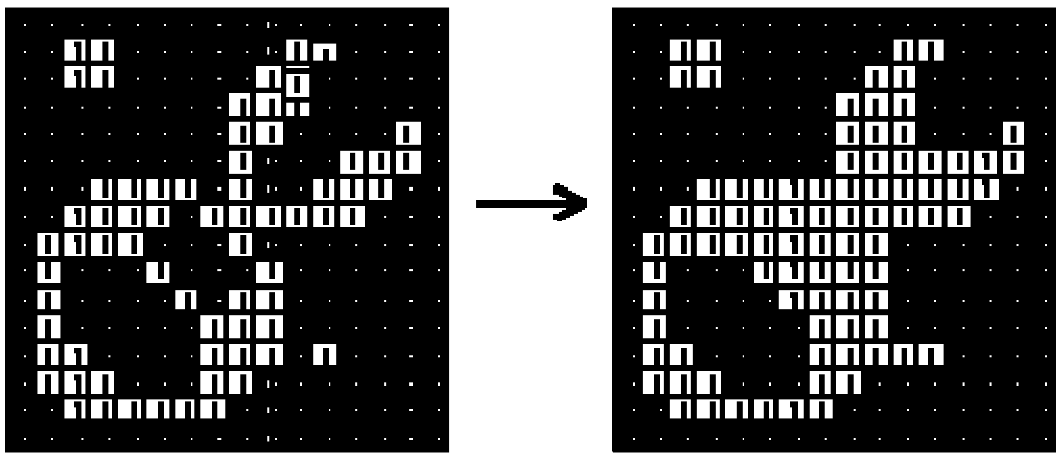
- Closing Illustration:



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

- Closing Illustration:



Chapter 9
Morphological Image Processing

- Opening and Closing

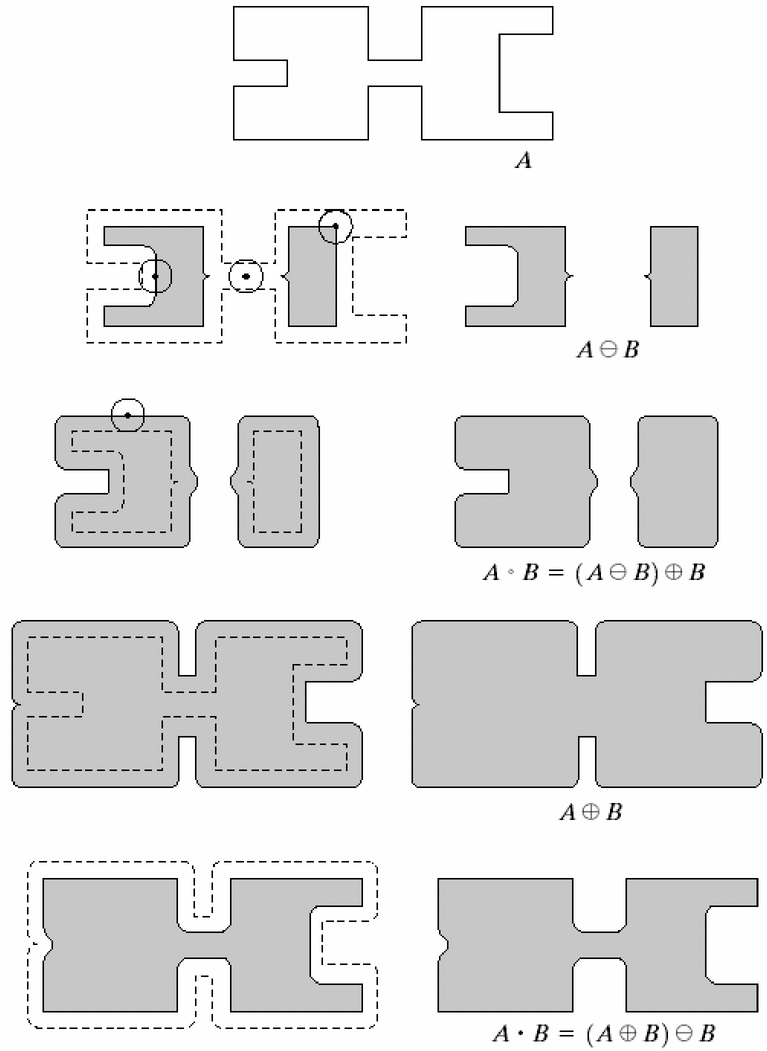


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

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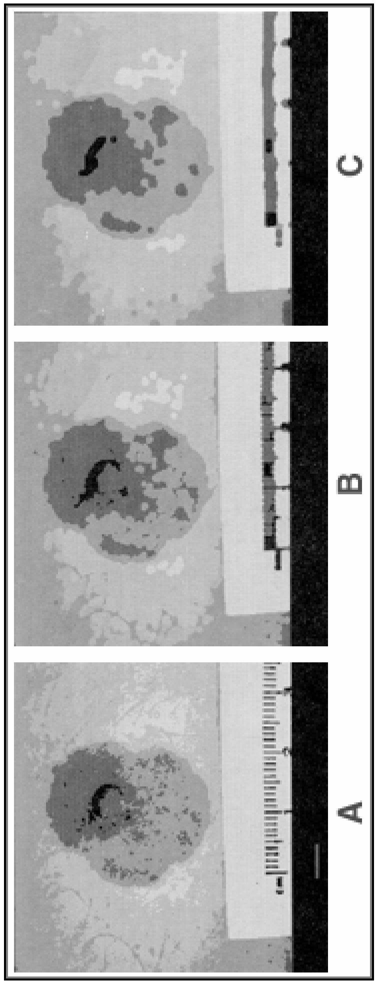
Morphological Image Processing

- Medical Application:

Closing 5×5

Opening 5×5

Original Segmentation

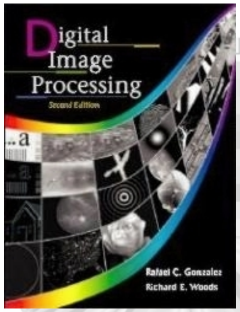


Chapter 9
Morphological Image Processing

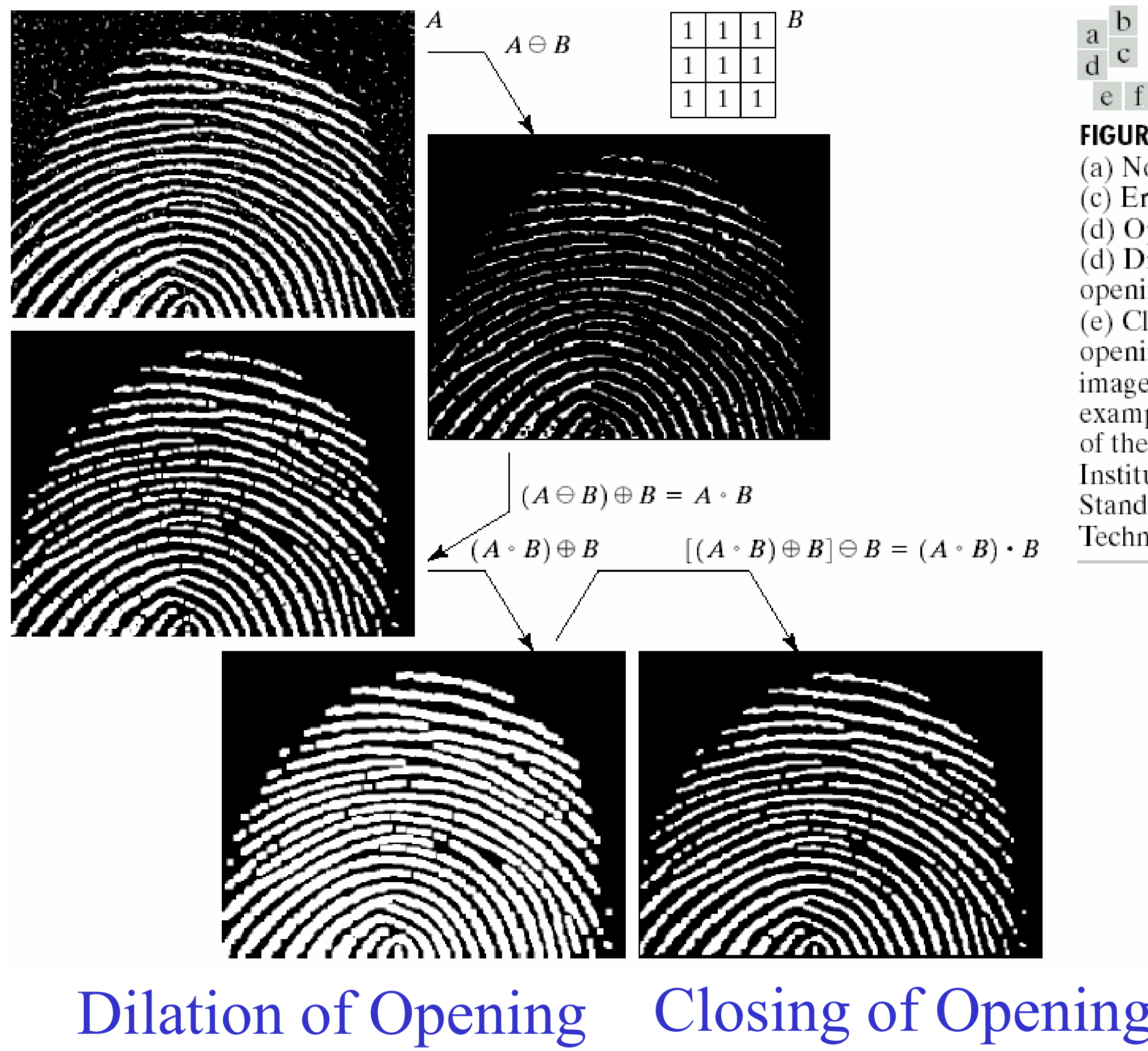
- Opening and Closing Duality:

$$\left(A \bullet B\right)^c = \left(A^c \circ \hat{B}\right)$$

- Opening Properties:
  - $A \circ B$ is a subset (subimage) of A
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  - $(A \circ B) \circ B = A \circ B$ ↔ Multiple apply has no effect.
- Closing Properties:
  - A is a subset (subimage) of $A \bullet B$
  - If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$
  - $(A \bullet B) \bullet B = A \bullet B$ ↔ Multiple apply has no effect.

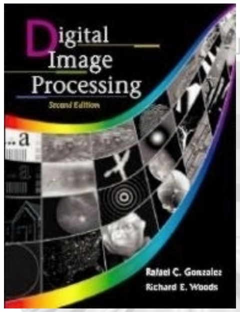


- Noise Reduction



a b
d c
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)



Chapter 9
Morphological Image Processing

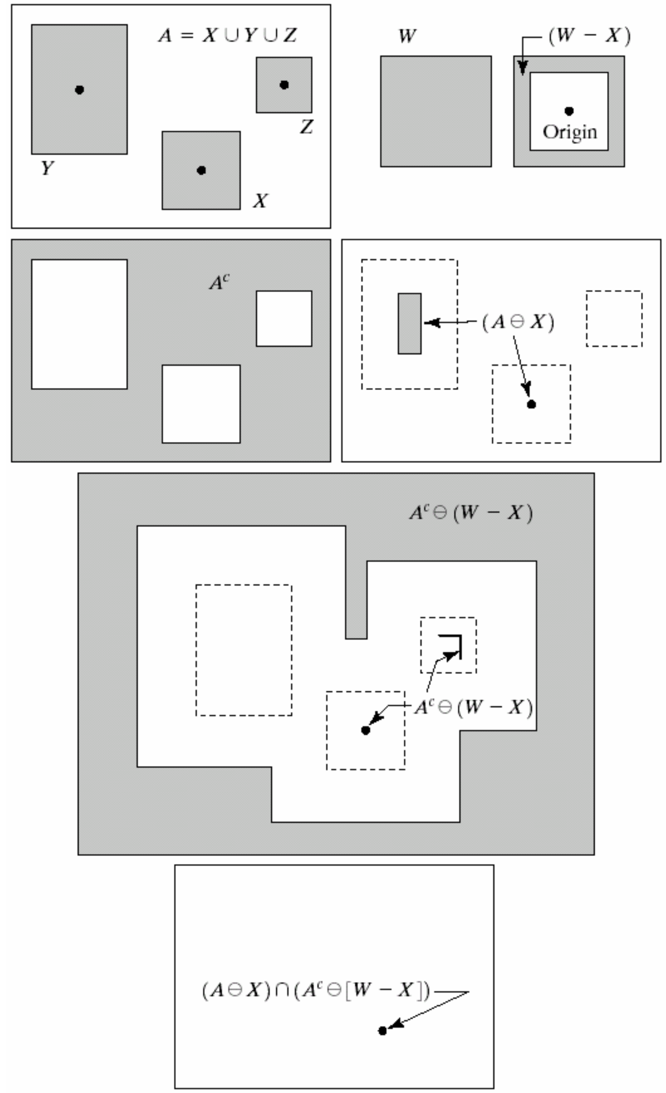
- Hit-or-Miss
  - Shape Detection
  - X-Y-X shape
  - X enclosed by W

$$A*B=(A\Theta X)\cap\left[A^c\Theta(W-X)\right]$$

$$A*B=(A\Theta B_1)\cap\left[A^c\Theta B_2\right]$$

$$A*B=(A\Theta B_1)-\left(A\oplus \hat{B}_2\right)$$

  - B_1: Object related
  - B_2: Background related



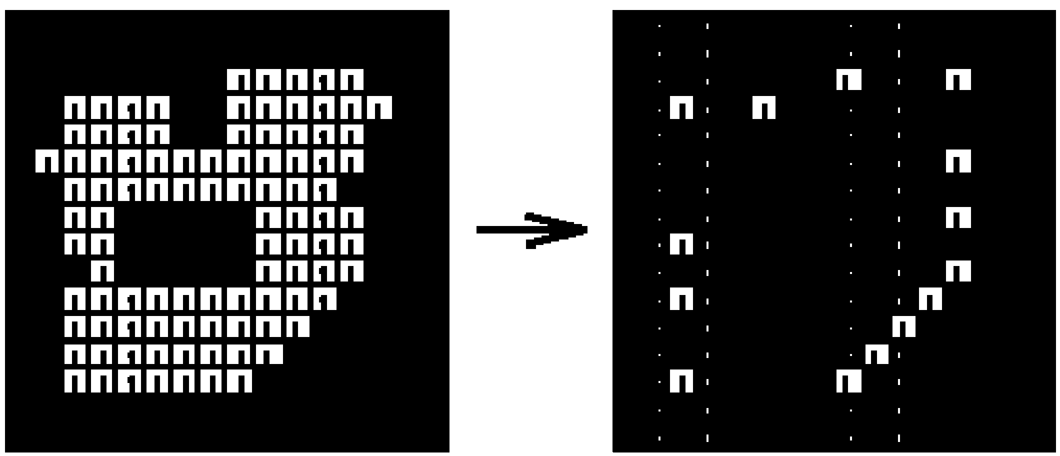
- Hit-or-Miss:
  - Another application:
    - Corners

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

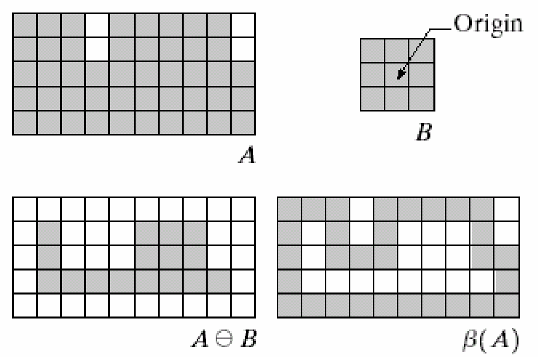
	0	0
1	1	0
	1	

0	0	
0	1	1
	1	



- Morphological Operator Applications:
  - Boundary Extraction:

$$\beta(A) = A - (A \ominus B)$$



a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Eroded
Difference

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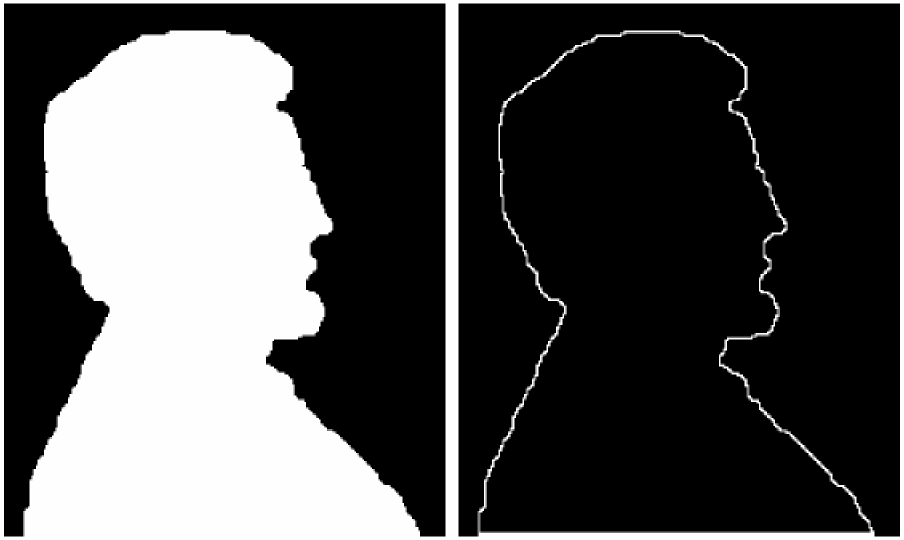
Morphological Image Processing

- Boundary Extraction:



a b

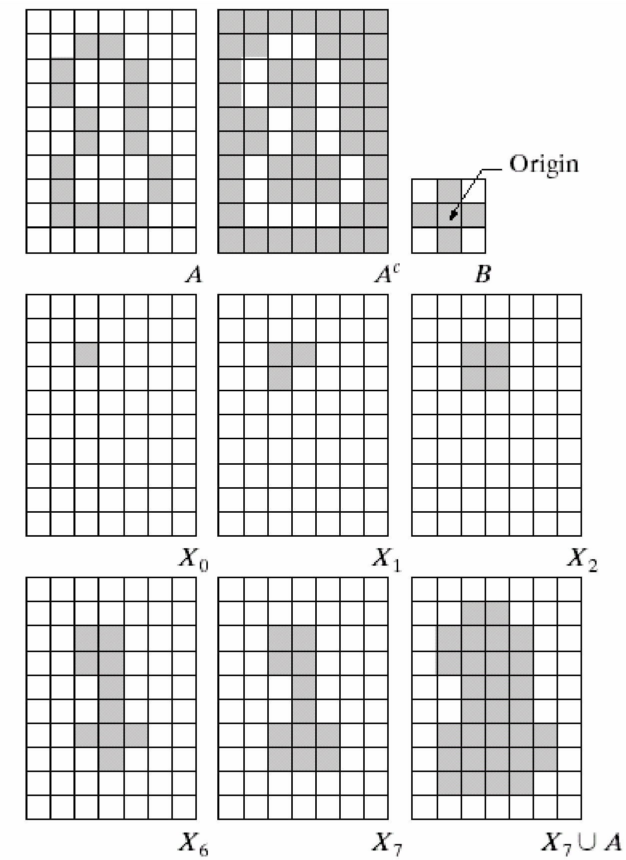
FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

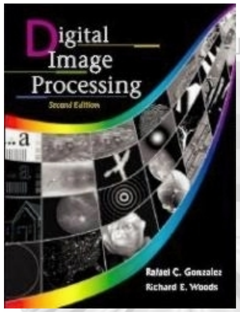
- Region Filling:
  - Start form *p* inside boundary.

$$X_0 = p$$

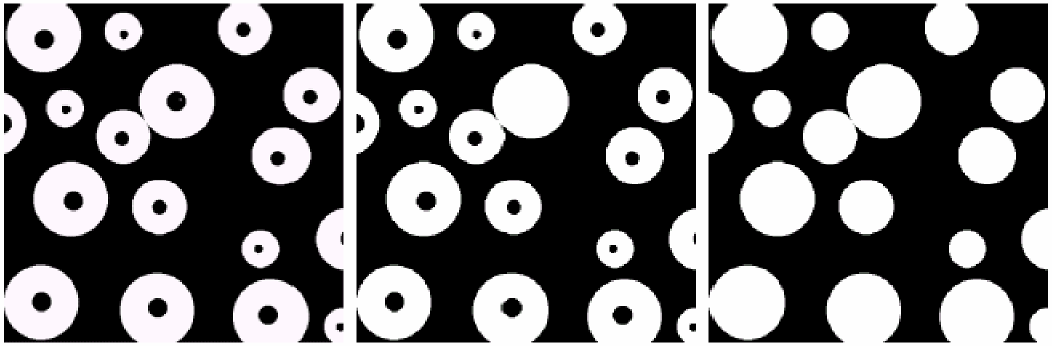
$$X_k = \left(X_k \oplus B\right) \cap A^c$$

$$\text{Until: } X_{k+1} = X_k$$





- Region Filling Example:
  - Semi-automated to cancel reflection effect



a b c

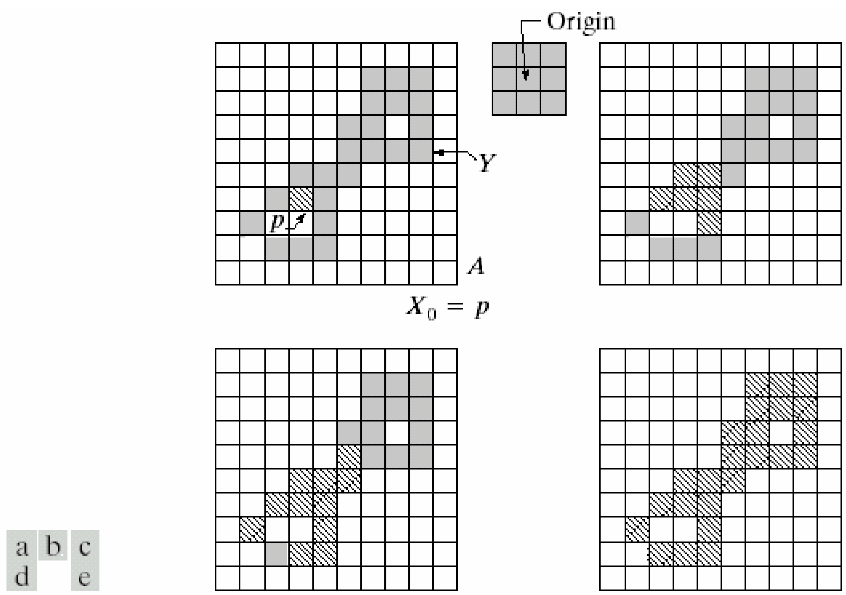
FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

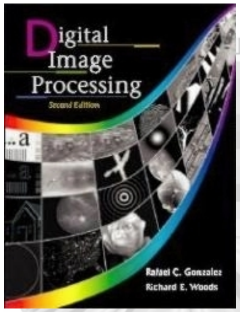
- Connected components Extraction:
  - Start from p belong to desired region.

$$X_0 = p$$

$$X_k = (X_k \oplus B) \cap A$$

Until: $X_{k+1} = X_k$





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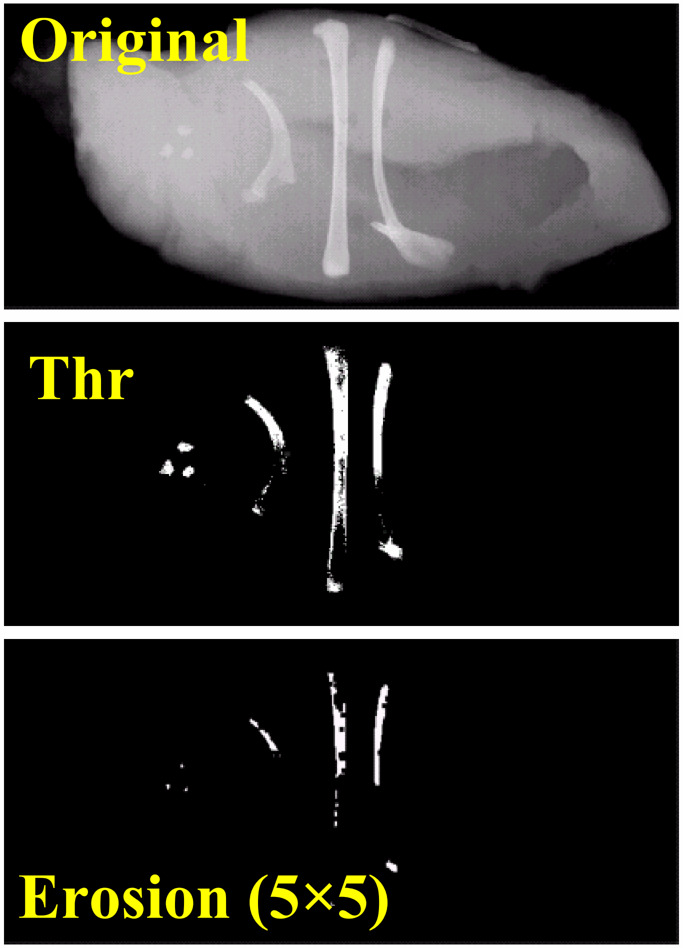
a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

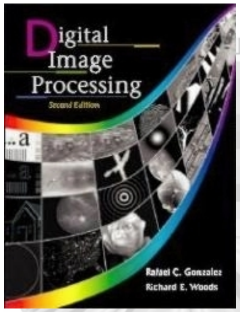


Connected Components with size

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Chapter 9
Morphological Image Processing

- Convex Hull of S:
  - Smallest Convex set H, containing S
    - Define Four basic structural elements, B^i, i=1,2,3,4
      - $X_0^i = A$
      - $X_k^i = \left(X_k^i * B^i \right) \cup A \quad i = 1,2,3,4 \quad \text{and} \quad k = 1,2,3,\cdots$
      - $C(A) = \bigcup_{i=1}^{4} D^i, \quad D^i = X_{converged}^i$



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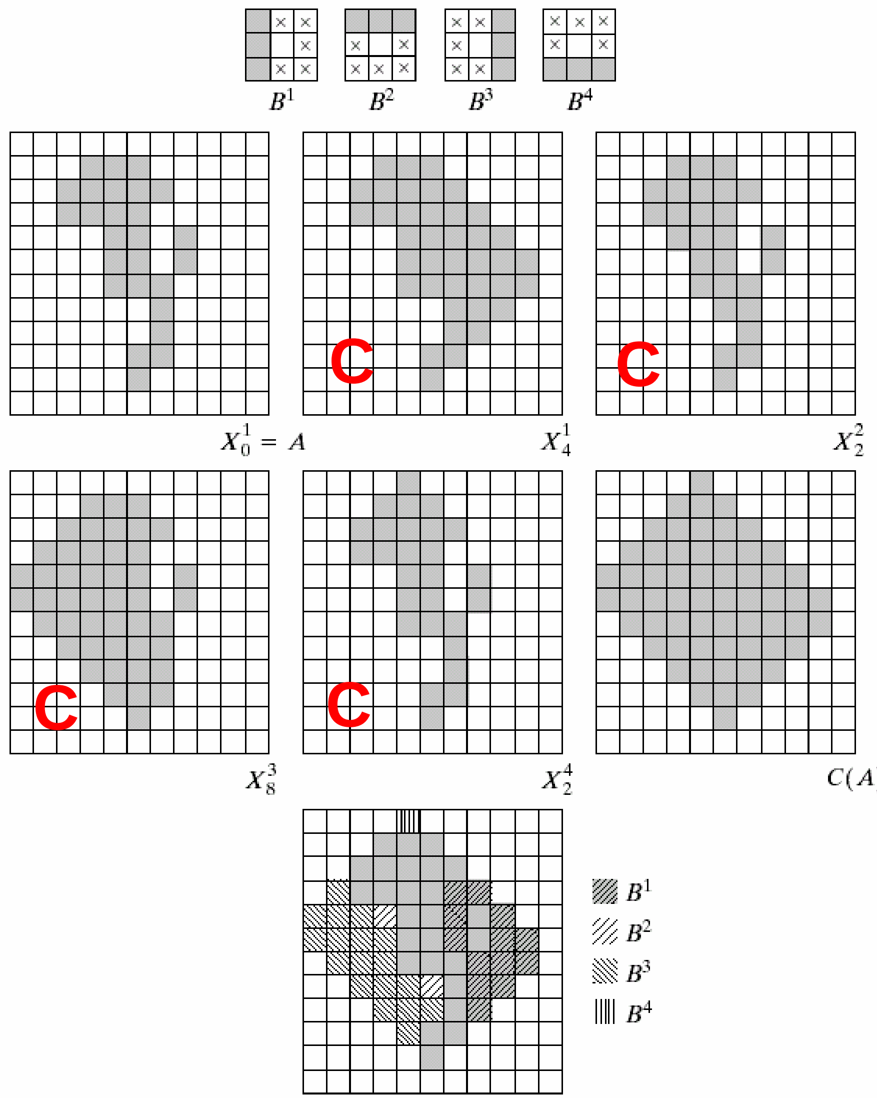
a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



Converged: C

C

- Shortcoming of previous algorithm:
  - Grow more than minimum required convex size.
  - Limit to vertical-horizontal expansion.

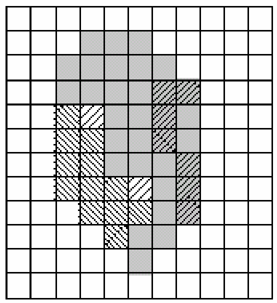


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

- Thinning:

$$A \otimes B = A - (A * B) = A \cap (A * B)^c$$

Another approach:

$$\{\mathbf{B}\} = \{B^1, B^2, \cdots, B^n\}$$

$$A \otimes \{\mathbf{B}\} = ((\cdots((A \otimes B^1) \otimes B^2)\cdots) \otimes B^n)$$

Repeat until convergence

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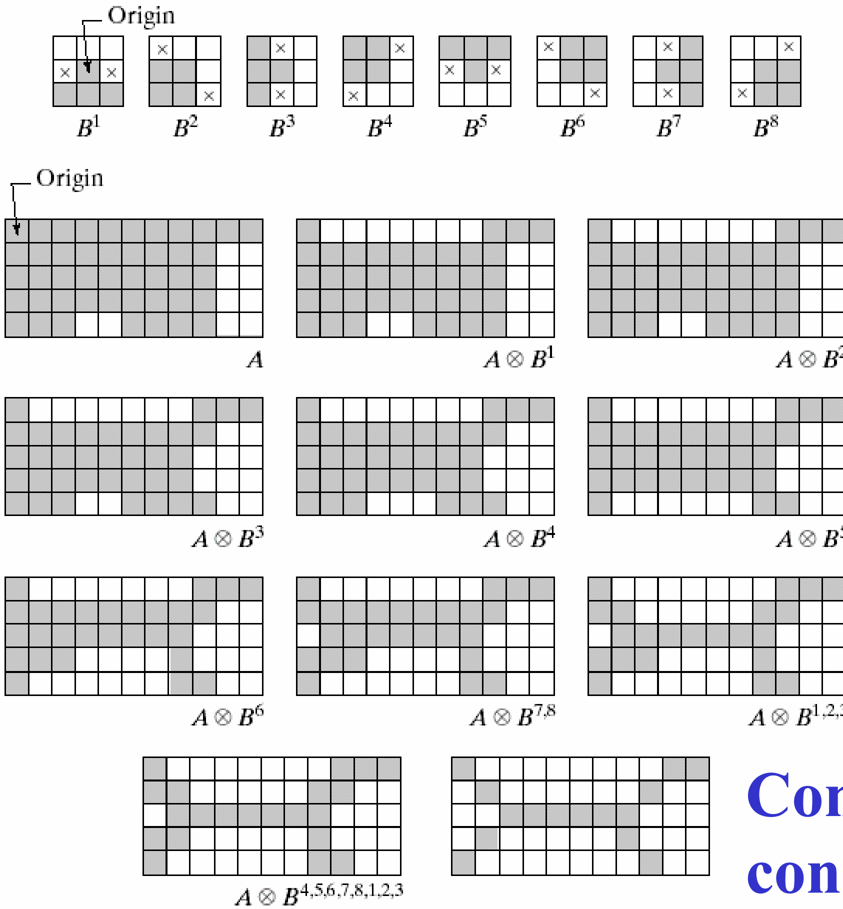
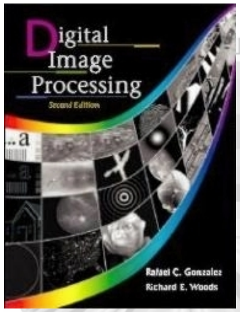
Morphological Image Processing



Convert to m-connection

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.



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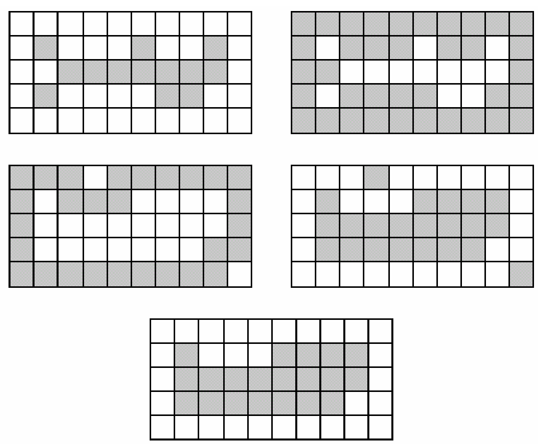
- Thickening:

$$A \odot B = A \cup (A * B)$$

$$A \odot \{\mathbf{B}\} = ((\cdots((A \odot B^1) \odot B^2)\cdots) \odot B^n)$$

- Structural elements are as before.

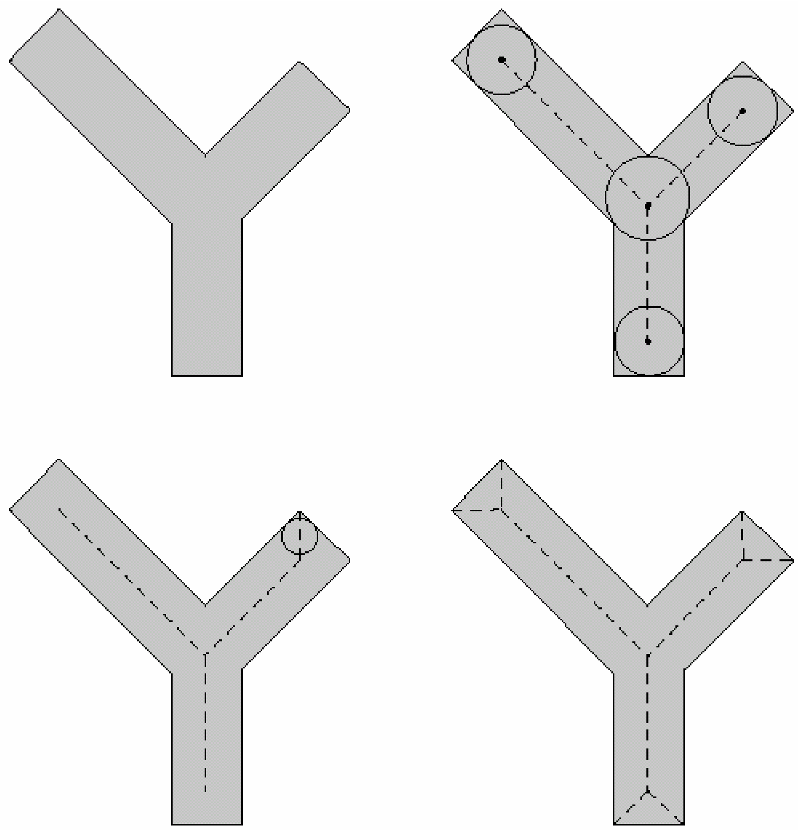


a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

- Skeletonization A with notation S(A):
  - For z belong to $S(A)$ and $(D)_z$, the largest disk centered at z and contained in A, one can not find a larger disk containing $(D)_z$ and included in A.
  - Disk $(D)_z$ touches the boundary of A at two or more different points.

- Skeleton by example:



a b
c d

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

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- Formulation:

$$S(A)=\bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A)=(A\Theta kB)-(A\Theta kB)\circ B, \quad \circ : \text{Opening}$$

$$(A\Theta kB)=((\cdots((A\Theta B)\Theta B)\cdots)\Theta B): \quad k \text{ times}$$

$$\text{K=max}\{\text{k} \mid (A\Theta kB)\neq\varnothing\}$$

Reconstruction:

$$\text{A=}\bigcup_{\text{k=0}}^{\text{K}}(S_k(A)\oplus kB)$$

$$(A\oplus kB)=((\cdots((A\oplus B)\oplus B)\cdots)\oplus B): \quad k \text{ times}$$

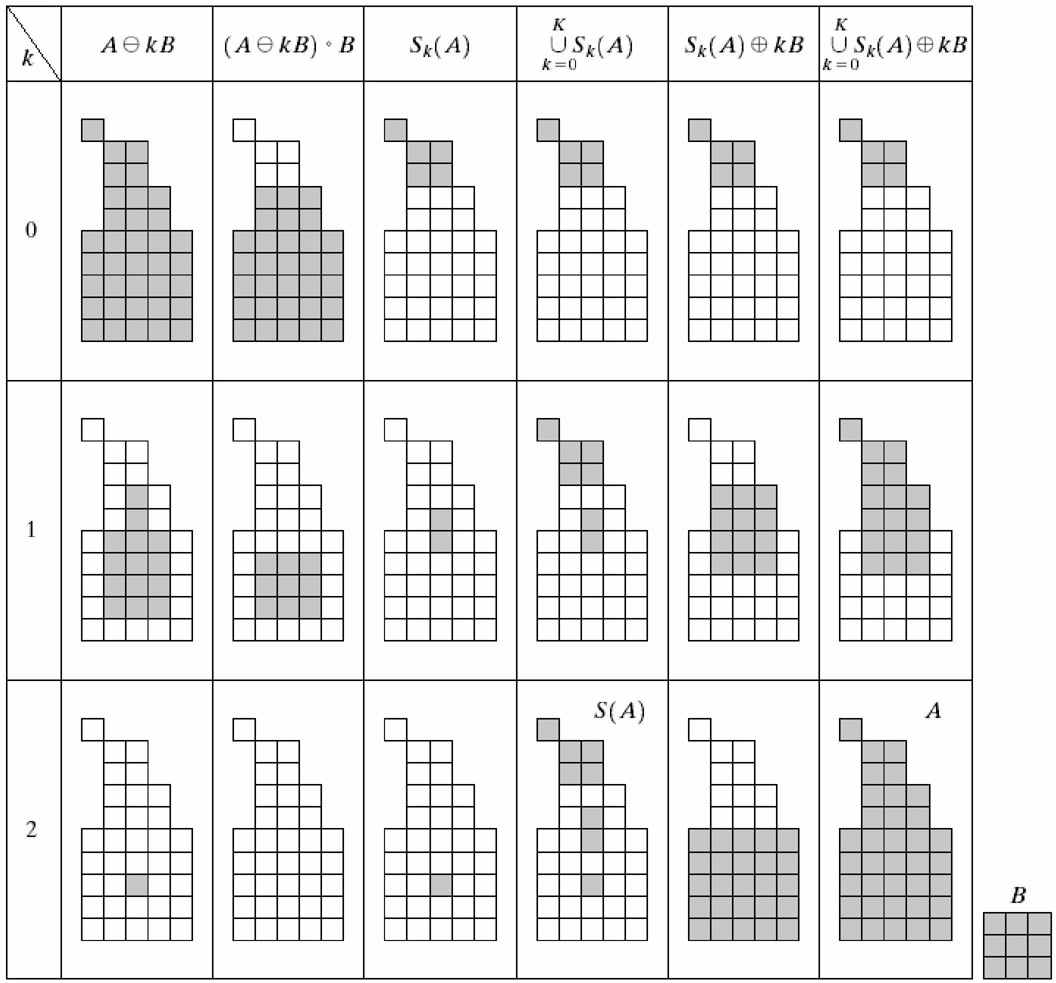

FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

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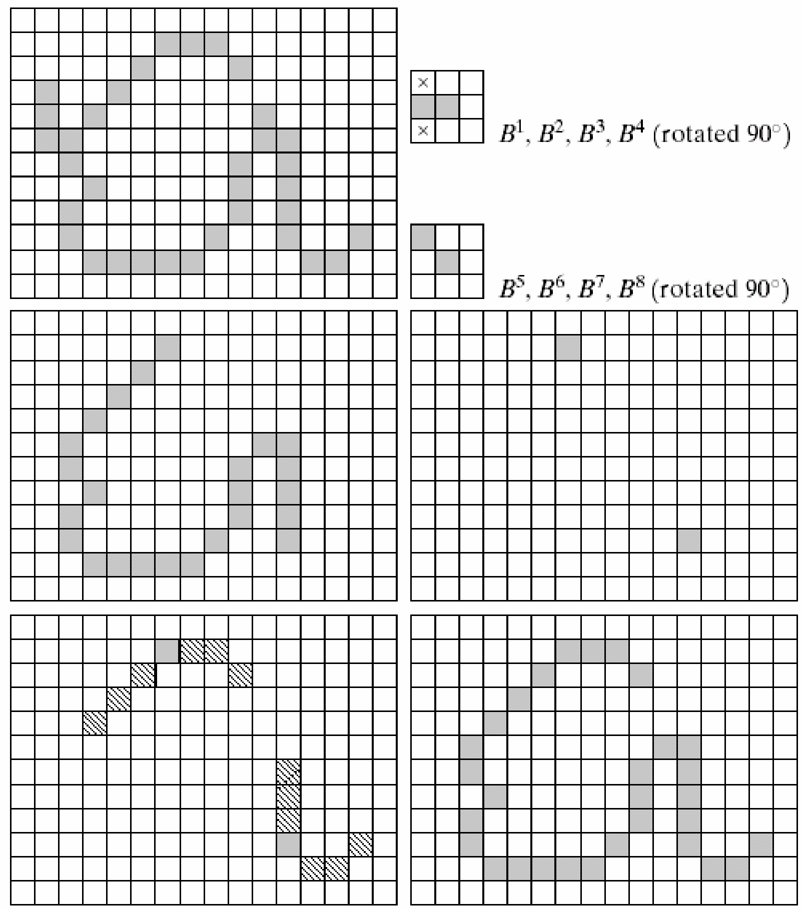

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

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TABLE 9.2 Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

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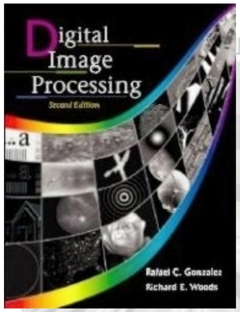
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

TABLE 9.2 Summary of morphological results and their properties. *(continued)*



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Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

TABLE 9.2 Summary of morphological results and their properties. *(continued)*

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Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

TABLE 9.2
Summary of morphological results and their properties.
(continued)

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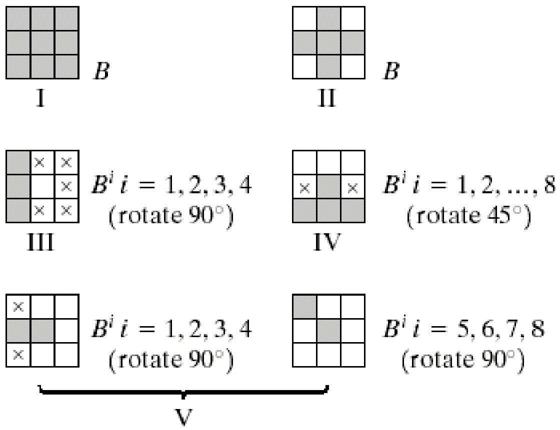

FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

- Extension to Gray-Level images:
  - f(x,y): the input image
  - b(x,y): a structuring element (a subimage function)
  - (x,y) are integers.
  - **f** and **b** are functions that assign a gray-level value (real number or real integer) to each distinct pair of coordinate (x,y)

- Extension to Gray-Level images:
  - Dilation:
    - D_f and D_b are the domains of f and b, respectively.

$$(f \oplus b)(s,t) = \max\{ f(s-x, y-t) + b(x,y) \mid (s-x),(t-y) \in D_f ; (x,y) \in D_b \}$$

    - condition (s-x) and (t-y) have to be in the domain of **f** and (x,y) have to be in the domain of **b** is similar to the condition in binary morphological dilation where the two sets have to overlap by at least one element

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- Dilation Similarity with Convolution:
  - f(s-x) : f(-x) is simply f(x) mirrored with respect to the original of the x axis. the function f(s-x) moves to the right for positive s, and to the left for negative s.
  - Max operation replaces the sums of convolution
  - Addition operation replaces with the products of convolution.
- General effect
  - If all the values of the structuring element are positive, the output image tends to be brighter than the input
  - Dark details either are reduced or eliminated, depending on how their values and shapes relate to the structuring element used for dilation.

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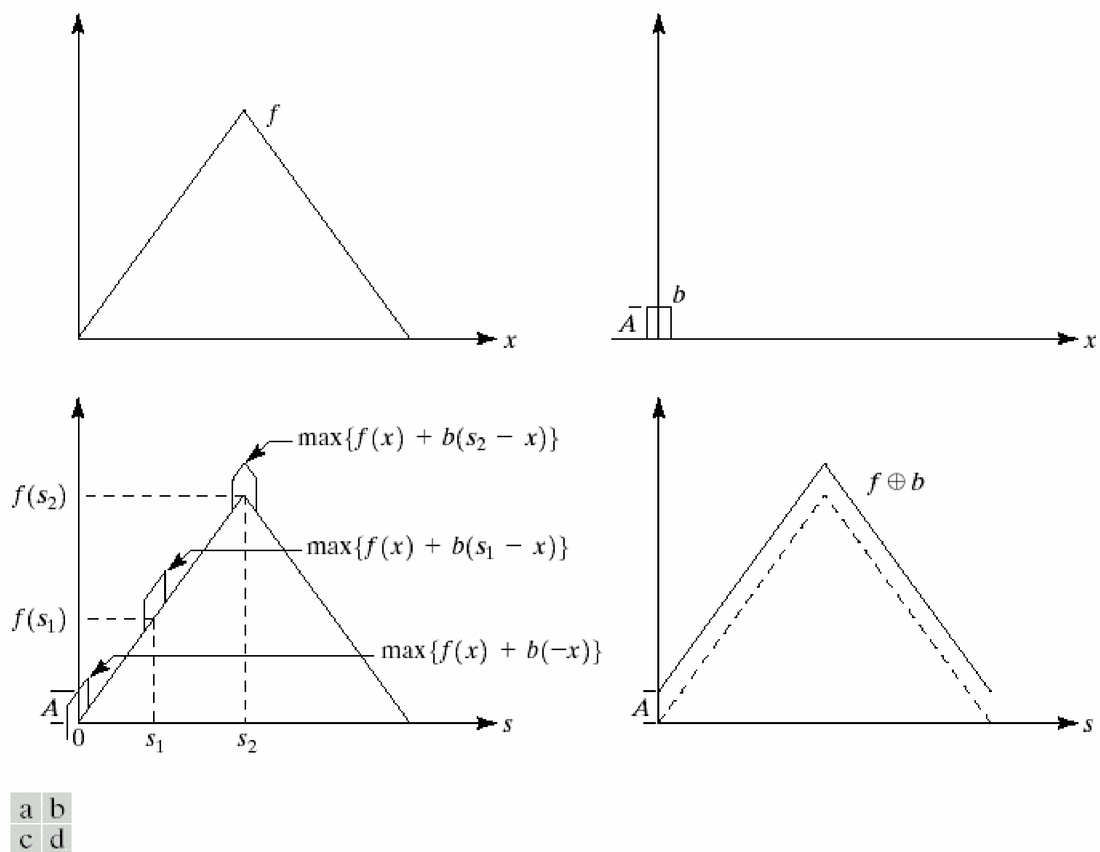
- 1D example:

$$(f \oplus b)(s) = \max\{f(s-x)+b(x) \mid (s-x)\in D_f;\, x\in D_b\}$$



- Erosion:

$$(f \ominus b)(s,t) = \min\{f(s+x, y+t) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

- Condition (s+x) and (t+y) have to be in the domain of f and (x,y) have to be in the domain of b is similar to the condition in binary morphological erosion where the structuring element has to be completely contained by the set being eroded.

- Similarity to 2D correlation
  - f(s+x) moves to the left for positive s and to the right for negative s.
- General effect
  - If all the elements of the structuring element are positive, the output image tends to be darker than the input
  - The effect of bright details in the input image that are smaller in area than the structuring element is reduced, with the degree of reduction being determined by the gray-level values surrounding the bright detail and by the shape and amplitude values of the structuring element itself.

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- 1D case:

$$(f \ominus b)(s) = \min\{ f(s+x) - b(x) \mid (s+x) \in D_f ; x \in D_b \}$$

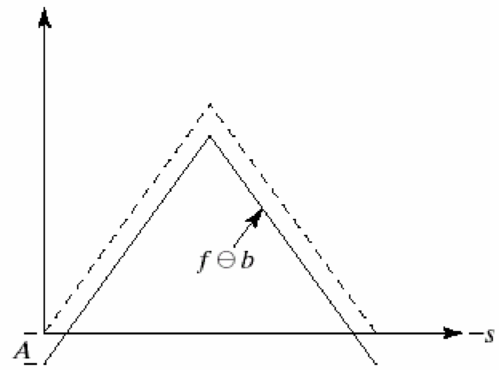


FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

- Erosion-Dilation Duality:

$$(f \Theta b)^c(s,t) = (f^c \oplus \hat{b})(s,t)$$

where

$$f^c = -f(x,y) \text{ and } \hat{b} = b(-x,-y)$$

- Dilation-Erosion

Dilation

Original

Erosion

- Opening-Closing:
  - Same (binary) relation with Dilation-Erosion:

$$f \circ b = (f - b) \oplus b$$

$$f \bullet b = (f \oplus b) - b$$

$$(f \bullet b)^c = f^c \circ \hat{b}$$

- 1D Example:

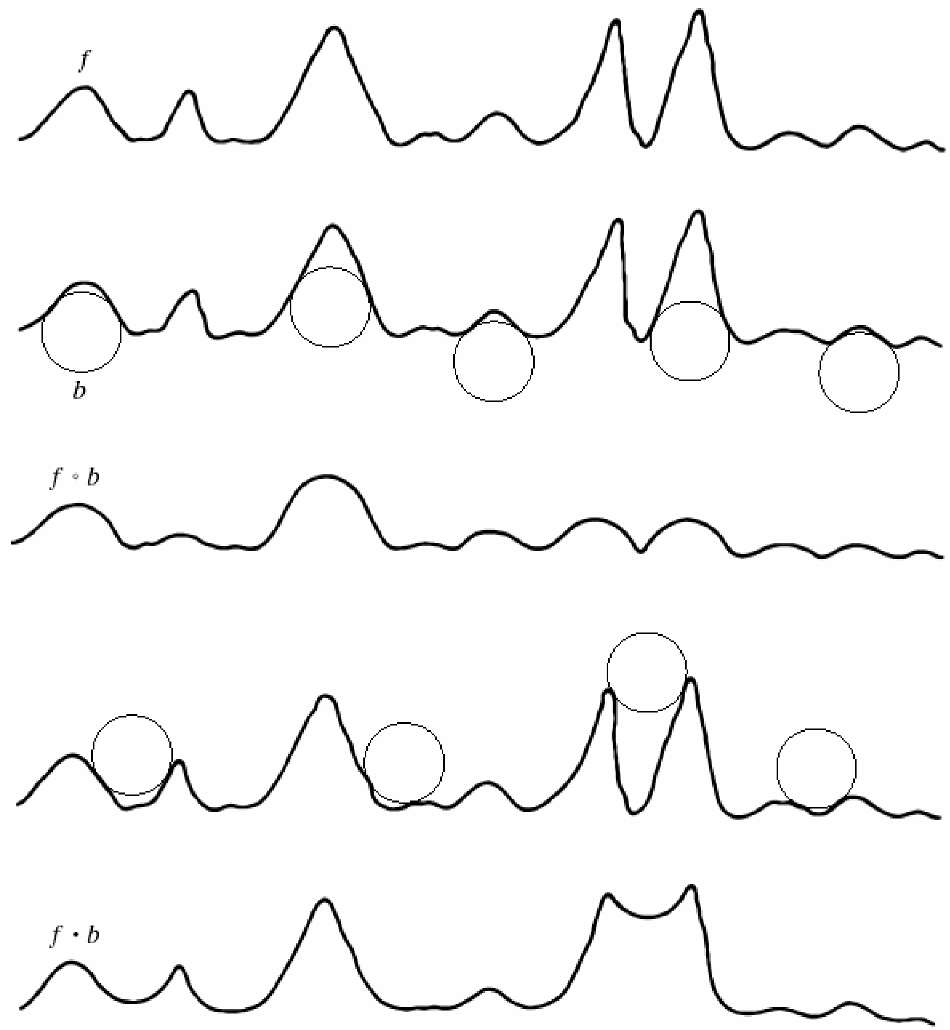
One scan line

Rolling ball for opening

Opening results

Rolling ball for Closing

Closing Results



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- Opening-Closing Properties:
  - Opening:

$$(i) \quad (f \circ b) \dashv f$$

$$(ii) \quad \text{if } f_1 \dashv f_2, \text{ then } (f_1 \circ b) \dashv (f_2 \circ b),$$

$$(iii) \quad (f \circ b) \circ b = (f \circ b)$$

  - Closing:

$$(i) \quad f \dashv (f \bullet b)$$

$$(ii) \quad \text{if } f_1 \dashv f_2, \text{ then } (f_1 \bullet b) \dashv (f_2 \bullet b)$$

$$(iii) \quad (f \bullet b) \bullet b = (f \bullet b)$$

$e \dashv r$ indicates that the domain of e is a subset of the domain of r, and also that $e(x,y) \leq r(x,y)$ for any (x,y) in the domain of e

- Opening
  - The structuring element is rolled underside the surface of f
  - All the peaks that are narrow with respect to the diameter of the structuring element will be reduced in amplitude and sharpness
  - So, opening is used to remove small light details, while leaving the overall gray levels and larger bright features relatively undisturbed.
  - The initial erosion removes the details, but it also darkens the image.
  - The subsequent dilation again increases the overall intensity of the image without reintroducing the details totally removed by erosion.

- Closing
  - The structuring element is rolled on top of the surface of f
  - Peaks essentially are left in their original form (assume that their separation at the narrowest points exceeds the diameter of the structuring element)
  - So, closing is used to remove small dark details, while leaving bright features relatively undisturbed.
  - The initial dilation removes the dark details and brightens the image
  - The subsequent erosion darkens the image without reintroducing the details totally removed by dilation

- Opening Closing Example

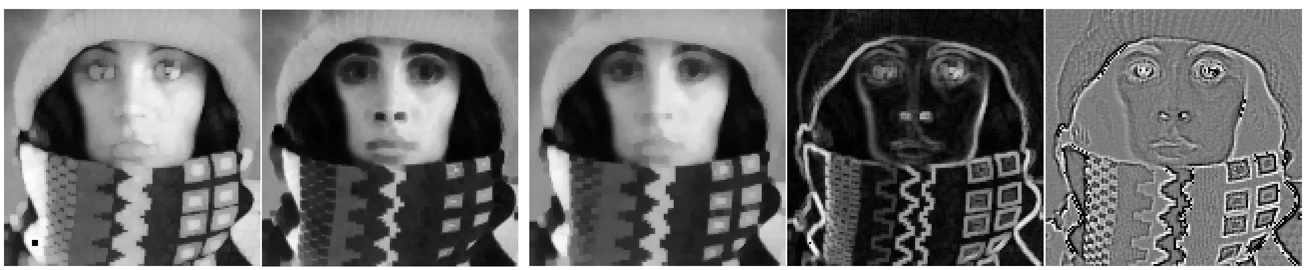
a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Opening: Decreased size of small bright details. No changes to dark region

Closing: Decreased size of small dark details. No changes to bright region

- Gray Level Morphological Examples:
  - Smoothing: $g = ((f \circ b) \bullet b)$
  - Gradient: $g = (f \oplus b) - (f \Theta b)$
  - Laplacian: $g = (f \oplus b) + (f \Theta b) - 2f$



Dilation Erosion Smoothing Gradient Laplacian

- Smoothing

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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- Gradient:

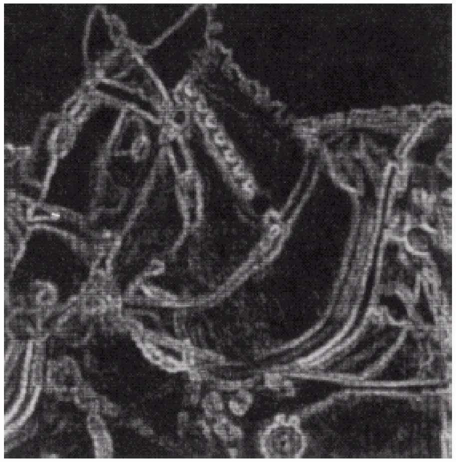

FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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- Top-hat: $h = f - (f \circ b)$
  - Enhancing details in presence of shades

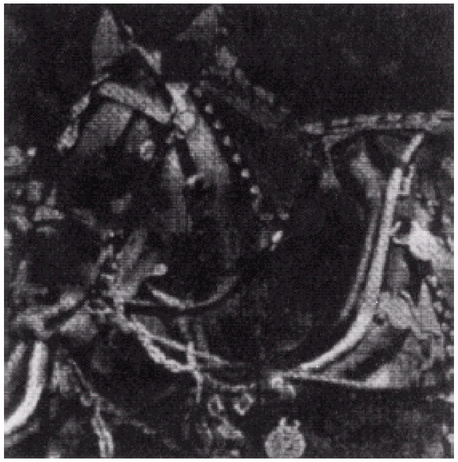

FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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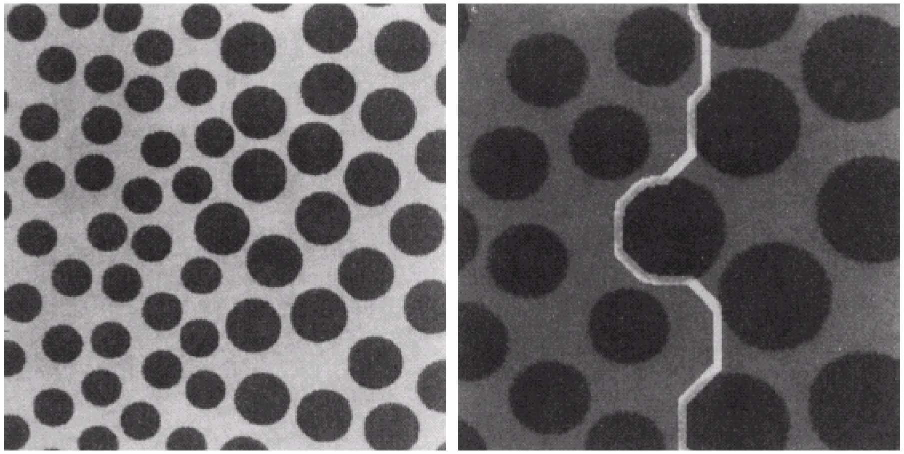

a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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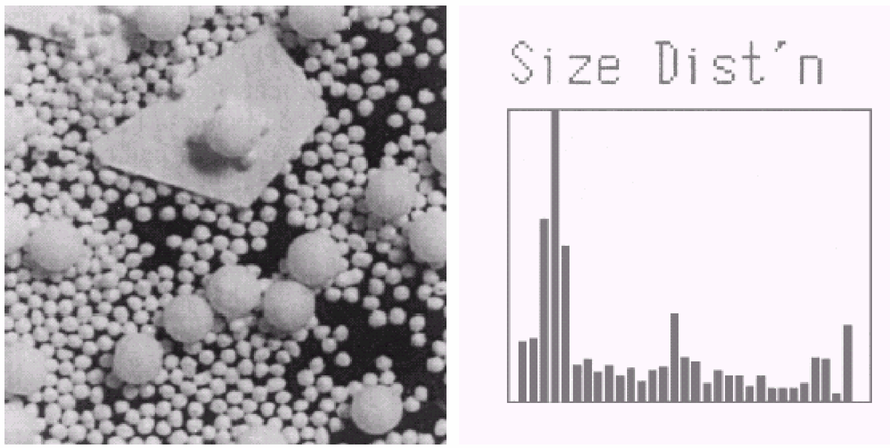

a b

FIGURE 9.36 (a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)