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Forecasting model of global stock index by stochastic time effective neural network

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ABSTRACT

In this paper, we investigate the statistical properties of the fluctuations of the Chinese Stock Index, and we study the statistical properties of HSI, DJI, IXIC and SP500 by comparison. According to the theory of artificial neural networks, a stochastic time effective function is introduced in the forecasting model of the indices in the present paper, which gives an improved neural network – the stochastic time effective neural network model. In this model, a promising data mining technique in machine learning has been proposed to uncover the predictive relationships of numerous financial and economic variables. We suppose that the investors decide their investment positions by analyzing the historical data on the stock market, and the historical data are given weights depending on their time, in detail, the nearer the time of the historical data is to the present, the stronger impact the data have on the predictive model, and we also introduce the Brownian motion in order to make the model have the effect of random movement while maintaining the original trend. In the last part of the paper, we test the forecasting performance of the model by using different volatility parameters and we show some results of the analysis for the fluctuations of the global stock indices using the model.

1. Introduction

Recently, some progress has been made in the work done on the fluctuations of the Chinese stock market, for example, see Ji and Wang (2007) and Li and Wang (2006). In the present paper, we investigate the statistical properties of the fluctuations of the indices of the Chinese Stock Exchange, and study the statistical properties of HSI, DJI, IXIC and SP500 by comparison. A predictive model of the stock prices is constructed using the theory of artificial neural networks and a stochastic time effective function, further the data from the Chinese stock markets are analyzed in the model. China has Stock A and Stock B in the stock markets. The indices of Stock A and Stock B play an important role in the Chinese stock markets, and the database of the indices is from the website www.sse.com.cn.

Recently, the properties of the fluctuations of the stock markets have been studied in many research fields, for example, see Azoff (1994), Nakajima (2000), Pino, Parreno, Gomez, and Priore (2008), Shtub and Versano (1999) and Wang (2007). Artificial neural networks are one of the technologies that have made great progress in the study of the stock markets. Usually stock prices can be seen as a random time sequence with noise, artificial neural networks, as large-scale parallel processing nonlinear systems that depend on their own intrinsic link data, provide methods and techniques that can approximate any nonlinear continuous function, without a priori assumptions about the nature of the generating process, see Pino et al. (2008) and Shtub and Versano (1999). They have good self-learning ability, a strong anti-jamming capability,and have been widely used in the financial fields such as stock prices, profits, exchange rate and risk analysis and prediction. Although the historical data have a great influence on the investors' positions, the degree of impact of the data depends on the date at which they occurr (or time), we get a high level effect of the data when they are very near the current state. Furthermore, we also introduce the Brownian motion in the model (see Wang, 2007), in order to make the model have the effect of random movement while maintaining the original trend. We test the forecasting performance of the model by using different volatility parameters, and we show some results of the analysis for the fluctuations of the global stock index using the model. In this work, the forecasting model is developed to estimate the level of returns on Stock A Index of China. In Section 3, the results show that the forecasting model can predict the index behavior better in a short time interval than in a long time interval, and show the different performances of HSI, DJI, IXIC and SP500 by comparison, see Abhyankar, Copeland, and Wong (1997), Austin, Looney, and Zhuo (1997) Balvers, Cosimano, and McDonald (1990).

In this paper, forecasting based on neural network involves two major steps, data preprocessing and structure design. In the pretreatment stage, the collected data should be normalized and properly adjusted, in order to reduce the impact of noise in the stock markets. At the design stage, different data training sets, validations and data processing will cause the different results, see Breen,

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Glosten, and Jagannathan (1990) and Campbell (1987). In stock markets, the environment and behavior of the markets may change greatly, for example see the Chinese stock markets in 2007. As a result, the data in the data training set should be time-variant, reflecting the different behavior patterns of the markets at different times. If all the data are used to train the network equivalently, the network system may not be consistent with the development of the stock markets, see Chenoweth and Obradovic (1996, 1996), Cybenko (1989) and Demuth and Beale (1998). Especially in the current Chinese stock markets, stock market trading rules and management systems are changing rapidly, for example, the daily price limit (now 10%), shareholding reformation, the direct investment of Hong Kong stock markets, the reorganization of A share, B share and H share, and the establishment of financial derivatives such as futures and options. Therefore, using the historical data of the past it is difficult to reflect the current Chinese stock markets' development. However, if only the recent data are selected, a lot of useful information will be lost which the early data hold. In the financial model of the present paper, a promising data mining technique in machine learning is proposed to uncover the predictive relationships of numerous financial and economic variables. Considering the above-mentioned financial situation, this paper presents an improved neural network model, the stochastic time effective series neural network model: each historical datum is given a weight depending on the time it occurs in the model, and we also use the probability density functions to classify the various variables from the training samples, see Desai and Bharati (1998), Duda, Hart, and Stork (2001) and Elton and Gruber (1991).

2. Methodology for stochastic time effective function

2.1. Introduction of stochastic time effective function

In this section, first we describe a three-layer BP neural network model (see Azoff, 1994), which is shown in Fig. 1. The neural network model includes three layers: input layer, hidden layer and output layer. In the model, the proper number of the hidden layer nodes requires validation techniques to avoid under-fitting (too few neurons) and over-fitting (too many neurons). Generally, too many neurons in the hidden layers, and, hence, too many connections, produce a neural network that memorizes the data and lacks the ability to generalize.

Suppose that a three-layer neural network has neurons, and for any fixed neuron $n(n = 1, 2, \ldots, N)$, the model has the following structure: let $\{x_i(n) : i = 1, 2, \ldots, p\}$ denote the set of input of neurons, $\{y_j(n) : j = 1, 2, \ldots, m\}$ denote the set of output of the hidden layer neurons; V_i is the weight that connects the node i in the input layer neurons to the node j in the hidden layer, W_j is the weight that connects the node j in the hidden layer neurons to the node k in the output layer; and $\{o_k(n) : k = 1, 2, \ldots, q\}$ denote the set of output of neurons. Then the output value for a unit is given by the following function

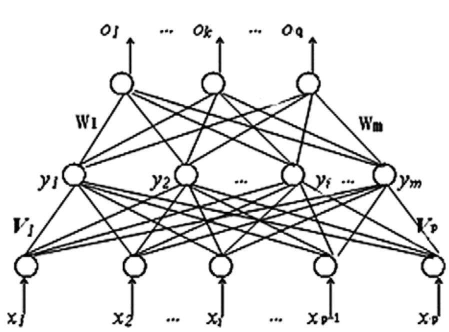


Fig. 1. Three-layer neural network.

$$y_j(n) = f\left(\sum_{i=1}^{p} V_i x_i(n) - \theta_j\right), \qquad o_k(n) = f\left(\sum_{i=1}^{p} W_j y_j(n) - \theta_k\right)$$

where θ_j and θ_k are the neural thresholds, $f(x) = \frac{1}{1+e^{-x}}$ is the sigmoid activation function. Let $T_k(n)$ be the actual value of the data sets, then the error of the corresponding neuron k to the output is defined as $\varepsilon_k = T_k - o_k$. In this paper, the error of the output is defined as $\varepsilon = \frac{\varepsilon_k^2}{2}$, then the error of the sample $n(n = 1, 2, \ldots, N)$ is defined as

$$e(n,t) = \frac{1}{2}\ \phi(t) \sum_{k=1}^{q} (T_k(n) - o_k(n))^2$$

where $\phi(t)$ is the stochastic time effective function. Now we define $\phi(t)$ as follows:

$$\phi(t_1 - t_n) = \frac{1}{\tau}\ \exp\left\{\int_{t_1}^{t_n} \mu(t)dt + \int_{t_1}^{t_n} \sigma(t)dB(t)\right\}$$

where $\tau(> 0)$ is the time strength coefficient, t_1 is the current time or the time of the newest data in the data set, and t_n is an arbitrary time point in the data set. $\mu(t)$ is the drift function (or the trend term), $\sigma(t)$ is the volatility function, and $B(t)$ is the standard Brownian motion (Wang, 2007). The stochastic time effective function implies that the recent information has a stronger effect for the investors than the old information. In detail, the nearer the events happened, the greater the investors and markets affected. And the impact of data follows the time exponential decay, see Nakajima (2000) and Pino et al. (2008). Then the total error of all the data training sets in the set output layer with the stochastic time effective function is defined as

$$\begin{aligned} E &= \frac{1}{N}\sum_{n=1}^{N} e(n,t) \\ &= \frac{1}{N}\sum_{n=1}^{N} \frac{1}{\tau} e^{\int_{t_1}^{t_n} \mu(t)dt + \int_{t_1}^{t_n} \sigma(t)dB(t)} \sum_{k=1}^{q} \frac{1}{2}(T_k(n) - o_k(n))^2. \end{aligned}$$

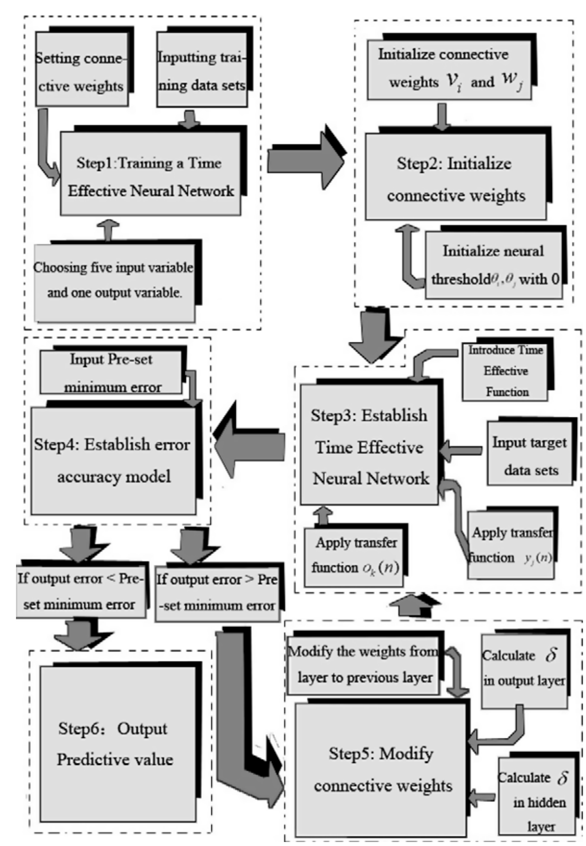


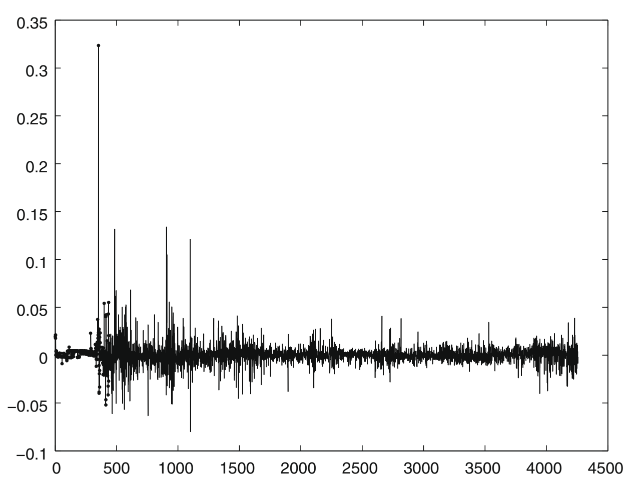
Fig. 2. Procedure followed in the stochastic time effective model.

Correlations

		A	B	HSI	DJI	IXIC	SP500
A	Pearson Correlation	1	.885 (**)	.629 (**)	.463 (**)	.108 (**)	.329 (**)
	Sig. (2-tailed)		.000	.000	.000	.000	.000
	N	3744	3744	3744	3744	3744	3744
B	Pearson Correlation	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Sig. (2-tailed)	.000		.000	.000	.000	.000
	N	3744	3744	3744	3744	3744	3744
HSI	Pearson Correlation	.629 (**)	.817 (**)	1	.744 (**)	.607 (**)	.708 (**)
	Sig. (2-tailed)	.000	.000		.000	.000	.000
	N	3744	3744	3744	3744	3744	3744
DJI	Pearson Correlation	.463 (**)	.658 (**)	.744 (**)	1	.798 (**)	.973 (**)
	Sig. (2-tailed)	.000	.000	.000		.000	.000
	N	3744	3744	3744	3744	3744	3744
IXIC	Pearson Correlation	.108 (**)	.458 (**)	.607 (**)	.798 (**)	1	.894 (**)
	Sig. (2-tailed)	.000	.000	.000	.000		.000
	N	3744	3744	3744	3744	3744	3744
SP500	Pearson Correlation	.329 (**)	.623 (**)	.708 (**)	.973 (**)	.894 (**)	1
	Sig. (2-tailed)	.000	.000	.000	.000	.000	
	N	3744	3744	3744	3744	3744	3744

** Correlation is significant at the 0.01 level (2-tailed).

Fig. 3. Related coefficients.



2.2. Procedure followed in the stochastic time effective model

Note that the training objective of stochastic time effective neural network is to modify the weights so as to minimize the error between the network's prediction and the actual target. When all the training data are data (that is $t_1 = t_n$), the stochastic time effective neural network is the general neural network model. In Fig. 2, the training algorithms procedures of stochastic time effective neural network are shown, which are as follows:

Step 1: Train a stochastic time effective neural network by choosing five kinds of stock prices in the input layer: daily opening price, daily closing price, daily highest price, daily lowest price and daily trade volume, and one price of the stock prices in the output layer: the closing price of the next trading day. Then set the connective weights, and input the training data sets.

Step 2: At the beginning of data processing, connective weights V_i and W_j follow the uniform distribution on $(-1,1)$, and let the neural threshold θ_k, θ_j be 0.

Step 3: Introduce the stochastic time effective function ϕ_t in the error function $e(n,t)$. Choose different volatility parameters. Give the transfer function from the input layer to the hidden layer and the transfer function from the hidden layer to the output layer.

Step 4: Establish an error acceptable model and set pre-set minimum error. If the output error is below pre-set minimum error, go to Step 6, otherwise go to Step 5.

Step 5: Modify the connective weights: Calculate backward for the node in the output layer:

$$\delta_o(n) = \frac{1}{\tau}\, e^{\int_{t_1}^{t_n}\mu(t)dt + \int_{t_1}^{t_n}\sigma(t)dB(t)} o(n)[o(n) - T(n)][1 - o(n)]$$

Calculate δ backward for the node in the hidden layer:

$$\delta_h(n) = \frac{1}{\tau}\, e^{\int_{t_1}^{t_n}\mu(t)dt + \int_{t_1}^{t_n}\sigma(t)dB(t)} o(n)[1 - o(n)]\sum_{h'} W_j \delta_{h'}(n)$$

where $o(n)$ is the output of the neuron n, $T(n)$ is the actual value of the neuron n in the data sets, $o(n)[1-o(n)]$ is the derivative of the sigmoid activation function and h' is each of the nodes which connect with the node h and in the next hidden layer after node h. Modify the weights from the layer to the previous layer:

$$W_j(n+1) = W_j(n) + \eta\delta_o(n)y(n) \quad \text{or}$$
$$V_j(n+1) = V_j(n) + \eta\delta_k(n)x(n),$$

where η is the learning step, which usually takes constants between 0 and 1.

Step 6: Output the predictive value.

3. Experiment analysis

3.1. Selection and preprocessing of data

In this paper, we select the data of Stock A Index (SAI) and Stock B Index (SBI) of China for each trading day in a 18-year period, that is from December 19, 1990 to June 7, 2008, which are from the Shanghai and Shenzhen Stock Exchange. And we also choose the data of HSI, DJI, IXIC and SP500 by contrast. First, we study the statistical properties of the returns of the index using the stochastic time effective neural network model, and then study the relativity between the Chinese stock indices and foreign country stock indices.

Fig. 3 presents the related coefficients between SAI, SBI, HSI, DJI, IXIC and SP500. In Fig. 3, we can see that the related coefficients between SAI and SBI, HSI, DJI, IXIC and SP500 are 0.885, 0.629, 0.463, 0.108, 0.329; and the related coefficients between SBI and SAI, HSI, DJI, IXIC and SP500 are 0.885, 0.817, 0.688, 0.458, 0.623.

In the model of this paper, we suppose that the network inputs include five kinds of data, daily opening price, daily closing price, daily highest price, daily lowest price and daily trade volume, and the network outputs include the closing price of the next trading day.

Fig. 4 presents the plot of the time sequence log return of SAI, SBI, IXIC and SP500. We denote the price sequence of SAI, SBI, IXIC and SP500 of time t by $S(t)$ $(t = 0,1,2\ldots)$, then $R(t)$ denotes the logarithm of return rate, respectively, by

$$R(t) = \ln\left(\frac{S(t+1)}{S(t)}\right) = \ln\left(1 + \frac{\Delta S(t)}{S(t)}\right).$$

In Fig. 4, we can see that the prices of the index fluctuate wildly, and this indicates that there is a big noise in the data that causes difficulty in forecasting. Thus, we should go through data preprocessing before forecasting, so the data are normalized as follows:

$$S(t)' = \frac{S(t) - minS(t)}{maxS(t) - minS(t)}.$$

Similarly to (5), the normalized values of the above-mentioned five kinds of data can also be given.

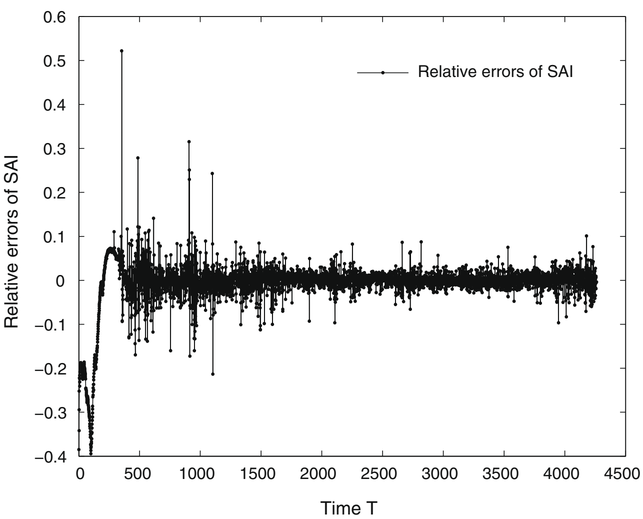
3.2. Training stochastic time effective neural network

Data sets are divided into two parts, data training set and data testing set. We collect the data of SAI in 1990–2006 as the training set and the data of SAI in 2007–2008 as the testing set. According to the procedures of the three-layer network introduced in Section

Table 1
Comparison of errors of different dates for SAI, SBI, HSI, DJI, IXIC and SP500 ($(\mu(t), \sigma(t))$ is $(1,1)$).

Time	Actual	Predictive	Error
(a) SAI			
90/12/20	104.39	144.57	−0.384
91/05/10	108.53	149.06	−0.373
06/09/20	1820.8	1830.3	−0.005
07/02/13	2973.4	2974.0	−0.0001
(b) SBI			
92/02/24	124.65	128.88	−0.034
93/12/24	93.14	87.64	0.059
05/07/11	59.88	60.8308	−0.02
06/11/15	108.27	106.37	0.017
(c) HSI			
01/07/31	12316.7	12123.8	0.016
02/01/29	11014.2	10712.2	0.027
06/10/11	17862.8	17849.5	0.001
07/09/26	26430.2	26230.8	0.008
(d) DJI			
91/12/23	3022.6	2948.1	0.025
92/07/07	3295.2	3340.8	−0.014
06/07/26	11102.5	11125.5	−0.002
07/12/28	13365.9	134129	−0.004
(e) IXIC			
91/11/08	548.08	540.3	0.014
92/03/05	621.97	635.6	−0.022
06/03/07	2268.38	2290.0	−0.010
07/05/29	2572.06	2559.6	0.005
(f) SP500			
91/01/03	321.91	339.06	−0.053
92/02/04	413.85	408.38	0.013
06/10/31	1377.94	1376.05	0.001
07/02/05	1446.99	1448.47	−0.001

2, the number of neural nodes in the input layer is 5, the number of neural nodes in the hidden layer is 20 and the number of neural nodes in the output layer is 1, and the threshold of the maximum training cycles is 100 and the threshold of the minimum error is 0.0001. We take the $\mu(t)$ (the drift parameter) and $\sigma(t)$ (the volatility parameter) $(\mu(t), \sigma(t))$ to be (1,0), (1,1) and (1,2). While $(\mu(t), \sigma(t))$ is (1,0), the model has the effect of only time effective function; while $(\mu(t), \sigma(t))$ is (1,1), the model has the effect of both time



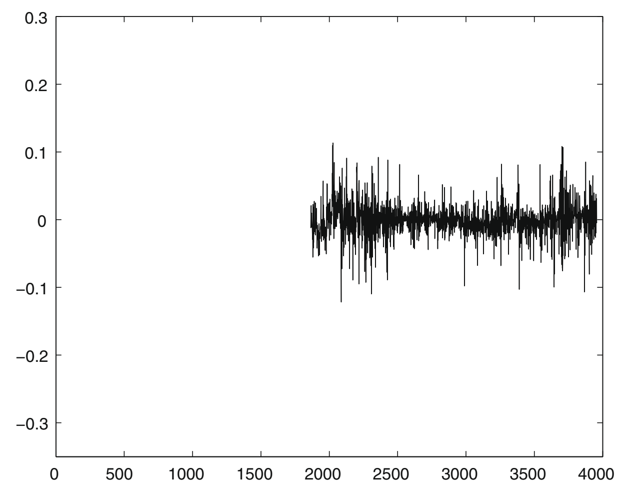


Table 2
Predictive values and errors of time effective neural network model of SAI by different $\sigma(t)$.

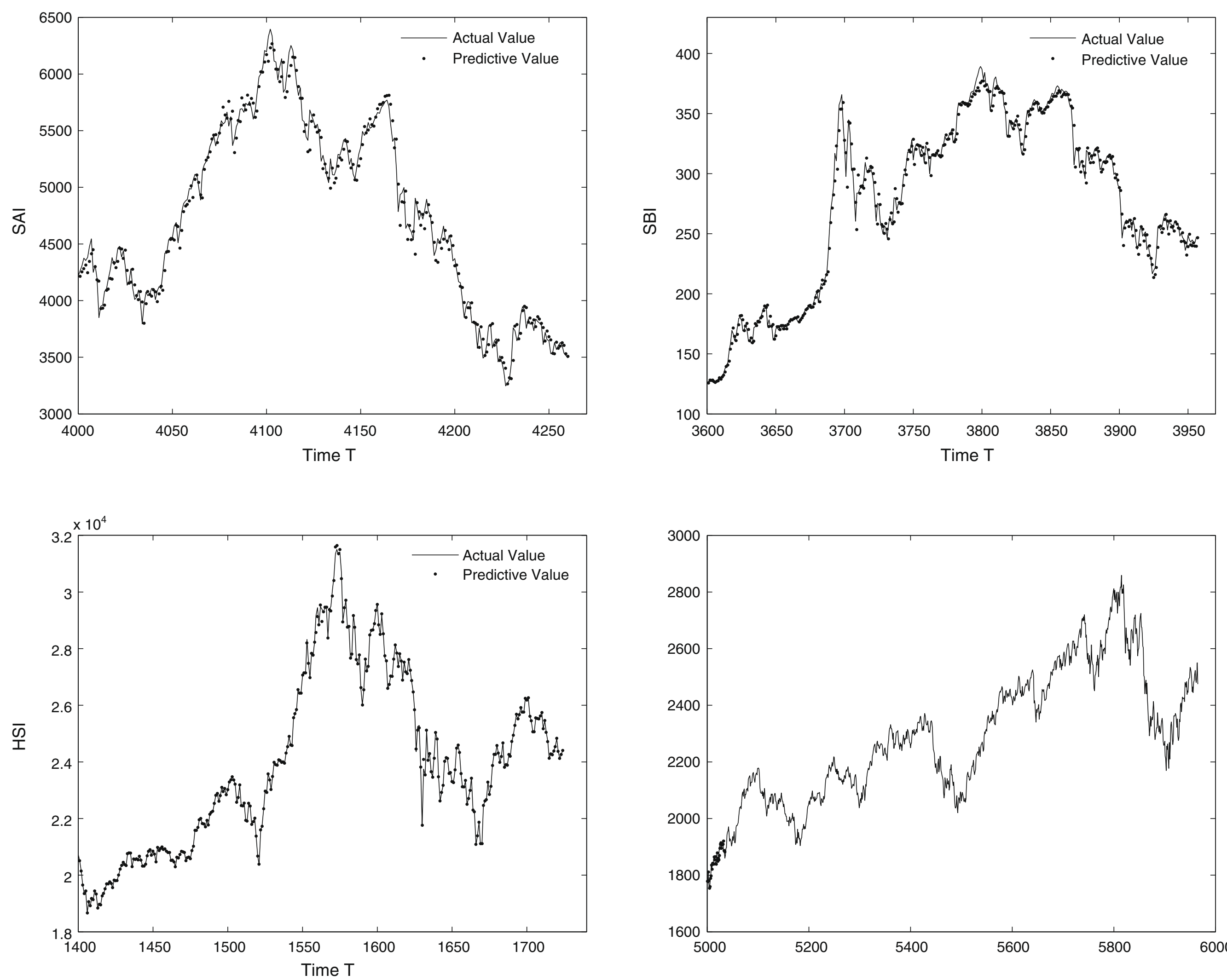
Time	Actual	Predictive	Error
(a) $\sigma(t) = 1$			
08/06/03	3605.859	3627.168	−0.0059
08/06/04	3535.809	3604.064	−0.0193
08/06/05	3516.219	3530.171	−0.0039
08/06/06	3493.189	3507.505	−0.0040
(b) $\sigma(t) = 0$			
08/06/03	3605.859	3655.273	−0.014
08/06/04	3535.809	3689.849	−0.044
08/06/05	3516.219	3559.069	−0.012
08/06/06	3493.189	3502.137	−0.003
(c) $\sigma(t) = 2$			
08/06/03	3605.859	4060.890	−0.126
08/06/04	3535.809	4034.350	−0.141
08/06/05	3516.219	4001.172	−0.138
08/06/06	3493.189	3968.767	−0.136

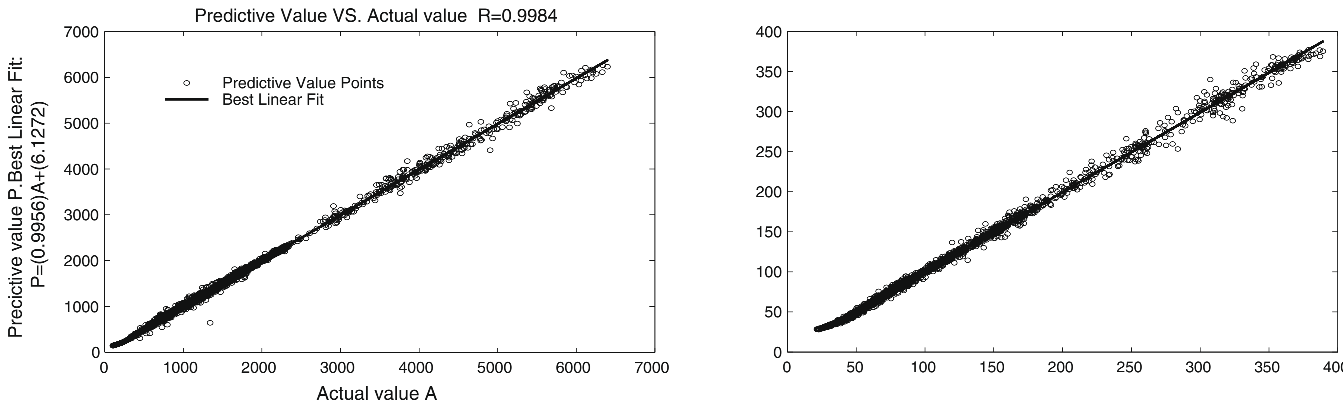
Table 3
Average relative errors for SAI, SBI, DJI, IXIC, HSI and SP500 of different $\sigma(t)$.

	SAI	SBI	DJI	IXIC	HSI	SP500
(a) $\sigma(t) = 1$						
I	0.026	0.0212	0.0388	0.0152	0.0104	0.0074
II	0.0688	0.0262	0.0697	0.0366	0.0112	0.0077
III	0.013	0.0141	0.0058	0.0078	0.0109	0.0078
(b) $\sigma(t) = 0$						
I	0.0766	0.0284	0.0128	0.2143	0.0322	0.0345
II	0.1637	0.0391	0.201	0.7247	0.0379	0.045
III	0.0514	0.0148	0.0103	0.0328	0.0247	0.0366
(c) $\sigma(t) = 2$						
I	0.2458	0.1041	0.043	0.1443	0.0636	0.0897
II	0.8686	0.1276	0.083	0.3466	0.0875	0.2256
III	0.0353	0.0985	0.034	0.1041	0.0347	0.0265

effective function and normal randomization; and while $(\mu(t), \sigma(t))$ is $(1, 2)$, the model has the effect of intensive randomization.

In Table 1 the parts of training errors of different trading dates for (a) SAI, (b) SBI, (c) HSI, (d) DJI, (e) IXIC and (f) SP500 are given. It can be clearly seen that during the years 1991 and 1992, the relative error is larger than that in the year 2007, this clearly shows the effect of time effective function. Furthermore, the gap between the relative error of SAI and SBI is much more greater than the relative error of the foreign stock markets. So we can conclude that the value of the historical data in the foreign stock markets is greater than that in the Chinese stock markets, this means that the Chinese stock markets fluctuate more sharply than the foreign markets.





Predictive Value Points
Best Linear Fit

Fig. 7. Regression of the predictive values and actual values.

Fig. 5 shows the fluctuations of the time sequence of relative errors during the years 1991 to 2007 for the prices of SAI, SBI, HSI, DJI, IXIC and SP500. In these plots, 0 represents the farthest data to the current date, and the larger t (date) represents the date that is nearer to the current date. Fig. 5 also clearly indicates that the stochastic time effective neural network model can be realized by assigning different weights to the data of different times. Time sequence of relative errors of (b) SBI and (d) DJI in Fig. 5 clearly reflects the model of randomization by the effect of the Brownian motion. And we also conclude that SBI is similar to DJI, and that SAI is similar to HIS. By coincidence, the conclusion is supported by the relativity analysis shown in Fig. 3.

In order to test the validity of the volatility parameter $\sigma(t)$, we take $\sigma(t)$ to be 0 or 2. If $\sigma(t)$ is 0, then the model has no effect of randomization but has only the effects of the time effective function. If $\sigma(t)$ is 2, then the model has intensive effect of the wild fluctuation.

In Table 2, the predictive values and relative errors of SAI by the

In Table 3, I stands for the average relative error, II stands for the average relative error of the first 1000 days in the data sets, and III ... test 100 days in the data sets. In Table 3, the time ... is clearly expressed. Take the rela- ... average relative error ... 88% and

the average relative error of the latest 100 days is 1.3%. So the latest data are more valuable than the historical data of the past in the stock market.

In Table 3, the global stock indices errors of the different values $\sigma(t)$ are also given. Take the relative error in SAI for example, while $\sigma(t)=1$, the average relative error is 2.6%; while $\sigma(t)=0$, the average relative error is 7.66%; while $\sigma(t)=2$, the average relative error is 24.58%. So this implies that the appropriate volatility parameter is beneficial for the prediction of the stochastic time effective neural network, and that it is widely applicable to the global stock markets.

In Fig. 6, the comparison between predictive values and the actual values in SAI, SBI, HSI and IXIC by the stochastic time effective neural network model is shown.

In Fig. 7, by using the linear regression method, we compare the predictive values of stochastic time effective neural network model with the actual values in SAI (a), SBI (b), NSDK (c) and SP500 (d). Through the regression analysis, different linear equations in SAI ... the linear

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And the correlation coefficient $R = 0.9978$; the linear equation for NSDK(c) is

$$P\ (\text{predictive value}) = (1.085) \times A\ (\text{actual value}) + (0.9992).$$

And the correlation coefficient $R = 0.9988$ and the linear equation for SP500 (d) is

$$P\ (\text{predictive value}) = (0.9991) \times A\ (\text{actual value}) + (0.2898).$$

And the correlation coefficient $R = 0.9991$. So we test the accuracy of the results of the forecasting from another angle.

4. Conclusions

This paper introduces a new stochastic time effective function to model a stochastic time effective neural network model. The effectiveness of the model has been analyzed by performing a numerical experiment on the data of SAI, SBI, HSI, DJI, IXIC and SP500, and the validity of the volatility parameters of the Brownian motion is tested. Further, the present paper shows some predictive results on the global stock indices using the stochastic time effective neural network model.

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