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Chain Rule Assignment

1) $f(z) = \log_e(1+z)$ [Where $z = x^T x$, $x \in \mathbb{R}^d$]

$\Rightarrow$ If $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{bmatrix}$, then $x^T = [x_1 \ x_2 \ x_3 \ \cdots \ x_d]$

$\therefore x^T x = [x_1^2 \ x_2^2 \ x_3^2 \ \cdots \ x_d^2]$

Now, applying chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \frac{dz}{dx}$$

$$= \frac{d}{dz}(\log_e(1+z)) \cdot \frac{d}{dx}(x^T x)$$

$$= \frac{1}{1+z} \cdot 1 \cdot \frac{d}{dx}(x_1^2 + x_2^2 + \cdots x_d^2)$$

$$= \frac{1}{1+z} \cdot (2x_1 + 2x_2 + \cdots + 2x_d)$$

$$= \frac{2}{1+z} \sum_{i=1}^{d} x_i$$

[Ans]

(2) $f(z) = e^{-\frac{z}{2}}$ [where $z = g(y)$, $g(y) = y^T S^{-1} y$, $y = h(x)$, $h(x) = x - \mu$

$\Rightarrow$ Using the chain rule,

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x} \quad \text{--- (1)}$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left(e^{-z/2}\right) = -\frac{e^{-z/2}}{2}$$

Now, $\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(y^T S^{-1} y\right)$

$$= \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) S^{-1} (y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{(y^T S^{-1} + h S^{-1})(y+h) - y^T S^{-1} \cdot y}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(y^T S^{-1} + y S^{-1} + h S^{-1})}{\cancel{h}}$$

$$= \lim_{h \to 0} y S^{-1} + y S^{-1} + h S^{-1}$$

$$= y^T S^{-1} + y S^{-1}$$

$$\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}(x - \mu) = 1$$

$\therefore$ from ①

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$= -\frac{e^{z/2}}{2} \cdot (y^T S^{-1} + y S^{-1}) \cdot 1$$

$$= -\frac{e^{\frac{z}{2}}}{2S}(y^T + y) \quad [\text{Ans}]$$