

MA1001 - LINEAR ALGEBRA

Dr. P. Venugopal

Professor of Mathematics
School of Science & Humanities
Shiv Nadar University Chennai

What is Algebra?



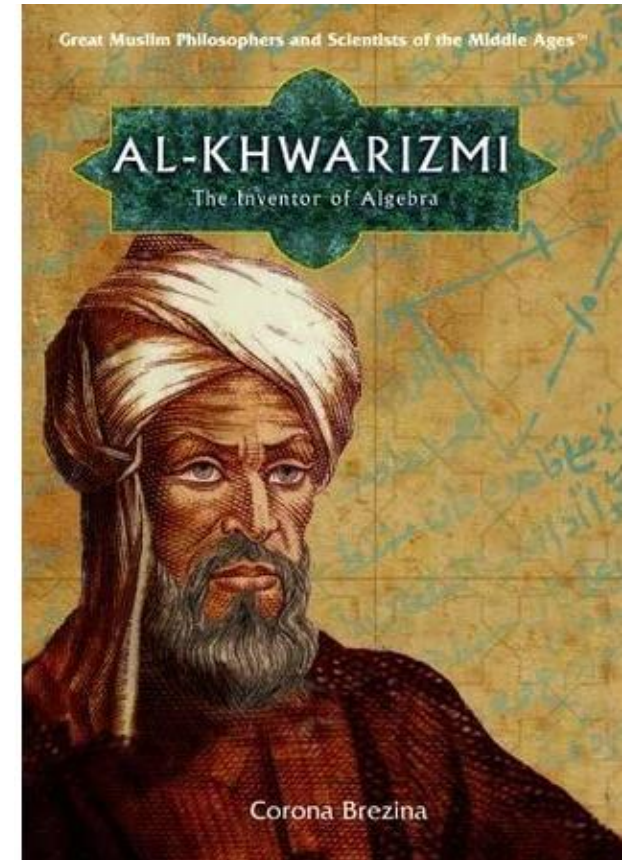
The sum of the ages of 5 children born at the intervals of 3 years each is 50 years. What is the age of the youngest child?



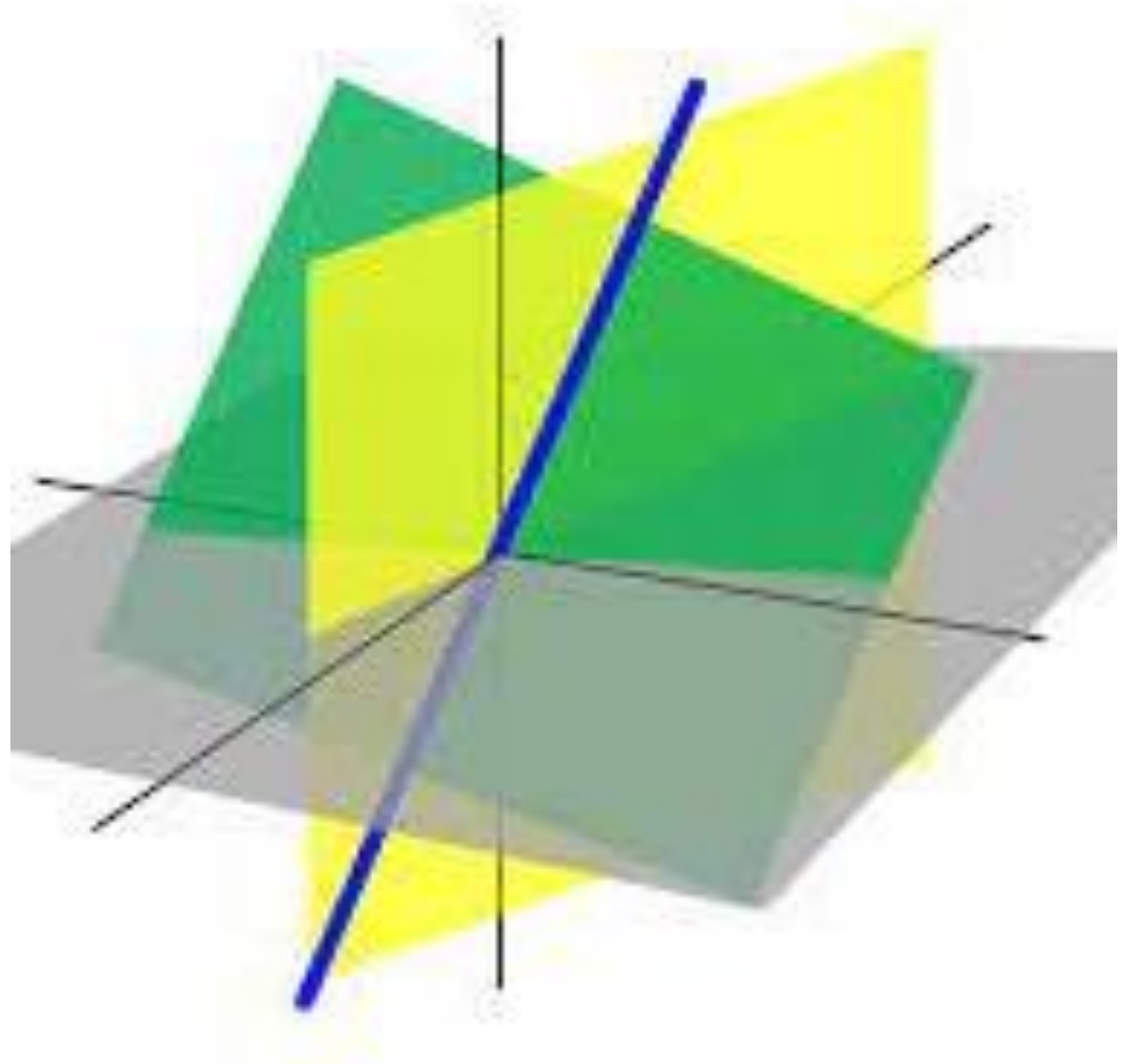
The cost of 2 apples and a mango is Rs.50, whereas the cost of an apple and 2 mangoes is Rs.70. Find the cost of an apple and a mango.



- Algebra is the branch of mathematics that helps to represent problems or situations using mathematical expressions.
- Algebra is the study of mathematical symbols and the rules for manipulating these symbols.
- **Al-Khwarizmi** – Father of Algebra
- The word “algebra” derived from the title of his book, **Kitab al-Jabr**.
- The Arabic word **al-jabr**, which means “**the reunion of broken parts**”.



What is LINEAR ALGEBRA?



Linear algebra is a branch of mathematics that deals with vectors, vector spaces (also called linear spaces), linear transformations, and systems of linear equations.



Polynomial function

A **polynomial** is an expression consisting of coefficients and variables.

$$f(x) = \boxed{a_n x^n} + a_{n-1} x^{\boxed{n-1}} \boxed{+} \dots + a_1 x + \boxed{a_0}$$

Diagram illustrating the components of a polynomial expression $f(x)$:

- $a_n x^n$ is labeled **Term** (blue arrow).
- $n-1$ is labeled **Exponent** (red arrow).
- $+$ is labeled **Operator** (purple arrow).
- a_0 is labeled **Constant** (orange arrow).

Polynomial

$$p(x) = 2x^5 - 3x^4 + 2x^3 + 5x^2 - x + 8$$

Diagram illustrating the components of the polynomial $p(x)$:

- The entire expression is labeled **Polynomial** (black arrow).
- The highest power, 5, is labeled **Degree** (red arrow).
- The terms are identified by their degrees:

 - $2x^5$ is the **Quintic term** (red arrow).
 - $-3x^4$ is the **Quartic term** (blue arrow).
 - $2x^3$ is the **Cubic term** (green arrow).
 - $5x^2$ is the **Quadratic term** (orange arrow).
 - $-x$ is the **Linear term** (blue arrow).
 - 8 is the **Constant term** (pink arrow).

Identify the Polynomial function?

$$1. f(x) = \frac{1}{2}x^2 - 3x^4 - 7$$

$$2. f(x) = x^3 + 3^x$$

$$3. f(x) = 6x^2 + 2x^{-1} + x$$

$$4. f(x) = -0.5x + \pi x^2 - \sqrt{2}$$

Answer:

1 and 4 are polynomial functions.

Linear function

A **linear function** is generally a polynomial function whose degree is utmost 1 or 0. Geometrically, it represents a straight line in a graph.

What is a linear function?

Linear

$$5x - 3y = 7$$

$$x = 9$$

$$6s = -3t - 15$$

$$y = \frac{1}{2}x$$

Not linear

$$7a + 4b^2 = -8$$

$$y = \sqrt{x+5}$$

$$x + xy = 1$$

$$y = \frac{1}{x}$$

Zero polynomial

A **zero polynomial** is one where all the coefficients are equal to zero. So, the degree of the zero polynomial is either undefined, or it is **set equal to -1** .

Linear system of equations

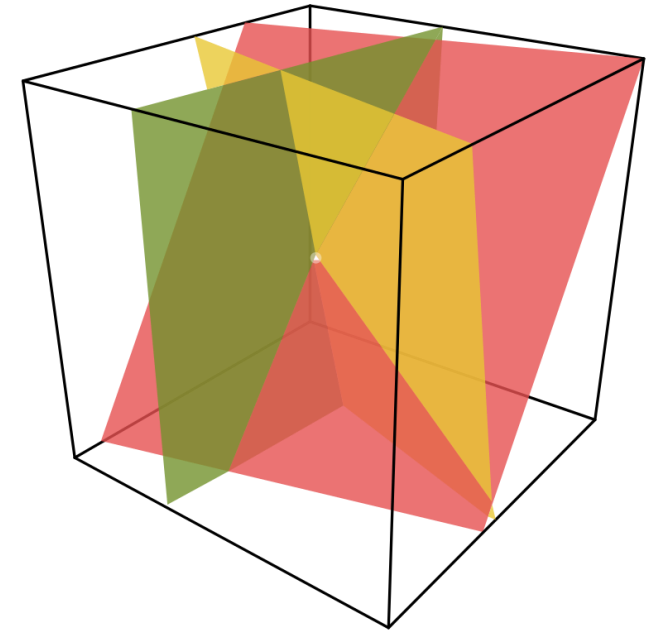
A System of Equations has **two or more equations in one or more variables**.

Example: 3 equations in 3 variables

$$2x + y - 2z = 3$$

$$x - y - z = 0$$

$$x + y + 3z = 12$$



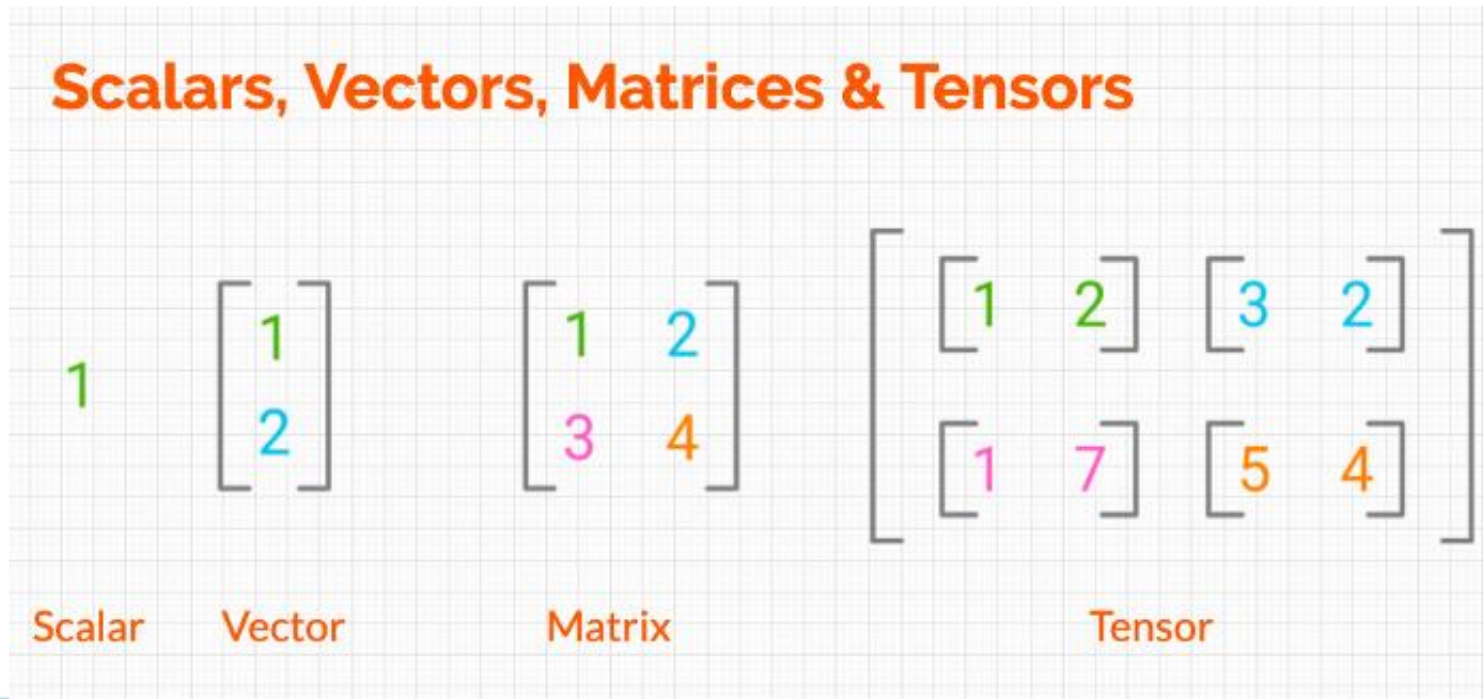
A linear system with three variables determines a collection of [planes](#).

The intersection point is the solution.

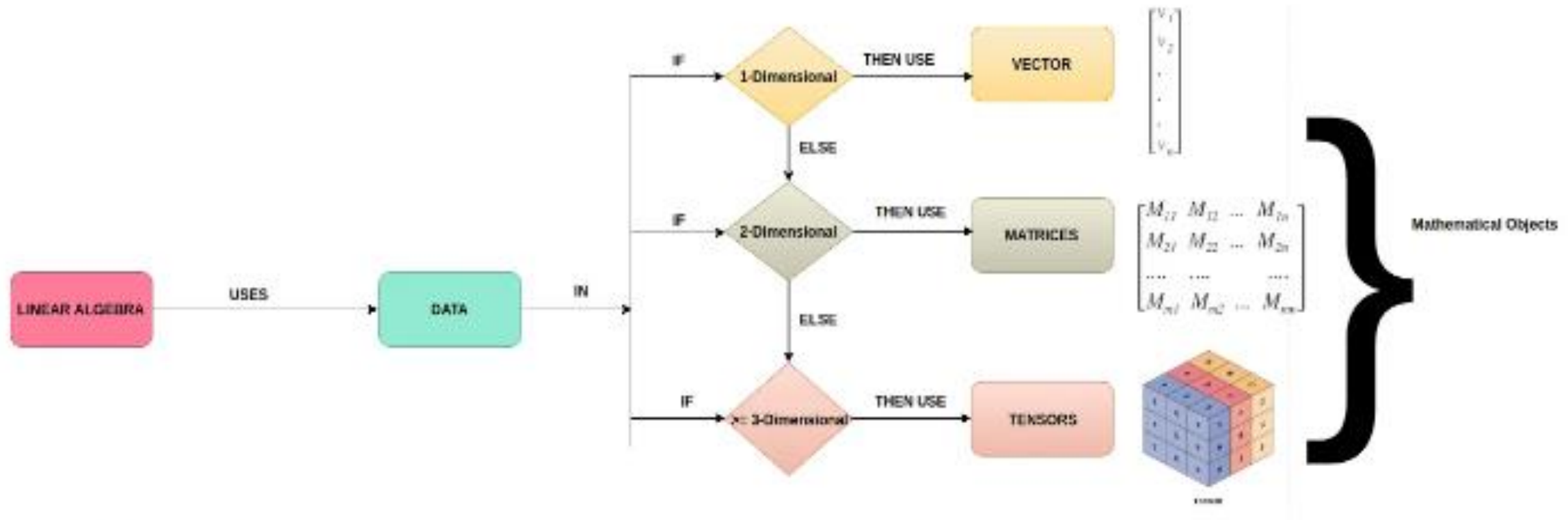
Linear Algebra

Linear algebra is the branch of mathematics concerning **linear equations** such as linear functions and their representations through matrices and vector spaces.

Linear algebra is about combinations. In other words, it's about using arithmetic (linear) processes on a column of numbers (vectors), and then combining several vectors together to create matrices. Similarly, several matrices can be combined together in order to create tensors, which are used in the library called *TensorFlow*.



Linear Algebra or Mathematical objects are Vectors, Matrices and Tensors. Depend upon the dimensions of the data one must choose the right object to store and process



How Mathematical or Linear Algebra Objects(Vector, Matrices and Tensor) used in AI to store the different dimensions of Data.

Why to study linear algebra?

- Linear Algebra is a branch of mathematics that is extremely useful in data science and machine learning.
- Linear algebra is the most important math skill in machine learning. Most machine learning models can be expressed in matrix form.
- A dataset itself is often represented as a matrix.
- Linear algebra is used in data preprocessing, data transformation, and model evaluation.
- Linear algebra is used for creating an algorithm which can reduce the dimension of data. By using the power of dimensionality reduction, we can visualize higher dimensional data and understand their trends.

What is AI?



Artificial Intelligence

- "It is a branch of computer science by which we can create intelligent machines which can behave like a human, think like humans, and be able to make decisions."
- Artificial Intelligence is composed of two words **Artificial** and **Intelligence**, where Artificial defines "*man-made,*" and intelligence defines "*thinking power*", hence AI means "*a man-made thinking power.*"

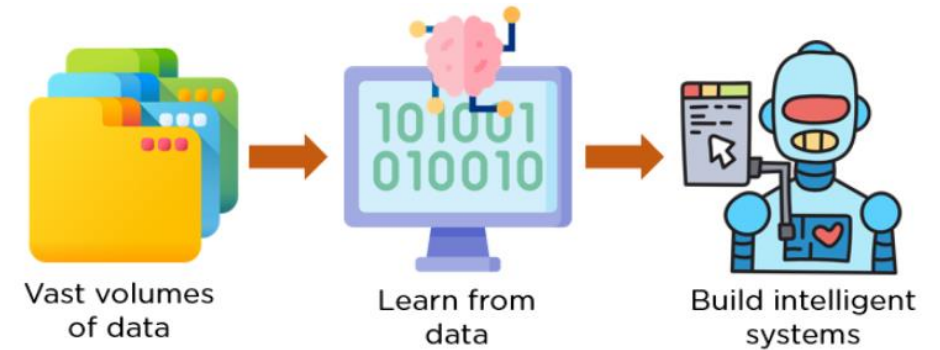
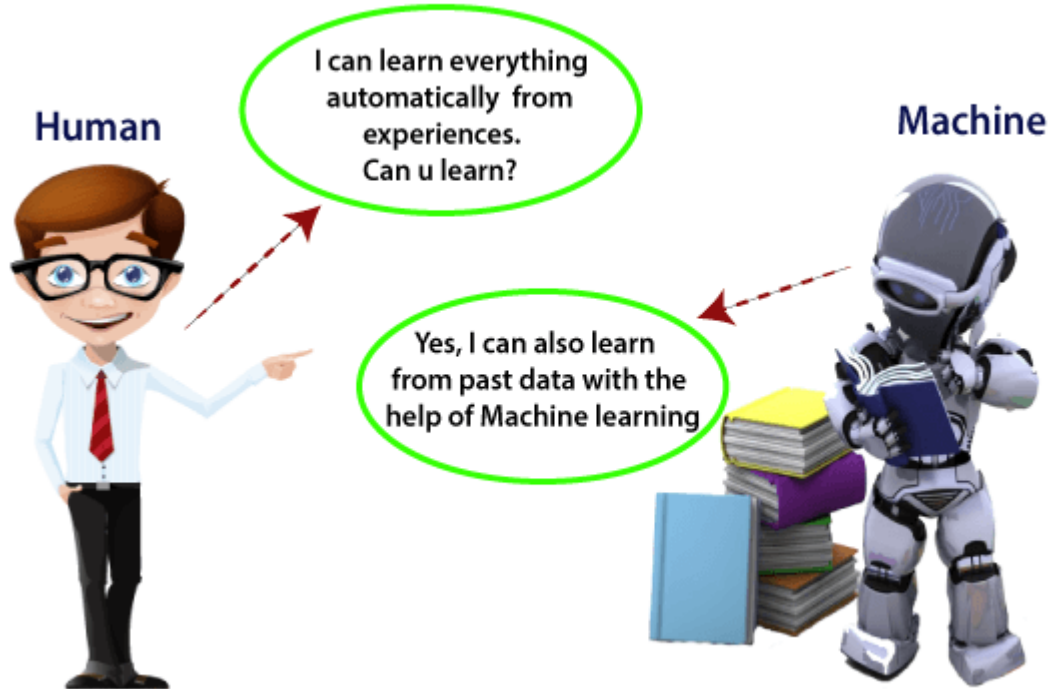


What is Machine Learning?



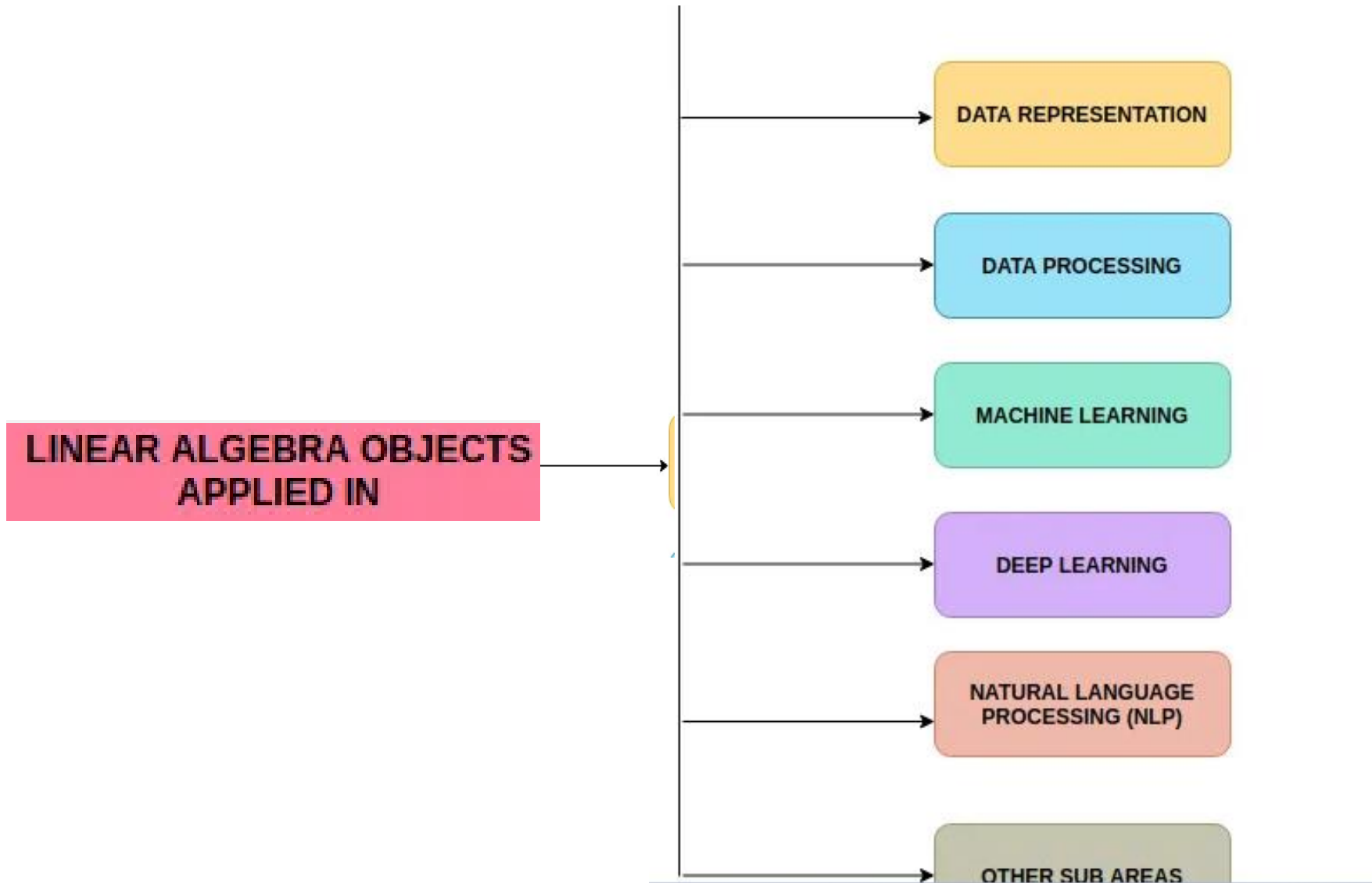
Machine learning

Machine learning enables a machine to automatically learn from data, improve performance from experiences, and predict things without being explicitly programmed.



The term machine learning was first introduced by **Arthur Samuel** in **1959**.

Application of Linear Algebra in Computer Science



UNITS

Unit 1. VECTOR SPACES

Semigroup, group, ring, field (Definitions and examples)–vector space, subspace, linear independence and dependence-basis and dimension.

Unit 2. LINEAR TRANSFORMATION

Linear transformation-range space and null space -rank and nullity-dimension theorem.

Unit 3. EIGEN VALUES AND EIGEN VECTORS

Matrix representation of linear transformation-eigenvalues and eigenvectors of linear transformation.

Unit 4. INNER PRODUCT SPACES

Inner product and norms-properties-orthogonal, orthonormal vectors - Gram Schmidt orthonormalization process.

Unit 5. MATRIX DECOMPOSITION

QR decomposition - Singular value decomposition -Least square approximations.

Friedberg A.H, Insel A.J. and Spence L, Linear Algebra, Prentice Hall of India, New Delhi, 2004.

