Section 1 - Vector space

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Scalars and vectors:

A quantity that has only magnitude, but no direction is known as scalar.

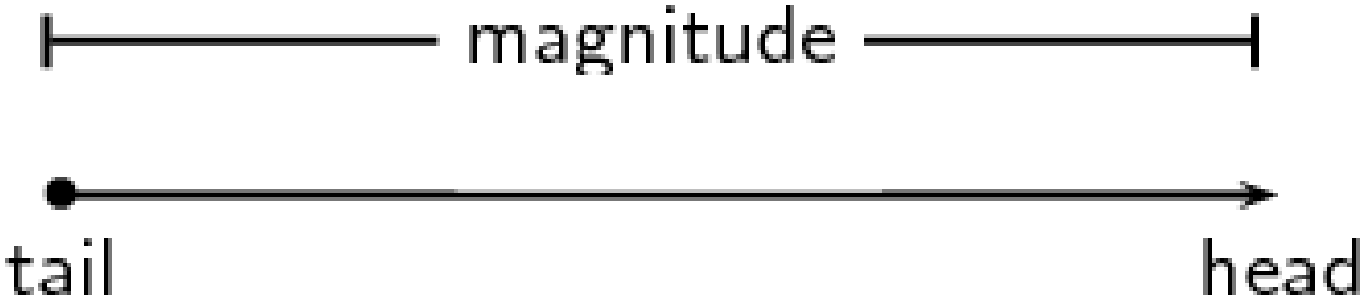
A quantity that has magnitude and acts in a particular direction is described as vector.

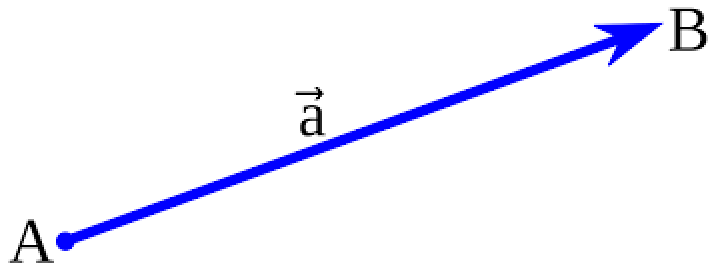
Examples of Vectors and Scalars

Vector	Scalars
Thrust	Length
Displacement	Temperature
Weight	Voltage
Acceleration	Time
Momentum	Power
Drag	Area
Lift	Energy
Movement	Pressure
Velocity	Speed

Graphical representation of vectors

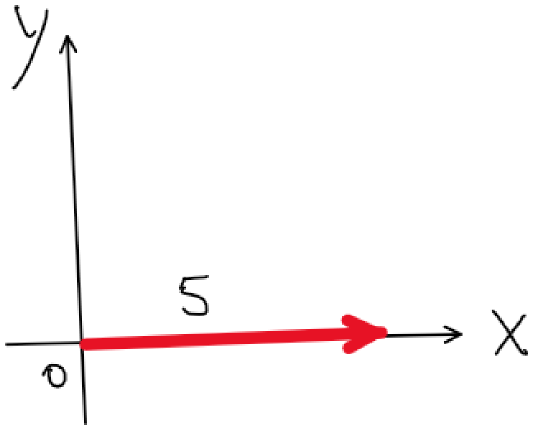
Vectors can be represented graphically as **arrows**. The length of the arrow indicates the magnitude of the vector.





Matrix representation of vectors

An object moving 5 km/hour in the east direction is a vector quantity.

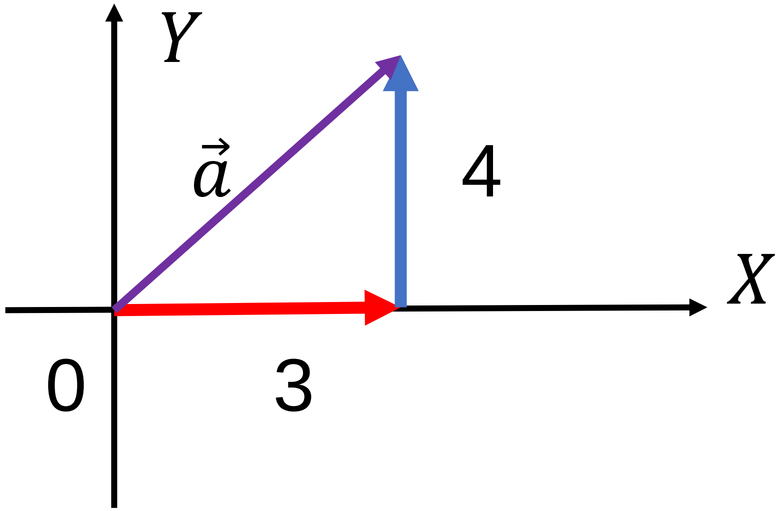


This is represented as $\vec{v} = (5, 0)$ --- **row vector** $\vec{v} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ --- **column vector**

Horizontal Direction

Vertical direction

$$\vec{a} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$



$$\text{Magnitude} = \sqrt{3^2 + 4^2} = 5$$

$$\vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- A vector represents a set of numbers written in a column (row) is called as column (row) vector.
- The individual numbers in a vector are called components/ entries/ elements.
- If a vector has n elements, it is called as n dimensional vector or n-tuple.

Addition of vectors

To add 2 vectors, add the corresponding components.

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

Example:

$$\vec{a} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}, \vec{b} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$$

$$\vec{a} + \vec{b} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

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Multiplication of a vector by a scalar

$$\alpha(x, y) = (\alpha x, \alpha y)$$

Example:

$$\vec{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$3\vec{a} = 3\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

Magnitude is increased by 3 and direction remains the same.

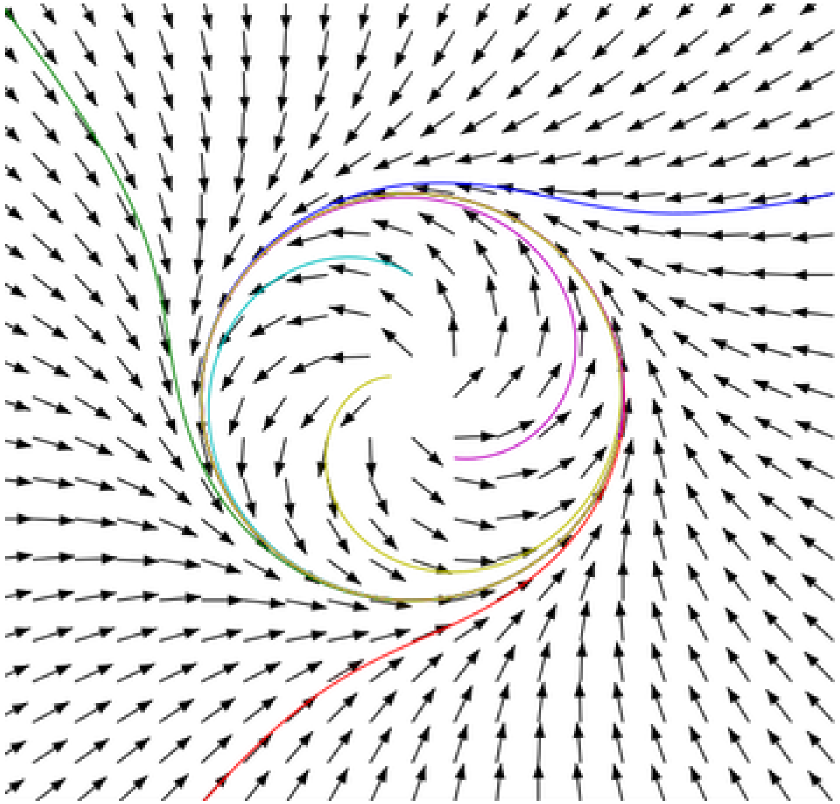
$$-\vec{a} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

Magnitude is same, direction is reversed.

Note: Scalar multiplication is just scaling.

Vector Space

A vector space is a set of vectors together with the rules for vector addition and scalar multiplication.



Definition of Vector Space

A non-empty set V is said to be a vector space over a field F, if, there are two binary operations vector addition (+) and scalar multiplication (·), satisfying the following axioms.

Let u, $v, w \in V$ and $\alpha, \beta \in F$,

For Vector Addition

1. Closure: $u + v \in V$

2. Commutative: $u + v = v$ +u

3. **Associative**: $(u + v) + w = u + (v + w)$

4. **Existence of identity element**: There exists an element $0 \in V$ such that $v + 0 = 0 + v = v$

5. **Existence of inverse**: There exists an element $v^{-1} \in V$ such that v $+ v^{-1} = v^{-1} + v = 0$

For Scalar Multiplication

6. **Closure**:For $\alpha \in F, v \in V, \alpha v \in V$

7. **Distributive**: For $\alpha \in F$, and $u, v \in V$,

$$\alpha \cdot (u+v) = \alpha \cdot u + \alpha \cdot v$$

8. **Distributive**: For $\alpha, \beta \in F$, and $u \in V$

$$(\alpha+\beta) \cdot u = \alpha \cdot u + \beta \cdot u$$

9. **Associative**: For $\alpha, \beta \in F$, $v \in V$, $(\alpha\beta)v = \alpha(\beta v)$

10. **Unitary law**: For $1 \in F$ such that $v \cdot 1 = 1 \cdot v = v$

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Note:

- The elements of the field F are called *scalars*. and the elements of the vector space V are called *vectors*.
- The vector space V over a field F is denoted as $V(F)$.
- An n-tuple is a list of n numbers $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ where $\alpha_1, \alpha_2, \cdots, \alpha_n \in F$ are called the entries or components of the n-tuple.
- The set of all n-tuples with entries from a field F is denoted by F^n or $V_n(F)$.
- The elements 0 and 1 are called identity elements for addition and multiplication, respectively.
- The element $v^{-1}$is called as multiplicative inverse.

Examples of vector space:

- The space R^n consists of all column vectors with n components, is a vector space.
- In particular, R is a one-dimensional vector space, which is nothing, but a line contains only one component, R^2 is a two-dimensional vector space, representing xy-plane in general, contains only two components, R^3 is a three-dimensional vector space, contains three components and so on.
- $\{0\}$ is a vector space. It is said to be zero vector space.

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