UNIT 1 : Vector Space

Lecture 1: Binary Operator

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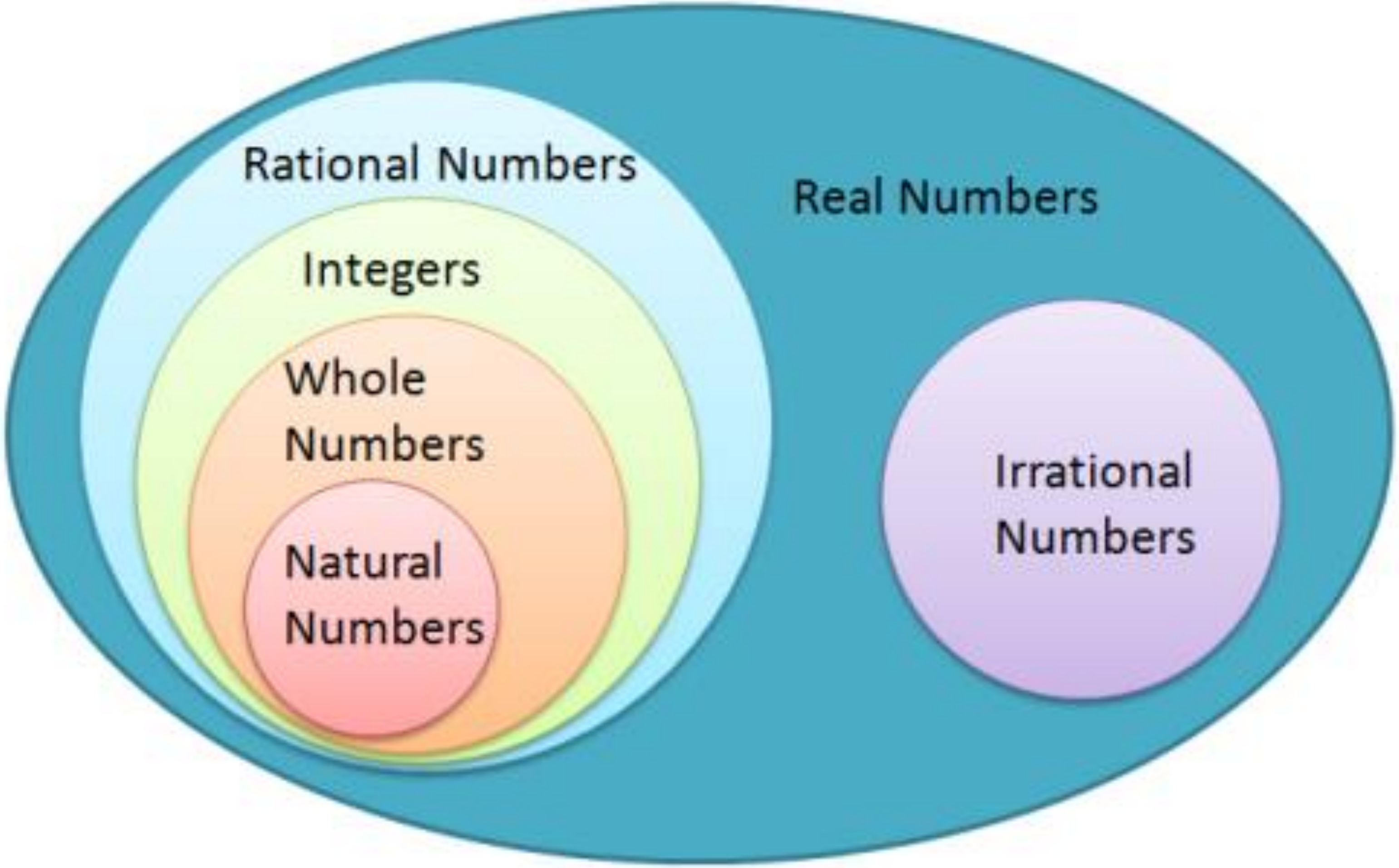
Number System



Set

A *set* is a collection of elements or numbers or objects, represented within the curly brackets { }.

Binary Operation

A *binary operation* or *dyadic operation* $*$ is a rule for combining two elements (called operands) a and b of a set S to produce another element $a * b$ of the set S.

Let S be a non-empty set and $*$ is said to be a *binary operation* on S, if $a * b$ is defined for all $a, b \epsilon S$.

That is, the binary operation $*$ is an operation performed on a set S and is given by $* : S * S \rightarrow S$.

Note:

It is denoted by $(S,*)$.

Properties of Binary Operations

1. Closure property:

An operation $*$ on a non-empty set A has closure property, if $a, b \in A, \Rightarrow a * b \in A$.

2. Commutative property:

A binary operation $*$ on a nonempty set A is commutative if $a * b = b * a$, for all $a, b \in$ A.

3. Associative property:

The associative property of binary operations hold if, for a non-empty set A, we can write

$(\mathrm{a} * \mathrm{b}) * c = a *(b * c)$ for all $a, b, c \in A$.

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4. Distributive property:

Let $+$ and . be two binary operations defined on a non-empty set A. The binary operations are distributive if

$$a.(b + c) = (a\,.\,b) + (a\,.\,c) \text{ or}$$

$$(b + c).\,a = (b\,.\,a) + (c\,.\,a).$$

5. Identity property:

Let A be a non-empty set and $*$ be the binary operation on A. An element e is the identity element of the set A, if $a * e = a = e * a,$ for all $a \in A$.

If the binary operation is addition $(+)$, $e = 0$ and for multiplication(.), e $=1$.

6. Inverse property:

Let A be a non-empty set and $*$ be the binary operation on A. An element b is said to be the inverse element of $a \in A$, if $a * b = b * a = e$, for all $a, b \in A$.

In this case, we can write b as a^{-1}.

This implies $a * a^{-1} = a^{-1} * a = e$

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Thank you