

Pushdown Automata (PDA)

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PDA

A pushdown automaton is a way to implement a context-free grammar in a similar way we design DFA for a regular grammar. A DFA can remember a finite amount of information, but a PDA can remember an infinite amount of information.

Basically, a pushdown automaton is –
"Finite state machine" + "a stack"

PDA

A pushdown automaton has three components –

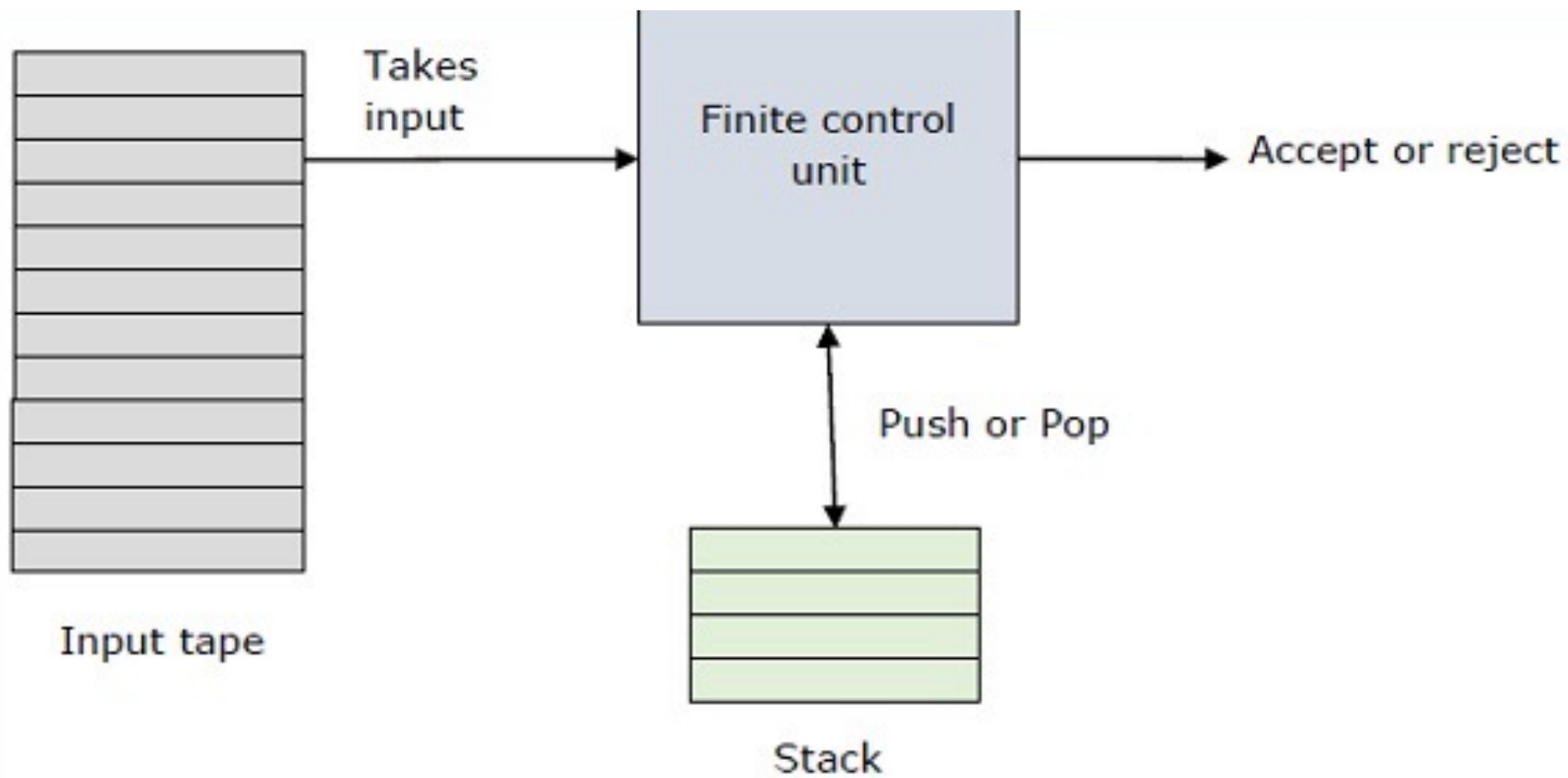
- an input tape,
- a control unit, and
- a stack with infinite size.

PDA

A stack does two operations –

- **Push** – a new symbol is added at the top.
- **Pop** – the top symbol is read and removed.

PDA

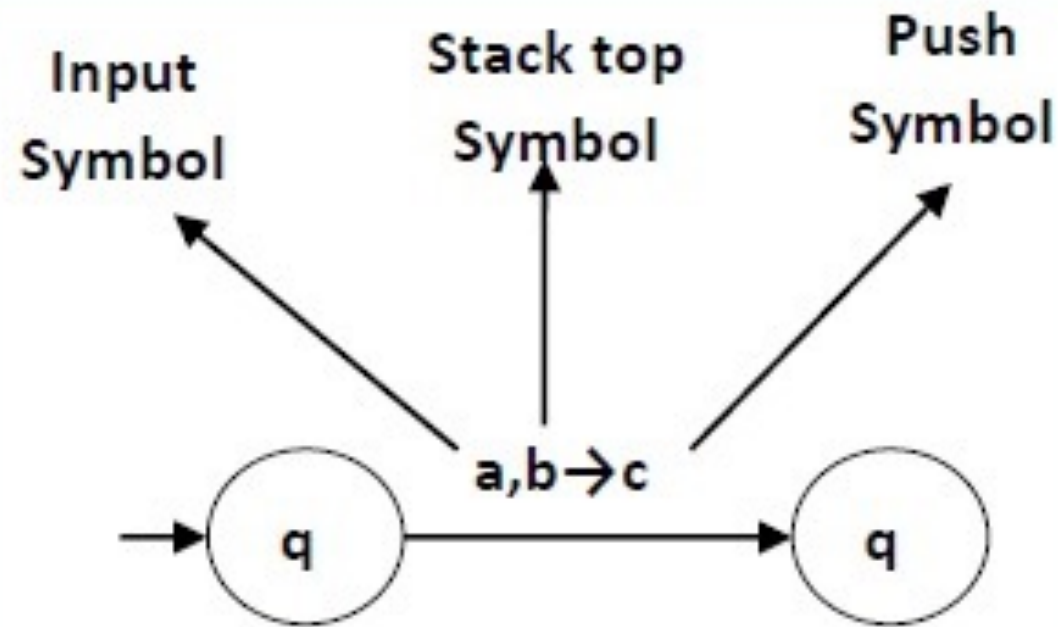


PDA

A PDA can be formally described as a 7-tuple $(Q, \Sigma, S, \delta, q_0, I, F)$ –

- **Q** is the finite number of states
- Σ is input alphabet
- **S** is stack symbols
- δ is the transition function: $Q \times (\Sigma \cup \{\epsilon\}) \times S \times Q \times S^*$
- **q₀** is the initial state ($q_0 \in Q$)
- **I** is the initial stack top symbol ($I \in S$)
- **F** is a set of accepting states ($F \subseteq Q$)

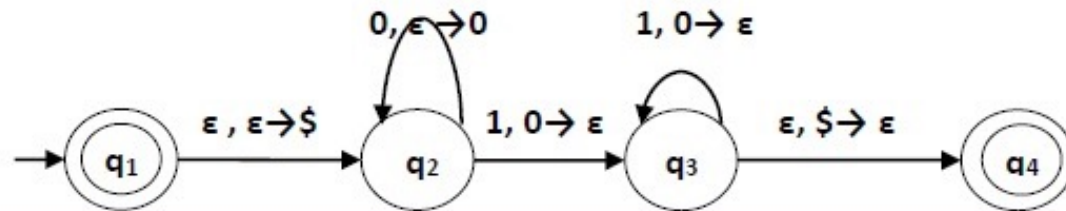
PDA



This means at state q_1 , if we encounter an input string a and top symbol of the stack is b , then we pop b , push c on top of the stack and move to state q_2 .

Example

Construct a PDA that accepts $L = \{0^n 1^n \mid n \geq 0\}$



PDA for $L = \{0^n 1^n \mid n \geq 0\}$

This language accepts $L = \{\epsilon, 01, 0011, 000111, \dots\}$

Here, in this example, the number of **a** and **b** have to be same.

Initially we put a special symbol **\$** into the empty stack.

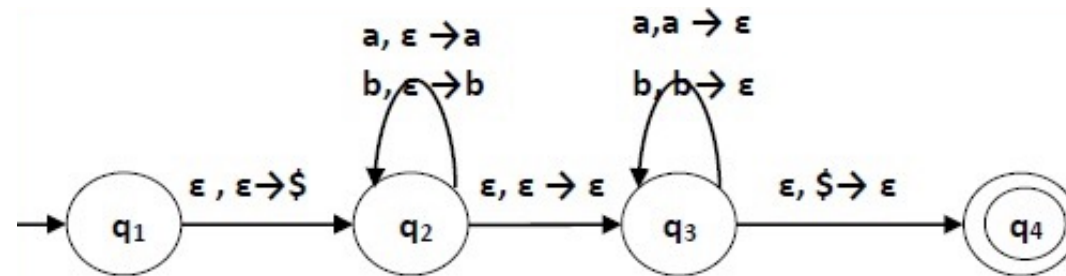
Then at state q_2 , if we encounter input 0 and top is Null, we push 0 into stack. This may iterate. And if we encounter input 1 and top is 0, we pop this 0.

Then at state q_3 , if we encounter input 1 and top is 0, we pop this 0. This may also iterate. And if we encounter input 1 and top is 0, we pop the top element.

If the special symbol **\$** is encountered at top of the stack, it is popped out and it finally goes to the accepting state q_4 .

Example

Construct a PDA that accepts $L = \{ ww^R \mid w = (a+b)^* \}$



PDA for $L = \{ ww^R \mid w = (a+b)^* \}$